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ABSTRACT

A semi-analytical dual-porosity simulator for unsaturated flow in fractured rock masses has been developed. Fluid flow between the fracture network and the matrix blocks is described by analytical expressions that have been derived from approximate solutions to the imbibition equation. These expressions have been programmed into an unsaturated flow simulator, TOUGH, as a source/sink term. Flow processes are then simulated using only fracture elements in the computational grid. The modified code is used to simulate flow along single fractures, and infiltration into pervasively fractured formations. Comparisons are made with simulations carried out using discretization of both the fractures and matrix blocks. The new semi-analytical code is shown to be accurate, and typically requires an order of magnitude less computational time.

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INTRODUCTION

A potential site for an underground radioactive waste repository is in Nevada at Yucca Mountain, in a region consisting of highly-fractured, volcanic tuff.[1] In order to characterize the site for the purposes of determining its suitability for a repository, it is necessary to develop methods for modeling and predicting the flow of water in unsaturated, fractured rock masses having low matrix permeability. Such models will also be useful in assessing the travel-times required for radionuclides from the repository to reach the saturated zone below the water table. In order to study the highly transient flow processes that may occur, we need to be able to treat flows in tuffaceous geological units that consist of two intermingled networks of porosity: a (relatively) high-permeability, low storativity fracture network, and low-permeability, high-storativity matrix blocks. Existing unsaturated-flow simulators, such as TOUGH[2], can analyze such dual-porosity systems by discretizing both the fracture systems and matrix blocks, and solving the flow equations numerically in each system. Numerical simulations of fully-discretized systems such as these require a large number of computational cells, and consequently a large amount of computer time. We attempt to mitigate these problems by modifying the TOUGH simulator so as to treat flow between the fracture network and matrix blocks by an analytical expression, eliminating the need to discretize the matrix blocks. This modified code, which performs dual-porosity simulations in a much more computationally efficient manner, can be used to study transient flow processes that may be expected to occur at Yucca Mountain.

DUAL-POROSITY MODELS

For processes which occur on a sufficiently slow time scale, it is often assumed that the fractured rock mass can be treated as an equivalent porous medium, with an effective permeability that is a weighted average of the permeabilities of the fracture network and the matrix blocks.[3] This approach assumes that the matrix blocks are always in local equilibrium with their surrounding fractures; it is therefore capable of

simulating processes that occur slowly enough that pressure equilibrium can be achieved between the fractures and matrix blocks. However, the time required for fracture/matrix equilibration is inversely related to the permeability of the matrix blocks, and is consequently very large for a system such as the fractured tuffs at Yucca Mountain. Numerical simulations of imbibition into blocks of Topopah Spring tuff have shown[4] that the equilibration time will be on the order of months for matrix blocks that are a few tens of centimeters on a side, and longer if the blocks are larger. For highly-transient processes, such as the infiltration that would occur after a precipitation event, the "dual-porosity" nature of the medium must therefore be accounted for. In a dual-porosity medium, the fractures provide most of the high permeability of the rock mass, whereas most of the fluid storage takes place in the relatively low-permeability matrix blocks. The complex behavior of dual-porosity systems arises from the fact that there are different time scales corresponding to diffusion of water in the fracture network and in the matrix blocks.

Transient flow in fractured/porous media could be treated numerically by explicitly modeling each fracture and each matrix block. The detailed geological information required for such analyses are usually unavailable, and, moreover, analyses of a large-scale system might require a prohibitively large number of computational grid-blocks. This would be feasible in for analyzing flow in a reservoir with a relatively small number of large matrix blocks,[5] but would become impractical for a system such as the unsaturated zone at Yucca Mountain. For example, of the number of distinct matrix blocks in the vicinity of the potential repository site is easily estimated to be on the order of millions, if one assumes fracture spacings on the order of one meter.[6]

A more commonly used conceptual model of a dual-porosity system assumes the existence of two overlapping continua, the fracture continuum and the matrix continuum. Flow is assumed to take place not only through the fractures, but also between the fractures and the matrix.[7,8] A relatively efficient numerical implementation of this

type of model is the MINC[9] (Multiple INteracting Continua) method, in which the representative matrix blocks are discretized into nested elements, thus treating flow within each matrix block as being mathematically one-dimensional. The matrix continuum in MINC-type simulations is broken up into computational gridblocks that may be much larger than the actual matrix blocks; the appropriate way to handle this is explained by Pruess and Narasimhan.[9] MINC-type simulations require many fewer computational cells than do discrete-fracture simulations, but still require large numbers of matrix gridblocks. We have found that accurate treatment of transient effects with the MINC method requires that the representative matrix block associated with each computational cell of the fracture continuum must itself be broken up into about ten nested grid blocks. Hence, MINC simulations of transient processes will require about ten times the number of computational cells needed for quasi-steady-state simulations. Since the CPU time required by most numerical simulators grows at a rate at least directly proportional to the number of computational cells, simulation of large-scale transient processes in fractured rocks would be computationally burdensome.

We have taken the approach of incorporating into a numerical simulator analytical expressions[4,10] for absorption of water into matrix blocks, which then plays the role of a source/sink coupling term for the fractures. A related approach for treating single-phase fluid flow and heat transfer in fractured reservoirs has been taken by Pruess and Wu.[11] They modeled the fracture/matrix interactions not by an analytical expression, but by an ordinary differential equation, the solution of which involves very little computational effort. These approaches are somewhat in the spirit of the MINC method, since they assume that the fractures surrounding each individual matrix block are in local equilibrium with each other, and so imbibition into the matrix block commences simultaneously from its entire outer boundary. Our analytical expression accounts for factors such as block geometry, initial saturation, and the hydrological properties of the

matrix blocks. Different absorption expressions have been developed for media which have relative permeability and capillary pressure curves of the van Genuchten-Mualem or Brooks-Corey type. The accuracy of these interaction terms have been verified through numerical simulations of imbibition into single matrix blocks of various shapes and sizes.[4] For either form of the characteristic curves, evaluation of the fracture/matrix flow coupling term requires an insignificant amount of CPU time. The computational meshes used in our simulations then consist only of "fracture elements", whose permeability, porosity, and other properties are those of the fracture system, averaged over a suitably large representative elementary volume.[12] The matrix blocks are not explicitly discretized or represented in the computational mesh, but are accounted for through the source/sink coupling term. The only geometrical properties of the matrix blocks that are required as input to the simulations are their volumes and their surface areas. Flow from the fractures into the matrix blocks at any given location is assumed to begin when the liquid saturation in the fractures at that location reaches some nominal value which is taken to signal the arrival of the liquid front. After this time, the volumetric flux of water into the matrix block is calculated from our analytical expressions, as a function of the matrix block properties, initial saturation, and elapsed time since the start of imbibition into that particular block.

FRACTURE/MATRIX FLOW INTERACTION EXPRESSIONS

In a large-scale flow process occurring in an unsaturated dual-porosity medium, the saturations and pressures will vary with time in both the fractures and the matrix blocks. However, due to the much larger permeability of the fractures, diffusion of liquid will typically occur much more rapidly in the fractures than in the matrix blocks. Hence, if a liquid saturation front is diffusing through the fracture network, we expect that the time it takes to travel past a single matrix block will be very small compared with the characteristic time of diffusion within that block. This allows us to make the approximation that any given matrix block is surrounded by fractures that are

either ''ahead of'' or ''behind'' the saturation front in the fracture network. We therefore assume that the boundary conditions for the matrix block, which are provided by the surrounding fractures, are of the step-function type. That is to say, the saturation in the fracture is at its initial value S_i for $t<t_o$, and then abruptly jumps to some higher value, say S_o, for $t>t_o$. In a sense, we have partially uncoupled the problem, which allows us to solve the diffusion equation in the matrix block under boundary conditions that are assumed to be ''known''.

Hence the basic problem that must be solved in order to develop analytic expressions for the fracture/matrix flow interactions is that of water imbibing into an initially unsaturated matrix block, under the previously-discussed step-function boundary conditions. Under the assumption that the gas phase is infinitely mobile, the flow of liquid water in an unsaturated porous medium can be described by the highly nonlinear Richards' equation:[13]

$$\operatorname{div}\left[\frac{k k_r(\psi)}{\mu\phi}\operatorname{grad}\psi(x,t)\right] = \frac{\partial S(x,t)}{\partial t} \quad . \tag{1}$$

In eq. (1), ψ represents the pressure potential of the water in the matrix block. It is positive in regions of full saturation, where it is equivalent to the usual (hydrostatic) pressure used in fluid mechanics; it is negative in regions of partial saturation. The saturation S represents the fraction of pore space that is filled with water. S and ψ are related through the capillary pressure relation, the precise form of which depends on rock type (see below). The parameter k is the permeability of the matrix block under fully-saturated conditions, μ is the viscosity of the pore water, and ϕ is the porosity of the matrix block. $k_r(\psi)$ is the dimensionless relative permeability function, which quantifies the decrease in the permeability to water due to the fact that some of the pores are occupied by air; it is typically a strongly increasing function of S.

There are various assumptions and simplifications inherent in eq. (1) which require some discussion. Most rocks and soils are hysteretic with regards to capillary pressure, which means that the $S(\psi)$ relationship depends on whether drainage or imbibition is occurring, and on the past saturation history of the rock. For most of the processes that we are interested in, the saturations vary monotonically, and so hysteresis can be ignored. Hence, we assume that S is a single-valued function of ψ, with no dependence on past values. Eq. (1) also assumes that as the liquid water imbibes into the matrix, it is not impeded by the air that is initially in place. This assumption is known to be correct for flow into an unbounded medium, since the air can escape ahead of the advancing liquid front. For flow that is assumed to be entering a finite-sized matrix block, it is possible that air will be trapped in a pocket at the center of the block; however, this problem will be ignored in the present discussion.

Finally, eq. (1) also neglects gravity, which otherwise would lead to an additional gravitational potential term ρgz to be added to the pressure potential ψ. Gravity can be neglected in the early stages of imbibition, when capillary gradients are large relative to the gravitational gradient. At longer times, the capillary gradients have been smoothed out, and the gravitational gradient predominates. For imbibition into a finite-sized block, the relative effects of the these two gradients are quantified by the ratio of the block size to the "sorptive length", which was defined by Philip[14] in terms of the integral of the relative permeability function with respect to ψ. Zimmerman et al.[4] showed that, using hydrological parameters believed to be appropriate for the Topopah Spring welded tuffs at Yucca Mountain,[15] the sorptive length is about 8 m. This means that gravity can be ignored in any matrix block whose diameter is much less than 8 m. Since fracture spacings in the Topopah Spring unit are thought to be on the order of tens of centimeters,[6] we can assume that imbibition into matrix blocks is dominated by capillary forces.

Each rock has its own set of ''characteristic functions'' that describe the relationships between S, ψ and k_r. Two sets of characteristic functions that are often used in modeling the hydraulic behavior of the volcanic tuffs at Yucca Mountain are those that were proposed by Brooks and Corey,[16] and Mualem[17] and van Genuchten.[18] In this paper, we will use the van Genuchten-Mualem functions to model the hydrological behavior of the tuffs; these functions are given by

$$S(\psi) = S_r + (S_s - S_r)[1 + (\alpha|\psi|)^n]^{-m} , \tag{2}$$

$$k_r(\psi) = \frac{\{1-(\alpha|\psi|)^{n-1}[1+(\alpha|\psi|)^n]^{-m}\}^2}{[1+(\alpha|\psi|)^n]^{m/2}} , \tag{3}$$

where α is a scaling parameter that has dimensions of 1/pressure, and m and n are dimensionless parameters that satisfy $m=1-1/n$, $n>2$.[19] S_r is the residual water saturation, at which the liquid phase becomes immobile, while S_s, which is usually very close to 1.0, is the saturation at which the matric potential goes to zero.

Eq. (1) is essentially a diffusion equation, with the conductance and capacitance related to the characteristic functions $k(\psi)$ and $S(\psi)$. The nonlinearity of these characteristic functions renders exact solutions to eq. (1) nearly impossible to obtain, even for simple geometries such as one-dimensional flow into an unbounded region. In order to construct a dual-porosity model that is fairly general, we need expressions for the rate of imbibition into finite, irregularly-shaped matrix blocks. To develop such expressions, we have used the following approach. First, we used the integral method to develop approximate expressions for one-dimensional imbibition into an unbounded formation.[10,20] Next, we extended these solutions to regular geometries such as spherical, cylindrical, or slab-like matrix blocks.[4] Finally, we developed scaling laws that allow us to extend these results to irregularly-shaped blocks, based on knowledge of

their volumes and surface areas.[4]

The basic problem of imbibition into a matrix block can be formulated as follows. Consider a matrix block occupying a region of space Ω, with boundary $\partial\Omega$. Initially, the block is at some partial saturation S_i, which corresponds to some potential $\psi_i<0$. At some time t_o the saturation front in the fracture network passes the matrix block, after which the matrix block has the boundary condition $\psi_w=0$ on the boundary $\partial\Omega$ (see Fig. 1). (The solution for the case where $\psi_w>0$ can be derived[10] from the solution for $\psi_w=0$ by a simple mathematical transformation.) For both the Mualem-van Genuchten and Brooks-Corey characteristic curves, we have used the integral method to develop fairly accurate solutions[10,20] to this problem for short times, when the process behaves like imbibition into a semi-infinite medium. For van Genuchten-Mualem media, our integral method solution yielded imbibition rates that were asymptotically accurate for large values of the van Genuchten parameter n, and for high initial saturations. In the simulations described in this paper, we have used a modification of this result which does not predict precisely the correct behavior for the imbibition rate in the limit as $S_i \rightarrow S_s$, but which is more accurate at lower values of the initial saturation. According to this approximation, the short-time cumulative volumetric flux into a block whose characteristic functions are of the Mualem-van Genuchten type is given by

$$Q = A\left[\frac{2k\phi m^{4/3}(S_s-S_i)\,(t-t_o)}{\alpha\mu(S_s-S_r)}\right]^{1/2}, \qquad (4)$$

where Q is the cumulative volumetric flux, A is the outer surface area of the block, and $m=1-1/n$. A convenient way to quantify the ability of a matrix block to absorb water is through the parameter known as the sorptivity, which is defined (for small values of the elapsed time, before boundary effects become important) by

$\Sigma = Q/A\sqrt{(t-t_o)}$. As an illustration of the accuracy of expression (4), Fig. 2 shows the sorptivity of a Mualem-van Genuchten porous medium, as predicted by eq. (4) and by numerical solution of the governing equation (1). This approximation to the sorptivity is seen to be very accurate as long as the initial saturation is not too close to S_s. Since n is restricted[19] to the range $n>2$, and the sorptivity curves for $n>10$ all lie not very far above the $n=10$ curve, Fig. 2 essentially covers all possible cases. Experience with dual-porosity simulators for single-phase flow shows that the overall hydrologic behavior of a field is somewhat insensitive to errors in treatment of the fracture/matrix flux term, as evidenced by the wide use of the quasi-steady-state Warren–Root[8] model. In this regard we note that Updegraff et al.[21] have recently developed a numerical dual-porosity model for unsaturated flow which utilizes a quasi-steady-state fracture/matrix interaction term, in which the instantaneous flowrate dQ/dt is taken to be directly proportional to the difference in liquid-phase pressure between the fracture and matrix block. Their model also differs from ours in that they allow flow between matrix blocks, whereas we assume that only the fracture network is interconnected.

Eq. (4) shows that, for small times, the cumulative flux is equal to $\Sigma A\sqrt{(t-t_o)}$. The total flux into the block, at sufficiently long times after the arrival of the saturation front in the adjacent fractures, will be $Q_\infty = \phi V(S_s - S_i)$. Hence the fractional uptake in the blocks, defined by Q/Q_∞, will initially be equal to $\Sigma A\sqrt{(t-t_o)}/\phi V(S_s - S_i)$. In other words, at small times, $Q/Q_\infty = \sqrt{\tau}$, where τ is defined by

$$\tau = \left[\frac{(A/V)\Sigma}{\phi(S_s - S_i)}\right]^2 (t-t_o) \, . \qquad (5)$$

We then make the assumption that Q/Q_∞ will be uniquely determined by the variable τ, for all values of t. This implies that the effect of block geometry will be accounted for by the surface-to-volume ratio, A/V. For Mualem-van Genuchten media, we have

found that this dependence can be approximated by

$$\frac{Q}{\phi V(S_s - S_i)} = \sqrt{\tau} - 0.24\tau \,, \qquad (6)$$

where τ is given by eq. (5), and Σ is defined by $\Sigma = Q/A\sqrt{(t-t_o)}$, in conjunction with eq. (4). Eq. (6) is used until $\tau = 6.25$, when the right-hand-side equals 1, at which time the matrix block is fully saturated; thereafter, the instantaneous flux is zero. This expression has been found to adequately account for the effect of block shape, as well as the effect of initial saturation.[4]

The fracture/matrix interaction term discussed above was derived under the assumption that there is no "skin" effect at the outer edge of the matrix blocks. Fracture skin, in analogy with the damaged rock material that is often found around wellbores, would be a thin region at the outer edges of the matrix blocks in which the permeability is much less than within the rock matrix itself. Fracture skin may be caused, for example, by deposition of minerals which have been transported by water flowing through the fractures. The effect of the fracture skin would be to cause a delay in the rate at which water is imbibed by the matrix block.[22] This delay cannot be accounted for by altering the numerical value of the sorptivity Σ, since a relationship of the form $Q = \Sigma A\sqrt{(t-t_o)}$ will no longer hold. The existence of fracture coatings at Yucca Mountain, and their effect on matrix imbibition, requires further study. The model described in this paper does not, at this time, account for fracture coating effects.

DUAL-POROSITY SIMULATOR

We have implemented expression (6) for fracture/matrix flow as a modification to the TOUGH^2 code, which is an integral finite-difference code that can simulate the

flow of liquid water, water vapor and air in porous or fractured media. The TOUGH code contains provisions for sources/sinks of mass and heat, which are calculated in the subroutine QU. The sources/sinks are often used to account for fluid that is injected or withdrawn from a borehole that penetrates one of the computational cells. We have modified this subroutine so as to include a new type of sink, which represents liquid water flowing into the matrix blocks. The magnitude of the instantaneous flux of the sink associated with each computational cell is computed using eqs. (4-6). The subroutine QU calculates a "generation" term G for each cell, at each timestep, which represents the average flux for the source/sink over the time interval $t_n \le t \le t_{n+1}$. For fracture/matrix leakage, this generation term is calculated from eq. (6) as

$$G = \frac{Q(\tau_{n+1}) - Q(\tau_n)}{\tau_{n+1} - \tau_n} , \qquad (7)$$

where the dimensionless time τ is calculated from eqs. (4) and (5). Leakage from a given fracture element is assumed to begin when the saturation in the fracture reaches some nominal value that is taken to mark the arrival of the saturation front. Each computational fracture element will therefore have its own value of t_o, and its own value of the dimensionless imbibition time τ. In the simulations described below, we use a saturation value of $S_i + 0.90(S_s - S_i)$ to signal the arrival of the liquid front in the fractures. This indicates the time at which 90% of the ultimate saturation increase has occurred. Further numerical simulations are planned to study the sensitivity of the computed results on this parameter.

As described above, fracture/matrix flow is assumed to begin abruptly based on the "arrival" of the saturation front in the fractures. Eq. (4) shows, however, that the instantaneous flux into a matrix block, dQ/dt, will drop off as $(t - t_o)^{-1/2}$, implying that

the flux is very large soon after t reaches t_o. This was found to cause a drastic decrease in the allowable size of the timesteps taken by the TOUGH program. The initially large value of the flux is, however, obviously an artifact of the assumption that fracture/matrix flow begins abruptly. We have found that this difficulty can be avoided by including a multiplicative "damping factor" in our calculation of the generation term G, thus forcing the instantaneous flux to begin at zero and then steadily approach the rate given by eqs. (4-6). An appropriate damping term is $\exp[-t_o/(t-t_o)]$, which goes to zero for small values of $(t-t_o)$, and approaches unity as t increases. Since the very large instantaneous fluxes predicted by eq. (4) occur only over a relatively small initial time period, this damping has very little effect on the overall behavior of the fracture/matrix system.

FLOW ALONG A FRACTURE WITH LEAKAGE TO THE MATRIX

One basic problem which has much relevance to understanding the hydrological behavior of the Yucca Mountain tuffaceous rocks is that of water flowing along a fracture, with leakage into the adjacent matrix, which is assumed to be semi-infinite in extent.[23,24] Consider the configuration shown in Fig. 3, in which water is flowing along a horizontal fracture located in a permeable formation. This model would also apply to the early stages of flow along a vertical fracture.[4] Flow into the fracture is driven by the imposed potential at the $y=0$ boundary. For the matrix blocks, we use the hydrological parameters that have been estimated for the Topopah Spring Member of the Paintbrush Tuff (Miocene) at Yucca Mountain,[15] which are $k=3.9\times10^{-18}\,m^2$, $\alpha=1.147\times10^{-5}\,Pa^{-1}$, $\phi=0.14$, $S_s=0.984$, $S_r=0.318$, and $n=3.04$. Very little experimental data exists from which to determine the appropriate characteristic curves for single fractures, at Yucca Mountain or elsewhere. For the fracture, we will use the curves that were derived by Pruess et al.[25] using a mathematical model of a fracture as being a rough-walled channel. Although these curves are not based on direct measurements, the parameters in the model were to some extent conditioned by data from

Yucca Mountain. The values of the hydrological parameters for the fracture are taken to be $k=1.1\times10^{-11}$ m^2 (per fracture), $S_s=1.0$, $S_r=0.0$, $\alpha=6.07\times10^{-4}$ Pa^{-1}, and $n=2.89$. This fracture permeability corresponds to an average aperture on the order of a few tens of microns, according to the cubic law, after including modifications to account for the effect of roughness and contact area.[26,27] The fracture was discretized into 14 elements, of successive lengths 1m, 2m, 4m, etc. The temperature was taken to be 20°C, and the initial capillary pressure was taken to be -1.013×10^5 Pa (1 atm), which corresponds (through the capillary pressure functions) to an initial matrix saturation of 0.6765, and an initial fracture saturation of 0.0004. Note that for the fractures and matrix blocks initially to be in equilibrium their capillary pressures must be equal, which in general does not imply equal saturations. Since the average fracture aperture is typically much larger than the average pore diameter in the matrix block,[25] a capillary suction that is sufficient to almost completely dry out the fracture will cause only moderate drying of the matrix.

We have solved this problem using both the modified version of TOUGH, and also using TOUGH without the source/sink expressions, but with ten grid blocks extending into the matrix adjacent to each fracture element. In order to estimate the effect of leakage into the matrix, we have also solved this problem under the assumption that the matrix blocks are totally impermeable. When using the modified code, we input a very small value of A/V for the matrix blocks, in order to simulate the unbounded region adjacent to the fracture. When solving the problem with explicit discretization of the fracture and matrix regions, the matrix elements must be extended sufficiently far into the the formation so as to effectively simulate a semi-infinite region. In this simulation the matrix elements were extended 1 m away from the fracture. The saturation profile in the fracture after an elapsed time of 10^5 s is shown in Fig. 4, for both methods of calculation, and for the case of an impermeable matrix. The instantaneous flux into the fracture from the $y=0$ inlet is shown in Fig. 5, where

the theoretically predicted[28] transition from $t^{-1/2}$ to $t^{-1/4}$ behavior can be clearly seen. The accuracy of the new semi-analytical dual-porosity TOUGH code for this problem is excellent, and the decrease in the amount of CPU time required for the simulation, from 51.22 s to 6.42 s on a Solbourne (Series 5) computer, was about a factor of eight. This decrease is seen to be roughly equivalent to the decrease in the number of computational cells, which was from 154 to 14, or a factor of eleven. Hence the decrease in computational time afforded by the new method will be extremely significant, in absolute terms, for larger-scale problems that necessarily require large numbers of computational cells.

VERTICAL INFILTRATION INTO A FRACTURED FORMATION

Another hydrological problem of interest for the site characterization process at Yucca Mountain is that of infiltration of liquid water into a pervasively fractured formation, under the influence of both capillary forces and gravitational forces (see Fig. 6). This infiltration can be modelled as occurring under conditions of constant pressure at the surface, or constant flux. We will consider infiltration to occur under "barely-ponded" conditions that correspond to a very small positive potential, which we will take to be zero; The effect of a positive pressure (potential) head h at the surface will be negligible compared to capillarity as long as $\rho g h \ll 1/\alpha$, which for most fractured rocks will be true as long as the head is no more than a few centimeters.[10] This type of infiltration would occur, for example, after a brief but intense rainfall event. If we assume uniform properties in the horizontal plane, this can be modeled as a one-dimensional problem that is very similar to the leaky-fracture problem discussed above. The only differences in the simulations are the inclusion of a gravitational gradient, and the use of a finite value for A/V, which simulates a finite-size matrix block. Assuming a spacing of 20 cm between fractures,[6] the appropriate value of A/V is $10\,m^{-1}$.

A detailed mathematical analysis of this problem, including asymptotic results for small and large times, has been given by Nitao and Buscheck.[28] They in effect linearized the flow equations in the matrix, and used a Green–Ampt[13] model for flow in the fractures. This latter idealization is equivalent to assuming that the fractures are smooth-walled, without any roughness or contact areas. The numerical simulations carried out by Nitao and Buscheck[28] indicated that neglect of capillary gradients in the fracture itself may have a large effect on the overall behavior of the fracture/matrix system at *small* times. In light of this result, we treat the fractures as porous media in their own right, with their own relative permeability and capillary pressure curves.

Fig. 7 shows the saturation profile in the fractures, as a function of depth below the surface, after an elapsed infiltration time of 10^5 s. The effect of gravity is to cause the liquid front to extend much farther down into the formation than it would have under the influence of capillarity alone (compare Figs. 7 and 4). The agreement between the fully-discretized TOUGH solution and the solution obtained with our new source/sink method is again quite close. The reduction in CPU time, from 81.13 s to 9.07 s, was in this case a factor of nine. Because the saturation profiles shown in Fig. 7 refers only to the fractures, these profiles do not directly reflect the amount of water that has leaked into the matrix blocks. The total volumetric imbibition into the fractured/porous formation can be measured by examining the instantaneous flux into the first fracture grid block. Comparison of the instantaneous fluxes into the formation also shows (see Fig. 8) very close agreement between the results of the two methods.

CONCLUSIONS

A semi-analytical dual-porosity model for unsaturated flow in fractured/porous media has been described. An approximate analytical solution for imbibition of liquid water into a matrix block has been incorporated into the numerical simulator TOUGH, to act as a source/sink term for the fracture elements. In its present stage of

development, the analytical treatment of matrix imbibition is applicable only to the wetting phase of an infiltration process. The modified code has been tested on the problem of flow along a single horizontal fracture, as well as vertical infiltration into a fractured formation under constant-head boundary conditions. In both cases the new method gives very close agreement with simulations carried out by explicitly discretizing the matrix blocks, while yielding a substantial savings in CPU time. Since the new method permits an order of magnitude decrease in the number of computational cells, the process of creating the grid and the input file for the TOUGH simulations is greatly simplified. Both of these factors allow much more efficient simulation of unsaturated flow processes in fractured/porous formations.

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Figure Captions

Fig. 1. Schematic diagram of problem of imbibition of water from saturated fractures into an initially unsaturated matrix block.

Fig. 2. Normalized sorptivity $\Sigma/[k\phi/\alpha\mu(S_s - S_r)]^{1/2}$ of a Mualem-van Genuchten porous medium, as a function of initial saturation, for two values of n. Curves are from numerical solutions of eq. (1), data points are from the approximate expression (4).

Fig. 3. Schematic diagram of flow along a single horizontal fracture, from a constant-pressure inlet, with transverse leakage into the matrix.

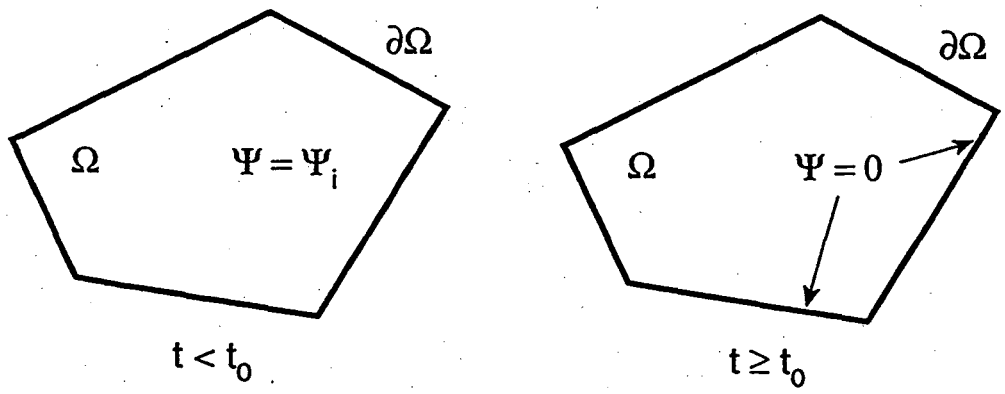
Fig. 4. Saturation profile in the fracture, during flow along a single horizontal fracture located in the Topopah Spring unit, after an elapsed time of 10^5 s. Results for a fracture in an impermeable matrix are shown for comparison. Parameters used in the simulations are listed in the text.

Fig. 5. Instantaneous flux into the fracture, per unit depth of fracture perpendicular to page in Fig. 3, for the horizontal fracture flow problem.

Fig. 6. Schematic diagram of vertical infiltration into a medium containing parallel, equally-spaced of fractures of aperture h and spacing distance 2L.

Fig. 7. Saturation profile in the fracture network, during vertical infiltration into a fractured formation, after an elapsed time of 10^5 s. Results for case of an impermeable matrix are shown for comparison. Parameters used in the simulations are listed in the text.

Fig. 8. Instantaneous flux at the surface of the fractured/porous formation, for the vertical infiltration problem.



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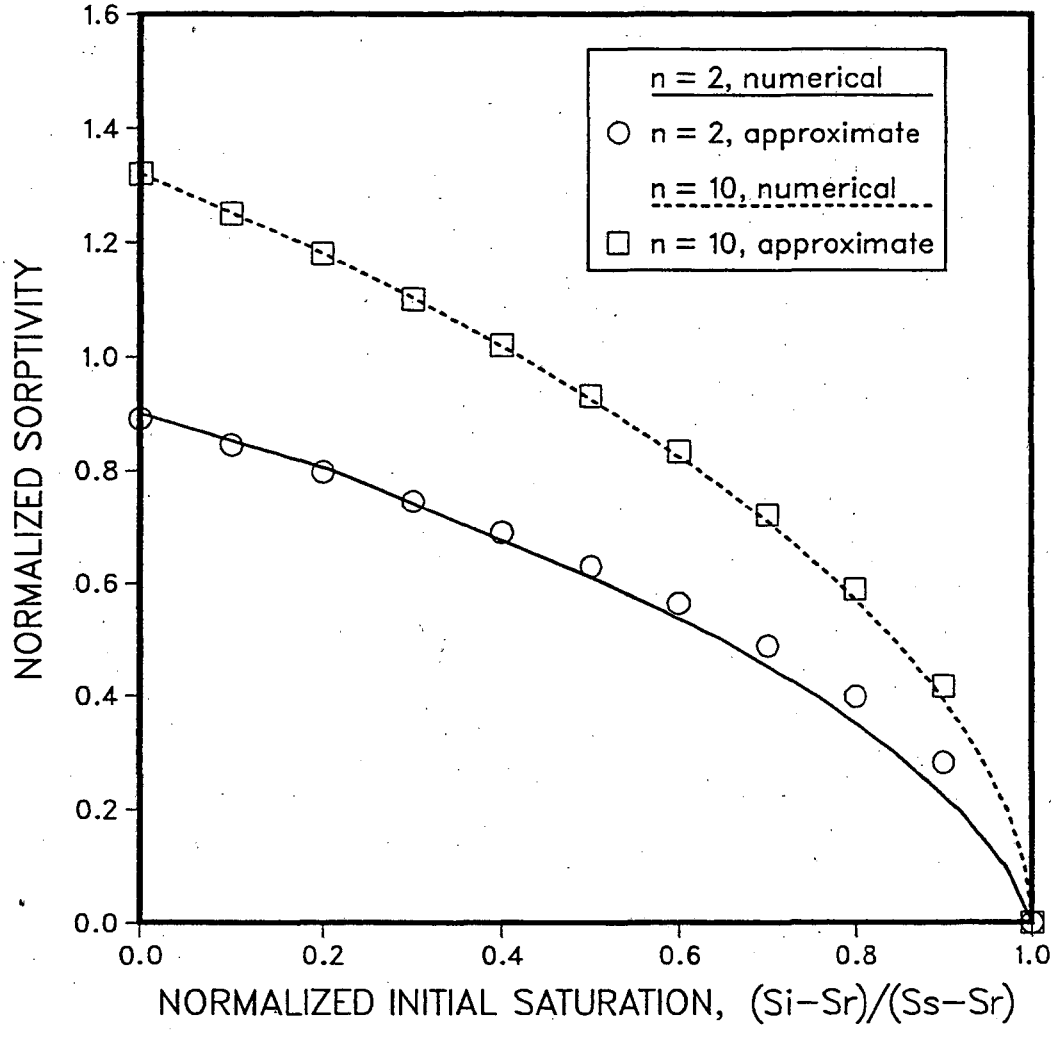

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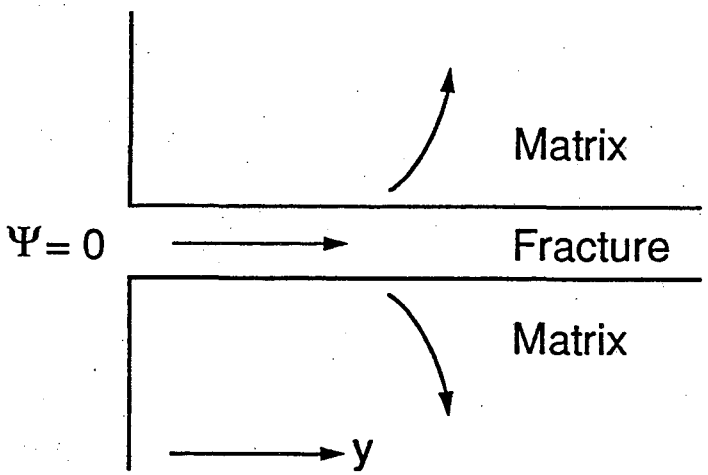

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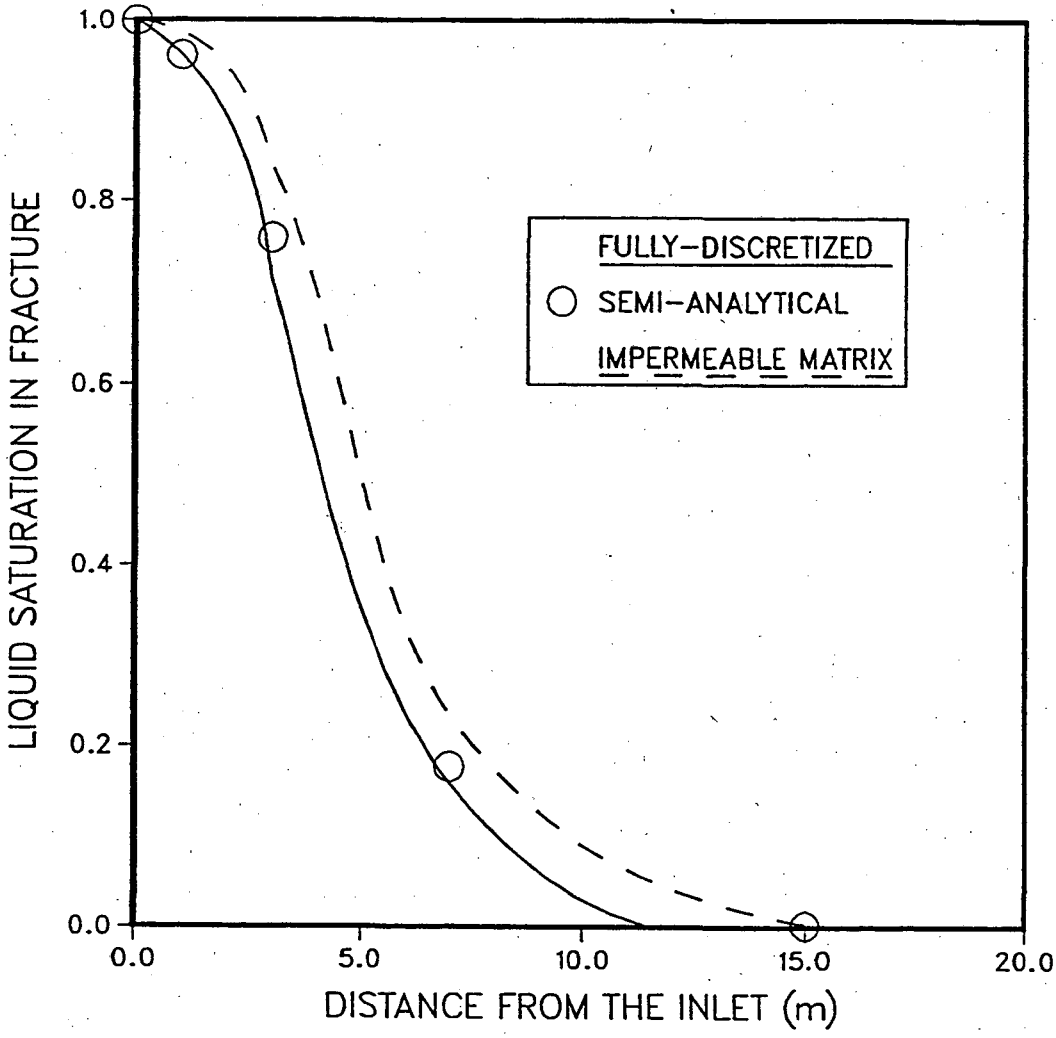
Fig. 3. Schematic diagram of flow along a single horizontal fracture, from a constant-pressure inlet, with transverse leakage into the matrix.



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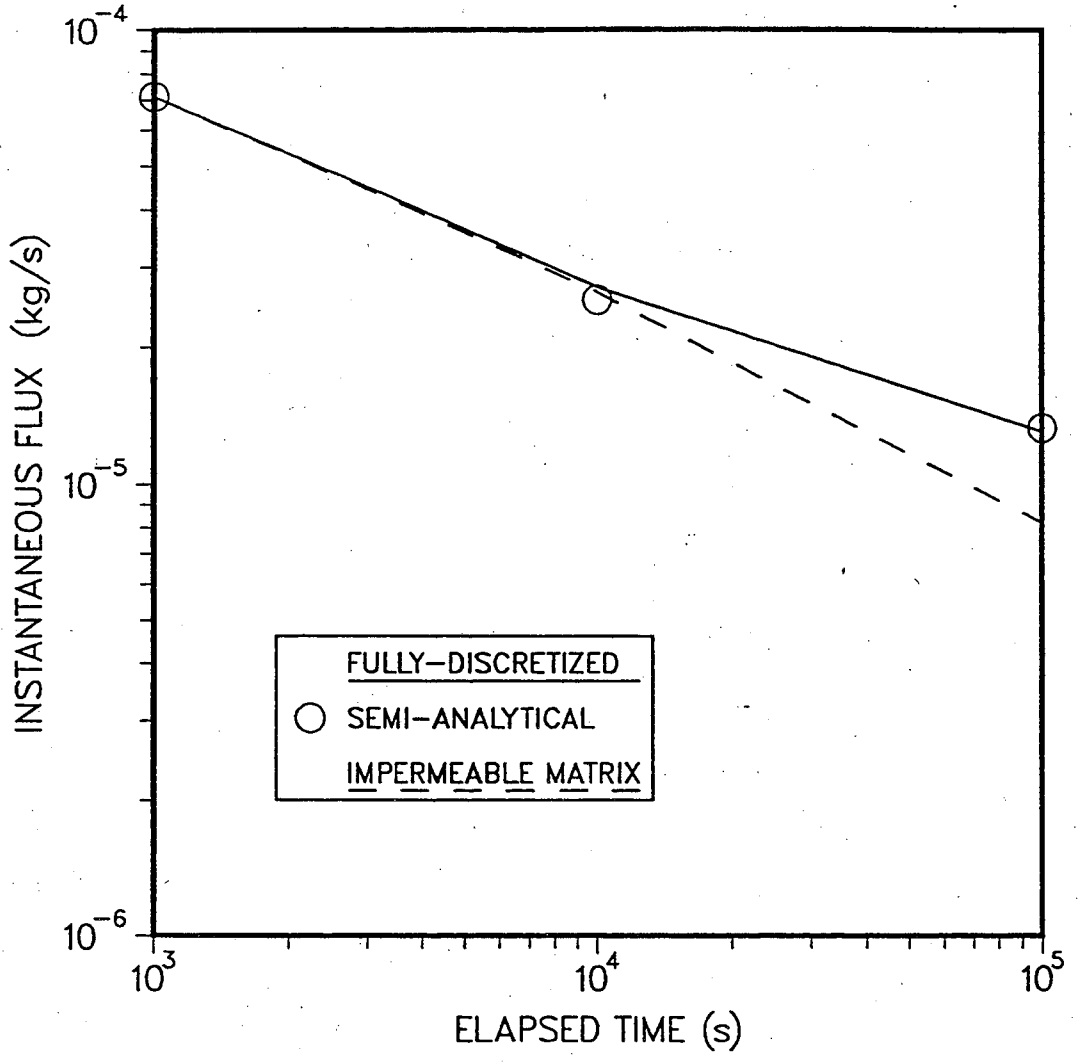

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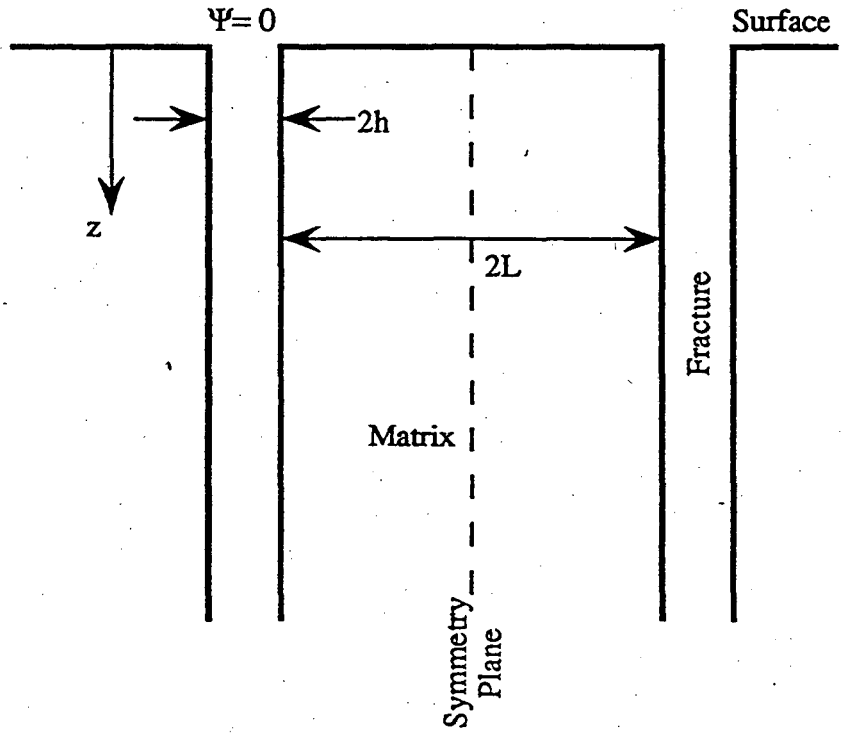

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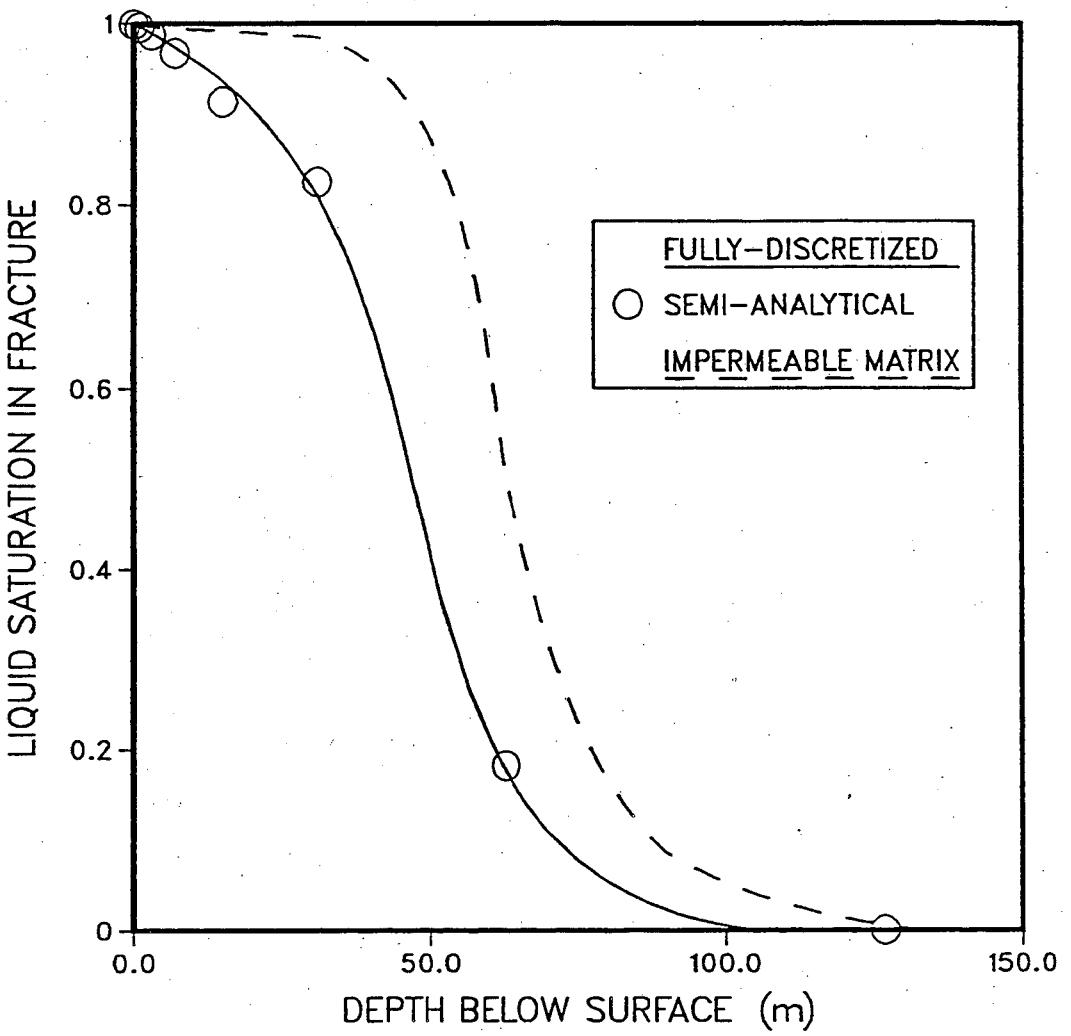

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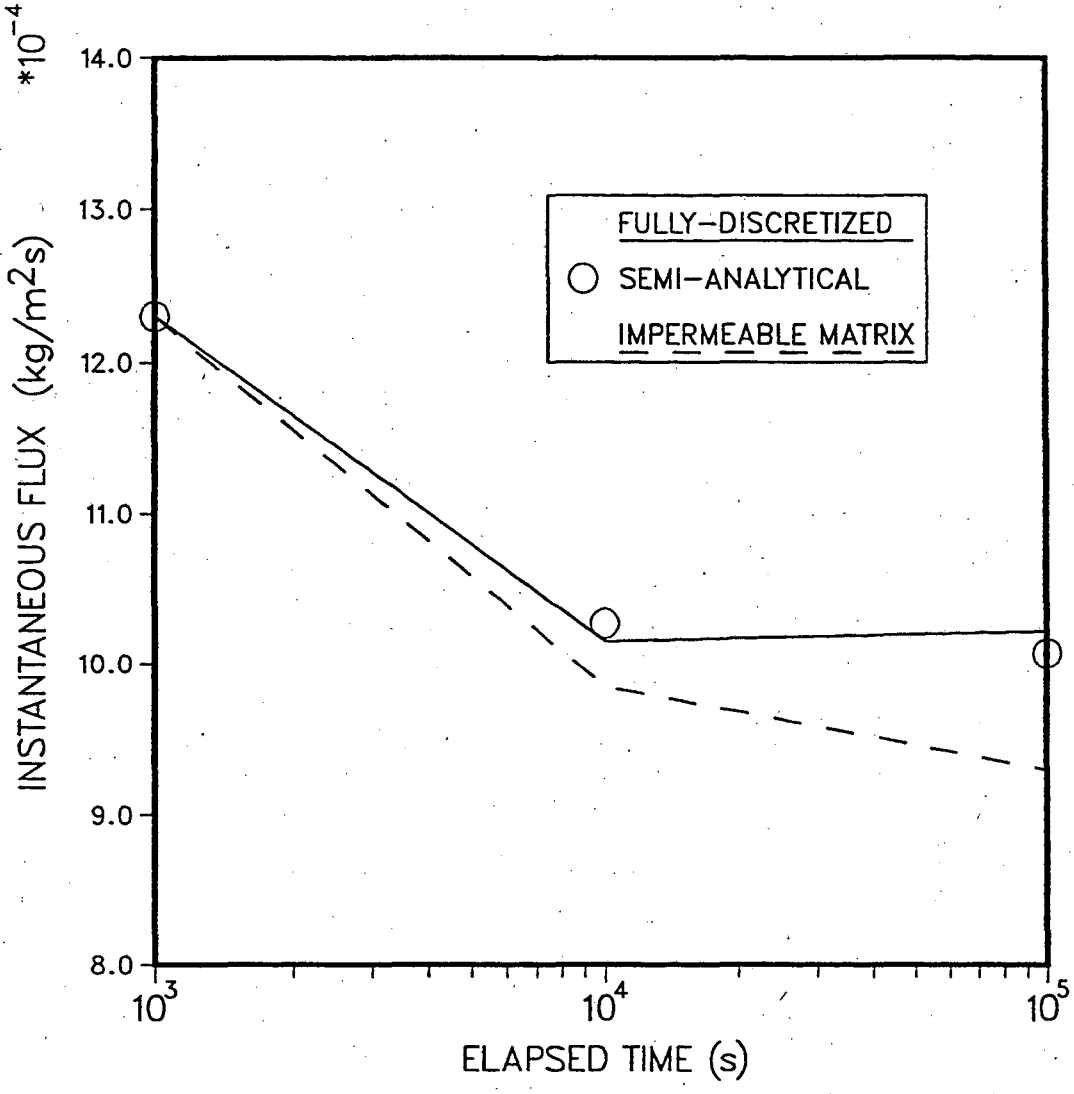

Fig. 8. Instantaneous flux at the surface of the fractured/porous formation, for the vertical infiltration problem.

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