Title
ALPHA-ALPHA SCATTERING IN THE RANGE 36.8 TO 47.3 MeV

Permalink
https://escholarship.org/uc/item/0m4630j3

Authors
Conzett, Homer E.
Igo, George
Shaw, Harlan C.
et al.

Publication Date
1959-07-15

UCRL -8829

UNIVERSITY OF CALIFORNIA

Ernest O. Lawrence Radiation Laboratory

BERKELEY, CALIFORNIA

DISCLAIMER

UCRL-8829

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California
Contract No. W-7405-eng-48

ALPHA-ALPHA SCATTERING IN THE RANGE 36.8 TO 47.3 MEV

Homer E. Conzett, George Igo, Harlan C. Shaw and Rodolfo J. Slobodrian

July 15, 1959

Printed for the U. S. Atomic Energy Commission

ALPHA-ALPHA SCATTERING IN THE RANGE 36.8 TO 47.3 MEV

Homer E. Conzett, George Igo, Harlan C. Shaw and Rodolfo J. Slobodrian

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 15, 1959

Abstract

Absolute differential cross sections for the elastic scattering of alpha particles from helium have been obtained at 36.8, 38.8, 40.8, 41.9, 44.4, 46.1, 47.1, and 47.3 Mev. Measurements have been made at intervals of 2 degrees over an angular range from about 15 degrees to beyond 90 degrees in the c.m. system. The angular distribution shows a single minimum at 65 degrees at the two lowest energies, and two minima, at about 35 and 70 degrees, at the other energies.

ALPHA-ALPHA SCATTERING IN THE RANGE 36.8 TO 47.3 MEV*

Homer E. Conzett, George Igo,† Harlan C. Shaw and Rodolfo J. Slobodrian§

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 15, 1959

Introduction

The elastic scattering of alpha particles from helium has been investigated in detail in the energy range from 0.4 to 23 Mev,[1,2,3] at 30 Mev,[4] and from 23.1 to 41.2 Mev.[5,6] The 0.4-to-23-Mev data have been analyzed by the various authors in terms of the method of partial waves, and the nuclear phase shifts that were determined have given evidence for several virtual excited states in the compound nucleus, Be^8. The alpha-alpha differential cross sections are very strongly energy-dependent, and consequently data at several neighboring energies are needed to explore resonance effects in the phase shifts. Since it is expected that data at more closely spaced energy intervals above 23 Mev will become available soon,[6] we have explored the region from 37 to 47 Mev. It should then be possible to determine the phase shifts smoothly as functions of the energy up to 47 Mev.

In addition, at energies above 34.7 Mev. there can be other end products of the interaction of two alpha particles, namely, $Li^7 + p$, $Be^7 + n$, $He^4 + t + p$, $He^4 + He^3 + n$, and $Li^6 + d$. These reactions may proceed through the compound nucleus, Be^8, or through a direct-interaction mechanism.

---

* Work done under auspices of the Atomic Energy Commission.

† Summer 1957 visitor from Department of Physics, Stanford University, California.

§ Comision Nacional de Energia Atomica, Buenos Aires, Argentina.

Consequently, the technique of optical-model analysis may be very useful. The optical-model parameters determined from alpha-alpha scattering along with those determined for the elastic scattering of alpha particles from other elements should enable one to get an estimate of the size of the alpha-particle, a quantity which contributes to the size parameter determined by the optical-model analysis for other elements.

In addition to the optical-model calculations being carried out by one of us (G. I.), a partial-wave analysis of the data has been started. The results of both analyses will be reported in subsequent papers.

Experimental Procedure

The general experimental details concerning the scattering chamber, the beam-monitoring, and the energy-measuring methods have been described earlier.[7] The external 48-Mev alpha-particle beam of the Crocker Laboratory 60-inch cyclotron was directed down an evacuated pipe and through sufficient absorber, placed at the entrance to the scattering chamber, to degrade it to the desired energy. An Al absorber was used except for the three lowest energies, at which Be was used in order to reduce the loss from multiple scattering. Following the absorber was a carbon collimation system consisting of three apertures of 1/8, 1/16, and 3/16 in. diameter, separated by 10-1/2 and 6-1/2., respectively. Their respective functions were to (a) stop most of the excess beam, (b) define the cross-sectional area of the beam, and (c) act as an antiscattering baffle. Following the last collimator was a 0.001-in. Al vacuum foil (for the two 47-Mev runs, a 0.0005-in. Ni foil was substituted) through which the beam passed from the evacuated pipe into the helium-filled scattering chamber. The foil was 4-1/4 in. from the center of the chamber.

After passing through the chamber, the beam was collected in a Faraday cup and integrated. The mean energy of the beam was determined by measuring the range of the alpha particles in Al absorbers with the scattering chamber evacuated. The absorbers were located immediately in front of the Faraday cup. A plot of beam intensity at the Faraday cup versus thickness of Al absorber was used to determine the thickness of Al absorber corresponding to one-half maximum beam intensity. Range-energy tables[8] based on experimental proton range-energy data[9] were used.

These tables use projected ranges, and hence can be used directly to determine beam energy from thickness of Al absorber at half-maximum beam intensity. A correction to the energy was made for the length of path traversed by the alpha-particles in helium gas before arriving at the center of the scattering chamber. An estimate of the energy spread in the beam was obtained from the measured integral range-curve. The width of the differential range-curve derived from it, reduced by the theoretical width due to range straggling alone, gave the range spread in the beam itself. This corresponds to a 0.75% energy spread in the incident beam.

The elastically scattered alpha-particles were detected with a CsI (Tl) scintillator placed approximately 10 in. from the center of the chamber. A collimating system, consisting of a 0.125-in.-diam. aperture immediately in front of the scintillator and a 0.133-in.-wide vertical slit placed 6.75 in. in front of the aperture, defined the target volume and the solid angle subtended by the counter. The angular resolution achieved with this geometry was 1.4 deg. The counter collimation system in the scattering chamber was aligned radially by optical methods.

At laboratory angles less than about 12 deg., the counter and its associated mounting obscured the Faraday cup from the direct beam. Under these conditions, another scintillation counter was used as a beam monitor. It was placed outside the chamber at a fixed laboratory angle near 19 deg. It viewed the chamber through a window port with defining apertures attached, so that it detected alpha particles scattered only from the central region of the scattering chamber. Calibration was done directly by recording its total counts and the corresponding charge collected in the Faraday cup while running at large angles where there was no possibility of the counter obscuring the Faraday cup.

Pulses from the scintillation counter were amplified and fed into a 10-channel pulse-height analyzer. Figure 1 shows a typical spectrum obtained from this counter. At the higher energies, the background was quite low except at angles near the minima in the cross section. At the lower energies, however, the correction for background became quite appreciable.

In order to provide a check on the consistency of the results, the data were taken at alternate angles in such a way that each angular distribution was taken at least twice. In addition, several points on each curve were remeasured one or more times to ensure that no electrical or mechanical drift was present in the apparatus. Since the scattering of identical particles imposes angular-distribution symmetry with respect to

90 deg. c.m., some check-points were taken at c.m. angles larger than 90 deg.

Second-order geometry corrections to the cross sections were made, based on the calculation of Critchfield and Dodder.[10] These corrections became as large as 35% at the narrow minima of the angular distributions. At other angles they were of the order of 1% or less.

The pressure of the helium gas in the chamber was measured with a mercury manometer, and the temperature was measured with a thermometer in thermal contact with the chamber.

Errors

The charge-collecting system was calibrated against voltage and capacitive standards and was estimated to have an over-all accuracy of ± 0.50% relative, and ± 0.75% absolute, for each cross section.

The helium gas used in the scattering chamber was supplied by Liquid Carbonic Co. from the U. S. Bureau of Mines bottling service at Amarillo, Texas, and was specified to have less than 0.005% of impurity, mainly air and hydrogen. This amount was negligible. The measurement of temperature and pressure of the helium gas in the chamber was estimated to introduce an absolute error in the determination of the density of the helium gas varying from ± 0.3% to ± 1.0%. This depended mainly on the helium gas pressure for each run, and the percentage accuracy with which it could be determined.

The angular position of the counter with respect to the beam direction was reproducible to ± 0.1 deg. in the laboratory system (or ± 0.2 deg. in the c.m. system). This affects the calculation of the cross section through a tan θ_{lab} factor, causing a relative error varying from ± 0.54% at a laboratory angle of 20 deg. to ± 0.36% at 45 deg.

The calculation of the solid angle subtended by the counter telescope was affected mainly by the accuracy of measurements of the 0.125-in.-diam. collimation aperture directly in front of the crystal. The resulting absolute error in the solid angle was estimated to be ± 3%. The calculation of the effective length of the target volume of helium gas also depends mainly on the accuracy of measurement of the crystal collimation aperture. Since this is a linear quantity, the resulting absolute error is only ± 2%.

At each angle, the energy spectrum was obtained with sufficient points above and below the peak corresponding to elastically scattered alpha particles to enable the background curve to be well determined. This curve was then extrapolated into the elastic peak, and those counts arising from background were subtracted from the total counts in the peak. The background correction was quite small, and generally negligible at the higher energies, except at angles near minima in the cross section. Below 41.9 Mev, however, the size of this background correction became considerably larger, and, consequently, at the lower energies the error associated with this background correction accounts for most of the tabulated relative error on each cross section.

At the low pressures of helium gas in the chamber, the possibility of the loss of some beam from the Faraday cup by multiple scattering in the gas was negligible, and was calculated to be less than 0.1%.

The errors listed in Table 1 are relative errors. The absolute errors were compounded separately and result in a ± 3.8% error which should be applied to the ordinates of each bombarding energy.

Results

The measured differential cross sections are tabulated with their relative errors in Table 1 and plotted in Figs. 2 and 3. Angles and cross sections are in the c.m. system. The single prominent minimum seen at 36.85 and 38.83 Mev, gives way to two minima at the higher energies. This transition from one to two minima with increasing energy is also present in the 12-to-23-Mev alpha-alpha data, where resonance scattering from a virtual excited state (4+) around 11 Mev in Be^8 is observed.[11]

Acknowledgments

We are grateful to Prof. A. C. Helmholz for helpful discussions and support during the course of this work and to Drs. D. J. Prowse and G. W. Farwell for sending us their results prior to publication. Two of us (G.I. and R.S.) wish to also thank the other staff members of the Lawrence Radiation Laboratory who made it possible for us to participate in this work.

We wish to express our appreciation to the crew of the 60-in. cyclotron under the direction of William B. Jones for their assistance during the cyclotron runs.

Table I

Differential cross sections for alpha-alpha scattering

E_{lab} = 36.85 ± 0.21 Mev (147.64 ± 1.5 mg/cm^2 Al)				E_{lab} = 38.83 ± 0.07 Mev (161.73 ± 0.50 mg/cm^2 Al)			
θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)			θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)		
				11.7	979	±	42
13.7	1391	±	11	13.7	910	±	28
15.7	1116	±	12				
17.7	971	±	9	17.7	717	±	8
19.7	818	±	12				
21.7	691	±	10	21.7	620	±	7
23.7	626	±	8				
25.7	505	±	6	25.7	444	±	5
27.7	425	±	5	27.7	383	±	4
29.7	351	±	5	29.7	324	±	4
31.7	290	±	4	31.7	275	±	3
33.7	241	±	3	33.7	242	±	3
35.7	189	±	3	35.7	217	±	3
37.7	159	±	3	37.7	187	±	2
39.7	132	±	2	39.7	162	±	2
41.7	108	±	2	41.7	136	±	2
43.7	97.7	±	1.4	43.7	120	±	2
45.7	86.9	±	1.8	45.7	102	±	2
47.7	79.8	±	1.2	47.7	82.7	±	1.0
49.7	70.4	±	1.3	49.7	60.2	±	1.3
51.7	61.0	±	1.0	51.7	48.8	±	0.8
53.7	49.1	±	1.1	53.7	32.8	±	0.8
55.7	41.7	±	0.7	55.7	26.1	±	0.6
57.7	30.8	±	0.8	57.7	14.7	±	0.5
59.7	20.4	±	0.6	59.7	10.7	±	0.3
61.7	11.4	±	0.4	61.7	7.90	±	0.58
63.7	4.80	±	0.29	63.7	4.09	±	0.25
65.7	4.65	±	0.41	65.7	4.10	±	0.56
67.7	7.72	±	0.50	67.7	5.54	±	0.38
69.7	18.3	±	0.7	69.7	8.95	±	0.57
71.7	30.4	±	0.9	71.7	12.8	±	0.5
73.7	56.4	±	1.2	73.7	18.0	±	0.8
75.7	76.3	±	1.5	75.7	25.8	±	0.6
77.7	101	±	2	77.7	29.7	±	1.0
79.7	129	±	2	79.7	32.8	±	0.8
81.7	160	±	3				
83.7	175	±	4	83.7	37.6	±	1.4
85.7	199	±	3	85.7	40.0	±	1.6
87.7	202	±	4	87.7	42.5	±	1.7
89.7	209	±	3	89.7	40.8	±	1.4
91.7	194	±	5	91.7	43.9	±	1.9
93.7	192	±	3	93.7	40.9	±	2.2
95.7	179	±	4	95.7	35.5	±	2.2
97.7	150	±	4	97.7	32.7	±	2.3
99.7	115	±	5	99.7	24.8	±	1.9

Table I (Continued)

E_{lab} = 40.77 ± 0.05 Mev (176.04 ± 0.40 mg/cm^2 Al)			E_{lab} = 41.90 ± 0.02 Mev (184.56 ± 0.15 mg/cm^2 Al)		
θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)		θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)	
			12.2	2197	± 43
			14.0	1824	± 35
			16.0	1466	± 23
			17.8	1132	± 17
			19.8	954	± 18
			21.6	786.1	± 11.6
			23.4	569.4	± 5.6
			25.2	392.3	± 3.5
27.3	177	± 2	27.0	268.3	± 2.9
			28.8	173.8	± 1.7
31.3	29.5	± 1.1	30.6	94.4	± 1.1
33.3	12.3	± 0.9	32.4	45.0	± 0.6
35.3	8.82	± 0.54	34.2	16.20	± 0.31
37.2	33.2	± 1.2	36.2	12.89	± 0.30
			38.0	26.79	± 0.32
39.3	66.6	± 1.4	39.8	50.9	± 0.7
41.3	105	± 2	41.8	83.0	± 0.8
43.3	137	± 3	43.6	114.7	± 1.2
			45.8	143.0	± 1.5
47.3	184	± 4	47.8	167.6	± 1.8
51.3	182	± 4	50.0	184.4	± 1.8
53.2	172	± 3	52.0	184.7	± 1.8
55.3	153	± 5	54.2	171.2	± 1.6
			56.2	145.0	± 1.6
59.3	89.4	± 1.1	58.4	120.3	± 1.3
61.3	60.3	± 1.7	60.6	86.2	± 0.9
63.3	32.2	± 1.5	62.8	51.5	± 0.6
65.3	14.0	± 0.7	65.0	25.66	± 0.34
67.3	5.31	± 0.58	67.0	7.77	± 0.18
69.3	4.39	± 0.55	69.2	2.22	± 0.15
71.3	14.2	± 1.1	71.4	9.83	± 0.25
73.3	26.9	± 1.1	73.6	31.2	± 0.5
75.3	57.6	± 2.3	75.8	61.8	± 0.8
77.3	91.2	± 2.7	78.0	95.1	± 1.3
79.3	121	± 3	80.2	144.8	± 1.7
81.3	176	± 4	82.2	187.5	± 2.4
83.3	184	± 4	84.4	223.4	± 2.6
85.3	213	± 4	86.4	243.9	± 3.3
87.3	235	± 4	88.4	256.7	± 3.5
89.3	246	± 4	90.4	268.1	± 4.2
			92.4	258.3	± 4.0
			94.4	230.0	± 5.8
			96.4	207.7	± 4.3
			98.4	159.2	± 5.6
			100.4	117.4	± 4.6

Table I (Continued)

E_{lab} = 44.41 ± 0.02 Mev (204.33 ± 0.15 mg/cm^2 Al)		E_{lab} = 46.12 ± 0.02 Mev (218.24 ± 0.15 mg/cm^2 Al)	
θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)	θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)
12.2	1820 ± 37	12.2	1319 ± 35
14.0	1602 ± 33	14.0	1012 ± 18
16.0	1263 ± 25	16.0	857 ± 15
17.8	1058 ± 21	17.8	666 ± 12
19.8	905 ± 15	19.8	580.0 ± 10.1
21.6	711.4 ± 14.9	21.6	456.1 ± 6.9
24.8	382 ± 5	25.8	224 ± 3
26.8	258 ± 3	27.8	147 ± 2
28.8	176 ± 2	29.8	95.1 ± 1.2
30.8	121 ± 2	31.8	48.8 ± 0.6
32.8	64.4 ± 0.8	33.8	23.7 ± 0.3
34.8	28.4 ± 0.3	35.8	15.3 ± 0.2
36.8	13.6 ± 0.2	37.8	17.4 ± 0.2
38.8	14.2 ± 0.2	39.8	31.7 ± 0.4
40.8	26.0 ± 0.3	41.8	50.8 ± 0.6
44.8	72.9 ± 0.7	43.8	75.2 ± 0.8
45.8	87.6 ± 0.9	45.8	93.4 ± 1.0
47.8	112.6 ± 1.1	47.8	115.5 ± 1.3
50.0	134.4 ± 1.3	50.0	131.3 ± 1.4
52.0	144.2 ± 1.4	52.0	137.9 ± 1.5
54.2	143.5 ± 1.4	54.2	137.5 ± 1.5
56.2	139.9 ± 1.4	56.2	125.4 ± 1.4
58.4	124.9 ± 1.2	58.4	110.1 ± 1.2
60.8	91.6 ± 0.9	59.8	85.6 ± 1.0
		61.8	53.7 ± 0.7
64.8	38.8 ± 0.4	63.8	34.4 ± 0.5
		65.8	15.5 ± 0.2
66.8	19.2 ± 0.3	67.8	4.13 ± 0.08
		69.8	2.07 ± 0.06
70.8	1.79 ± 0.04	71.8	9.7 ± 0.2
		73.8	25.7 ± 0.4
74.8	20.5 ± 0.4	75.8	52.9 ± 0.6
		77.8	84.7 ± 1.0
78.8	62.3 ± 0.7	79.8	118 ± 2
82.2	103.8 ± 1.4	81.8	136 ± 2
		84.4	181.7 ± 2.1
86.4	151.2 ± 1.9	86.4	203.9 ± 2.2
88.4	160.0 ± 2.3	88.4	216.2 ± 2.3
90.4	165.1 ± 2.3	90.4	219.4 ± 2.4
92.4	153.6 ± 2.3	92.4	206.5 ± 2.4
94.4	134.7 ± 2.6	94.4	192.4 ± 2.3

Table I (Continued)

E_{lab} = 47.10 ± 0.04 Mev
(226.43 ± 0.40 mg/cm^2 Al)

θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)	
21.5	368	± 3
23.5	275	± 3
25.5	196.6	± 1.8
27.5	127.5	± 1.4
29.5	77.4	± 0.7
31.5	46.0	± 0.7
33.5	26.5	± 0.4
35.5	18.6	± 0.3
37.5	22.2	± 0.4
39.5	35.5	± 0.5
41.5	54.2	± 0.9
43.5	75.1	± 1.3
45.5	98.9	± 1.6
47.5	117.7	± 1.8
49.5	131.6	± 2.0
51.5	142.5	± 2.1
53.5	138.3	± 2.1
55.5	130.3	± 2.1
57.5	112.9	± 1.9
59.5	93.2	± 1.6
61.5	69.4	± 1.2
63.5	42.4	± 0.7
65.5	21.4	± 0.4
67.5	6.53	± 0.16
69.5	1.65	± 0.07
71.5	7.63	± 0.20
73.5	23.6	± 0.4
75.5	51.2	± 0.9
77.5	83.5	± 1.5
79.5	122.9	± 1.3
81.5	156.7	± 2.8
83.5	187.3	± 3.2
85.5	216.7	± 3.5
87.5	234.2	± 2.1
89.5	238.9	± 2.1
91.5	232.0	± 3.0
93.5	206.8	± 1.9
95.5	181.2	± 2.0
97.5	145.2	± 1.6
99.5	108.3	± 1.6
101.5	68.1	± 0.8
103.5	38.2	± 0.7
105.5	13.5	± 0.5

E_{lab} = 47.28 ± 0.03 Mev
(227.93 ± 0.25 mg/cm^2 Al)

θ_{cm} (deg.)	$(d\sigma/d\Omega)_{cm}$ (mb/sterad)	
18.3	639	± 10
20.3	437	± 5
22.3	336	± 4
24.3	228	± 3
26.3	155.0	± 2.1
28.3	104.5	± 1.2
30.3	59.3	± 1.3
32.3	34.0	± 0.6
34.3	24.5	± 0.6
36.3	17.8	± 0.3
38.3	27.3	± 0.7
40.3	43.1	± 0.8
42.3	67.0	± 1.4
44.3	85.6	± 1.3
46.3	111.1	± 2.2
48.3	120.9	± 1.5
50.3	136.9	± 2.5
52.3	132.8	± 1.7
54.3	135.1	± 3.2
56.3	114.9	± 1.6
58.3	108.1	± 2.1
60.3	74.7	± 1.0
62.3	46.2	± 1.0
64.3	29.0	± 0.5
66.3	11.2	± 0.3
68.3	2.19	± 0.15
70.3	2.15	± 0.21
72.3	9.6	± 0.4
74.3	28.3	± 0.6
76.3	55.3	± 0.9
78.3	88.6	± 1.6
80.3	123.5	± 1.6
82.2	151.3	± 2.5
84.3	176.1	± 2.8
86.3	205.9	± 2.8
88.3	216.5	± 2.3
90.3	222.0	± 2.6
92.3	207.9	± 2.2
94.3	185.1	± 2.2
96.3	152.7	± 2.4
98.3	124.0	± 2.2
100.3	84.1	± 2.1

Figure Legends

1. Pulse-height spectrum of alpha-particles scattered from helium at $E_{lab} = 41.9$ Mev; $\theta_{lab} = 32.5^\circ$.
2. Differential cross sections for elastic alpha-alpha scattering at laboratory energies from 36.85 to 41.90 Mev.
3. Differential cross sections for elastic alpha-alpha scattering at laboratory energies from 44.41 to 47.28 Mev.

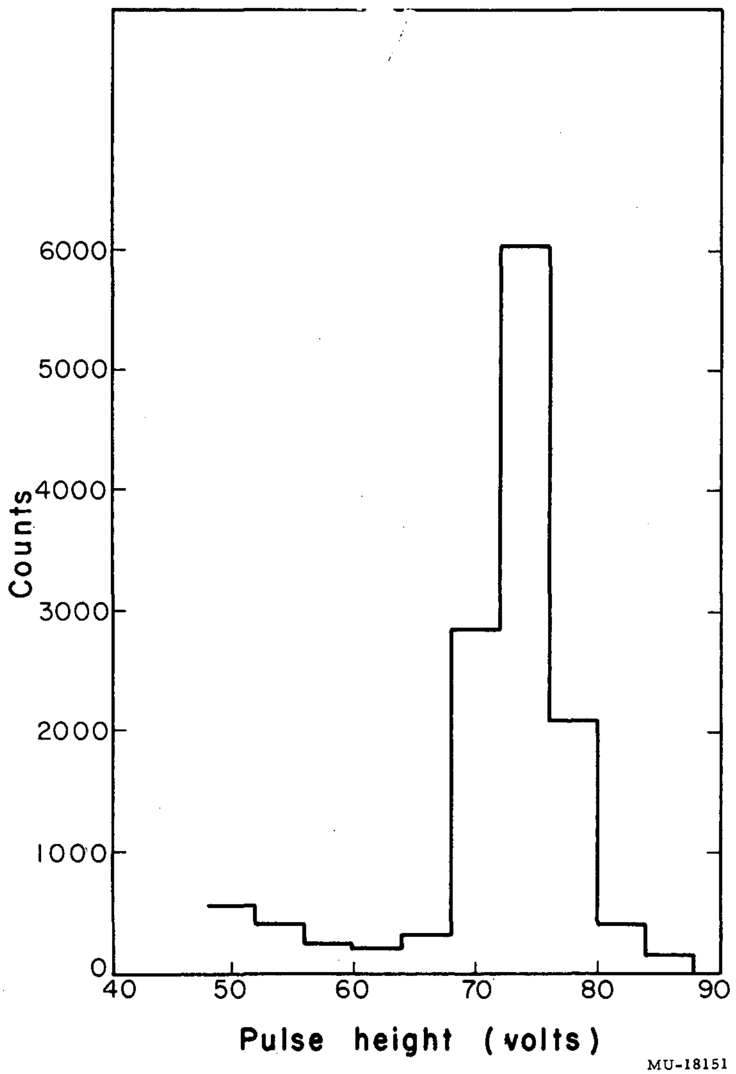
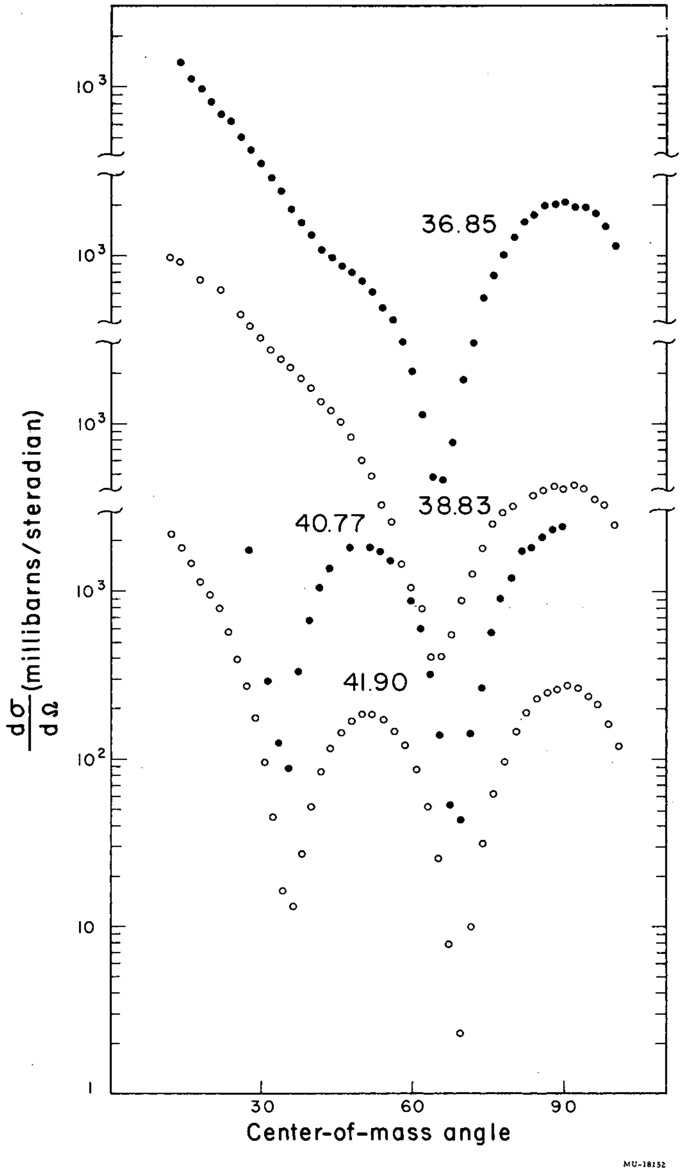
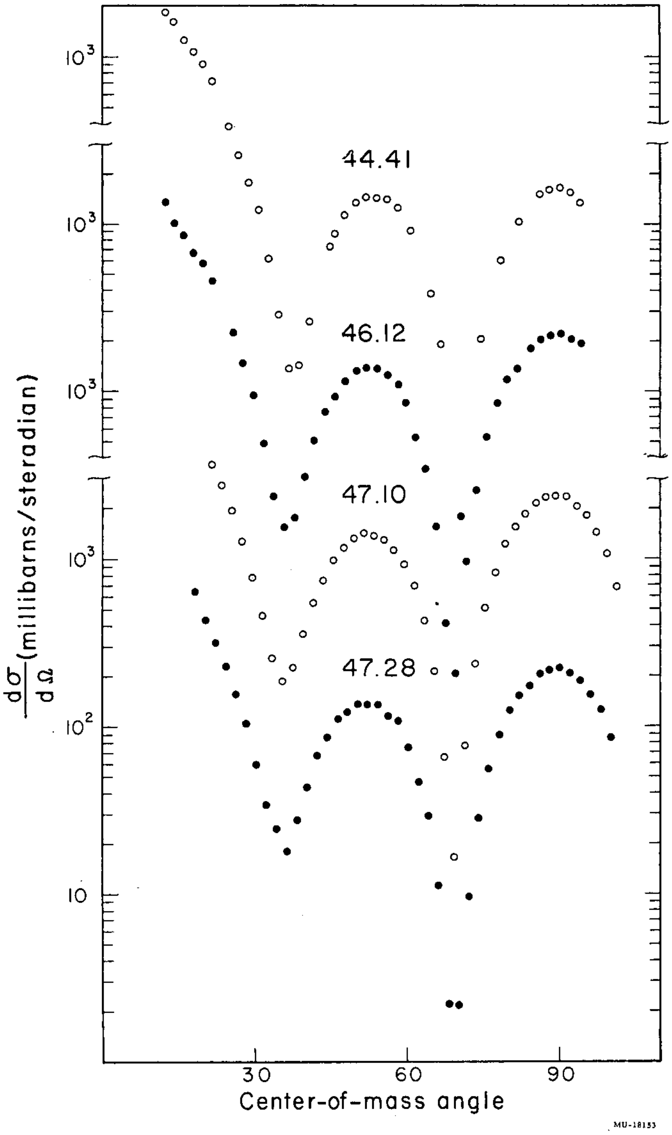

6000
5000
4000
3000
2000
1000
0
Counts
40
50
60
70
80
90
Pulse height (volts)
MU-18151

Fig. 1



dσ/dΩ (millibarns/steradian)
10³
10³
10³
10³
10²
10
1
30
60
90
Center-of-mass angle
36.85
38.83
40.77
41.90
MU-18152

Fig. 2



44.41
46.12
47.10
47.28
10^3
10^3
10^3
10^3
10^2
10
$\frac{d\sigma}{d\Omega}$(millibarns/steradian)
30
60
90
Center-of-mass angle
MU-18153

Fig. 3

References

1. M. P. Heydenburg and G. M. Temmer, Phys. Rev. 104, 123 (1956), 0.4 to 3 Mev.
2. Russell, Phillips, and Reich, Phys. Rev. 104, 135 (1956), 3 to 6 Mev.
3. Nilson, Kerman, Briggs, and Jentschke, Phys. Rev. 104, 1673 (1956), 12 to 22.9 Mev.
4. E. Graves, Phys. Rev. 84, 1250 (1951).
5. Burcham, Gibson, Prowse and Rotblat, Nuclear Phys. 3, 217 (1957).
6. Dennis J. Prowse, University of California at Los Angeles, private communication; George W. Farwell, University of Washington, Seattle, Wash., private communication.
7. R. E. Ellis and L. Schecter, Phys. Rev. 101, 636 (1956).
8. Ellis, Summers-Gill, and Vaughn, Lawrence Radiation Laboratory, Berkeley, Calif., private communication.
9. Bichsel, Mozley, and Aron, Phys. Rev. 105, 1788 (1957).
10. C. L. Critchfield and D. C. Dodder, Phys. Rev. 75, 419 (1949).
11. Nilson, Jentschke, Briggs, Kerman, and Snyder, Phys. Rev. 109, 850 (1958).