How Impasses Enable Subjects to Discover the Relevant Properties of the Problem : Problem Space as a Space of Properties

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Abstract

Two main ideas are proposed in this article. Richard and Tijus (in press) shown that problem solving can be explained by object properties that subjects take into account during the solving process. Stable properties are those which can not be modified by an action (for instance, an object's size, shape, etc.) and unstable properties are those which can be modified by an action (for instance, an object's location). Our purpose is that the problem space (Newell & Simon, 1972) can be described by state properties and that this description permits explaining the subjective distance (in the subject's mind) between two states. We suggest that similarity between state properties guides a subject's search through the problem space and can lead subjects through irrelevant paths. We think that in this condition, the well known beneficial effect of impasse situations consists in the fact that they permit subjects to discover the relevant properties of objects, problem constraints, and goal properties. Two experiments are proposed here. Results obtained in the first experiment show that working on impasse situations before solving the problem improves performance. Results of the second experiment show that working on impasse situations allow subjects to discover the relevant properties of a problem space, and that the benefit can be extended to all problems sharing the same problem space (which naturally contain the same impasses), even if their initial and final states are different. These results shed some light on the beneficial effects of impasses in problem solving.

Introduction

Skill acquisition can naturally be seen as the successive elimination of errors. How are errors unlearned and what are the cognitive mechanisms that enable people to detect and correct errors? Impasse situations have been studied in different domains of psychology. Great attention has been accorded to these situations in psychology of learning (Anderson, 1981, 1983; Ohlsson, 1983; Thorndike, 1913), in problem solving domain (Richard, 1981, 1994 ; Vanlehn, 1991), in piagetian literature (Cellerier & Inhendler, 1990), and in AI (Langley, 1985; Ohlsson, 1983). In summary, we know the importance of impasses in learning, discovering, thinking and reasoning, but what exactly is the beneficial effect of these situations ?

In problem solving, impasses have almost always been considered as favorable situations in which positive as well as negative subgoals are generated (Newell & Simon, 1972; Richard, 1981). Impasses are also considered as favorable situations in which relevant features of problems are discovered (Hammond, Converse, Marks, & Seifert, 1993; Ohlsson, 1996).

Our purpose is to consider (i) impasses as situations in which relevant properties of the objects involved in actions can be discovered, and (ii) as situations which enable subjects to enrich their initial representation of the problem and of the goal state.

As Richard and Tijus (in press) explain, each object has two kinds of properties : stable properties which define the object itself (size, shape, etc.), and unstable properties which are location properties. An action modifies these properties. In the case of classical transformation problems, an action almost always modifies location properties, excepting for some TOH isomorph (changing-size problems, Kotovsky, Hayes, & Simon, 1985).

Problem Space, as defined by Newell and Simon (1972) is the objective space in which new states are generated after each legal move. In the case of transformation problems (i.e. the Tower of Hanoi, Missionnaries and Cannibals, etc.), a state is defined both by the objects and their location. Thus, states are differentiated by a change in location of the objects. We suggest that the properties of the objects can define a state and the problem space. Thus, a Problem Space can be considered as a space of properties, whose modification generates new states. The initial and final states, as well as impasse situations, can also be defined by a set of properties. For instance, in the classical Tower of Hanoi problem (with 3 disks), the initial state can be defined by a set of stable and unstable properties : the big, the medium and the little (stable property) disks, are in position A (unstable property). The final state is differentiated from the initial state by a change in the unstable properties : the big, the medium and the little (stable properties) on the position C (unstable property).

This description permits characterizing each state by its properties, and thus explaining the subjective distance (in the subject's mind) between two states. Because they share some properties, two states may be thought close to each other even though they are actually very far apart in the objective problem space. With this view, as soon as one or several properties are considered, the problem space - usually considered as a homogeneous space - becomes a set of distinct areas of states. Each area can be considered as a category of states sharing properties (Zamani, Bernard, & Richard, 1998).

Our claim is that properties shared by different states guide a subject's behaviour and can lead him through irrelevant paths. For instance, in some states, subjects may believe that they are quite near the goal state only because the current state shares some properties with the goal state. However, in the objective space, these two states can be very far from the another and many moves might be needed in order to change the current state into the goal state. We believe that these properties can guide subjects through the objective problem space. For instance, subjects may attempt to reach states where some properties of the goal state are present. However, the path may prove irrelevant (unable to provide real progress through the problem space towards the goal). Thus, the subjective distance between two states can be described by differences between their properties. We think that this is why impasse situations are beneficial: they allow subjects to discover all the other properties of the goal that subjects believed to be able to realize and the observation that it is in fact impossible. This leads subjects to notice overlooked goal properties.

We suggest that a subject's goal may be different from the real goal. In problem solving literature, authors have almost always considered that, during the solving process, subjects have a good representation of the goal state. Results we obtained in our laboratory show that at the beginning of the solving process, indeed sometimes until the end, a subject's goal can be very different from the objective goal state. That is, among all the properties which define the goal state, subjects may take only a limited set into account and thus, a subject's (representation of the) goal may be more restricted than the real final goal. The representation of the goal that has guided a subject's solving process can be inferred from protocols and verbalizations. Inappropriate goals are often responsible for impasses. In order to clarify this point, we will first briefly present problems we have studied. They are taken and adapted from the Passalong test (Alexander, 1935), which is a test of practical intelligence.

The apparatus is a kind of rickshaw game (figure 1) and consists of 9 problems composed of blue figures (located at the top of the playing board) and red figures (located at the bottom of the board). Although unknown in problem solving literature, Passalong problems are classical transformation problems, i.e. similar in every way to TOH, Missionaries and Cannibals, etc.

The problem space is exactly the same for problems 7, 8 and 9 (Bernard, personal communication). However, problems 8 and 9 are the most difficult ones in the test[1]. Figure 2 shows some impasses belonging to problems 7, 8 and 9.

Verbalizations collected during impasse situations, when subjects tried to solve problem 8, show that the subjects' goal almost always consisted in moving the red rectangle (the horizontal one) to the top of the board, and of moving "all the other pieces" or the "the blue ones" to the bottom.

[1] As we have shown (Zamani & Richard, submitted) this difficulty is due to the fact that analogy is recognized between problem 6 (very easy to solve) and problem 7, whereas the analogy is not recognized between problem 7 and problems 8 and 9. In fact, solving problems 8 and 9 by the expert path requires joiningthe rectangles from the beginning of the solving process and placing them just as they are in problem 7. Once the rectangles have been joined, the problem can be solved either by turning clockwise or counterclockwise until the horizontal rectangle is moved to the top of the board. This procedure is generally used in problems 6 and 7, but as the analogy between these problems and problems 8 and 9 goes unrecognized, problems 8 and 9 are very difficult to solve.

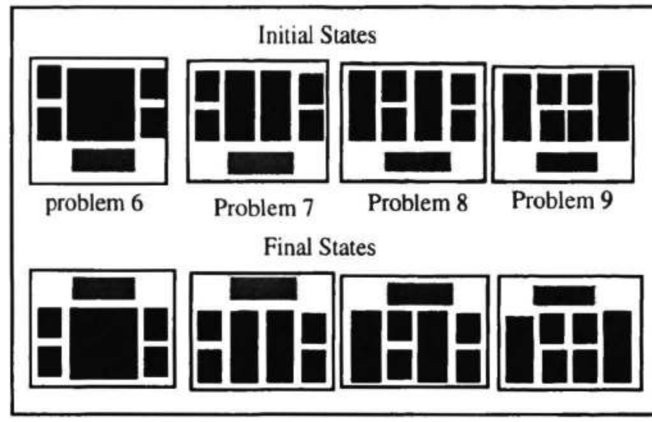

Figure 1: Initial and Final States of Problems 6,7,8 and 9 of the Passalong Test.

This representation of the goal state is very limited as compared to the final goal that subjects have to attain (see figure 1), given that expressions "all the other pieces" or "the blue ones", level the necessary distinction to be made between the "blue" objects according to their specific properties. These properties are discovered in impasse situations. For instance, when subjects arrived at impasse n°1 (figure 2), they almost always explained that they got stuck because they wanted to move the horizontal rectangle but that the empty space available was not sufficient for doing so. This impossibility led subjects to realize that moving the horizontal rectangle requires creating a horizontal empty place (two adjacent places), something which cannot be obtained as long as the *vertical rectangle* remains above. On the other hand, the *two little squares* (figure 2, impasse n°1 and its corresponding favorable state), placed side by side in a horizontal position, do allow moving the horizontal rectangle. This contrast shows the kind of distinction that needs to be made between objects according to their properties: rectangles are different from squares. Further, subjects in impasse situations did learn that *satisfying a goal depends on the specific properties of the objects* and that the objects cannot be treated in the same way. Thus, moving the horizontal rectangle requires creating a horizontal empty place with two little squares. This is a relevant property which is valuable for all states in the problem space which lead to the goal state and, once this property has been learned, subjects were able to infer the appropriate goal structure which consists in avoiding states in which a vertical rectangle figures above the horizontal rectangle. Likewise, in impasse n°2 (figure 2), subjects have the opportunity to learn that moving the horizontal rectangle creates a horizontal space which can only be filled with two squares, not with one vertical rectangle. Thus, in order to avoid impasses, two squares should always figure immediately below the horizontal rectangle.

These impasse situations also led subjects to change their representation of the goal. As long as a subject's goal just consisted of "moving the horizontal rectangle" to the top of the board, the problem was impossible to solve. Impasses permitted subjects to realize that in order to move the horizontal rectangle, it was necessary to put the right objects to the right places. Among "all the blue pieces", there were rectangles and squares, with their specific properties, some of which allow reaching some subgoal but not others.

Moreover, subjects noticed that they had to arrange all of these objects in some order to reach the goal state. This does not mean that they had not previously noticed the objects or that they had never analyzed the goal state. But rather that specific properties of the objects and their relation to subgoals were not taken into account until impasses were been encountered. Impasse situations permitted subjects to discover the relevant properties of the objects in order to generate new and more relevant subgoals.

The experiments presented here aimed at showing that learning relevant properties from impasses is possible, even without any action. In the first experiment, we aimed at showing that studying impasse situations before solving a problem increases performance as compared to cases when subjects solve problems directly. This shed some light on the beneficial effects of impasses in problem solving. If studying impasses allow subjects to discover the relevant properties of a problem space, the benefit should extend to all problems sharing this same problem space (which naturally contain the same impasses), even if their initial and final states are different. The second experiment aimed at studying this point: subjects learned impasses belonging to problem 8 without solving it, and they then solved problem 9.

Experiment 1

Method

Participants 40 volunteers participants, in different fields at the University of Paris 8, participated in the experiment.

Material On a computer screen, we presented problems 6,7, 8 and 9 of the Passalong Test (see figure 1). A computer program checked the legality of the intended moves and placed the figures in the desired positions whenever the move was legal. When it was not, nothing happened and the subject had to attempt a new move. Every move as well as latency between moves was recorded by the computer. The final state of each problem was visible

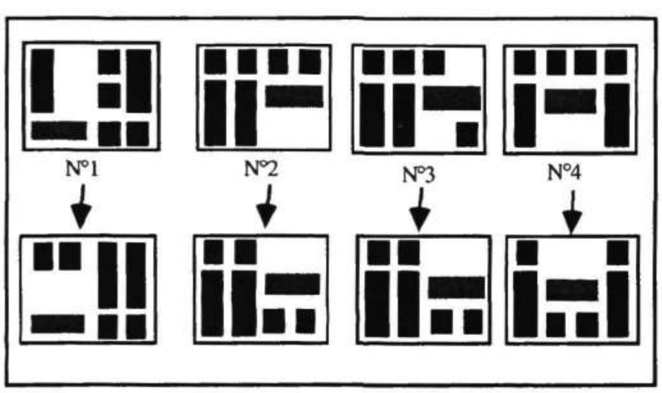

Figure 2: Four of the most frequent impasses in problems 8 and 9 of the Passalong test (the first line) and their corresponding favorable situations in which the horizontal rectangle can be moved to the top of the board (second line). In the first impasse, the two last actions consisted in moving the little squares to the right. In impasse 2, the last action consisted on moving the horizontal rectangle up. In impasse 3, the last action consisted in moving the little square placed above the horizontal rectangle to the left, and in impasse 4, the last action consisted in moving the horizontal rectangle up.

on the screen the entire time subjects were solving the problem.

Procedure Participants were tested individually in an isolated room. There were 2 experimental groups (20 subjects per group). One group (hereafter called G1) solved problems 6 and 7 first. Then initial and final states of problem 8 were shown but subjects were not asked to solve the problem. Instead, four illustrations of the most frequent impasses for problem 8 (shown in figure 2) were presented to the subjects. Each illustration contained the impasse situation and the two last moves which led to the impasse. Subjects were invited to explain why the situation was an impasse, which objects were the most blocking ones, and where these objects had to be in order to make the goal pursed attainable. Subjects worked for an average of 84 seconds on each impasse. At the end of the presentation phase, they were invited to solve problem 8.

The second group (hereafter called G2) solved problems 6, 7, and 8. Groups were compared as to their performance on solving problem 8.

Analysis Method Although we were interested in the properties discovered in each impasse situation, analysis here will only concern the behavior (performance) in solving problem 8. Dependant variables were: 1) number of impasses, 2) success or failure (failure was defined as having solved problem 8 in 80 or more moves, that is 4 times the number of moves required by the expert path), 3) number of subjects who reached favorable states in order to attain the final state (there are two important states which permit moving the horizontal rectangle to the top of the board and attaining the final state. If these situations are reached, then the path through the final state is not complicated. However, reaching these states seems to be very difficult. The most favorable situation is reached by joining the two rectangles (see footnote in the Introduction part) and it requires only 5 moves. Thus, in our analysis, the number of moves required to reach a favorable state varies from 5 to 80 moves. Above 80 moves, we considered that subjects had failed to attain a favorable state. This variable is called hereafter the "Favourable situation rate"), and 4) number of moves made by subjects to reach the favourable situations.

Results

All subjects in the first group were able to discover relevant properties in impasse situations. Results of the analysis of verbalization are not presented here. Results of the behavior analysis for problem 8 are presented in table 1. It should be noticed that groups did not differ as to performance on problems 6 and 7.

As it can be seen in table 1, the percentage of success increased when subjects worked on impasse situations before solving problem 8. This difference was significant, Fisher's Exact Probability, $p < .001$. In the same way, a greater number of subjects in G1 were able to attain the favorable situations which lead to the final goal. This difference was again significant, Fisher's Exact Probability, $p = .02$. The number of moves required to reach the favorable situations also differed between the groups. As explained above, to attain the most favorable situation for reaching the final goal, the two rectangles must be joined, something which requires only 5 moves (see Analysis method). As results show, 35 %

of subjects in G1 were able to join the rectangles within 5 to 16 moves and 70 % of subjects were able to do so within 40 moves, while only 30 % of subjects in G2 did as well. This difference was significant by a one-tailed Kolmogorov-Smirnov test, $D = 8$, $p < .025$.

Given the insufficient number of subjects in G2, we were not able to make a statistical test for the number of impasses encountered by subjects who succeeded in solving the problem in each group (only 4 subjects in G2 solved the problem). However, it can be noticed that the number of impasses encountered by subjects who solved the problem in each group was not very high.

Discussion

We hypothezised that working on impasse situations improves performance in solving the problem. Results obtained are in accordance with this hypothesis. In fact, subjects who worked on impasse situations were able to discover properties of situations which are unfavorable for solving the problem (for instance, having one vertical rectangle above the horizontal rectangle is not a good situation). These properties permitted subjects to avoid unfavourable situations. We emphazise that the beneficial effect of studying impasse situations consists in the fact that subjects were able to create relevant negative and positive subgoals.

As it has been shown, a greater number of subjects in G1 attained favorable situations and they did it more quickly during the solving process than did subjects in G2. Analysis of protocols also showed that the average number of moves made to solve the problem was about 47 in G1 (i.e. nearly two times the expert path). Indeed, subjects did make some useless moves, but were not far from the expert path. This was also observed for subjects in G2, though the total number of subjects who solved the problem in this group was low as compared to G1.

Analysis of *a posteriori* interviews with subjects who solved problem 8 (G2) show that the majority of these subjects had a single main goal until the very end of the solving process. This goal was simply to move the horizontal rectangle to the top of the board.

They did not change their goal even after encountering impasses. Subjects explained that the problem was difficult because none of the actions they made permitted moving the horizontal rectangle. No subject was able to tell us conditions necessary for moving the horizontal rectangle (for instance, having two squares above it). No subgoal was reported by these subjects. The poor performance of these subjects as compared to those of G1 clarifies the beneficial effect of studying impasses before solving the problem.

Table 1: Performance on solving problem 8, subjects in G2 solved it directly, subjects in G1 examined four impasses before solving it.

	Groups	
Measures	G1	G2
Success rate	.75	.20
Favourable situation rate	.75	.35
Number of impasses		
M	2.13	2.5
SD	.53	2.6

These results also suggest that studying impasses without making moves has a more beneficial effect than encountering impasses during the solving process.

The next experiment aimed at examining whether studying the impasses was also beneficial for solving a more distant problem. That is, a problem sharing the same problem space, but with different initial and final states.

Experiment 2

Method

Participants 40 volunteers participants, in different fields at the University of Paris 8, participated in the experiment.

Material The material was the same as in experiment 1.

Procedure There were 2 experimental groups (20 subjects per group). One group (hereafter called G1) solved problems 6 and 7 first. Then initial and final states of problem 8 were shown but subjects did not solve the problem. Instead, four illustrations of the most frequently encountered impasses in problem 8 (shown in figure 2) were presented to subjects. The same procedure as in experiment 1 was used to get subjects to work on impasse situations. At the end of the presentation phase, subjects were invited to solve problem 9. The second group (hereafter called G2) solved problems 6, 7, 8 and 9. Groups were compared as to their performance in solving problem 9.

Analysis Method The method of analysis was the same as in experiment 1, except for the fact that the thresholds for analysis were fixed at 14 (the minimum number of moves necessary to reach a favorable state) to 140 moves (4 times the expert path).

Results

Again, all subjects in the first group were able to discover the relevant properties in impasse situations. Results of the analysis of verbalizations are not presented here. The results of the behavior analysis for problem 9 are presented in table 2. Note that the groups did not differ as to performance on problems 6 and 7.

As can be seen in table 2, a greater number of subjects in G1 solved problem 9 than subjects in G2, Fisher's Exact Probability, $p = .03$. Likewise, number of subjects who reached a situation favorable for attaining was also higher in G1 than in G2, Fisher's Exact Probability, $p = .01$. The low number of subjects who solved the problem in G2 (only 5 subjects), eliminated the possibility of making a statistic test of the number of impasses. However, the groups showed nearly the same performance.

The number of moves required to reach the favorable situations also differed between the two groups. Attaining the most favorable situation (joined rectangles) requires only 14 moves. As results show, 50 % of subjects in G1 were able to join the rectangles within 74 to 81 moves, while only 5 % of subjects in G2 did as well.

This difference was significant by a one-tailed Kolmogorov-Smirnov test, $D = 9$, $p < .025$. Although 74 moves would seem to indicate poor performance, it is better than not being able to reach any favorable situation at all, which was the case for the vast majority of subjects in G2. In

Table 2: Performance on problem 9, after having solved problem 8 (G2), or after examining the impasses of problem 8, without solving it (G1).

	Groups	
Measures	G1	G2
Success rate	.55	.25
Favourable situation rate	.60	.25
Number of impasses		
M	10	12.4
SD	2.5	2.19

addition, problem 9 is a very difficult problem and subjects rarely succeed in solving it by the expert path.

Discussion

In this experiment we aimed at studying whether having participants analyse impasses of a given problem would improve performance in solving another problem belonging to the same problem space.

Results show that when subjects studied impasse for problem 8 without solving it, their performance on problem 9 was better than performance of subjects who solved it after having solved problem 8. The success rate and the number of subjects who reached the favourable states in order to attain the goal, differed significantly between groups.

Subjects who worked on impasses before solving problem 9 were also able to reach favorable states more quickly than subjects who solved problems 8 and 9. As explained above, problem 9 is a very difficult problem. During the experiment, we noticed that even when subjects were able to discover the relevant properties which allow solving the problem, they had difficulty in establishing priorities. For instance, the property which leads to take the expert path occurs when the two rectangles are joined (as in problem 7). Another favorable property occurs when two squares are below and two squares are above the horizontal rectangles. But which property is most important? In fact, the first property leads to the second (i.e. when the rectangles are joined from the beginning of the solving process, the squares are necessarily below and above the horizontal rectangle). But it is very difficult to infer this relation and subjects were not able to do so during the analysis of the impasses. Thus, during the solving process, they had difficulty in prioritizing these properties related to subgoals. Impasses occurred in these cases. Subjects in G1 made many irrelevant moves and encountered many impasses. However these impasses were local impasses, that is, they did not lead subjects away from the expert path. This also explains the great number of moves required to reach favorable situations. Analysis of protocols show that these impasses almost always concerned subgoals situations. Though subjects had difficulty in simultaneously organinzing the different subgoals, the path they pursued was the relevant one. On the other hand, subjects in G2 did not show the same pattern of performance.

Results again suggest that studying impasses without making moves has a more beneficial effect than encountering impasses during the solving process.

General Discussion

Two main ideas were proposed in this article. According to Richard and Tijus (in press), objects have two kind of properties : stable and unstable properties. Stable properties are those which define the objects in a problem and which can not be modified by an action. Unstable properties are those which can be modified by an action. For instance, in the case of classical transformation problems, unstable properties are almost always location properties, whereas stable properties are usually an object's size, shape, etc.

The first idea proposed here consisted in describing the problem space as a set of stable and unstable properties. In this way, all the states of the problem state can be described by object's properties. This description permits characterizing each state by its properties, and thus explaining the subjective distance (in the subject's mind) between two states. Because they share some properties, two states may be thought close to each other even though they are actually very far apart in the objective problem space. We suggest that with this view, as soon as one or several properties are considered, the problem space - usually considered as a homogeneous space - becomes a set of distinct areas of states. Our claim is that properties shared by different states guide a subject's behaviour and can lead him through irrelevant paths. This is the case when the properties subjects focus on at the beginning of the solving process are irrelevant. For instance, if subjects take only a limited set of goal properties into account, they may try to reach the states where these properties are present. However, the path they thus take may be an irrelevant one for solving the problem. Thus, impasses occur.

The second idea concerned impasse situations more directly. In the experiments reported in this article we showed that working on impasse situations before solving a given problem increases performance. Moreover, this beneficial effect was also observed for more distant problem, that is a problem belonging to the same problem space. These results suggest that it is possible to learn the relevant properties of a problem space through studying its impasses and that this learning is transferable to all problems inside this space.

We suggest that the well known beneficial effect of impasse situations is due to the fact that they allow subjects to discover the relevant properties of the objects involved in actions and to enrich their representations of the problem and the goal. Impasse situations are favorable situations because they permit subjects to discover that in order to attain a goal, the objects to work with must be selected because their properties are what will allow satisfying the conditions which are necessary for attaining the goal. The specific properties of objects are related to subgoals. Thus, discovery of properties has an effect on subgoal structure modification.

We think that subjects construct subgoals according to properties they have already taken into account. When subgoals are based on irrelevant properties, they lead to impasses. If subjects analyze impasses they can discover all the other properties of the goal (or the subgoal) that they failed to take into account. In other words, impasses are beneficial because they allow subjects to discover the relevant prerequisites and to make necessary detours in order to reach the goal.

In terms of state properties, impasses enable subjects to notice that the properties they have taken into account were irrelevant. This lead subjects to discover other object properties. Generally, these properties are the functional properties relevant to solve the problem. In this case, we consider that properties taken into account can be modified, i.e. a property's weight (the cognitive importance of a property) may change during the learning process.

Moreover, we think that the benefits of the impasse situations may depend on the subject's subgoal structure. Results we obtained in our laboratory show that subjects with elaborate subgoal structure (even if irrelevant) benefit more often from impasses than subjects without subgoals.

Impasses permit subjects to realize that properties they have taken into account are irrelevant. But impasses rarely indicate which properties are relevant. This may be one reason why it is so difficult to conceptualize a problem during the solving process. In order to benefit from impasses, subjects have to think about the properties *which are necessary* in order to attain a goal, i.e. in terms of *favorable states*. This leads subjects to pay attention to properties they have neglected. In the experiments reported here, when subjects worked on impasse situations before solving the problem, they were invited to explain which objects were the most problematic and where these objects had to be placed in order to make the goal pursued attainable. This led subjects to think in terms of favorable states and favorable properties. Such analysis rarely occurs when subjects are acting and we think that this is why the analysis of impasses without acting was beneficial in our experiment.

These results are interesting in that they allow explaining the pathways subjects take through the objective problem space, from the point of view of the properties a subject takes into account at each moment of the solving process.

References

Anderson, J.R. (1981). *Cognitive skills and their acquisition.* Hillsade, NJ: Erlbaum.

Anderson, J.R. (1983). *The architecture of congition.* Cambridge, MA: Harvard University Press.

Hammond, K., Converse, T., Marks, M., & Seifert, C.M. (1993). Opportunism and Learning. *Machine Learning, 10,* 279-309.

Inhelder B., & Cellerier G. (Eds.) (1992). *Le cheminement des découvertes de l'enfant.* Lausane. Delachaux et Niestlé.

Kotovsky K., Hayes J.R. & Simon H.A. (1985). Why are some problems hard ? Evidence from Tower of Hanoï. *Cognitive Psychology, 17,* 248-294.

Langely, P. (1985). Learning to search : From weak methods to domain specific heuristics. *Cognitive Science, 9,* 217-260.

Newell, A., & Simon, H. (1972). *Human problem solving.* Englewood Cliffs, NJ: Prentice Hall.

Ohlsson, S. (1996). Learning From Performance Errors, *Psychological Review, 103,* 241-262.

Richard, J.F. (1982). Planification et organisation des actions dans la résolution du problème de la Tour de Hanï par les enfants de 7 ans, *Année Psychologique, 82,* 307-336.

Richard, J.F. (1994). Résolution de problème. In M. Richelle, M. Robert, and J. Reguin (Eds.). *Traité de Psychologie Expérimentale.* Paris, PUF.

Richard, J.F., & Tijus, C.A. (in press). Modelling affordances of objects in problem solving. *Special Issue of Analise Psicologica about Cognition and Context.*

Richard, J.F., Zamani, M. (1996). L'application des modèles de résolution de problème à l'analyse des tests. *Psychologie Française, 41,* 77-88.

Thorndike, E.L. (1913). Educational psychology : *The psychology of learning* (Vol. 2). New York: Teacher College, Clumbia University.

Vanlehn, K. (1991). Toward a theory of impasse-driven learning. In H. Mandl, and A. Lesgold (Eds.). *Learning Issues for Intelligent Tutoring Systems.* NewYork: Springer Verlag.

Zamani, M., & Richard, J.F. (submitted). Objects Encoding, Goal Similarity, and Analogical Transfer.

Zamani, M., Bernard, J.M., & Richard, J.F. (1998). *Problem Space: a set of distinct areas of states, not an unitary and homogeneous space* (Tech. Rep. CNRS UPRES-A 7021). Saint Denis: University of Paris 8, Laboratory of Cognitive Psychology.