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The Cyclical Behavior of Interest rates

Abstract

This paper investigates the behavior of the term structure of interest rates over the business cycle. In contrast to the simple change in aggregate economic activity used in previous research, we use a more appropriate measure of the business cycle: the deviation of aggregate economic activity from its potentially stochastic trend. Stochastically detrending Gross Domestic Product (GDP) by Watson's [1986] UC-ARIMA methodology significantly improves the term spread's informativeness regarding future economic activity. We also investigate the implications of the UC-ARIMA representation of aggregate consumption dynamics for a linear consumption based model of the term structure. The presence of an unobserved but independent cyclical component in aggregate consumption also allows for the more efficient estimation of consumption asset pricing models.

1 Introduction

The notion of systematic variation in interest rates over the business cycle is a familiar one in economics and finance. Beginning with the work of Kessel [1965] describing the variation of long and short term interest rates with respect to economic activity, numerous authors have investigated the relationship between the term structure of interest rates and changes in fundamental macroeconomic variables such as gross domestic product (GDP), consumption, and inflation.

More recent theories of the term structure, like Cox, Ingersoll, and Ross [1985], are based on a general equilibrium approach in which real interest rates depend, through the representative investor's marginal utility, on consumption. Faced with changing consumption opportunities, the representative investor uses financial assets to smooth out lifetime consumption. Given specific assumptions on the representative investor's utility function as well as the dynamics of the underlying state variables, explicit bond pricing formulae are obtained which link the behavior of long and short term interest rates. Harvey [1988], for example, uses this framework to investigate the relationship between real interest rates and consumption changes, the latter taken to be a proxy for the business cycle.

However, Lucas [1976] more precisely defines the business cycle as the deviation of aggregate output from its potentially stochastic trend. Such a definition underlies the real business cycle literature in macroeconomics (see, among others, Kydland and Prescott [1982]) as well as the construction of recent business cycle indicators (Boldin [1994]). The purpose of this paper is to investigate, both theoretically and empirically, the implications of this more appropriate measure of the business cycle on the behavior of interest rates.

Recognizing that the business cycle is unobservable, we use Watson's [1986] UC-ARIMA (unobserved components ARIMA) methodology to stochastically detrend U.S. macroeconomic data and estimate its corresponding cyclical component. UC-ARIMA models are nested cases of ARIMA models. However, as noted by Watson, the estimated versions of these alternative business cycle specifications may display different properties at different frequencies. We find that the term spread of nominal interest rates is significantly related to the UC-ARIMA cyclical component of GDP, while there exists no relationship whatsoever with GDP's ARIMA cyclical component.

We further investigate the relationship between interest rates and the business cycle in the context of a consumption based equilibrium model of the term structure of real interest rates. We do so by incorporating our business cycle measure into Harvey's [1988] term structure framework. This allows us to theoretically demonstrate a more precise relationship between the real term structure and aggregate consumption's cyclical component. We also consider the empirical plausibility of various real term structure models. For example, assuming aggregate consumption's cyclical component follows an AR(1) specification results in the procyclical behavior of the real term spread which is inconsistent with extant empirical evidence. However, higher order real term structure models are shown to be consistent with observed countercyclical behavior.

Finally, we investigate the maximum likelihood estimation of consumption asset pricing models (Hansen and Singleton [1983]). Following Watson's UC-ARIMA approach, our multivariate estimation methodology explicitly recognizes the presence of an independent but unobservable cyclical component in aggregate consumption. We compare this estimation methodology to that proposed by Hansen and Singleton and find that the precision of the estimated coefficients, in particular, the coefficient of relative risk aversion, increases dramatically. In addition, we provide new insights into the lack of variability in aggregate consumption which plagues the performance of consumption asset pricing models.

The plan of this paper is as follows. In Section 2 we introduce a more precise measure of the business cycle by stochastically detrending U.S. GDP data. We demonstrate that the superiority of the term spread in forecasting the UC-ARIMA estimated business cycle, as opposed to the ARIMA estimated business cycle or simple growth rates in GDP. Assuming that the representative individual's utility function is characterized by constant relative risk aversion and that the logarithm of aggregate consumption is normally distributed, Section 3 develops a model of real interest rates. Corresponding real term structure models are also derived for the case in which consumption's unobservable but independent cyclical component follows an autoregressive process. In Section 4 we use a multivariate Kalman filter to simultaneously estimate aggregate consumption's cyclical component as well as its relationship to interest rates. We find a statistically significant relationship between real bond returns and our estimate of the business cycle. Section 5 concludes the paper.

2 The Term Structure of Interest Rates and the Business Cycle

Interest rates should vary with the business cycle. Intuitively, if an investor expects a recession (expansion), he or she will be more (less) willing to demand a bond today, thereby decreasing (increasing) prevailing interest rates. As a consequence, the current term structure of interest rates should provide information about the business cycle. For example, Harvey [1988] finds that yield spreads can reliably forecast the growth in the real consumption of nondurables and services. Estrella and Hardouvelis [1991] also find the slope of the yield curve forecasts future growth in real GNP.

However, growth rates in consumption and other macroeconomic variables are imprecise measures of the business cycle. Most macroeconomic time series trend over time and the business cycle is more properly viewed as stationary deviations about this potentially stochastic trend (Lucas [1976]). Simply first differencing a macroeconomic time series which contains both growth (non-stationary) and cyclical (stationary) components confounds the contribution of each (Stock and Watson [1988]). As a result, by imprecisely measuring the business cycle, previous empirical research may not have thoroughly characterized the relationship between interest rates and the business cycle.

2.1 Estimating the Business Cycle

Empirical evidence suggests that macroeconomic time series are integrated of order one (I(1)); that is, their first differences are stationary. However, as noted by Beveridge and Nelson [1981], integrated series contain a stochastic trend. In particular, any ARIMA$(p, 1, q)$ model can be represented as a stochastic trend plus a stationary component.

In this paper we recognize that while the logarithm of a macroeconomic variable, y_t, is itself observable, its additive components, the trend, τ_t, and the cycle, c_t, are individually unobservable:

$$y_t = c_t + \tau_t \ . \tag{1}$$

The cycle c_t is assumed characterized by a stationary autoregressive process,

$$\Phi(L)c_t = e_t^c \ , \qquad var(e_t^c) = \sigma_c^2, \tag{2}$$

where $\Phi(L)$ denotes a polynomial in the lag operator L. The trend τ_t is assumed to follow a random walk with drift,

$$\tau_t = \mu + \tau_{t-1} + \epsilon_t^\tau \ , \quad var(\epsilon_t^\tau) = \sigma_\tau^2 \ . \tag{3}$$

This specification implies that the macroeconomic variable's logarithmic average growth rate is constant at μ. We assume that ϵ_t^τ and ϵ_t^c are normally distributed. More significantly, we follow Watson [1986] and decompose the observed series using a UC-ARIMA model which assumes that the trend and stationary innovations are uncorrelated

$$cov(\epsilon_t^\tau, \epsilon_{t-k}^c) = 0 \quad \forall k.$$

That is, the economic factors giving rise to trend innovations are assumed to be unrelated to the economic sources of business cycle movements. In contrast, if we assume that ϵ_t^τ and ϵ_t^c are perfectly correlated then, from Beveridge and Nelson [1981], an ARIMA model obtains.

2.2 Data

To investigate the relationship between interest rates and the cyclical component of aggregate economic activity requires a comprehensive measure of economic activity. To that end, we use quarterly seasonally adjusted data on Gross Domestic Product (GDP) over the sample period 1960:1 to 1992:4, expressed in constant (1987) dollars. We take the natural logarithm of this series as our measure of aggregate economic activity.

The interest rate data are quarterly observations on 3-, 6-, 9-, and 12-month Treasury bills from CRSP's Fama file, and on 1-, 2-, 3-, 4-, and 5-year Treasury bonds from CRSP's Fama-Bliss file. The interest rates are based on an average of bid and ask prices. We align the data so that the interest rate corresponding to a particular quarter is given by that rate quoted on the last date of the quarter and, as such, corresponds to the consumption flow over the quarter. To deflate returns in a particular quarter, we use the logarithmic change in the Consumer Price Index (CPI) from the final month in that quarter to the month in which the return is realized.

2.3 Univariate Estimation of the Cyclical Component

We cast the UC-ARIMA model in state-space form and evaluate the resultant log-likelihood function using the Kalman filter initialized at a vague prior (see Harvey [1989a]). Since a warm-up period of 16 quarters is assumed, estimation begins in the first quarter of 1964. In addition, given maximum likelihood parameter estimates, the Kalman filter estimates the business cycle at each time point in our sample, with the cycle estimated at time t based only on information available up to time t.

Results of the maximum likelihood estimation of the UC-ARIMA model for log real GDP are as follows[1]:

$$\begin{aligned} y_t &= c_t + \tau_t, & & (4) \\ c_t &= \underset{(0.178)}{1.67897}\, c_{t-1} - \underset{(0.180)}{0.72118}\, c_{t-2}, & \sigma_c = 0.0040882, & \quad (5) \\ \tau_t &= \underset{(0.0007)}{0.0082472} + \tau_{t-1}, & \sigma_\tau = 0.0070415, & \quad (6) \end{aligned}$$

with the following summary statistics

SE=0.0088752,

[1]A specification analysis confirms that log real GDP's cyclical component follows an AR(2) process. In particular, we estimated the UC-ARIMA model assuming different AR(p) specifications, p =1, 2, 3, 5, and 10. We selected that specification which maximized the Akaike Information Criterion (AIC) defined by $2(Loglikelihood - p)$. The same model is chosen by Watson [1986] for GNP, although for a slightly different sample period.

Log-likelihood= 383.488.

Figure 1 plots the UC-ARIMA estimated business cycle. Notice that this estimated business cycle appears broadly consistent with recent U.S. macroeconomic behavior. In particular, the estimated business cycle implies a significant economic expansion in the mid-1960s, and significant economic downturns in the mid-1970s, the early 1980s, as well as the late 1980s.

Since the UC-ARIMA model is a special case of an ARIMA specification, we also fit an ARIMA model to the log real GDP series. The results over the 1964:1 to 1992:4 sample period are as follows:

$$\Delta y_t = \underset{(0.0010276)}{0.00173872} + \underset{(0.088096)}{0.293234}\ \Delta y_{t-1}, \quad (7)$$

with the following summary statistics

SE=0.00886467.

Log-likelihood= 384.591.

In comparison, notice that the ARIMA model fits the log real GDP series marginally better as measured by its lower standard error as well as its higher maximized log-likelihood value. However, the resultant estimated business cycle, plotted in Figure 2, displays a more erratic behavior which does not appear to be consistent with the long-run persistence characterizing the business cycle.[2]

[2]Our estimation results for quarterly log real GDP data are similar to Watson's results for quarterly log real GNP data over his 1949 to 1984 sample period. In particular, Watson fits a UC-ARIMA model with an AR(2) cyclical component as well as an ARIMA(1, 1, 0) model. As noted by Watson, the ARIMA(1, 1, 0)

As these figures make clear, the ARIMA and UC-ARIMA business cycle models have quite different long-run properties. To see this more clearly, we follow Watson and compare the moving average representations implied by the two models. For the ARIMA model the sum of the corresponding moving average coefficients is 1.415, while for the UC-ARIMA model this sum is only 0.811.[3] In other words, according to the ARIMA model, a one-unit innovation will eventually increase log real GDP by 1.415, while the same innovation eventually increases log real GDP by only 0.811 according to the UC-ARIMA model. Therefore, the ARIMA model predicts an impact nearly twice as large, generating a more erratic cyclical component. As we shall see below, this result will have significant implications for the long term informativeness of the term spread.

2.4 Misspecification of Previous Tests

The presence of a stochastic trend in log real GDP has implications for testing the relationship between real returns and expected changes in log real GDP. This empirical analysis has been traditionally carried out (Harvey [1988], [1989b]) by regressing ex post realized changes in aggregate economic activity against either a measure of real interest rates or term spreads. However, in the presence of a stochastic trend, these tests may be adversely affected by consequent measurement errors.

Without loss of generality, we illustrate this by considering one period changes in log real

model implies that it is inappropriate to decompose the change in log real GDP into an independent random walk and stationary component. However, when we include the 16 quarters of warm-up data in our ARIMA estimation, giving a sample period of 1960:1 to 1992:4, an ARIMA(2, 1, 0) model obtains. More significantly, the business cycle estimated by the ARIMA(2, 1, 0) model is almost indistinguishable from the ARIMA(1, 1, 0) estimated business cycle.

[3]Lippi and Reichlin [1991] show that in UC-ARIMA models this sum is constrained to be less than 1.

GDP. From (1), (2), (3), we have:

$$y_{t+1} - y_t = \mu + E_t[c_{t+1} - c_t] + e^c_{t+1} + e^\tau_{t+1} \ . \tag{8}$$

Notice that in the presence of a stochastic trend, the ex post change in log real GDP will be a noisy proxy for the predictable component $E_t[c_{t+1} - c_t]$ since it includes both disturbance terms e^c_{t+1} and e^τ_{t+1}. The latter source of measurement error is related to the change in trend and may be quite relevant. Recall that in fitting the UC-ARIMA model to the real log GDP series, the standard deviation of e^τ is approximately twice as large as the standard deviation of e^c. Over longer horizons, say j periods, the measurement error, $\sum_j e^\tau_{t+j}$, will be increasingly important as its variance grows linearly with the length of the forecast horizon j. By contrast, in the UC-ARIMA specification, e^τ_t and e^c_t are independent, implying that the unanticipated change, e^τ_t, may be eliminated by exclusively relying on the estimated cycle.

2.5 Univariate Evidence

Figures 1 and 2 also plot the spread between 5-year and 1-year default-free nominal interest rates against the UC-ARIMA and ARIMA estimated business cycles, respectively. The countercyclical behavior of this term spread is especially evident in the case of the UC-ARIMA estimated cycle.

To confirm this, Table 1 presents univariate regression evidence investigating the term spread's informativeness regarding future economic activity using our quarterly log real GDP data. If the term spread provides information about the business cycle, a more precise measure of the business cycle should result in a more accurate assessment of this relationship.

We run regressions of the form:

$$ts_t = a_j + b_j \Delta c^j_{t,t+i} + u_t, \tag{9}$$

where ts_t is the term spread prevailing at quarter t , and $\Delta c^j_{t,t+i}$ the i-quarter change from quarter t in the jth measure of the business cycle. The term spread is measured by the difference between 5-year and 1-year default-free rates. We use i-quarter changes in the level of log real GDP (Panel A), as well as i-quarter changes in the ARIMA (Panel B) and UC-ARIMA (Panel C) business cycle estimates.

As can be seen in Table 1, the informativeness of the term spread varies across these alternative measures of the business cycle. In general, we expect a positive relationship between the term spread and subsequent changes in economic activity as the steepening (flattening) of the term structure implies that an expansion (recession) is imminent.

From Panel A, consistent with previous evidence, we see a statistically significant relationship between the term spread and subsequent changes in log real GDP. The regressions' adjusted R^2s initially increase with the forecasting horizon, peak in the vicinity of 8-quarters, then subsequently decline. In contrast, Panel B indicates either a negative or non-existant relationship between the term spread and subsequent changes in the ARIMA business cycle estimate. For all of the forecasting horizons, the adjusted R^2s in Panel B are much lower than those of Panel A, indicating that the term spread provides more information about changes in the level of log real GDP than changes in the ARIMA estimates of the business cycle.

Turning our attention to Panel C of Table 1, we see a statistically significant positive relationship between the term spread and subsequent changes in the UC-ARIMA business cycle estimate. The adjusted R^2s increase with the forecasting horizon throughout so that at 10 quarters the term spread explains approximately 50% of the subsequent change in this business cycle estimate. The goodness of fit of the regressions in Panel C, as measured by their adjusted R^2s, exceed those of panel A at all forecasting horizons beyond 6 quarters, indicating that the term spread provides reliable information about subsequent long run changes in economic activity as proxied by the UC-ARIMA business cycle estimate.

To explore the empirical relationship between the term spread and the contemporaneous business cycle, Table 2 presents the results of regressing various term spreads against the change in log real GDP (panel A), as well as the ARIMA (Panel B) and UC-ARIMA (Panel C) estimated business cycles. Notice that a statistically significant countercyclical relationship is evident only in the case of the UC-ARIMA model. No relationship whatsoever exists in the case of the ARIMA model, while a procyclical relationship obtains for changes in log real GDP.

3 The Term Structure of Real Interest Rates and the Business Cycle

Our empirical evidence suggests a statistically significant relationship between the term spread and the UC-ARIMA estimated business cycle. We now explore the nature of the theoretical relationship between the term structure of real interest rates and the business cycle in the context of a consumption based framework which explicitly recognizes the presence of an independent cyclical component in consumption dynamics.

We consider a discrete-time exchange model (Lucas [1978]) where the representative investor solves

$$\max_{C_t} E_t\left[\sum_{j=0}^{\infty} U(C_{t+j})\right] . \tag{10}$$

with $U(C_{t+j})$ denoting the utility of consumption at time $t+j$. From the representative investor's first order conditions, the price at time t of a real zero coupon bond paying one unit at time $t+j$ is given by:

$$B_t(t+j) = E_t\left(\frac{U'_{C_{t+j}}}{U'_{C_t}}\right), \tag{11}$$

where U'_{C_s} denotes marginal utility.

To provide a simple yet testable model linking changes in consumption to real returns, we follow Harvey [1988] and make the following two assumptions which are retained throughout: conditional on information available at time t, I_t, (i) the logarithm of aggregate consumption, y_{t+j}, is normally distributed, with the variance of $y_{t+j} - y_t$ denoted by $\sigma_y^2(j)$; (ii) the representative investor has utility function $U(C_{t+j}) = e^{-\delta j}\ (C_{t+j}^{(1-\alpha)} - 1)/(1-\alpha)$, where α is the coefficient of relative risk aversion and δ is the rate of patience.[4]

Given assumptions (i) and (ii), the yield at time t on a j-period zero coupon bond, $r(t,j)$ is:

$$\begin{aligned} r(t,j) &= -\frac{\ln B_t(t+j)}{j} \\ &= \delta + (\alpha/j)E[y_{t+j} - y_t|I_t] - \frac{1}{2}(\alpha^2/j)\sigma_y^2(j). \end{aligned} \tag{12}$$

Expression (12) represents a simple linear version of a consumption asset pricing model. More generally, Breeden [1986] demonstrates that (12) holds as a second order approximation if consumption is not lognormally distributed.

[4]The implications of observed asset returns for the parameters of the utility function in (ii), without distributional assumptions, have been tested extensively by, among others, Hansen and Singleton [1982], and Ferson and Harvey [1992].

Unfortunately, expression (12) is not a term structure formula per se, as it does not specify how interest rates of different maturities are related, nor does it link real interest rates to the business cycle. To provide both these additional implications, we adopt the UC-ARIMA model for consumption dynamics. In particular, the stochastic model given by expressions (1), (2), and (3) is consistent with expression (12), since the logarithm of consumption is normally distributed.

From (1) and (3) it is clear that

$$\begin{aligned} r(t,j) &= -\frac{ln B_t(t+j)}{j} \\ &= \delta + \alpha\mu - \frac{\alpha^2}{2}\sigma_\tau^2 + (\alpha/j)E[c_{t+j} - c_t|I_t] - \frac{1}{2}(\alpha^2/j)\sigma_c^2(j), \end{aligned} \tag{13}$$

where $\sigma_c^2(j) = var(c_{t+j} - c_t)$. Expression (13) implies that real interest rates (as well as returns) are related to the expected change in the cyclical component, *not* on (log) consumption itself. The intuition behind this result is that under constant relative risk aversion and homoscedastic optimal consumption, the representative investor smooths only the fluctuations of consumption around its trend, and not the random walk component itself.

Once a particular autoregressive specification for the cyclical component is chosen, a term structure formula obtains which parametrizes interest rates for any maturity. This analysis extends the discussion of Campbell [1986] by allowing a stochastic trend in consumption and by computing explicit term structure formulae.

However, in this framework, the consol rate will be constant at $\ell = \delta + \alpha\mu - (\alpha^2/2)\sigma_\tau^2$. This constant consol rate can be avoided by assuming that the consumption drift, μ, follows a stochastic process (say, a random walk), or by simply assuming that μ is not known by the investor who then must estimate it every period.[5] In either case, the spread $r(t,i) - r(t,j), i > j$, will be a linear function of current and past values of the cycle, and will not depend on the consol rate ζ.

3.1 The Countercyclical Behavior of the Term Spread

Since under the autoregressive cyclical specification, expression (2), the expectation in expression (13) may be written as a linear function of current and past values of the cyclical component, real interest rates may also be expressed as a linear function of known values footnoteA related model making a similar point is that of Balvers, Cosimano and McDonald [1990], although its implications are derived only for stock returns. Under the additional assumptions of logarithmic utility and a single input, time varying Cobb-Douglas technology, they derive an equilibrium model in which the logarithm of production (output) follows the process $y_{t+1} = a + \mu t + \rho y_t + \varepsilon_{t+1}$ and aggregate consumption is a constant fraction of output. Due to the predictability of output, the real interest rate will be a linear function of the contemporaneous value of the logarithm of output (or consumption). The Markovian structure of its evolution implies that the current value of y is the only relevant information for evaluating its expected value.

In contrast, we argue that the trend of consumption is stochastic, and that the real interest rate depends only on the predictable cyclical component of output (or consumption). of

[5]The possibility that the representative investor does not observe the exogenous cyclical and permanent components in consumption implies that the investor, like the econometrician, will filter information regarding the state of the economy in a Bayesian fashion. The term structure will depend on the estimated cycle, and the variance term $\sigma_y^2(j)$ will now be larger as it will include the uncertainty surrounding the cycle and trend estimates. This variability will become constant as time elapses. If we carry out a two-step estimation procedure in which we first estimate the UC-ARIMA model (1), (2), (3), and then test the relationship between interest rates and the resultant consumption cycle, we will not be faced by an errors in variable problem. The estimate of the cycle obtained by the econometrician coincides with that of the representative investor and, as such, is the correct state variable to be used. Models with unobservable state variables include, among others, Williams [1977], Dothan and Feldman [1986], Gennotte [1986], Feldman [1992].

c_t:

$$r(t,j) = \ell - \frac{1}{2}(\alpha^2/j)a_p(j) + (\alpha/j)\sum_{i=1}^{p} b_{p_i}(j)c_{t+1-i}, \tag{14}$$

where $a_p(j) = var[c_{t+j} - c_t]$, and p is the order of the autoregressive process. Given the stationarity of this process, $\lim_{j\to\infty} a_p(j)/j = 0, \lim_{j\to\infty} b_{p_i}(j)/j = 0$.

Under these assumptions, we require a non-Markovian representation of the cyclical component (i.e. an autoregressive process of order $p > 1$) to generate the countercyclical behavior of the term spread observed empirically.[6]

The key to understanding why higher order term structure models imply countercyclical term spread behavior, at least for finite maturities, lies in the behavior of the terms $E[c_{t+j} - c_t|I_t]/j$ and $(1/j)a_p(j)$ to which j-period interest rates are proportional to. In particular,

[6] For the special case $p = 1$, that is, consumption's cyclical component follows an AR(1) process, we have (assuming complete information) the simple term structure formula

$$r(t,j) = -\frac{ln\ B_t(t+j)}{j} = \ell - 1/2\ (\alpha^2/j)a_1(j) + (\alpha/j)b_1(j)c_t, \tag{15}$$

$$\text{where} \quad a_1(j) = \sigma_c^2(1-\rho^{2j})(1-\rho^2)^{-1},$$
$$b_1(j) = -(1-\rho^j),$$

and $b_1(j) < 0, \lim_{j\to\infty} a_1(j)/j = 0$, $\lim_{j\to\infty} b_1(j)/j = 0$. Under incomplete information, we would simply add a constant to $a_1(j)$.

Expression (15) corresponds to Vasicek's [1977] term structure model in which bond prices are lognormally distributed and interest rates follow an AR(1) process. This term structure formula will obtain from expression (12) if the logarithm of consumption, in addition to being normally distributed, is also assumed to be Markovian with a stationary distribution. If the logarithm of consumption is not stationary, the term structure will necessarily be flat.

The AR(1) model implies that the real term structure is steeper at business cycle peaks and flatter at business cycle troughs. This is inconsistent with the empirical evidence of Kessel and others that the term spread tends to narrow at cyclical peaks and widen at cyclical troughs. To see the procyclical behavior of the term spread in the AR(1) model, note that the difference between the real rate for maturity $i > 1$ and the one period real rate is given by:

$$r(t,i) - r(t,1) = (\alpha^2/2)\left(a_1(1) - \frac{a_1(i)}{i}\right) + \alpha\left(\frac{(i-1)-(i\rho-\rho^i)}{i}\right)c_t\ ,$$

where the coefficient on c_t is > 0. Given the assumption of constant σ_c^2 and σ_τ^2, the AR(1) model's real term structure is upward sloping at a business cycle peak ($c_t > 0$) and vice versa. Similarly, Breeden's [1986] term structure model in a single good economy also implies procyclical behavior in the term spread.

consider the behavior of the first order term

$$E[c_{t+j} - c_t | I_t]/j \ .$$

If, for example, we are at a business trough (c_t negative), to generate an upward sloping term structure, we must expect business conditions (c_{t+j}) to improve *fast enough*, i.e. faster than j. If c_t follows an AR(1) process, this first order term is always decreasing in absolute value. In other words, under the AR(1) specification, we expect business conditions to improve, but not fast enough to generate an upward sloping term structure.

Therefore, under the assumptions that c_t is gaussian and homoscedastic, we require a non-Markovian process to generate expectations of a sufficiently strong business recovery. For example, an AR(2) specification is more appropriate. The behavior of $E[c_{t+j}|c_t, c_{t-1}]$ when c_t is negative is displayed in Figure 4. It is clear that while under the Markovian assumption this expectation increases at a decreasing rate, under the AR(2) specification it increases at an increasing rate for a number of quarters. As a result, $E[c_{t+j} - c_t|I_t]/j$ (and the term structure) will be increasing at the business trough.

4 Kalman Filter Estimation of the Consumption Asset Pricing Model

The consumption asset pricing model has traditionally been estimated by either the generalized method of moments (Hansen and Singleton [1982]), maximum likelihood estimation of an appropriately restricted vector autoregression (VAR) (Hansen and Singleton [1983]), linear regression (Harvey [1988]), or simple calibration (Mehra and Prescott [1985]).

Under our maintained assumptions of lognormality and constant relative risk aversion, Hansen and Singleton [1983] (H&S) posit a consumption asset pricing model in which returns are proportional, through the coefficient of relative risk aversion, to changes in the logarithm of real consumption. In their framework, the investor's information set, consisting of past changes in the log of consumption and past returns, is assumed to follow a VAR with gaussian disturbances. In particular, changes in the logarithm of real consumption are described by the VAR, while one period realized returns on the single asset are proportional to the change in the logarithm of consumption plus an error term. Defining Δy_{t+1} as the change in the logarithm of real consumption from quarter t to quarter $t+1$, and $R_{j,t+1}$ as the corresponding one-quarter holding period return on a zero coupon bond with maturity j, H&S consider the following

$$R_{j,t+1} = \mu_r + \alpha\, \Delta y_{t+1} + v^r_{t+1}. \tag{16}$$

$$\Delta y_{t+1} = \mu + \sum_{i=1}^{m} \phi_i\, \Delta y_{t+1-i} + \sum_{i=1}^{m} \gamma_i\, R_{j,t+1-i} + v^y_{t+1}. \tag{17}$$

The white noise disturbances v^r and v^y are normally distributed with constant covariance matrix Σ. The H&S procedure requires the estimation of this system by maximum likelihood.[7]

However, as previously argued, the decomposition of the changes in the logarithm of real consumption into independent temporary and permanent components implies that changes in real consumption are a noisy proxy for the relevant state variable. Since real rates and changes in consumption are both linearly related to the business cycle, this suggests the following statistical efficient bivariate strategy for estimating the parameters of the consumption asset pricing model.

[7] In the single asset example μ_r and μ may be concentrated out.

Consumption dynamics, defined by expressions (1), (2), and (3), may be written as [8]:

$$\Delta y_{t+1} = \mu + \Delta c_{t+1} + \epsilon^{\tau}_{t+1}, \tag{18}$$

$$\Delta c_{t+1} = \sum_{i=1}^{m} \rho_i \Delta c_{t+1-i} + \epsilon^{c}_{t+1}. \tag{19}$$

The theoretical asset pricing model provides an additional measurement equation which links the business cycle to interest rates. In particular, it can easily be seen from expression (13) that the term structure model implies a linear relationship between expected changes in the business cycle and expected real returns. For realized returns, we have

$$R_{j,t+1} = \mu_r + \alpha \Delta c_{t+1} + z_{t+1} \ , \tag{20}$$

where z is a white noise error, assumed gaussian. The effect of z on the estimation procedure will be discussed below.[9]

Our model may be estimated by maximum likelihood using the multivariate Kalman filter: equations (18) and (20) provide two measurement equations, while equation (19) is the transition equation. This estimation strategy explicitly recognizes measurement error due to the presence of an independent random walk component in the logarithm of real consumption and, furthermore, that the relevant explanatory variable, Δc, is not observable.[10]

According to our specification, real returns and changes in consumption have a common predictable factor, Δc. However, real rates and consumption are not cointegrated: while y

[8] Given equation (2), $c_t\Phi(L) = e^c_t$, where $\Phi(L)$ is a lag polynomial, we assume that we can factor $\Phi(L)$ so that $\Phi(L) = \hat{\Phi}(L)(1-L)$, and this yields (19),$c_t\hat{\Phi}(L)(1-L) = \epsilon^c_t$. See Stock and Watson [1991].

[9] In the case of nonlinear and/or non-gaussian models, (20) may be interpreted as the result of linearizing the model. However, more general numerical procedures could be used to handle a nonlinear relationship between returns, the business cycle, and stochastic trend, as well as incorporating different distributional assumptions (see Kitagawa [1987]). If only the normality assumption is violated, the Kalman filter remains the optimal linear estimator.

[10] In the case of a single asset, we can concentrate the parameter μ_r out of the likelihood function, while for more than one asset this parameter cannot be concentrated out as it will be a function of the consumption variance.

contains a stochastic trend and is stationary only in first differences, real rates do not contain a stochastic trend and are stationary in levels.

Stock and Watson ([1989], [1991]) also use the multivariate Kalman filter to estimate a common stationary component from several macroeconomic time series, including nominal interest rates. In contrast, we use the Kalman filter to estimate the parameters of a specific asset pricing model. In fact, we must allow for correlation between the error made in forecasting the business cycle, e^c, and the pricing error, z. However, we still maintain that the trend and cycle disturbances are independent.[11]

In comparison with H&S, our model includes an additional measurement equation with a disturbance component arising from the presence of a stochastic trend in consumption, e^τ, which may be filtered out. Also, in H&S's framework (expression (14)), the conditional expectation of Δy_{t+1} is a direct function of past asset returns. In our case, past asset returns indirectly influence the conditional expectation of Δc_{t+1} by determining past values of Δc.

[11]That is, $cov(e_t^\tau, e_{t-k}^c) = 0 \; \forall k$ and $cov(e_t^\tau, z_{t-k}) = 0 \; \forall k$, $cov(z_t, e_t^c) \neq 0$. The model, (18), (20), and (19), as it stands does not satisfy the assumption of independence between the transition and measurement equations required by the Kalman filter. This necessitates a modification of the standard filtering procedure. To see this, write the model in compact form as

$$\begin{aligned} Y_t &= \mathbf{Z}\alpha_t + \varepsilon_t, & (21) \\ \alpha_{t+1} &= \mathbf{T}\alpha_t + \eta_t, & (22) \end{aligned}$$

where the vector $Y_t = \left[(R_{j,t+1} - \mu_r) \; (\Delta y_{t+1} - \mu) \right]'$, $Var(\varepsilon) = H$ and $Var(\eta) = Q$, both diagonal matrices. In general, however,

$$G = E\left[\eta \varepsilon'\right] = \begin{bmatrix} \sigma_{zc} & 0 \\ 0 & 0 \end{bmatrix}.$$

To accommodate the correlation between ε and η, we follow Harvey [1989a, p. 113] and use the equivalent transition equation:

$$\alpha_{t+1} = \mathbf{T}^* \alpha_t + GH^{-1}Y_t + \eta_t^*, \tag{23}$$

where $\mathbf{T}^* = \mathbf{T} - GH^{-1}\mathbf{Z}$ and $\eta_t^* = \eta_t - GH^{-1}\varepsilon_t$. By construction, the error terms in the measurement and transformed transition equation are now independent, permitting the application of the Kalman filter in calculating the corresponding likelihood function.

4.1 Data

We use the logarithmic transform of quarterly seasonally adjusted data on the consumption of non-durables and services from the National Income and Product Accounts over the sample period 1964:2 to 1993:3, expressed in constant (1987) dollars. The interest rate data is as before.[12]

4.2 Empirical Results

A specification search resulted in an AR(3) model for log consumption's cyclical component, expression (19).[13] Prior to estimation, we de-meaned the observable series and concentrated both μ and μ_r out of the likelihood function. After normalizing σ_τ^2 to equal 1 (see Stock and Watson [1991]), we then estimated the parameter vector $\Psi = \{\sigma_c, \sigma_z, \rho_1, \rho_2, \rho_3, \alpha, \sigma_{cz}\}$. [14]

Table 3 reports the results of our maximum likelihood estimation for 3- and 12-month Treasury bills as well as for 3- and 5-year Treasury bonds. The average (annualized) real return on the sampled bills and bonds is between 1.8% and −5.7%, while the average (annualized) growth rate in real consumption is estimated to be 2.9%. The estimated coefficient

[12] The Fama file starts in 1964:2. We used deflated 3-month interest rates from the Federal Reserve Quarterly Bulletin as a proxy for returns in the warm-up period. These rates are not used in the subsequent estimation.

[13] The resulting model in the state-space form is:

$$\begin{bmatrix} R_{j,t+1} \\ \Delta y_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_r \\ \mu \end{bmatrix} + \begin{bmatrix} \alpha & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta c_{t+1} \\ \Delta c_t \\ \Delta c_{t-1} \end{bmatrix} + \begin{bmatrix} z_{t+1} \\ e^\tau_{t+1} \end{bmatrix}, \tag{24}$$

$$\begin{bmatrix} \Delta c_{t+1} \\ \Delta c_t \\ \Delta c_{t-1} \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 & \rho_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta c_t \\ \Delta c_{t-1} \\ \Delta c_{t-2} \end{bmatrix} + \begin{bmatrix} e^c_{t+1} \\ 0 \\ 0 \end{bmatrix}. \tag{25}$$

We then modify (25) to take into account the correlation between the transition and measurement disturbance.

[14] Otter [1988, Theorem 1] provides conditions for model identification.

of relative risk aversion, $\hat{\alpha}$, lies between 5.7, for 3-month bills, and 7.9, for 5-year bonds. Compared to previous research, the precision of these parameter estimates is remarkable. By comparison, Table 4 summarizes the corresponding results of the H&S estimation procedure, both without (Panel A) and with (Panel B) the use of past returns. Notice that in Table 4 the estimated coefficients of relative risk aversion are always statistically insignificant, while the standard errors of the bond return equations are consistently high.

While the fit of the return equation appears to be superior using our UC-ARIMA approach, the consumption equation performs slightly worse as the standard errors of changes in consumption are consistently higher in Table 3 than in Table 4. This follows from the fact that our estimation of the implied cyclical component is heavily influenced by the highly variable bond returns.

This result highlights a deficiency of consumption asset pricing models. The problem plaguing consumption asset pricing models is that the variability of aggregate consumption is considerably lower than the observed variability of real returns. The relevant explanatory variable in our model is consumption's cyclical component and unfortunately we estimate an implausibly volatile cyclical component. In other words, it is not consumption per se which is not variable enough but, more precisely, it is the variability of consumption's cyclical component which is too low to satisfactorily explain asset returns.

To see this clearly, consider Figure 5 where we plot the estimated cycle against the demeaned changes in consumption as well as real returns on the 3-month Treasury bill (Figure 5a) and real returns on the 3-year Treasury bond (Figure 5b). The estimated cyclical component represents the predictable variation in consumption required to explain observed bond returns. Notice that as the bond maturity increases from 3 months (Figure 5a) to 3 years (Figure 5b), the variability of returns increases, causing the variability of the estimated cyclical component to actually be higher than the variability of the original consumption series.

5 Summary and Conclusions

This paper investigates the behavior of the term structure of interest rates over the business cycle. As compared to previous research, we use a more appropriate measure of the business cycle: the deviation of aggregate economic activity (GDP and aggregate consumption) from its stochastic trend. Using this more precise measure, as opposed to the simple growth in aggregate economic activity, significantly improves the term spread's informativeness regarding future economic activity.

We exploit this measure of the business cycle in investigating a consumption based model of the real term structure. The empirical evidence of Kessel and others suggests that the spread between long and short rates of interest widens at cyclical troughs and narrows at cyclical peaks. However, we demonstrate that a non-Markovian model of the real term structure is required to capture this countercyclical behavior.

We also explore the implications of an unobservable but independent cyclical component in consumption dynamics on the estimation of consumption asset pricing models. Compared to previous maximum likelihood procedures which ignore the presence of this cyclical component, the coefficient of relative risk aversion is estimated far more precisely. However, the resultant fit of the consumption asset pricing model is not completely satisfactory as the required variability of consumption's cyclical component is implausibly high.

Table 1A

Estimation of the Regressions

$$ts_t = a_j + b_j \Delta c_t + u_t$$

Quarters	a	b	SE	R^2	N. Obs.
1	.00249	.31628	.00843	.10099	115
	(.00188)	(.09498)			
2	.00093	.26297	.00786	.19599	114
	(.00195)	(.07585)			
3	.00008	.21012	.00757	.23204	113
	(.00192)	(.06029)			
4	-.00071	.18066	.00729	.26102	112
	(.00190)	(.05013)			
5	-.00145	.16356	.00708	.29526	111
	(.00194)	(.04618)			
6	-.00209	.15026	.00698	.31236	110
	(.00193)	(.04140)			
7	-.00266	.13936	.00689	.32412	109
	(.00193)	(.03842)			
8	-.00326	.13216	.00683	.33211	108
	(.00195)	(.03512)			
9	-.00385	.12743	.00688	.32818	107
	(.00202)	(.03381)			
10	-.00446	.12340	.00693	.32207	106
	(.00216)	(.03372)			

This table reports the results of a regression of the term spread on subsequent changes in the logarithm of GDP. Changes are computed over an increasing number of quarters. The estimation period is 64-1 to 92-4. In parentheses are the Newey-West (1987) adjusted standard errors of the coefficients (20 lags).

Table 1B

Estimation of the Regressions

$$ts_t = a_j + b_j \Delta c_t + u_t$$

Quarters	a	b	SE	R^2	N. Obs.
1	.00467	-.16268	.00860	.06354	115
	(.00189)	(.03165)			
2	.00454	-.13816	.00827	.10990	114
	(.00184)	(.02066)			
3	.00438	-.10752	.00828	.08138	113
	(.00182)	(.01907)			
4	.00420	-.07948	.00829	.04804	112
	(.00180)	(.02515)			
5	.00409	-.06824	.00828	.03656	111
	(.00180)	(.02436)			
6	.00400	-.05330	.00833	.01994	110
	(.00181)	(.02469)			
7	.00389	-.03309	.00836	.00337	109
	(.00182)	(.02183)			
8	.00379	-.01699	.00838	-.00583	108
	(.00183)	(.02193)			
9	.00372	.00065	.00843	-.00952	107
	(.00185)	(.02338)			
10	.00363	.01860	.00845	-.00627	106
	(.00187)	(.02730)			

This table reports the results of a regression of the term spread on subsequent changes in the cycle component of GDP estimated from the ARIMA model. Changes are computed over an increasing number of quarters. The estimation period is 64-1 to 92-4. In parentheses are the Newey-West (1987) adjusted standard errors of the coefficients (20 lags).

Table 1C

Estimation of the Regressions

$$ts_{t,j} = a_j + b_j \Delta c_t + u_t$$

Quarters	a	b	SE	R^2	N. Obs.
1	.00169	.32149	.00886	.00782	115
	(.00191)	(.30016)			
2	.00455	.27484	.00865	.02708	114
	(.00185)	(.17405)			
3	.00444	.31591	.00827	.08501	113
	(.00176)	(.12779)			
4	.00434	.35243	.00772	.17469	112
	(.00167)	(.10644)			
5	.00430	.35948	.00729	.25428	111
	(.00160)	(.08703)			
6	.00425	.36462	.00688	.33137	110
	(.00154)	(.06812)			
7	.00418	.36293	.00650	.39824	109
	(.00148)	(.05418)			
8	.00410	.36196	.00617	.45501	108
	(.00144)	(.04310)			
9	.00405	.35387	.00606	.47849	107
	(.00143)	(.03936)			
10	.00399	.34681	.00600	.49160	106
	(.00143)	(.03519)			

This table reports the results of a regression of the term spread on subsequent changes in the cycle component of GDP estimated from the UC-ARIMA model. Changes are computed over an increasing number of quarters. The estimation period is 64-1 to 92-4. In parentheses are the Newey-West (1987) adjusted standard errors of the coefficients (20 lags).

Table 2A

Estimation of the Regressions

$$ts_t = a_j + b_j \Delta y_t + u_t$$

Spread	a	b	SE	R^2	N. Obs.
5yr.-1yr.	.00249	.31628	.00813	.10099	115
	(.00188)	(.09498)			
3yr.-1yr.	.00161	.21015	.00593	.09033	115
	(.00128)	(.06116)			
5yr.-2yr.	.00161	.18929	.00501	.10218	115
	(.00121)	(.06341)			

This table reports the results of a regression of different definitions of the term spread on one-quarter changes in the logarithm of GDP. The estimation period is 64-1 to 92-4. In parentheses are the Newey-West (1987) adjusted standard errors of the coefficients (20 lags).

Table 2B

Estimation of the Regressions

$$ts_{t,j} = a_j + b_j c_t + u_t$$

Spread	a	b	SE	R^2	N. Obs.
5yr.-1yr.	.00556	-.02873	.00906	-.00531	116
	(.00267)	(.04698)			
3yr.-1yr.	.00340	-.00954	.00632	-.00799	116
	(.00178)	(.02754)			
5yr.-2yr.	.00369	-.02637	.00539	-.00059	116
	(.00140)	(.02853)			

This table reports the results of a regression of different definitions of the term spread on the cycle component of GDP estimated from the ARIMA model. The estimation period is 64-1 to 92-4. In parentheses are the Newey-West (1987) adjusted standard errors of the coefficients (20 lags).

Table 2C

Estimation of the Regressions

$$ts_{t,j} \;=\; a_j + b_j c_t + u_t$$

Spread	a	b	SE	R^2	N. Obs.
5yr.-1yr.	.00196	-.43601	.00777	.25921	116
	(.00249)	(.12983)			
3yr.-1yr.	.00109	-.31613	.00534	.28097	116
	(.00173)	(.08869)			
5yr.-2yr.	.00158	-.21797	.00488	.17923	116
	(.00143)	(.08105)			

This table reports the results of a regression of different definitions of the term spread on the cycle component of GDP estimated from the UC-ARIMA model. The estimation period is 64-1 to 92-4. In parentheses are the Newey-West (1987) adjusted standard errors of the coefficients (20 lags).

Table 3

Kalman filter estimation of the model:

$$\Delta y_{t+1} = \mu + \Delta c_{t+1} + e^{\tau}_{t+1}$$
$$R_{j,t+1} = \mu_r + \alpha \Delta c_{t+1} + z_{t+1}$$
$$\Delta c_{t+1} = \rho_1 \Delta c_t + \rho_2 \Delta c_{t-1} + \rho_3 \Delta c_{t-2} + e^{c}_{t+1}$$

	3-month	12-month	3-year	5-year
α	5.731042	4.602343	5.636200	7.856265
	(1.73945)	(0.18277)	(0.02927)	(0.06229)
ρ_1	0.239837	0.842837	0.299911	0.292930
	(0.24157)	(0.07015)	(0.04590)	(0.27857)
ρ_2	0.214667	-0.479048	0.033919	-0.007196
	(0.23428)	(0.07541)	(0.01051)	(0.12731)
ρ_3	0.356732	0.460023	0.215169	0.217392
	(0.08105)	(0.06387)	(0.00431)	(0.11597)
σ_z	0.003569	0.016653	0.035992	0.046395
σ_c	0.000010	0.001220	0.000104	0.000072
$cov(e^c, z)$	0.87182	-0.27945	-0.36536	-0.28943
$SE(R)$	0.001059	0.023905	0.022257	0.025401
$SE(\Delta y)$	0.020803	0.019933	0.027837	0.028137
μ_r	0.018794	0.000344	-0.054824	-0.057087
μ	0.029078	0.029078	0.029078	0.029078
Log Lik.	-262.692413	-184.149200	-77.425812	-40.067268

In the model specification, Δy denotes the logarithm of consumption of nondurables and services; R is the one-quarter return on the j-maturity bond; Δc denotes the change in the unobservable cyclical component. The estimation is carried out on data from 1964:2 to 1992:3. To take account of the correlation between z and e^c, a modified transition equation is used in the estimation, and σ_c is the standard deviation of the modified equation. The variance of the trend disturbance e^{τ} is normalized to 1. $SE(R)$ and $SE(\Delta y)$ denote the sample standard deviation of the prediction error of the two observable series.

Table 4a

Hansen and Singleton (1983) ML estimation of the relationship between one-quarter real returns and changes in consumption (data from 64:2 to 92:3). The model used is:

$$\Delta y_{t+1} = \mu + a_1 \Delta y_t + a_2 \Delta y_{t-1} + a_3 \Delta y_{t-2} + v_{t+1}^y$$

$$R_{j,t+1} = \mu_r + \alpha \Delta y_{t+1} + v_{t+1}^r$$

	3-month	12-month	3-year	5-year
α	-0.921698	-0.367809	-1.645450	-2.191430
	(0.36907)	(0.60089)	(1.33460)	(1.79982)
a_1	0.346778	0.360372	0.318777	0.318729
	(0.09009)	(0.09620)	(0.09807)	(0.09815)
a_2	-0.018080	-0.009269	-0.006680	-0.002558
	(0.08200)	(0.09274)	(0.10061)	(0.09986)
a_3	0.278088	0.253571	0.299603	0.296512
	(0.09005)	(0.09997)	(0.10253)	(0.10277)
$SE(R)$	0.038192	0.056492	0.128324	0.176842
$SE(\Delta y)$	0.016906	0.016913	0.016898	0.016895
$cov(v^r, v^y)$	0.000311	0.000329	0.000493	0.000633

Table 4b

Hansen and Singleton (1983) ML estimation of the relationship between one-quarter real returns and changes in consumption (data from 64:2 to 92:3). The model used is:

$$\Delta y_{t+1} = \mu + a_1 \Delta y_t + a_2 \Delta y_{t-1} + a_3 \Delta y_{t-2} + b_1 R_{j,t+1} + b_2 R_{j,t-1} + b_3 R_{j,t-2} + v^y_{t+1}$$

$$R_{j,t} = \mu_r + \alpha \Delta y_{t+1} + v^r_{t+1}$$

	3-month	12-month	3-year	5-year
α	3.877890	1.459100	-1.213430	-1.771170
	(1.42011)	(0.54353)	(1.02963)	(1.44062)
a_1	-0.066778	0.120956	0.302318	0.302985
	(0.04809)	(0.11674)	(0.10424)	(0.10410)
a_2	0.076785	0.100428	0.041263	0.038847
	(0.04783)	(0.11355)	(0.10254)	(0.10169)
a_3	-0.010265	0.203831	0.274867	0.277278
	(0.03863)	(0.09729)	(0.10249)	(0.10421)
b_1	0.113327	0.103845	0.034989	0.022906
	(0.04451)	(0.03411)	(0.01462)	(0.01013)
b_2	0.013285	0.006199	0.034989	0.012619
	(0.02431)	(0.03877)	(0.01497)	(0.01101)
b_3	0.072394	0.055527	-0.003085	-0.002159
	(0.034915)	(0.033126)	(0.015529)	(0.010848)
$SE(R)$	0.085196	0.057145	0.126953	0.175525
$SE(\Delta y)$	0.019823	0.016443	0.016158	0.016264
$cov(v^r, v^y)$	-0.001629	-0.000370	0.000324	0.000475

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Figure 1

This figure illustrates the UC-ARIMA business cycle estimated with U.S. GDP data and the nominal term spread.

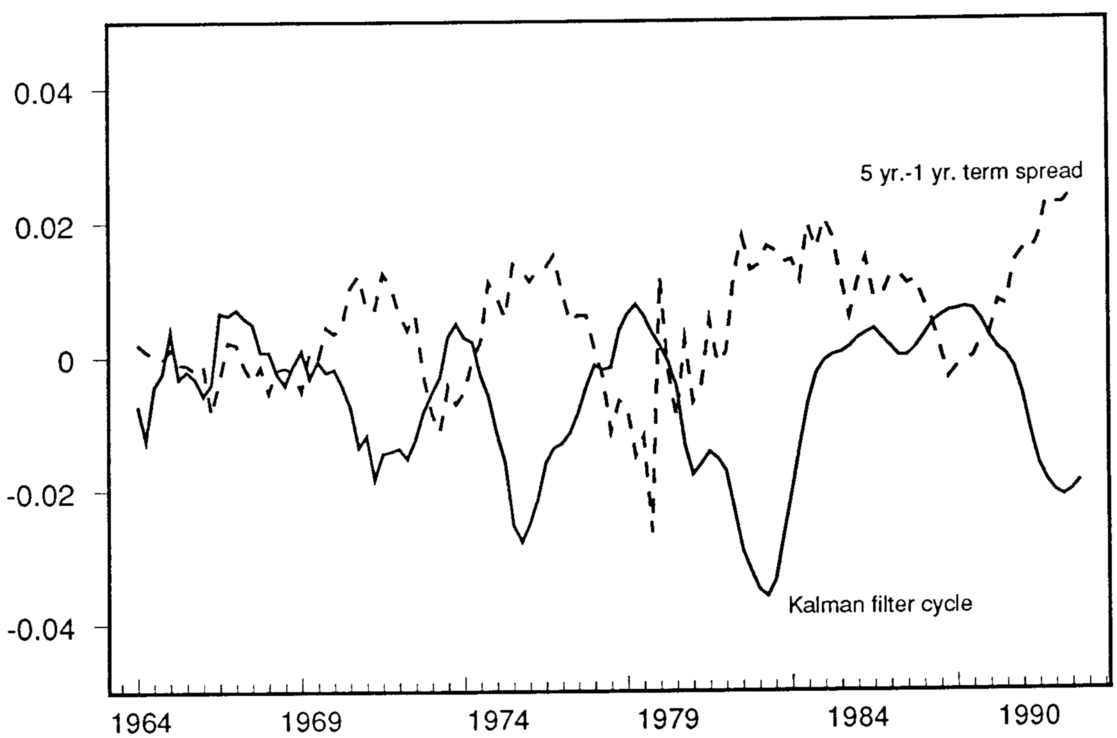

Figure 2

This figure illustrates the ARIMA business cycle estimated with U.S.GDP data and the nominal term spread.

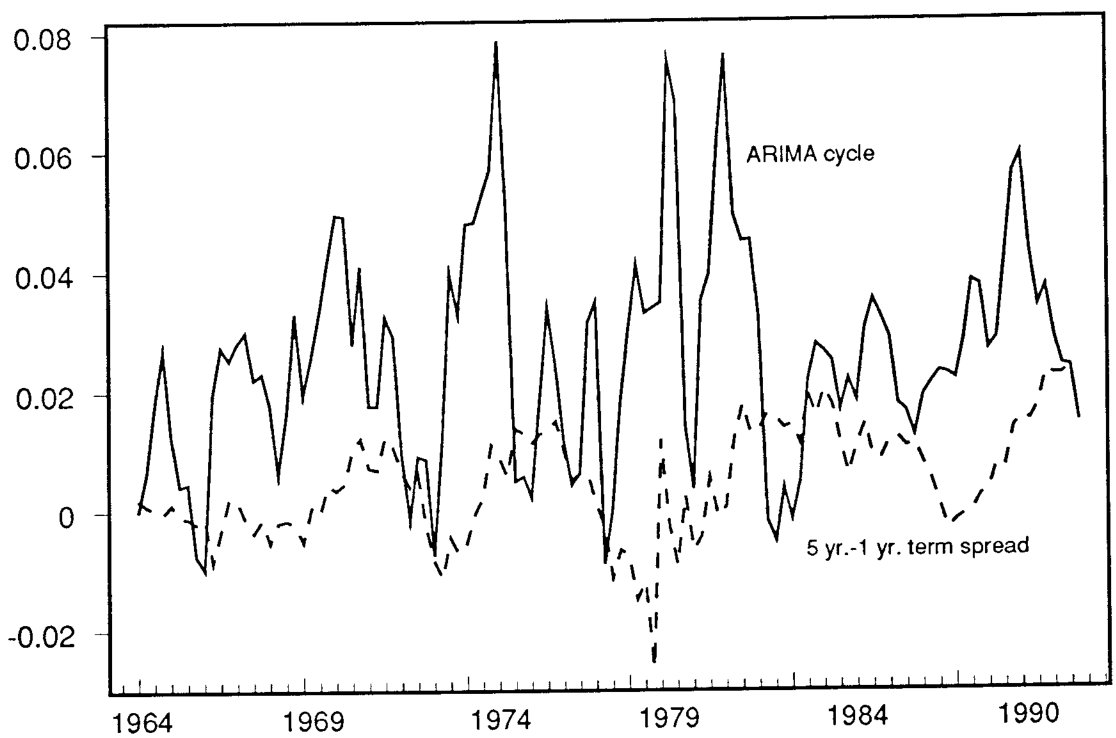

Figure 3

This figure illustrates one-quarter (annualized and demeaned) changes in the logarithm of U.S. GDP and the nominal term spread.

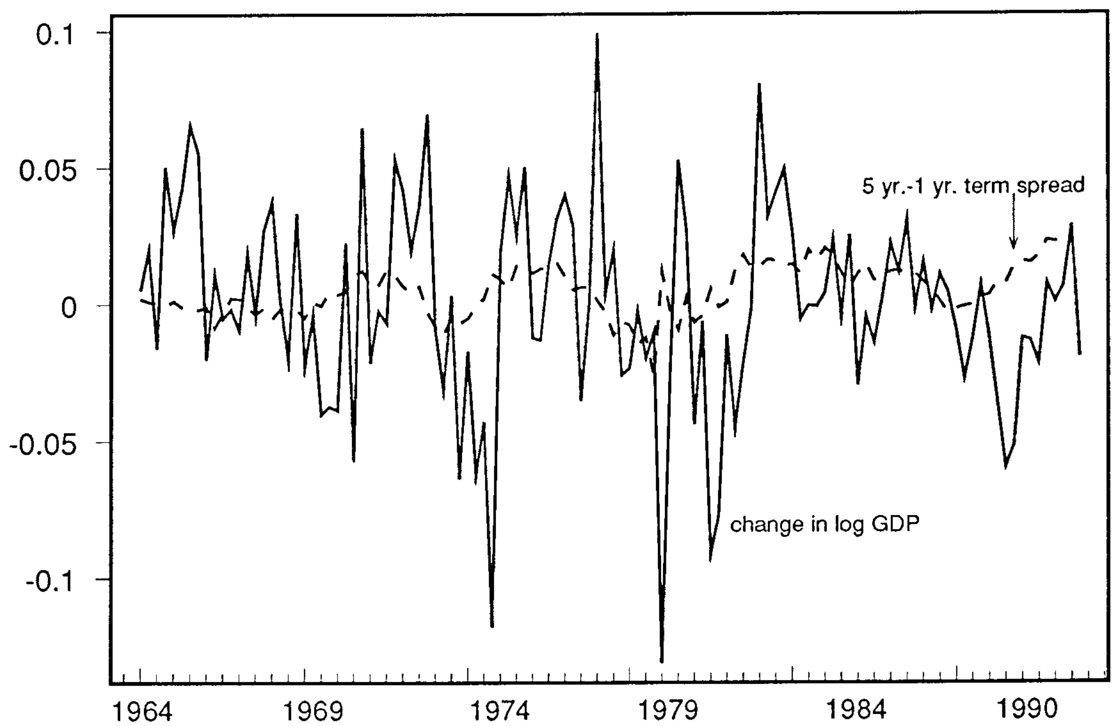

Figure 4

This figure illustrates the expected value of the consumption cycle at future points in time, starting with c_t=-0.012, and considering an AR(2) process with $\rho_1 = 1.77$ and $\rho_2 = -0.82$, and an AR(1) process with $\rho = 0.9$.

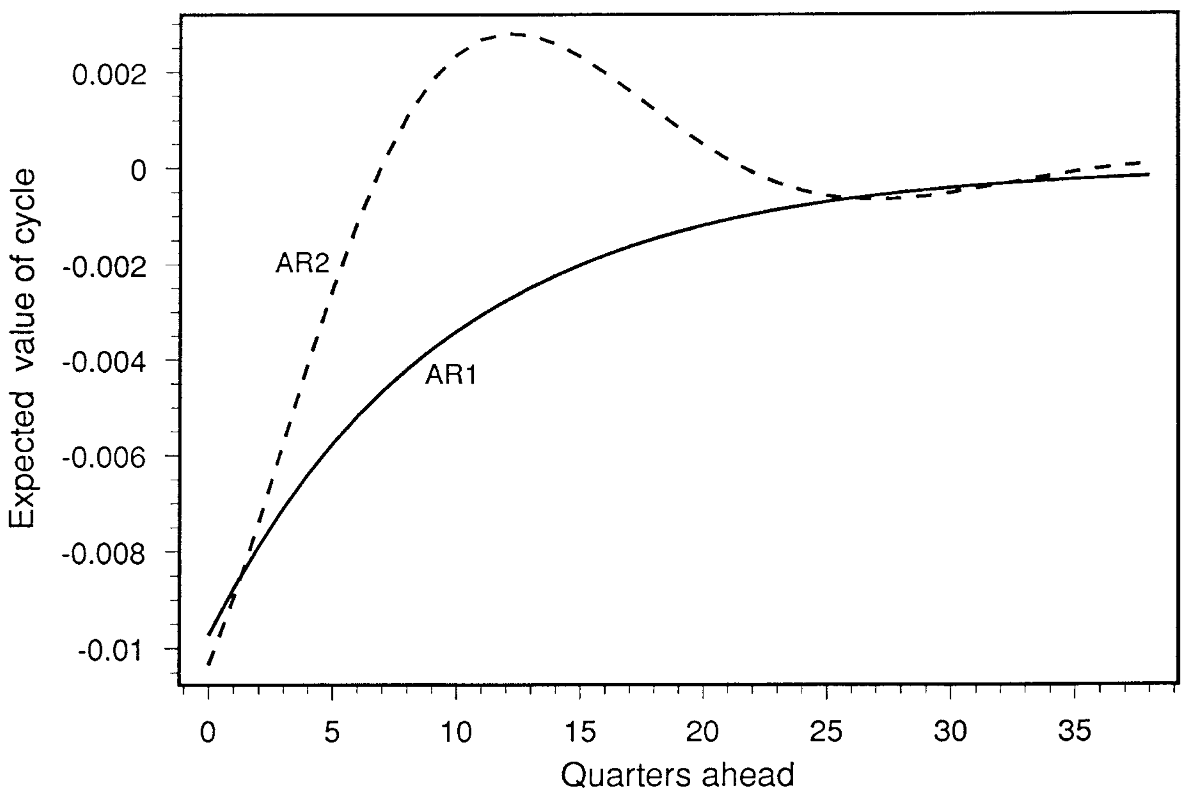

Figure 5a

This figure illustrates real returns on a 3-month bill, changes in the logarithm of consumption and the cyclical component estimated from the two series.

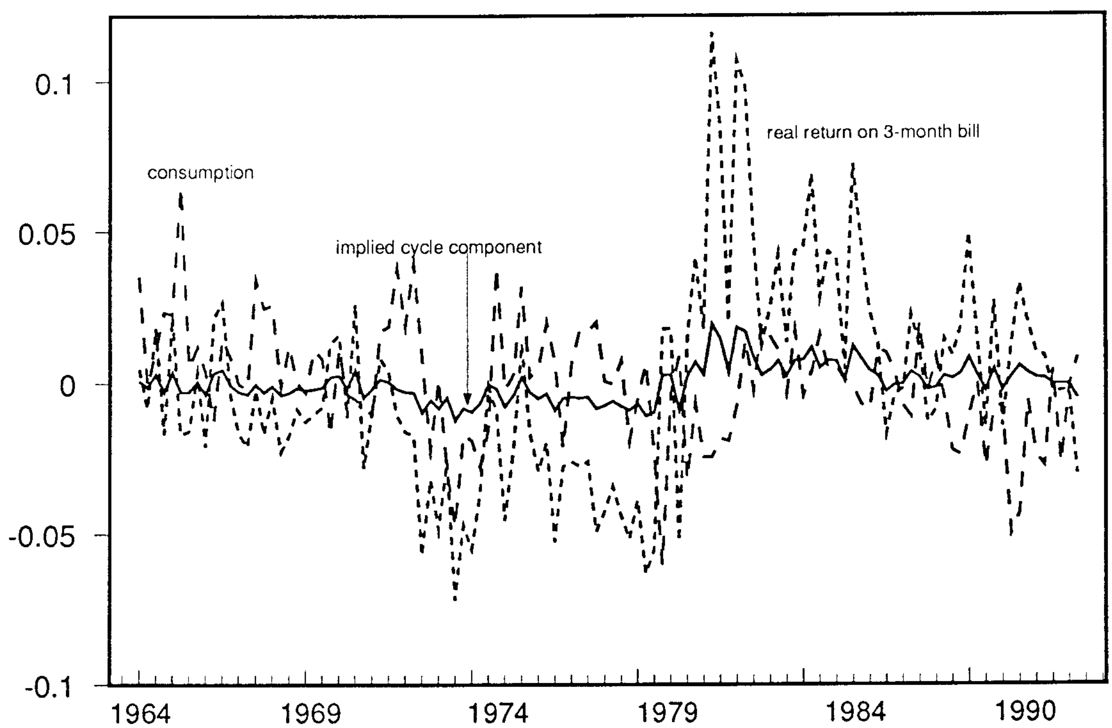

Figure 5b

This figure illustrates real returns on a 3-year bond, changes in the logarithm of consumption and the cyclical component estimated from the two series.