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Development of numerical estimation in Chinese preschool children

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ABSTRACT

Although much is known about the development of mental representations of numbers, it is not clear how early children begin to represent numbers on a linear scale. The current study aimed to examine the development of numerical estimation of Chinese preschoolers. In total, 160 children of three age groups (51 3- and 4-year-olds, 50 5-year-olds, and 59 6-year-olds) were administered the numerical estimation task on three types of number lines (Arabic numbers, dots, and objects). All three age groups took the test on the 0–10 number lines, and the oldest group also took it on the 0–100 and 0–1000 Arabic number lines. Results showed that (a) linear representation of numbers increased with age, (b) representation of numbers was consistent across the three types of tasks, (c) Chinese participants generally showed earlier onset of various landmarks of attaining linear representations (e.g., linearity of various number ranges, accuracy, intercepts) than did their Western counterparts, as reported in previous studies, and (d) the estimates of older Chinese preschoolers on the 0–100 and 0–1000 symbolic number lines fitted the two-linear and linear models better than alternative models such as the one-cycle, two-cycle, and logarithmic models. These results extend the small but accumulating literature on the earlier development of number cognition among Chinese preschoolers compared with their Western counterparts, suggesting the importance of cultural factors in the development of early number cognition.

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Introduction

Researchers have long been interested in mental representations of numbers and their development because such representations play an important role in mathematical cognition. To study mental representations, investigators have relied on Fechner's law that the magnitude of a sensation is a logarithmic function of objective stimulus intensity. This law is believed to describe representations of numerical as well as physical magnitudes (Bank & Hill, 1974; Dehaene, 1997). One of the widely used tasks to examine how the human mind represents numbers is the number line estimation task, in which participants are asked to place a given number on a straight line anchored with numbers at the two ends (e.g., 0 on the left end and 10 or 100 on the right end; Barth & Paladino, 2011; Dehaene, Izard, Spelke, & Pica, 2008; Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009; Muldoon, Simms, Towse, Menzies, & Yue, 2011; Opfer, Siegler, & Young, 2011; Siegler & Booth, 2004; Siegler & Opfer, 2003; Slusser, Santiago, & Barth, 2013; Whyte & Bull, 2008; Young & Opfer, 2011).

Empirical data from the number line estimation task have led researchers to propose several models of mental representations of numbers. For example, to explain the distance effect that was observed with infants and animals (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Starkey & Cooper, 1980), Dehaene (1997) proposed the *logarithmic model*, which suggests that infants exaggerate the distance between the small numbers and minimize the distance between the middle and large numbers. As an extreme case, Dehaene and his colleagues (2008) even found evidence of such a model in adults. The Mnundurucu people (adults as well as children) in the Amazon showed logarithmic representation of numbers on the 0–10 number line estimation task. This was true for both symbolic and nonsymbolic number lines. Because the Mnundurucu people have no structured mathematical language or formal schooling, the above results seem to further support the idea that logarithmic representation is the initial and intuitive way of mapping numbers to space. For Western children, however, this intuitive logarithmic representation is replaced by a linear representation at around 6 years of age (Case & Okamoto, 1996).

The strongest support for the *logarithmic-to-linear shift* hypothesis came from a large number of developmental studies carried out by Siegler and his colleagues (Booth & Siegler, 2006, 2008; Laski & Siegler, 2007; Opfer & Siegler, 2007; Opfer & Thompson, 2008; Opfer et al., 2011; Ramani & Siegler, 2008; Siegler & Booth, 2004; Siegler & Mu, 2008; Siegler & Opfer, 2003; Siegler & Ramani, 2008, 2009; Young & Opfer, 2011). They identified different ages at which American children move away from logarithmic representations and develop linear representations of different ranges of numbers. Specifically, they found that linearity in the 0–10 range was attained by preschoolers (at least for those who came from a middle-class background), that linearity in the 0–100 range was attained by some first graders and most second graders, and that linearity in the 0–1000 range was attained by some fourth and fifth graders and most sixth graders.[1]

During the past few years, researchers have also proposed alternative models of number representation. For example, Ebersbach and colleagues (2008) showed that a segmented linear regression model outperformed the logarithmic model to explain the performance of 5- to 9-year-old children on the 0–100 number line task. They further found that the breakpoint between the two linear segments of the model was associated with children's familiarity with numbers as assessed by a counting task. Similar to Ebersbach's model, but based on the decomposed representations of tens and units in two-digit number processing (Nuerk, Kaufmann, Zoppoth, & Willmes, 2004; Nuerk, Weger, & Willmes, 2001; Nuerk & Willmes, 2005; Wood, Nuerk, & Willmes, 2006), Moeller and colleagues (2009) proposed a *two-linear model* with the breakpoint between single- and two-digit numbers. They found evidence to support the two-linear model for first graders on the 0–100 number line task.

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[1] Siegler and colleagues studies also included control tasks such as a color board game, a circular board game task, and a numerical activity task because they were intervention studies. The control tasks typically showed much poorer indexes of linear representations at both the group and individual levels and both before and after the intervention (see Ramani & Siegler, 2008; Siegler & Ramani, 2009, for details).

More recently, Barth and Paladino (2011) proposed the *one- and two-cycle versions of the proportional power model* to account for the logarithmic-to-linear representation shift. The one-cycle model considers children's reliance on the two endpoints, which leads to overestimation of numbers below the midpoint (0.5) and underestimation of numbers above the midpoint. The two-cycle model assumes that children would rely on both endpoints plus the midpoint, which leads to two cycles with 0.25 and 0.75 as the midpoints and overestimation for numbers below 0.25 and between 0.50 and 0.75 but underestimation for numbers between 0.25 and 0.50 and above 0.75. They found that children's performance on the 0–100 number line fit the one-cycle model for 5-year-olds and fit the two-cycle model for 6- to 8-year-olds. The two-cycle model provided the best fit for 7- to 10-year-olds' median estimates on the 0–1000 number line and for 8- to 10-year-olds' median estimates on the 0–10,000 number line task (Barth & Paladino, 2011; Slusser et al., 2013).

Although much of the research on mental representations of numbers has focused on the general developmental patterns, a few studies have also examined whether the age of attainment of linear representation of numbers varies by children's family and cultural backgrounds. Siegler and Ramani (2008) found that preschoolers (mean age of 4.7 years) from middle-income family backgrounds showed more linear representations on the 0–10 number line than those from low-income family backgrounds (R^2_{lin} = .94 vs. .66; no indexes of fitting the data to a logarithmic curve were reported). In a cross-national study, Siegler and Mu (2008) found that American kindergarteners' estimates on the 0–100 number line were better fit by a logarithmic function, whereas Chinese kindergarteners' estimates were linear. Interestingly, when Chinese children were matched on arithmetic performance (rather than age) with Scottish children, Muldoon and colleagues (2011) found that Chinese children's (mean age of 4.5 years) number estimation was not more linear than the older Scottish children (mean age of 5.3 years) on the 0–10 number line (R^2 = .49 vs. .76 for mean linearity index, R^2 = .94 vs. .94 for median linearity index) and on the 0–100 number line (R^2 = .31 vs. .46 for mean linearity index, R^2 = .66 vs. .80 for median linear index).

Although much research has been conducted on the development of number representations, there is very limited research in the lower end of the age range. Most of the studies so far have used kindergarteners and older children and adults as participants. Only four studies included preschoolers: three in the United States (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009) and one in China and the United Kingdom (Muldoon et al., 2011). This lack of research on young children is especially relevant to Chinese children who begin preschool at 3 years of age and receive formal education about numbers. Previous studies have documented Chinese children's superior performance in mathematics across all age groups as compared with their Western counterparts (e.g., Campbell & Xue, 2001; Chen & Stevenson, 1995; Miller, Smith, Zhu, & Zhang, 1995; Stevenson, Chen, & Lee, 1993; Stevenson et al., 1990). Would Chinese 3- and 4-year-olds then have developed linear representations of numbers after a year or so of learning numbers in preschool? So far, no study has addressed this question. Furthermore, it is not clear whether concrete materials such as number of dots or candies would help children to have more accurate estimation of the number line. Finally, no study has investigated whether Chinese preschoolers' estimation of lines up to 100 and 1000 would fit logarithmic or linear or alternative models such as two-linear, one-cycle, and two-cycle models.

The current study aimed to systematically examine the development of number line estimation of Chinese preschoolers ranging from 3 to 6 years of age. Three number line estimation tasks were used: the symbolic number line with 0 and 10 at the two ends of the line as used in Dehaene's and Siegler's studies (Dehaene et al., 2008; Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009), the object number line (1 candy on one end and 10 candies on the other end), and the dot number line (1 dot on one end and 10 dots on the other end). The older preschoolers (5- and 6-year-olds), equivalent to American kindergarteners, completed two additional line estimation tasks: symbolic number lines of 0–100 and 0–1000.

Four specific hypotheses were tested. First, we hypothesized that with age Chinese children's number representations would show a systematic improvement in line estimation (i.e., a decrease in errors and an increase in linearity). This developmental trend was a downward extrapolation based on the early findings from older samples in the United States (e.g., Siegler & Booth, 2004). Second, we hypothesized that children's performance would be better on the number lines using dots and objects than on the symbolic number line because of children's familiarity with the

concrete materials over the symbolic numbers (Dehaene et al., 2008). Previous research showed that the nature of number representation depended on the type of quantity being represented. For example, Dehaene and colleagues (2008) found that their sample of American adults had linear representation of 0–10 symbolic numbers (in English and in Spanish, with the latter being a second language for the participants) and 1–10 dots, but not of 1–100 dots or 1–10 tones, both of which were logarithmically mapped. Third, although we expected similar developmental trends in Chinese children as in Western children, we expected that the shift from logarithmic to linear representation would occur earlier in Chinese children than has been found in American children because of the former's advantage in early mathematics (e.g., Campbell & Xue, 2001; Chen & Stevenson, 1995; Miller et al., 1995; Stevenson et al., 1990, 1993). This hypothesis was a downward extension of Siegler and Mu's (2008) findings from kindergarteners to preschoolers. In addition, it would reexamine Muldoon and colleagues' (2011) conclusion about cross-cultural differences in line estimation based on Chinese and Scottish children who were matched on mathematical performance but not on age. Finally, given the ongoing debate on the best model that describes children's representation of number lines of 0–100 and 0–1000 (Barth & Paladino, 2011; Barth, Slusser, Cohen, & Paladino, 2011; Moeller et al., 2009; Opfer et al., 2011; Slusser et al., 2013; Young & Opfer, 2011), we also attempted to fit our data from older preschoolers on those number lines to alternative models such as the one-cycle, two-cycle, and two-linear models. Given Chinese children's early advantage in mathematical cognition, we hypothesized that our data should fit a linear model at least for the 0–100 line but also possibly for the 0–1000 line.

Method

Participants

Participants of this study were 160 children from three grade levels of preschool: young, middle, and older preschoolers. The young (first-year) preschoolers comprised 51 children (27 boys and 24 girls, M_{age} = 4.1 years, SD = 3.99, range = 3.3–4.7). The middle (second-year) preschoolers comprised 50 children (34 boys and 16 girls, M_{age} = 5.0 years, SD = 3.16, range = 4.5–5.7). The older (third-year) preschoolers (equivalent to American kindergarteners) comprised 59 children (31 boys and 28 girls, M_{age} = 6.3 years, SD = 2.98, range = 5.8–6.7). Approximately 30% of the children had at least one parent with an advanced degree (master's or PhD), and nearly half (47%) of the children had one or both parents with a college education (from either a 2-year or 4-year institution). For 23% of the children, their parents' education did not extend beyond high school. All children were recruited from two average preschools in Beijing, China. Teachers reported not using number lines as part of their curriculum. Participation was voluntary, and neither children nor teachers received compensation.

Materials

Based on Dehaene's and Siegler's number line estimation tasks, three types of number lines with different number ranges were used for this study: 0–10, 0–100, and 0–1000, with the latter two types for the older preschoolers only.

The 0–10 number line

Three types of tasks were used in this study for the 0–10 number line: symbolic, object, and dot number line tasks. For the symbolic number line task, a 23-cm line was printed across the middle, with 0 at the left end and 10 at the right end. There were 16 such sheets of paper. The numbers from 1 to 9 except 5 were printed on 8 cards, one number per card. For the object and dot number lines, a 23-cm line was printed across the middle, with 1 candy/dot at the left end and 10 candies/dots at the right end. There were 14 such sheets of paper with dots and candies, respectively. Seven cards were printed with the numbers from 2 to 9 except 5 candies, one set per card, and seven comparable cards were made with dots.

The 0–100 number line

This task was the same as Siegler's number line estimation task (e.g., Siegler & Booth, 2004). A 23-cm line was printed across the middle, with 0 at the left end and 100 at the right end. There were 48 such sheets of paper. The numbers 3, 4, 6, 8, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90, and 96 were printed on 24 cards, one number per card.

The 0–1000 number line

Based on Siegler's 0–1000 number line estimation task (e.g., Opfer & Siegler, 2007; Siegler & Opfer, 2003), numbers below 100 were oversampled, with 4 numbers between 0 and 100 and 8 numbers between 100 and 1000. A 23-cm line was printed across the middle, with 0 at the left end and 1000 at the right end. There were 24 such sheets of paper. The numbers 6, 25, 71, 86, 144, 230, 390, 466, 683, 780, 810, and 918 were printed on 12 cards, one number per card.

Procedure

The experimenters were two Chinese female postgraduate students. Each child was tested two times: once in the morning and a second time in the afternoon within the same day. The test was administered individually, and each testing session lasted 18 to 22 min for the young and middle preschoolers and 35 to 40 min for the older preschoolers. The three types of tasks were presented using a Latin square design. Within each task, the number cards were presented in a random order. The instructions were the same as those used by Siegler and colleagues (e.g., Siegler & Booth, 2004). Specifically, the experimenter began the test by saying, "Today we're going to play a game with number lines. I'm going to ask you to show me where on the number line some candies/dots/numbers are. When you decide where the candies/dots/numbers go, please draw a line through the number line like this [making a vertical hatch mark]." Before each item, the experimenter said, "This number line goes from 0 at this end to 10/100/1000 at this end. If the number is N, where would you put it on this line?" Similar instructions were used for the dot and object number line tasks. Numbers 5, 50, and 500 as well as 5 dots and 5 objects were used as a warm-up exercise to help children understand the tasks. After the warm-up exercise, children were asked to mark the right location of the other numbers or the number of candies or dots with a pencil without feedback.

Results

Accuracy of estimation

To index accuracy of estimation, we calculated percentage absolute error (PAE) using the formula of (Estimate – Estimate Quantity)/Scale of Estimates (see Booth & Siegler, 2006, 2008; Muldoon et al., 2011; Slusser et al., 2013). As shown in Table 1, the mean PAE decreased systematically with age on all three 0–10 number line tasks. Young preschoolers had PAEs around 20%, middle preschoolers' PAEs ranged from 12% to 15%, and older preschoolers made fewer than 10% PAEs. One-way analysis of variance (ANOVA) revealed that the main effect of age group was significant for all three tasks, and post hoc tests (Fisher's least significant difference [LSD]) showed that all three age groups differed

Table 1
Mean percentage absolute errors of the three 0–10 number line tasks by age group.

Task type	Preschoolers			F	η_p^2	Post hoc comparisons
	Young	Middle	Older			
Symbolic numbers	.25 (0.10)	.15 (0.08)	.09 (0.03)	67.16[***]	.46	Y > M > O
Dots	.19 (0.07)	.12 (0.06)	.06 (0.03)	75.26[***]	.49	Y > M > O
Objects	.19 (0.07)	.12 (0.06)	.06 (0.03)	76.79[***]	.50	Y > M > O

Note. Y, young; M, middle; O, older.
[***] $p < .001$.

significantly from one another on all three tasks (see Table 1 for detailed statistics). The correlations between mean PAEs of the three types of the tasks were highly significant (see Table 2).

Mean PAEs of older preschoolers for the 0–100 and 0–1000 symbolic number lines were 11% and 17%, respectively. The correlation (Pearson's r) between the 0–100 and 0–1000 number tasks for older preschoolers was .76 ($p < .001$).

Fit of logarithmic and linear models

The median estimates for each number stimulus were used for logarithmic and linear regression analyses. As shown in Fig. 1, young preschoolers' median estimates fitted both the linear function and the logarithmic function about equally well (adjusted R^2 = .88 for linear vs. .79 for logarithmic line for symbolic numbers, adjusted R^2 = .88 vs. .86 for dots, and adjusted R^2 = .79 vs. .77 for objects). Following the procedure used by Siegler (see Siegler & Booth, 2004), a paired-samples t test of the fit indexes of linear versus logarithmic functions revealed no significant differences, $t(15) = 1.13$, $p = .28$, Cohen's $d = 0.28$, for symbolic lines; $t(13) = 0.204$, $p = .84$, Cohen's $d = 0.01$, for dots; and $t(13) = 0.16$, $p = .88$, Cohen's $d = 0.04$, for objects. (Note that for these and subsequent analyses, adjusted R^2 values were arcsine-transformed to approximate normal distribution before statistical tests.)

In contrast, middle and older preschoolers' patterns were significantly more linear than logarithmic. For the symbolic number line task, the adjusted R^2 for linear function was .98 for both middle and older preschoolers, which was significantly higher than that for logarithmic function (.83 for middle preschoolers and .85 for older preschoolers), $t(15) = 6.90$, $p < .001$, Cohen's $d = 1.72$, and $t(15) = 7.62$, $p < .001$, Cohen's $d = 1.91$, respectively. Similarly, the adjusted R^2 for linear function was also higher than that for logarithmic function for dots and objects in both middle and older preschoolers, R^2 = .99 versus .93, $t(13) = 4.41$, $p = .001$, Cohen's $d = 1.18$, for dots in middle preschoolers; R^2 = .99 versus .94, $t(13) = 2.68$, $p = .02$, Cohen's $d = 0.72$, for objects in middle preschoolers; R^2 = .99 versus .95, $t(13) = 3.93$, $p = .002$, Cohen's $d = 1.05$, for dots in older preschoolers; and R^2 = .99 versus .96, $t(13) = 3.14$, $p = .008$, Cohen's $d = 0.84$, for objects in older preschoolers. In terms of the slope of the median estimates for each number, there was a major change between the young and middle preschoolers, the latter of whom showed slopes close to the ideal 1.0 (see Fig. 1).

The above analysis shows that at the group level (i.e., median estimates across all children in each age group for each number), the mental representations of numbers for middle and older preschoolers were more linear than logarithmic, but for young preschoolers both functions fitted the data about equally well. Another way to examine age differences is to use individual children's linearity index (adjusted R^2), which would vary to a greater extent than the median estimate of the group (e.g., Booth & Siegler, 2006; Ramani & Siegler, 2008; Siegler & Booth, 2004; Siegler & Mu, 2008). Table 3 shows the mean linearity index by task and age group. A one-way ANOVA was performed to test age differences for each task. Results showed significant age effects, $F(2, 157) = 98.92$, $p < .001$, $\eta_p^2 = .56$, for symbolic numbers; $F(2, 157) = 93.03$, $p < .001$, $\eta_p^2 = .54$, for dots; and $F(2, 157) = 88.61$, $p < .001$, $\eta_p^2 = .53$, for objects. The estimates of the older preschoolers were more linear than those of the middle preschoolers, which in turn were more linear than those of the young preschoolers. In terms of the slopes of

Table 2
Correlations between mean percentage absolute errors of the three 0–10 number line tasks by age group.

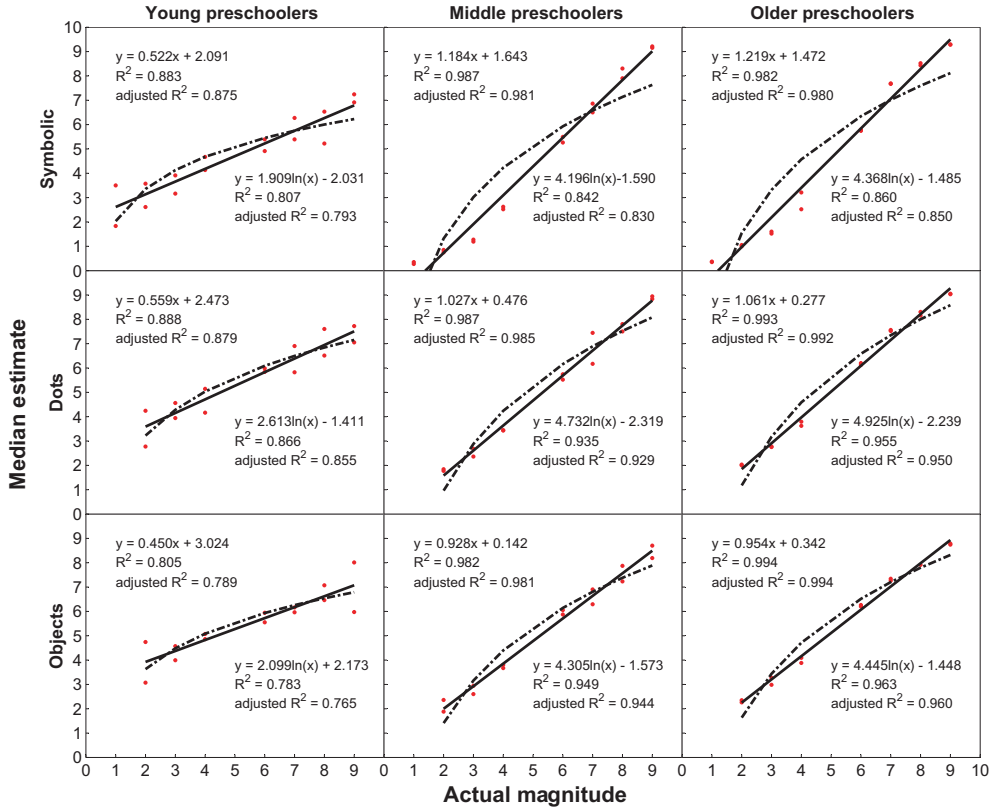| Preschoolers	Task type	Symbolic numbers	Dots
Young	Dots	.70[***]	–
	Objects	.59[***]	.62[***]
Middle	Dots	.66[***]	–
	Objects	.69[***]	.86[***]
Older	Dots	.44[***]	–
	Objects	.35[**]	.68[***]

[**] $p < .01$.
[***] $p < .001$.

Fig. 1. Logarithmic and linear models of median estimates of young, middle, and older preschoolers on the 0–10 object, dot, and symbolic number lines. The fit indexes for the linear model appear above the lines, and those for the logarithmic model appear below the lines.

Table 3
Mean linearity of the three 0–10 number line tasks by age group.

Task type	Preschoolers F			η_p^2	Post hoc comparisons
	Young	Middle	Older		
Symbolic numbers	.26 (0.37)	.75 (0.29)	.94 (0.04)	98.92[***]	.56	O > M > Y
Dots	.30 (0.36)	.74 (0.27)	.92 (0.06)	93.03[***]	.54	O > M > Y
Objects	.28 (0.33)	.70 (0.30)	.92 (0.10)	88.61[***]	.53	O > M > Y

Note. Y, young; M, middle; O, older.
[***] $p < .001$.

individual children's estimates, one-way ANOVA again showed significant age effects, $F(2, 157) = 86.11$, $p < .001$, $\eta_p^2 = .52$, for symbolic numbers; $F(2, 157) = 56.64$, $p < .001$, $\eta_p^2 = .42$, for dots; and $F(2, 157) = 44.55$, $p < .001$, $\eta_p^2 = .36$, for objects. Results indicated that, with age, the slopes of children's estimates approached the ideal 1.0 (see Table 4).

To examine whether individual differences in pattern of estimates were consistent across the three types of tasks, a correlation analysis was conducted. As in the analyses based on mean PAE, the fit indexes (mean adjusted R^2) were highly correlated across the tasks (see Table 5).

Table 4
Mean slope by linear function of the three 0–10 number line tasks by age group

Task type	Preschoolers			F	η_p^2	Post hoc comparisons
	Young	Middle	Older			
Symbolic numbers	0.36 (0.48)	0.99 (0.35)	1.19 (0.10)	86.11[***]	.52	O > M > Y
Dots	0.44 (0.40)	0.91 (0.35)	1.05 (0.14)	56.64[***]	.42	O > M > Y
Objects	0.39 (0.41)	0.82 (0.34)	0.94 (0.18)	44.55[***]	.36	O > M > Y

Note. Y, young; M, middle; O, older.
[***] $p < .001$.

Table 5
Correlations between mean linearity indexes across the three 0–10 number line tasks

Preschoolers	Task type	Symbolic numbers	Dots
Young	Dots	.67[***]	–
	Objects	.58[***]	.66[***]
Middle	Dots	.73[***]	–
	Objects	.75[***]	.84[***]
Older	Dots	.49[***]	–
	Objects	.43[**]	.50[***]

[**] $p < .01$.
[***] $p < .001$.

Figs. 2 and 3 show the data for the 0–100 and 0–1000 symbolic number line tasks of the older pre-schoolers. At the group level, median estimates also showed more linear than logarithmic representations, adjusted R^2 = .98 versus .80, $t(47)$ = 8.04, $p < .001$, Cohen's d = 1.16, for the 0–100 line; and adjusted R^2 = .94 versus .74, $t(23)$ = 4.63, $p < .001$, Cohen's d = 0.94, for the 0–1000 line. The slope of the group median estimates was closer to 1.00 for the 0–100 number line than for the 0–1000 number line. In terms of the mean fit indexes from individual participants' estimates, the mean adjusted R^2 was .78 for the 0–100 line and .65 for the 0–1000 line, both of which were significantly lower than adjusted R^2 = .94 for the 0–10 line, $F(2, 116)$ = 81.53, $p < .001$, η_p^2 = .58. The mean slope was .77 for the 0–100 line and .62 for the 0–1000 line, both of which were significantly lower than 1.19 for the 0–10 line, $F(2, 116)$ = 249.45, $p < .001$, η_p^2 = .81.

Fit of two-linear model and one- and two-cycle versions of proportional power model

We then fitted the data to alternative models. To examine the two-linear model, we used the point that separates single- and two-digit numbers as the breakpoint on the 0–100 number line and the point that separates two- and three-digit numbers as the breakpoint on the 0–1000 number line. We followed the procedure used by Moeller (e.g., Moeller et al., 2009). As shown in Figs. 2 and 3, on the 0–100 symbolic number line task, the adjusted R^2 was .98 for group median estimates and the mean adjusted R^2 across individuals was .81. The corresponding numbers for the 0–1000 symbolic number line task were .96 and .73.

Following the procedure used by Barth (e.g., Barth & Paladino, 2011; Slusser et al., 2013), we fitted the data to the one- and two-cycle models.[2] As shown in Fig. 2, on the 0–100 symbolic number line task, the adjusted R^2 of group *median* estimates was .97 for both the one-cycle and two-cycle models. The corresponding *mean*-adjusted R^2 values across individuals were .73 and .61 (not shown in Fig. 2). As shown in Fig. 3, on the 0–1000 symbolic number line task, the adjusted R^2 of group median estimates was .91 for

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[2] In addition to the original one- and two-cycle models, Barth and Paladino (2011) also added adapted versions of those models by adding parameter W. The adapted models were also applied to our data, and the results were virtually the same as those for the original models, with no changes in adjusted R^2 for the 0–100 line and differences of .05% to 1.5% for the 0–1000 line.
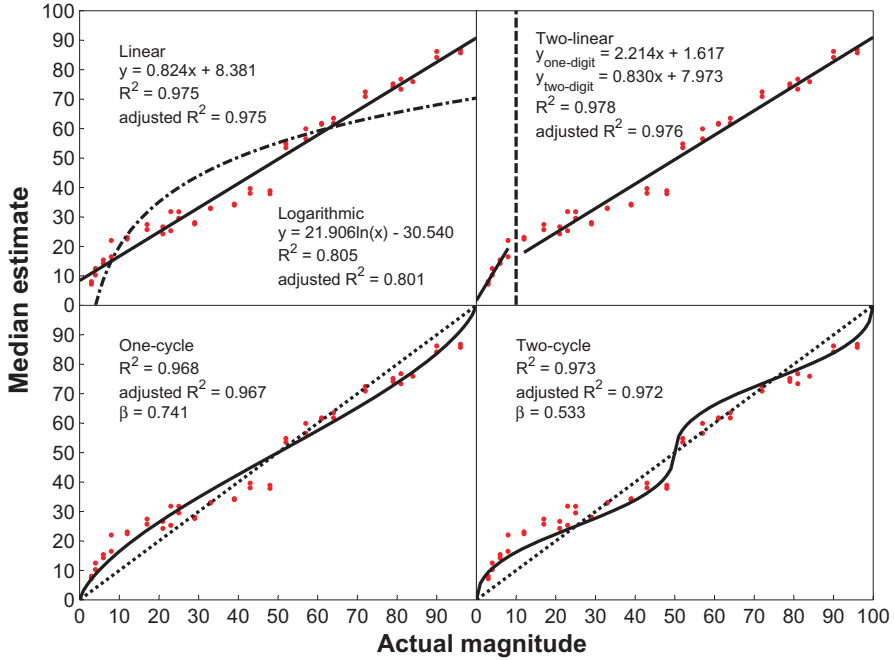
Fig. 2. Logarithmic, linear, two-linear, one-cycle, and two-cycle models of median estimates of 6-year-olds for the 0–100 symbolic number line.
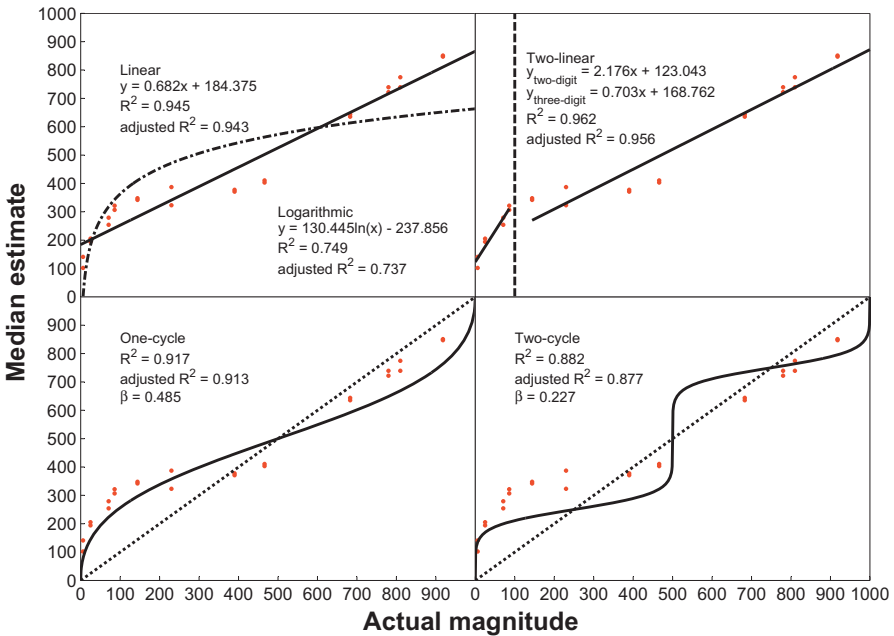

Fig. 3. Logarithmic, linear, two-linear, one-cycle, and two-cycle models of median estimates of 6-year-olds for the 0–1000 symbolic number line.

Table 6
Pairwise comparisons in mean R^2 of different models.

Range	Model	Linear	Logarithmic	Two-linear	One-cycle
0–100	Logarithmic	.24 (.02)***	–		
	Two-linear	−.04 (.01)**	−.28 (.02)***	–	
	One-cycle	.03 (.01)***	−.21 (.02)***	.06 (.01)***	–
	Two-cycle	.11 (.03)***	−.13 (.04)**	.15 (.03)***	.09 (.03)**
0–1000	Logarithmic	17 (.03)***	–		
	Two-linear	−.10 (.02)***	−.27 (.03)***	–	
	One-cycle	−.07 (.02)***	−.10 (.04)*	.17 (.03)***	–
	Two-cycle	.34 (.07)***	.17 (.09)	.44 (.07)***	.27 (.06)***

Note. Mean difference = column – row. Standard errors are in parentheses. Repeated-measures ANOVA with Games–Howell correction was used.
  * $p < .05$.
  ** $p < .01$.
  *** $p < .001$.

the one-cycle model and .88 for the two-cycle model. The corresponding mean-adjusted R^2 values across individuals were .54 and .25 (not shown in Fig. 3).

Direct comparisons across five models

To compare the fit indexes across the five models tested, the adjusted R^2 of individual children's estimates were subjected to repeated-measures ANOVA. Results showed a significant effect of model, $F(1.52, 84.98) = 43.22$, $p < .001$, $\eta_p^2 = .44$, for the 0–100 number line; and $F(1.34, 72.42) = 20.94$, $p < .001$, $\eta_p^2 = .23$, for the 0–1000 number line. Table 6 shows pairwise comparisons across the models. The two-linear model had the best fit, followed closely by the linear model. The one-cycle model was in the middle of the five models. The worst fitting model was either the logarithmic model (for the 0–100 line) or the two-cycle model (for the 0–1000 line).

Another way to compare across models is to show which of the five models fitted individual participants' data the best. For the 0–100 number line, the best-fitting model was the two-linear model for 59.3% of participants, the two-cycle model for 22.0% of participants, the linear model for 13.6% of participants, the one-cycle model for 5.1% of participants, and the logarithmic model for 0% of participants, whereas the order of the second best-fitting model was the linear model (45.8%), the one-cycle model (22.0%), the two-linear model (20.3%), the logarithmic model (6.8%), and the two-cycle model (5.1%). Similarly, for the 0–1000 number line, the order of the best-fitting model was the two-linear model (55.9%), the two-cycle model (13.6%), the logarithmic model (11.9%), the one-cycle model (10.2%), and the linear model (8.5%), whereas the order of the second best-fitting model was the linear model (40.7%), the two-linear model (25.4%), the logarithmic model (18.6%), the two-cycle model (8.5%), and the one-cycle model (6.8%).

Discussion

The current study aimed to examine the development of number representation among Chinese preschoolers from 3 to 6 years of age. We assessed the accuracy and linearity of Chinese preschoolers' estimates on three types of the 0–10 number lines (symbolic numbers, dots, and objects) and tested alternative models (i.e., the two-linear, one-cycle, and two-cycle models) for the 0–100 and 0–1000 symbolic number lines among older preschoolers.

Consistent with our first hypothesis, results from the 0–10 number lines showed a systematic age-related decrease in errors and an age-related increase in linearity of estimated magnitude. Young preschoolers' median estimates fitted the linear and logarithmic functions about equally well, whereas middle and older preschoolers' patterns were significantly more linear than logarithmic. These age differences were statistically significant; older preschoolers were more linear than middle preschoolers, who in turn were more linear than young preschoolers. These findings from Chinese preschoolers

generally supported Siegler's logarithmic-to-linear shift hypothesis, but with two important qualifications. First, we did not find that Chinese 3- and 4-year-olds showed a better fit to the logarithmic function than to the linear function. It is possible that after a year of preschool as well as other factors to be discussed in a later section, Chinese preschoolers may have learned enough about numbers to pass the logarithmic stage. Second, the current study did not find that older preschoolers' numerical representation changed gradually from the logarithmic model to the linear model as a function of increasing numerical range. Even on the 0–1000 line, which covers an unfamiliar range of numbers for them, older preschoolers' group median estimates already showed more linear than logarithmic representation. In future research, even larger, and thus more unfamiliar, number ranges such as the 0–10,000 number line task could be used to test the logarithmic-to-linear shift hypothesis with Chinese preschoolers.

Contrary to our second hypothesis, the three types of stimuli or number lines (symbolic, dots, and objects) showed the same pattern of results and had high intercorrelations. These results suggest that concrete materials did not seem to help Chinese children judge the location of a given number on the number line. One possibility is that after a year of preschool, Chinese children no longer rely on counting to estimate the quantity of objects; therefore, they try to directly map the values (Arabic numbers or quantity of objects) onto the response scale. Future research could test this speculation by closely observing children's behavior during the test (e.g., whether they were counting). Future research could also use unfamiliar quantities such as tones used by Dehaene and colleagues (2008) to assess whether Chinese preschoolers show logarithmic representations.

Our third hypothesis dealt with potential cross-cultural differences in early number cognition. We hypothesized that Chinese children should have an early advantage in number estimation because of their early advantage in mathematical performance. This hypothesis appeared to be supported. When comparing our results with those from previous studies of Western children, it appears that the estimates of Chinese preschoolers were more accurate and linear on the 0–10, 0–100, and 0–1000 number lines than their Western age mates (Booth & Siegler, 2006; Muldoon et al., 2011; Ramani & Siegler, 2008; Siegler & Booth, 2004; Siegler & Mu, 2008; Siegler & Ramani, 2008, 2009). Table 7 shows a summary of previous studies based on Western samples. The PAE, linearity, and slope on the 0–10 number lines of group median estimates found in our study were higher than those reported by Siegler and colleagues (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009) and the Scottish children (Muldoon et al., 2011). (It should be noted that the linearity and slope indexes from our young preschoolers were somewhat lower than those reported by Muldoon et al., 2011, on their sample of Chinese preschoolers, which can be explained by the fact that their sample was older [M_{age} = 4.5 years] than our young preschoolers.) Similarly, for the 0–100 number line, we found that the linearity indexes seemed much higher than those reported by Siegler and his colleagues for American kindergarteners (see Table 7) and, in fact, were similar to those for first and second graders in Siegler's American samples (Booth & Siegler, 2006; Laski & Siegler, 2007; Siegler & Booth, 2004; Siegler & Mu, 2008). Finally, for the 0–1000 number line, the linearity indexes were higher than those obtained for American second graders (Booth & Siegler, 2006; Opfer & Siegler, 2007; Siegler & Opfer, 2003).

With cautions duly noted about the difficulties in comparing data across studies (which will need to be conducted properly with a meta-analytical approach when more data are available from Chinese samples), the above discussion appears to suggest that Chinese preschoolers are likely to attain the landmarks of linear representations of numbers earlier than American and Scottish children. This may be attributed to several cultural factors (Geary, 1996; Zhou et al., 2007). First, Chinese children receive formal education about numbers early in preschool. According to the preschool and kindergarten education guideline for the city of Beijing, 3- and 4-year-old children would be taught how to count up to five objects and understand the quantities up to five objects (magnitude comparison). Second, Chinese parents give informal direct mathematics instruction and encourage mathematics-related activities such as counting fingers, stairs, and family members; solving arithmetic problems; and determining set sizes (Zhou et al., 2006, 2007). These activities about numbers would convey redundant kinesthetic, visual auditory, and temporal information about numerical magnitudes (Siegler & Mu, 2008). Third, Chinese language uses numbers in contexts that do not involve numbers in English, for example, names of days of the week (e.g., xingqi-yi ["weekday one" for Monday], xingqi-er ["weekday two" for Tuesday]) and months of the year (e.g., yi-yue ["one month" for January],

Table 7
Results from the studies of Siegler and colleagues

Range	Year published	Authors	Age of participants (years)	Participants from	Accuracy (%)	Group linearity (R^2)	Group slope	Mean linearity (R^2)	Mean slope
0–10	2008	Siegler & Ramani	4.6	Low-income (58% African American, 42% Caucasian)		.66	0.24	.15	0.26
				Upper middle class (77% Caucasian, 23% Asian)		.94	0.98	.60	0.70
	2008	Ramani & Siegler	4.8	Head Start centers in an urban area	28	.75	0.11	.17	0.21
				Number board game condition (50% African American, 43% Caucasian, 7% other)	28	.37	0.06	.15	0.13
				Color board game condition (55% African American, 40% Caucasian, 5% other)					
	2009	Siegler & Ramani	4.7	Head Start classrooms and two child-care centers for very low-income families	29	.22	0.03	.14	0.04
					29	.11	0.03	.15	0.09
				Linear board condition (40% African American, 53% Caucasian, 7% other)	28	.43	0.05	.16	0.12
				Circular board game (31% African American, 62% Caucasian, 7% other)					
				Numerical activities control condition (31% African American, 69% Caucasian)					
0–100	2003	Siegler & Opfer	7.9	Suburban school in an upper-middle-class area	19	.96			
	2004	Siegler & Booth	5.8	Middle and low income families (67% Caucasian, 32% African American, 1% Asian)	27	.49	0.64	.24	0.33
			6.9		18	.90		.59	0.58
			7.8		15	.95		.64	0.60
	2006	Booth & Siegler	5.8	Lower- to middle-income neighborhood (63% Caucasian, 33% African American, 2% Asian American)	24	.63		.36	
			6.8		12	.96		.77	
			7.9		10	.97		.87	

	2007	Lasiki & Siegler	6.1	Public schools; the percentages	15	.91	0.69	.66	0.63
			7.2	of children who were eligible for	13	.95	0.70	.74	0.67
			8.2	the free or reduced-fee lunch	10	.98	0.78	.88	0.76
				program were 33% and 17% (95% Caucasian)					
	2008	Siegler & Mu	5.7	Chinese children from	15	.95	0.63	.63	0.53
			5.6	preschools affiliated with	22	.72		.43	0.39
				university in China; American (12% Asian, 88% Caucasian)					
0–1000	2003	Siegler & Opfer	7.9	Suburban school in an upper middle-class area		.63			
	2006	Booth & Siegler	7.8	Public school in a middle-class area (96% Caucasian, 2% African American, 2% Indian American)	17	.91		.66	
	2007	Opfer& Siegler	8.2	Suburban school in a middle-class area	18	.80			

er-yue ["two month" for February]) (Kelly, Miller, Fang, & Feng, 1999). This feature of Chinese language may provide children with greater exposure to numbers in daily life than children speaking other languages. Moreover, the Chinese language uses a transparent ten-based number system. As Helmreich and colleagues (2011) found, a more transparent number word system would lead to better number representations. Therefore, the Chinese number word system may also contribute to Chinese preschoolers' better performance on the number line tasks.

Our fourth hypothesis dealt with the comparison across existing models of children's numerical representation. In addition to the logarithmic and linear models, we tested three alternative models—the two-linear, one-cycle, and two-cycle models—with the data from older preschoolers' performance on the 0–100 and 0–1000 lines. A comparison of the fit indexes showed that the two-linear model has the best fit, followed closely by the linear model. The other three models (the logarithmic, one-cycle, and two-cycle models) fared poorly. Our results were consistent with those of Ebersbach and colleagues (2008) and Moeller and colleagues (2009), who also found that the two-linear model outperformed the logarithmic model to explain the estimation performance of children on the 0–100 symbolic number line task. According to Moeller and colleagues, number magnitude representation of the 0–100 line develops, with age and experience, from two separate linear representations (one for the familiar one-digit numbers, which forms a steep linear line, and the other for the less familiar two-digit numbers, which forms a flatter line) into one single-linear representation. Given the closeness in fit indexes between the two-linear and linear models in our data, the transition to linear representation among Chinese preschoolers appears near the end. Finally, our results did not seem to support the one- and two-cycle models proposed by Barth and Paladino (2011). These models may be particularly effective in accounting for nonlinear (e.g., logarithmic at the lower end) representations (e.g., Barth & Paladino, 2011; Barth et al., 2011; Slusser et al., 2013). With Chinese children's number representation approaching the linear model, however, the one-cycle model loses its advantage and the two-cycle model simply fails to fit.

Several limitations of the current study need to be mentioned. As discussed earlier, this study could have used a larger number range such as 0–10,000 with the older preschoolers and the other number ranges with the younger preschoolers. It could have also used even younger children such as children who just entered preschool. The types of tasks could have been expanded by including more abstract quantities such as tones. In addition, although previous studies have shown associations between number estimation and mathematical performance, we did not measure other number skills in the current study. Finally, the comparison with Western children was based on a comprehensive review of previous studies. Studies with direct cross-cultural comparisons should be conducted in the future.

In conclusion, the current study found that Chinese children's accuracy and linearity of number estimates increased with age across three types of tasks. By the end of preschool years, these children already showed near-linear representations of number lines up to 1000. Consequently, they generally showed earlier onset of various landmarks of attaining linear representations than did the Western children as reported in previous studies. Results of the current study generally support Siegler's logarithmic-to-linear shift hypothesis. These results extended the small but accumulating literature on the earlier development of number cognition among Chinese preschoolers than among Western children, suggesting the importance of cultural factors in early number cognition. These results also imply that a better understanding of such cultural factors, especially those relevant to early number education (e.g., number education at preschool, family activities around numbers, a simplified ten-based counting system), will be important in closing the often reported cross-national gaps in mathematical achievement because early differences may be amplified during subsequent years.

Acknowledgments

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