## Title
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# RIDGE/CHANNEL PATH INTERDEPENDENCE IN DRAINAGE BASINS

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ABSTRACT. It is readily apparent from both map and field observation that the patterns of channels and ridges in fluvially eroded terrain are closely intertwined. The purpose of this paper is the formulation of some of the principles governing their relationship. Specifically, the paper models the sequence and length distribution of ridge paths on the basis of the sequence and length distribution of channel paths within a given drainage basin.

The modeling approach first codifies the channel and ridge networks within drainage basins in graph theoretical terms; it then takes advantage of the fact that ridge lines tend to run quasi-parallel to neighboring channels thus permitting the estimation of the former from the latter. Data were obtained from two drainage basins of the (environmentally homogeneous) Cumberland Plateau of Kentucky and the (environmentally complex) San Gabriel Mountains of southern California.

Comparison of the observed length values of ridge paths with their values as derived from channel path data demonstrate the intimate relation between channels and ridges, thereby providing support for the model as a formal description of the ridge-channel interdependence.

Key words: Channel networks; ridge networks; network geometry; graph theory.

### Introduction and outline

Whether observed on the map or in the field, fluvially eroded terrains tend to display a close and systematic interplay of channels and ridges. It is the purpose of this paper to substantiate this impression through the measurement and comparison of selected ridge and channel parameters followed by the formulation and testing of hypotheses about their interdependence.

In view of the conspicuous, wide spread, and seemingly repetitive and uniform interplay of channel and ridge patterns in maturely eroded areas the scarcity of research investigating their interrelation is puzzling. The first formal and rigorous conceptualization of the ridge/channel interplay seems to be the work by Warntz (1966, 1975) and Warntz and Woldenberg (1967) which was based on ideas published in the previous century (Caley 1859; Maxwell 1870). But although its level of rigor and abstraction is impressive there has been little subsequent research building on the concepts of the Warntz/Woldenberg formalism because their application leads to serious theoretical as well as practical difficulties (Mark 1979).

An alternative approach, based on the concept of interlocking networks, has led to the formulation of several mathematical relations between channel and ridge nodes, links, paths and subnetworks (Werner 1982, 1988, 1992). However, they refer to network topology only, and their implications for the spatial layout of ridge and channel patterns—if any—are uncertain. Finally, Abraham's (1980) study of the relation between channel link lengths and the angles formed by adjacent divide lines successfully investigates underlying geometric dependencies but does so only at the local level of individual channel links and bifurcations; the associated issue of ridge/channel relations at the level of entire networks is not adressed.

In this study both spatial and graph theoretic interdependencies will be investigated; specifically, we will define and examine the sequential order and length distribution of channel and ridge paths in drainage basins where length is defined in both geometric and topologic terms: As a topologic parameter length is measured by the number of links of the path in question, and as a geometric parameter it is measured in meters or some other unit of length in $R^2$. In concrete operational terms the input information we use consists of the number, the sequence and the lengths of all channel paths of selected natural drainage basins, with particular identification of those paths that drain the areas along the basin boundary. From this information estimates of the number, sequence and lengths of the ridge paths in the basin are derived (Figure 2). The last sections of the paper compare estimated and observed data of the length distribution of ridge paths (Figures 5–7) and close with some thoughts on the larger goals, the implications, and possible applications of this research.

The input data for our study were obtained from topographic maps; channels were identified through the contour crenulation method of Krumbein and Shreve (1970), and ridge lines were determined by the same method as adapted for this purpose by Werner (1988). To assess the accuracy of our estimation model but also its sensitivity to variation of external parameters we selected an area in eastern Kentucky representing a fairly homogeneous environment (for details see Krumbein and Shreve (1970)) and an area with a semi-arid climate and a highly complex geology in the Transverse Ranges of Southern California (Maxwell 1960). Following the Krumbein/Shreve technique individual channel networks and the boundaries of their respective drainage basins and subbasins were identified and measured using 1:24,000 U.S.G.S. topographic sheets.

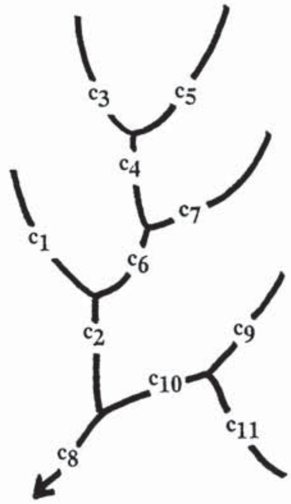


Fig. 1. Channel network of magnitude 6 showing the consecutive labeling of outer and inner links. Since each link defines exactly one node and one path the labeling applies to the network's nodes and paths as well.

## Graph theory description of channel networks

In graph theory terms a channel network can be represented as a trivalent planar rooted tree (Smart 1972), that is, the number of links incident on any network node is either one or three; the network can be drawn in the two-dimensional plane ($R^2$) without any two links intersecting; one of the network links with an end node of valency (or "degree") one is specified as the network root; and any two nodes are connected by exactly one network path. In line with established definitions we call the network root its outlet, we call the network nodes inner or outer nodes depending on whether they are of degree three or one; furthermore, we call the network links inner or outer links depending on whether their upstream end node is an inner or outer node, and we call all network paths originating or terminating in the outlet inner or outer outlet paths depending on whether their upstream end node is an inner or outer node. In the context of this paper only outlet paths will be considered, and for reasons of brevity we will henceforth refer to them simply as network paths keeping in mind that they start or end in the outlet. Further, the outlet node will be excluded from the set of all nodes on the grounds that it is not the upstream end node of a network link.

The number of outer links of a channel network is called its magnitude n. Since the outer links form an ordered set (Shreve 1966) we can label them sequentially and will do so using the notation $c_{2i-1}$ with $i = 1,..,n$, where $c_{2i-1}$ is the ith outer link to the left of the outlet link (for an illustration see Figure 1; it will become clear shortly why we choose 2i-1 as subscript rather than simply i). Each outer link $c_{2i-1}$ ends in an outer node which we designate with $P_{2i-1}$; furthermore, we label the outer network path connecting the link $c_{2i-1}$ and the outlet with $p_{2i-1}$. The outlet node—that is the point where the channel network is either connected to some larger network or else simply terminates, will be marked by the letter P. Finally, we call two outer links/nodes/paths adjacent whenever their subscripts differ by 2.

Consider now the adjacent outer channel paths $p_{2i-1}$ and $p_{2(i+1)-1} = p_{2i+1}$ with outer nodes $P_{2i-1}$, $P_{2i+1}$. In as much as the channel network is graph theoretically a tree, these paths will intersect by merging in a particular inner network node, and since the tree is trivalent they are the only outer paths to merge in this node. Furthermore, since there are n-1 inner nodes in a network of magnitude n, and since such a network has exactly n-1 pairs of adjacent outer paths $p_{2i-1}$, $p_{2i+1}$ with outer nodes $P_{2i-1}$, $P_{2i+1}$ where $i = 1,..,n-1$, it follows that there corresponds exactly one inner node to each such pair of adjacent outer nodes. This one-to-one relation permits us to label the node in which the two paths $p_{2i-1}$, $p_{2i+1}$ merge with $P_{2i}$. Given that (1) each inner node is the upstream end node of one inner link, that (2) each inner link is the upstream end link of one inner path, and that (3) these relations are one-to-one correspondences, it is reasonable to denote the inner link defined by the inner node $P_{2i}$ with $c_{2i}$ (Figure 1), and the inner path defined by $c_{2i}$ with $p_{2i}$. Evidently, the inner path $p_{2i}$ connecting $P_{2i}$ with the network outlet is simply the intersection of the outer paths $p_{2i-1}, p_{2i+1}$.

We can now summarize our deliberations in several statements and conclusions:

— In a network of magnitude n there are a total of 2n-1 nodes (excluding the outlet node) and the same number of links and (outlet) paths; each node is the upstream end node of exactly one network link, and each link is the upstream end link of exactly one such network path, with the n outer nodes corresponding to the n outer links and n outer paths, and the n-1 inner network nodes corresponding to the n-1 inner links and n-1 inner paths.

— The outer network links form a sequence $\{c_{2i-1} | i = 1,..,n\}$ and define corresponding sequences $\{P_{2i-1} \mid i = 1,..,n\}$, $\{p_{2i-1} \mid i = 1,..,n\}$ consisting of the outer nodes and paths of the network. Furthermore, any two adjacent outer paths $p_{2i-1}, p_{2i+1}$ intersect downstream in an inner node $P_{2i}$ which is the upstream node of the inner link $c_{2i}$ which in turn is the upstream end link of the inner path $p_{2i}$. All of these relations are one-to-one correspondences:

$$(P_{2i-1}, P_{2i+1}) \longleftrightarrow (c_{2i-1}, c_{2i+1}) \longleftrightarrow (p_{2i-1}, p_{2i+1}) \longleftrightarrow P_{2i} \longleftrightarrow c_{2i} \longleftrightarrow p_{2i}$$

— Thus, our particular labelling procedure places each inner link/node/path "between" exactly two adjacent outer links/nodes/paths; furthermore, links/nodes/paths form sequences $\{c_k\}/\{P_k\}/\{p_k\}$ where $k = 1,..,2n-1$, with odd and even indices referring to outer and inner links/nodes/paths respectively, and with the elements of any two sequences being one-to-one related.

— Finally, we label all outer links draining areas along the main basin boundary with $c_b$, where $b = 1,\ldots,w$ and $w \leqslant n$; clearly, the sequence of outer links $c_b$ is a subsequence of the sequence of all outer links $\{c_{2i-1} \mid i = 1,..,n\}$; equivalent statements hold for the sequences of corresponding outer nodes $P_b$ and outer paths $p_b$ as subsequences of all outer nodes and paths of the network. As to the value of w relative to n: Under environmentally homogeneous conditions the magnitude n of natural channel networks approximates a linear function of the corresponding drainage basin areas which, in turn, grow approximately as a power function (but not a square root function!) of the lengths of the basin boundaries. Hence, the value of w tends to be a power function of the network magnitude n (Werner 1982).

## Ridges

Identifying ridges on topographic maps or in the field is sometimes much more ambiguous than the identification of channels (for a detailed discussion see Werner (1988)). For this investigation we will choose a relatively safe method. Specifically we stipulate that

(1) every ridge begins with the crest formed by the drainage divide separating the catchment areas of the two upstream links created by a bifurcation in the channel network; on the map the ridge manifests itself as a sequence of cusps in the contour lines.
(2) every channel bifurcation, through the incision of its two upstream links, leads the to formation of such a ridge (Figure 2a).

We then follow the ridge until we reach the drainage basin boundary; it is this ridge line that, in the context of this paper, is referred to as an outer ridge path. To be sure, if one were to choose some other definition of the ridge concept there may be other ridges in the basin that are not part of the ridge paths identified above; but minimally, the ridge paths as defined for this investigation cover the entire basin area and constitute a comprehensive and representative subset of all ridge paths however they may be defined (Figure 2a).

To avoid repetition we will simply state that the definitions for inner paths as well as inner and outer links and nodes of ridges are equivalent to those previously formulated for channels. In graph theory terms a channel bifurcation is synonymous with an inner node of the channel network. Thus, there exists a one-to-one relation between inner channel nodes and outer ridge nodes which enables us to simply transfer the index structure devised for channel nodes/links/paths to the corresponding ridge nodes/links/paths. We note, however, that the ridges in a drainage basin usually form more than one network, the root of each net-

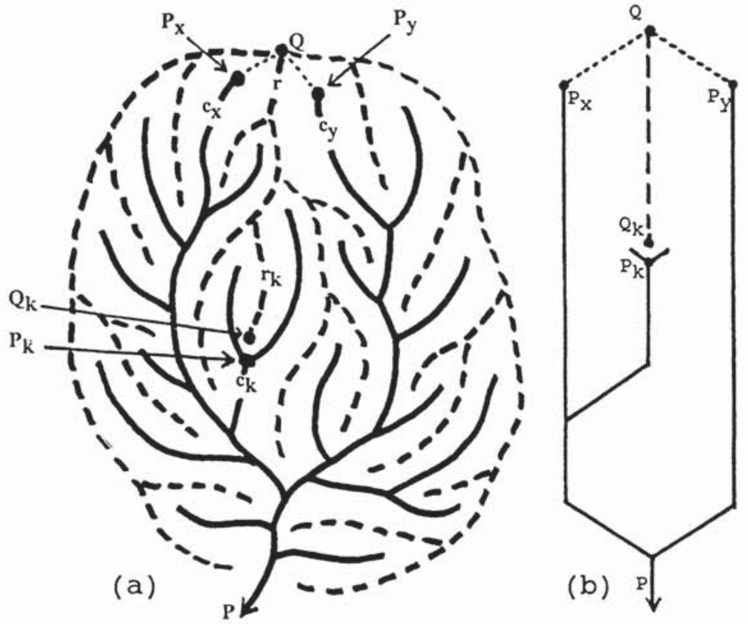


Fig. 2. **a**. Drainage basin and the channel and ridge networks contained in it. An outer ridge path $(QQ_k)$ and the corresponding inner channel path $(P_k,P)$ are "sandwiched" between a pair of outer channel paths $(P_x,P),(P_y,P)$. **b**. Schematic representation of same paths.

work being the ridge link in which one or more ridge paths are connected to the basin boundary; indeed, it is easy to see that the number of ridge networks in a drainage basin is w-1.

## Modeling the ridge/channel interdependency

Phrasing the preceding statements on channel bifurcations and the ridges they create in symbols, there corresponds to every inner node $P_k$ of the channel network an outer ridge node $Q_k$ which is the end node of an outer ridge link labelled $r_k$ which in turn is the end link of an outer ridge path labelled $q_k$ which merges with the main basin boundary in a ridge node labelled Q (Figure 2a; but note that, in so far as Q is the root node of one of the w-1 ridge networks in the drainage basin, it should be further specified by an appropriate subscript; however, again for reasons of simplicity, we have not done so). We recall that there exists to each inner channel node $P_k$ exactly one inner channel path $p_k$ connecting $P_k$ with the outlet node P; furthermore, our definition of outer ridge paths insures that the inner channel node $P_k$ and the outer ridge node $Q_k$ are not only one-to-one related but are also positioned in close geometric proximity to each other, with $Q_k$ occupying a location directly up slope from $P_k$. Thus, following the channel path $p_k$ from the outlet node P to $P_k$, from $P_k$ along the drainage divide line to $Q_k$, and continuing from $Q_k$ along the ridge path $q_k$ to Q we have traced a line that (1) connects the outlet with the basin boundary; that (2) nearly coincides with the two paths $p_k$, $q_k$ the only difference being the connecting divide segment $(P_kQ_k)$; and that, therefore, (3) approximates the combined lengths of $p_k$ and $q_k$ whether measured in topologic or geometric terms; this line we denote by $(pq)_k$.

Let r be the link connecting the outer ridge path $q_k$ with the main basin boundary in the ridge node Q. Following the notation introduced earlier the area along the basin boundary is drained by a sequence of outer channel links $c_b \mid b = 1,..,w$; of these let $c_x$ be the last link to the left of r and $c_y$ be the first to its right (that is to say, the boundaries of the two drainage areas of the outer channel links $c_x$, $c_y$ both contain the ridge node Q). In line with the definitions and symbols of the last section we label the upstream end nodes of the links $c_x$, $c_y$ with $P_x$ and $P_y$; likewise we denote with $p_x$, $p_y$ the two outer channel paths connecting $c_x$ and $c_y$ with the outlet node P. We denote with $(pq)_x$ and $(pq)_y$ the two lines which start in the network outlet and follow the paths $p_x$, $p_y$ up to their end nodes $P_x$, $P_y$ and from there on straight to the boundary point Q (Figure 2a,b). These lines nearly coincide with the paths $p_x$, $p_y$ and closely approximate their respective lengths. Examining the three lines $(pq)_k$,

$(pq)_x$, $(pq)_y$ we have constructed so far we note that they all start in the outlet and terminate in the same ridge node Q. In as much as

(1) by construction the line $(pq)_k$ is "sandwiched" between the lines $(pq)_x$ and $(pq)_y$;
(2) the lengths of the lines $(pq)_x$, $(pq)_y$ are approximately equal to those of the channel paths $p_x$, $p_y$; and
(3) the channel part of the line $(pq)_k$ is the inner channel path $p_k$,

it is possible to construct an estimate of the length of the outer ridge path $q_k$ as a function F of the lengths of the lines $(pq)_x$, $(pq)_y$ and the channel part of the line $(pq)_k$ (Figures 2a, b). It should be emphasized, however, that there is no logical necessity that would impose such a function, in whatever form, on the length value of $q_k$ unless one introduces specific assumptions about the spatial organization of drainage basins. Clearly, the quality of the estimates will be the better the smaller the differences between the respective circuities of the participating network paths.

Let L be the symbol for path length without specification as to whether it refers to the geometric or topologic measurement. As a first step we estimate the length of the line $(pq)_k$ through linear interpolation of the lengths of the lines $(pq)_x$ and $(pq)_y$; that, however, requires the definition of a measure of distance between channel paths which we resolve by assigning a dimensionless distance of one unit between any two paths occupying consecutive positions within the sequence $\{p_k \mid k = 1,..,2n\text{-}1\}$. Thus:

$$(|L(pq)_y - L(pq)_x|)(|y-x|)^{-1} \approx (|L(pq)_k - L(pq)_x|)(|k-x|)^{-1} \quad (1)$$

Substituting the approximation

$$L(pq)_k \approx L(p_k) + L(q_k) \quad (2)$$

into equation (1) gives us a first estimate of the length of the outer ridge path $q_k$:

$$L(q_k) \approx |(k-x)|(|L(pq)_y - L(pq)_x|)(|y-x|)^{-1} + L(pq)_x - L(p_k) \quad (3)$$

Substituting the lengths of the outer channel paths $p_x$ and $p_y$ as approximations of the lengths of the lines $(pq)_x$ and $(pq)_y$ produces a second estimate of the length of the ridge path $q_k$:

Fig. 3. Drainage basin of Laurel Creek, Kentucky. First order channels have been ommitted, and ridges have been restricted to those originating in the bifurcations of the reduced channel network.

$$L(q_k) \approx (|k-x|)(|L(p_y) - L(p_x)|)(|y-x|)^{-1} + L(p_x) - L(p_k) \quad (4)$$

Note that the variables on the right side of this equation refer to channel paths and their sequential order only while the dependent variable on the left represents the length values of individual ridge paths in the drainage basin under consideration. However, sequential order and length distribution of the channel paths constitute our input information which means that equation 4 accomplishes our objective of expressing the lengths of ridge paths in terms of the lengths of channel paths. Thus, we are now ready to test our model against data from natural drainage basins.

**Test results**

Figure 3 shows the channel network of Laurel Creek, a magnitude 164 tributary of Rockcastle Creek along the southern border of Lawrence County, Kentucky. To preserve readibility under drastic reduction in scale all first order tributaries of the network have been ommitted, and the only ridges included are those that result from the bifur-

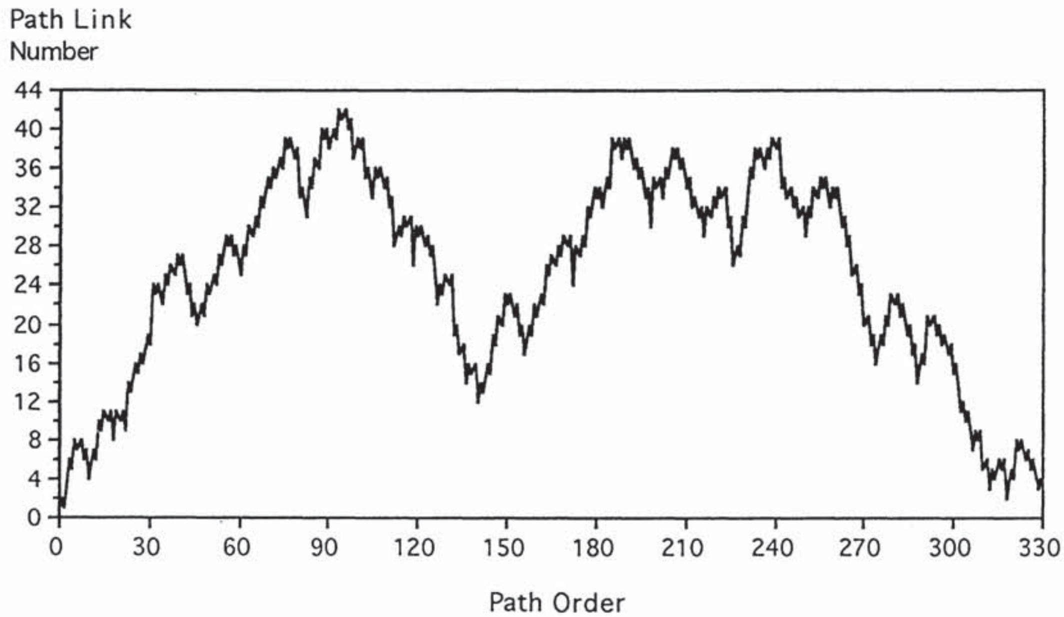


Fig. 4. Link number distribution of all channel paths terminating in the outlet of the Laurel Creek channel network. Paths are ordered along the horizontal axis according to the ordering principle shown in Fig. 1; the scale of the vertical axis refers to path link numbers. The information contained in Fig. 4 represents input data used in the estimation of the link numbers of the ridge paths of Fig. 5.

cations of this generalized channel network. Figure 4 shows the topologic input information for our model, that is, the length distribution of the channel paths of the network measured in numbers of links and ordered acccording to the position of their upstream end links relative to the network outlet. To clearly understand the informational content of Figure 4 it might be helpful to describe the topologic path length distribution of the network shown in Figure 1: the values of $p_i$, where $i = 1,2,3,..,10,11$, are, respectively, 3,2,5,..,2,3.

Since the lengths of network paths measured in meters and in link numbers are highly correlated, the geometric paths length distribution of Laurel Creek is almost identical to the distribution shown in Figure 4. To better facilitate the actual measurement of the raw data the relevant part of the U.S.G.S. topographic sheet (Milo Quadrangle) was enlarged by close to 20 percent. The unit of measurement used on the enlarged map was 6.25 mm for both channel and ridge paths, which corresponds to 127.4 meter in the field. The partial unit at the end of each path length measurement was estimated and counted as a fractional unit.

As a side observation it might be worthwhile to point out that together the two figures provide a nice example for the considerable independence of shape and organization of drainage basins: While the shape of the basin of the Laurel Creek is fairly compact and symmetric, the path length distribution of the network draining the basin is multi-modal and asymmetric.

Figure 5 shows the observed topologic length distribution of the outer ridge paths (solid line) in the Laurel Creek basin; superimposed is their distribution as estimated by equation 4 on the basis of the basin's topologic channel path data (dashed line) as displayed in Figure 4. Even though the correlation between the two distributions is faily high ($r = .9520$), the quality of fit is somewhat crude in detail as indicated by an average 22.6 percent deviation of the estimated from the observed data. The quality of fit is substantially better for the longer network paths, with the most serious discrepancies occurring among the smallest paths. The most successful performance of the estimation is probably its very close reproduction of the seemingly wild and unsystematic gyrations of the actual ridge path lengths.

The estimation procedure was repeated for

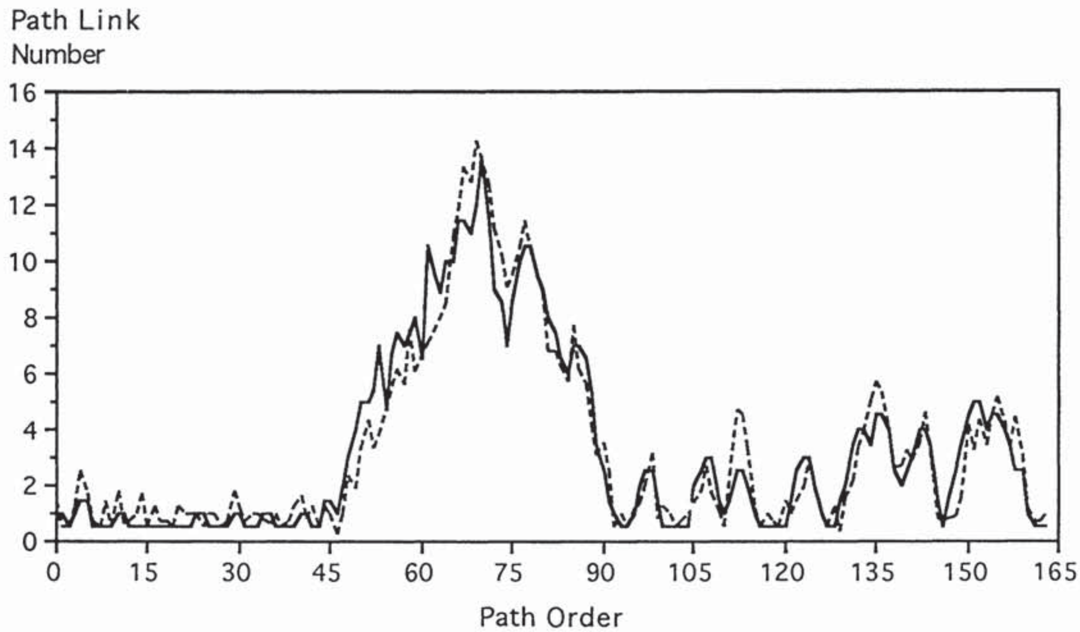


Fig. 5. Observed link number distribution (solid line) and estimated distribution (dashed line) of outer ridge paths in Laurel Creek drainage basin. The horizontal axis indicates the position of each path within the sequence of all outer ridge paths in the basin; the vertical axis measures the link numbers of the paths in units of two links.

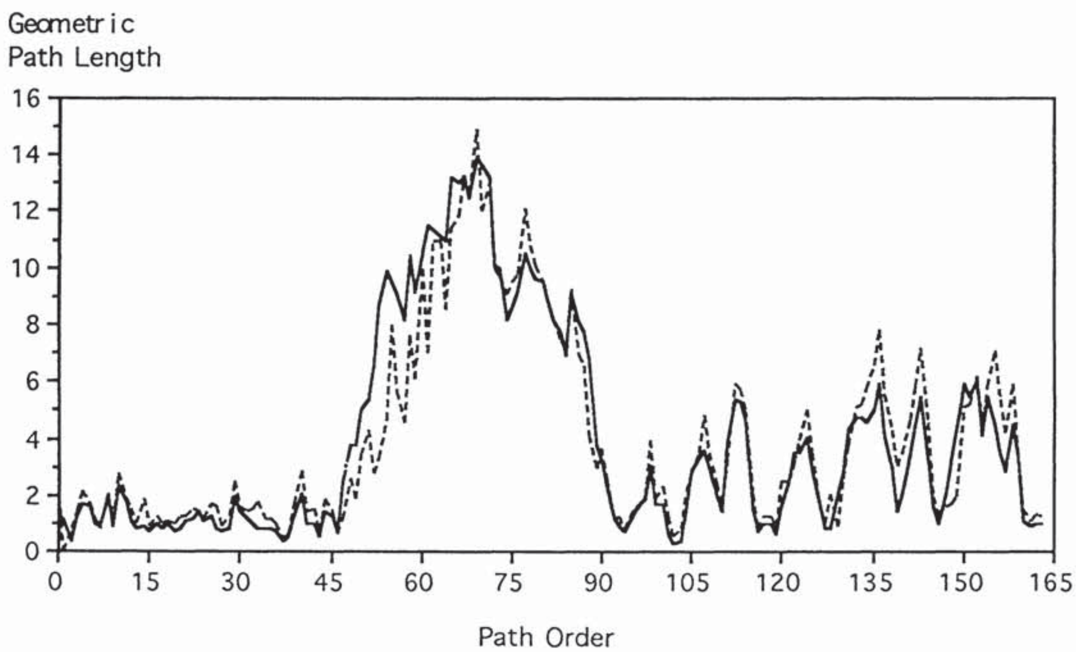


Fig. 6. Repetition of Fig. 5, except that path length is now measured in meters rather than number of links. The unit of measurement along the vertical axis is 254.8 meters.

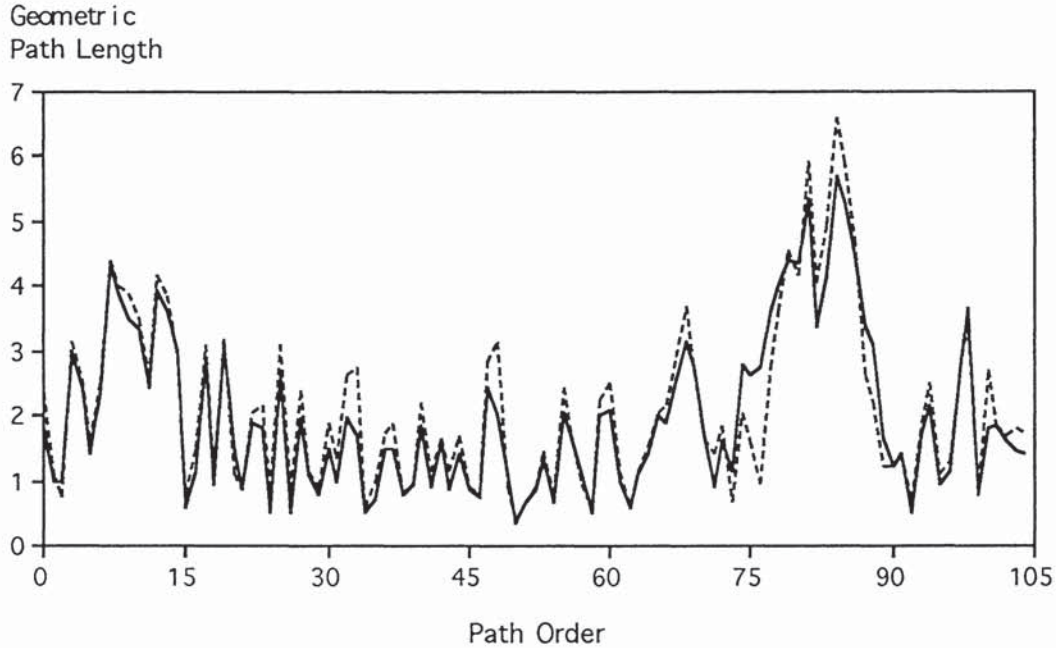


Fig. 7. Observed length distribution (solid line) and estimated length distribution (dashed line) of outer ridge paths in the drainage basin of Little Dalton Canyon, San Dimas Experimental Forest, California. As in the preceding figures the horizontal axis records the position of each path within the sequence of all outer ridge paths; the unit of measurement along the vertical axis is 197.8 meters.

geometric path length data and generated essentially the same results (Figure 6). In view of the high correlation between geometric path length and (topologic) path link number ($r = .9926$ for channel paths and .9837 for ridge paths) about the same results had to be expected. The correlation coefficient for the observed and estimated lengths of the outer ridge paths turned out to be $r = .9339$, with a mean 22.1 percent deviation of the estimations from the observations.

The second natural drainage basin that was chosen to test the ridge/channel path interdependence hypothesis as formulated in equation 4 is the catchment area of the upper reaches of the Little Dalton Canyon, San Dimas Experimental Forest, Los Angeles County, California. The area is drained by a magnitude 105 channel network; as was the case in the first example, close resemblance of the geometric and topologic path length distributions assured duplication of the geometric path length estimates by the link number estimates, and the latter are therefore ommitted. The observed and estimated distributions of the geometric lengths of the basin's outer ridge paths are shown in Figure 7; their correlation is $r = .9492$, and the mean deviation of the estimates is 15.1 percent. The unit of measurement used on the map corresponds to 98.9 meters in the field.

## Concluding comments

(1) The same process can readily be extended to estimate the lengths of the inner ridge paths on the basis of the lengths of the outer channel paths which originate between the links of ridge bifurcations; however, in as much as it would only mean an increase in the input and output data while repeating what is essentially the same methodology this extension was not made part of the study.

(2) The body of data consisting of both the lengths and the sequential order of the basin's channel paths contains implicitly the complete information of the channel network's topological configuration; if the lengths of both inner and outer ridge paths is estimated with sufficient accuracy, it is a simple matter to construct the correct and complete topology of all ridge networks in the basin.

(3) With some minor modifications and without

changing the principle of the model the entire process can be reversed and will then produce estimates of the channel path lengths on the basis of the length distribution of the ridge paths in the basin. Once again, the output information will permit the construction of the channel network topology provided that the estimates are sufficiently accurate.

(4) The representation of the lengths of ridge paths as a function of channel path lengths provides a measuring scale against which some of the output data will show up as conspicuous deviations from the observed values. These modeling residuals can guide the researcher and give rise to the investigation of local or regional particularities that otherwise might go unnoticed.

(5) This study should be seen as an incremental effort in the context of pursuing the much larger goal of a comprehensive model that describes in formal terms the patterns of channels, slopes, and ridges which together make up the fluvially eroded landscape.

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