Selecting between visuomotor lotteries to measure mental effort in risky decisions

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Abstract

It is intuitive to believe that humans take considerations of mental effort into account when making decisions. However, it has proved difficult to differentiate theories of mental effort in the absence of direct measurements of this psychological construct. Existing measurements of mental effort using response times and revealed preferences have low reliability. In this paper, we present a new experimental task - selecting between visuomotor lotteries using eye-tracking for sampling lotteries - that enables direct measurement of mental effort. Unlike response time-based measures, effort measurements in this task are not confounded by actual effort allocation. Unlike revealed preference-based measures, effort measurements in this task are acquired on a natural scale unitized by automatic visual selection processes. We also report results from a simple experiment conducted using this task, which reproduce existing findings of costly effort-aversion, and also demonstrate adaptive adjustment of mental effort.

Keywords: decision making; decisions from experience; information accumulation; mental effort

Introduction

Recent experiments have clearly demonstrated that humans find the exertion of mental effort aversive (Kool, McGuire, Rosen, & Botvinick, 2010), reducing effort application in cognitively demanding tasks (Gailliot et al., 2007). This aversion appears to be adaptive, since humans can also increase effort for larger rewards (Camerer & Hogarth, 1999). Such observations are consistent with adaptive allocation of mental effort, suggesting that humans allocate mental resources sensitive to the value of consequent outcomes.

The idea that people rationally factor anticipated cognitive effort into decisions about how to behave is extremely intuitive, and appealing on multiple theoretical grounds. Optimizing metabolic costs, governed to some degree by mental effort, is a clear target for natural selection (Christie & Schrater, 2015). Given resource limitations, such as working memory size, it is rational for humans to take them into account to make resource-rational decisions (Lieder & Griffiths, 2020). Well-known constraints on our capacity to undertake tasks in parallel imply that it is rational to attempt to optimize the costs of undertaking any single activity (Musslick & Cohen, 2021). It is also possible to model mental effort as the opportunity cost of processing information relevant for deciding what to do in a given situation (Shenhav et al., 2017).

However, it has proved difficult to test these theories directly, since it is difficult to measure cognitive effort directly, a predicament that Kool and Botvinick (2018) refer to as an 'econometric problem in mental effort research'. In particular, in the absence of direct measurements of mental resources being expended, it is not yet clear whether resource-rationality assumptions are to be treated simply as modelling devices, or as valid correlates of real psychological mechanisms (Rahnev, 2020).

In earlier work, researchers have tried to measure mental effort using either response times (Lieder et al., 2014), or revealed preferences (Shenhav et al., 2017). Neurological measures of mental effort, such as neural activity in dACC (Cavanagh & Frank, 2014), rely upon direct behavioral measures such as the ones described above for their validity, and so are not considered separately here.

Lieder et al. (2014) use response times as direct measurements of mental effort in developing a model of decision-making which treats such effort as one half of a trade-off (reward being the other) that resource-rational decision-makers use as a domain-general optimality principle (Lieder & Griffiths, 2020). However, response time measures are unreliable as measures of mental effort, since high RTs may indicate both high mental effort (someone is working hard on the task) or low mental effort (leisurely working on the task).

Kool et al. (2010) use revealed preferences to determine the effect of mental effort in a demand-selection task, wherein participants select between stimuli coding for hard and easy sequences containing multiple trials of different cognitive tasks (with task difficulty controlled by the frequency of task-switching) and showed that task-switching was aversive for participants, suggesting that they are sensitive to mental effort. In another study, Westbrook, Kester, and Braver (2013) use a continuum of difficulty in working memory tasks to operationalize mental effort, eliciting 'cost' measurements using monetary amounts needed to shift participants' preferences from easy to hard tasks. Revealed preferences are more reliable, but, being ordinal estimates, are coarse in granularity of measurement. While they can tell us whether participants found mental effort aversive, they are unable to tell us clearly *how* aversive they found it (Kool & Botvinick, 2018).

Thus, theorizing about the properties of mental effort is currently constrained by the inability to measure mental effort. In this paper, we present a new experimental task for measuring mental effort that avoids some of the problems that response time and preference-based elicitations face. Specifically, we design a task where observers may select to sample evidence from and then finally choosing either of two expected value-matched lotteries, with the presentation of the and procedure for final selection using a visuomotor estimation paradigm developed by Juni, Gureckis, and Maloney (2011). Unlike conventional visuo-motor lotteries, wherein evidence is sampled using key-presses symbolically associ-

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ated with outcomes, in our design, participants sample evidence from either graphically visualized by simply looking at it, with visual fixations mapped to sample generation using eye-tracking. Below, we describe this method of measuring cognitive effort, its advantages in eliminating some key confounds that bedevil existing mental effort estimation procedures, and some preliminary results obtained using it.

Selecting between visuomotor lotteries

As we describe above, the basic building block of our experimental task is the visuomotor lottery, proposed in Juni et al. (2011). In visuomotor lotteries, participants are required to estimate the location of a hidden target with the help of sequentially sampled location hints which appeared in the proximity of the target with some precision. Essentially, through these hints, participants sample the mean location of the hidden target from a bi-variate Gaussian distribution centered on the true location of the hidden circle and constructed a sampling distribution. Participants can sample as many times as they want before making a guess by pointing to a specific location within the target area. On a successful guess, they receive a reward. Every sampled hint reduces the reward by a fixed amount, thus making sampling explicitly costly.

Paralleling earlier task selection paradigms used for measuring mental effort (Kool et al., 2010; Westbrook et al., 2013), we design our task to require participants to choose between visuomotor lotteries, as illustrated in Figure 1. Participants saw two visuomotor lotteries in parallel on the same screen, and were free to sample from either of them before making a consequential guess for the hidden target in either one.

We make one lottery more difficult than the other by increasing the covariance of the distribution from which hints are sampled, thus necessitating more samples for accurate estimation. With lower accuracy, the probability of receiving the payoff decreases, but this can be offset by decreasing the cost of each sample. Thus, it is possible to design a choice between lotteries that is matched in terms of expected value, but with different mental effort requirements for either lottery. To ensure there is no additional cost due to of memory maintenance, all sampled hints stay on screen throughout each trial.

While the number of hints sampled for a lottery is a possible proxy for how much information a participant is looking for during a trial, operationalizing this variable using key presses, as in Juni et al. (2011) suffers from similar confounds as response times. Participants could generate fewer hints by virtue of paying great attention to the cost structure of the experiment, or draw many hints desultorily.

To reduce this confound in our task, hints are sampled in our task using eye movements indicating attention to either in the display. When a participant wants to generate a hint, they visually focus inside the box marking the borders of the corresponding visuomotor lottery, and retain their focus inside the box for a second. This design element ensures that sample generation remains coupled with attending to the accumulated information that is being presented on screen. By virtue of this coupling, people cannot sample hints without attending to the estimation task, and cannot perform the estimation task without sampling hints, thereby tightly coupling mental effort with task performance.

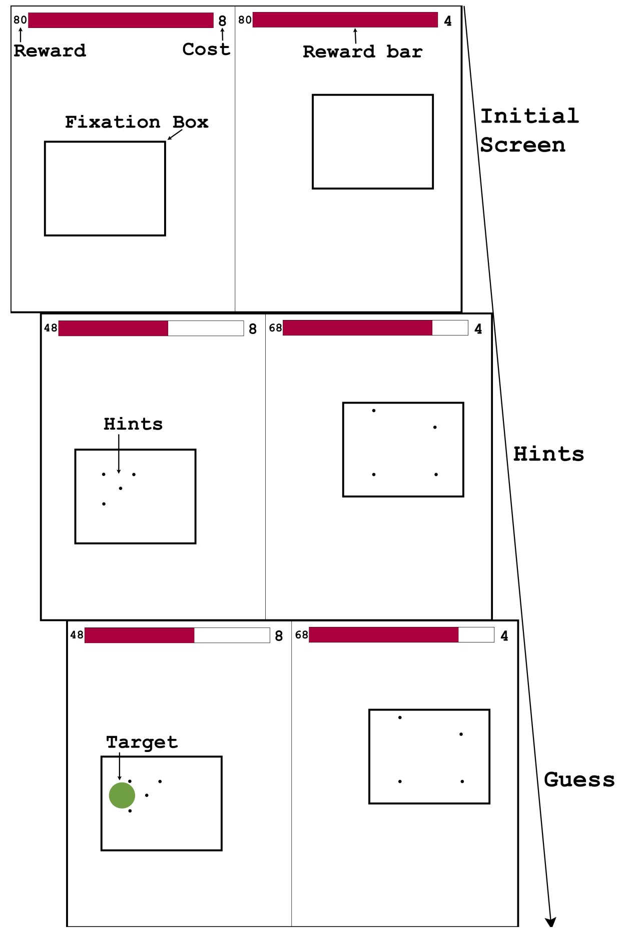

Figure 1: Illustration of the visuomotor selection task. In each trial, participants could sample hints from either using eye movements as many times as they wanted before making a final guess that completed the trial.

Designing Expectation-Matched Lotteries

As a starting point for experimenting with this task, we consider the special case where participants select between lotteries that are matched on expected value. For such a choice, a preference for selecting the easier would indicate a preference for lower cognitive effort, following similar demonstrations in other experimental paradigms (Kool et al., 2010).

We use the same analytic approach as (Juni et al., 2011) to calculate the expected reward or the expected gain as a function of number of samples (**EG(n)**) as follows:

$$EG(n) = P[hit/n](R - nC),$$

where R is the initial reward, C is the constant redacted

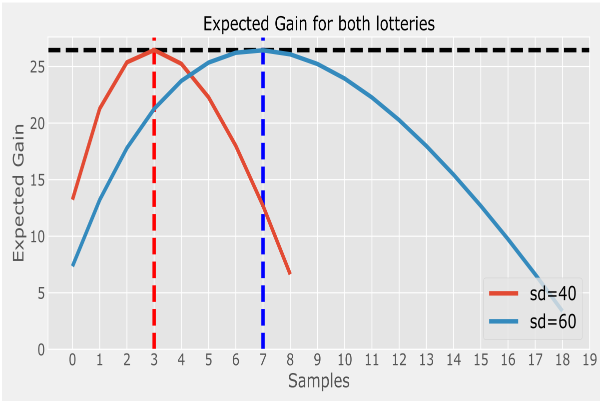

Figure 2: Expected value of both visuomotor lotteries used in our experiment over a range of sampling possibilities. The optimal number of samples for both lotteries are marked with dashed lines.

from reward with each sample and P[hit/n] or probability of target being in the middle of n samples is:

$$P[hit/n] = \int \int_T \phi(0, \Sigma) dxdy,$$

where T is the area of the hidden circle and ϕ is the probability density function of the multivariate Gaussian from which the samples are drawn with Σ as co-variance matrix

$$\Sigma(n) = \begin{bmatrix} \sigma^2(n) & 0 \\ 0 & \sigma^2(n) \end{bmatrix}$$

The EG(n) function plotted in Figure 2, with maximum expected reward visualized at peaks, clearly indicates the optimal number of samples for a specific pair of visuomotor lotteries we use in the experiment reported further below.

A Basic Experiment

Participants Ten university students participated in the experiment for monetary compensation. The rate of compensation included a fixed base rate plus a variable performance-dependent component. All participants reported perfect eyesight. Data was collected with approval from the university's IRB.

Apparatus The experiment was displayed on a 1920x1080 pixel screen split vertically to display two lotteries in parallel in a dark room. A standard PC mouse was used to click and guess the position of the target. An Eyelink 1000 eye tracker was used to record gaze data at 1000Hz. A head mount was used to fix the position of the head. The PsychoPy python library was used to create the stimuli and Pylink was used to integrate the eye tracker.

Stimuli A circle of radius 24 pixels was used as the target. The screen was split vertically into two equal sides. Each side displayed a bar on top, with a number on the left side of the bar indicating how much potential reward can the participant earn with respect to the corresponding lottery, which was 80 to begin with for both the lotteries. Hints were dots of 4 pixels radius. The standard deviation used for one of the multivariate Gaussian was 40 pixels(easy lottery), and it was 60 for the other one(hard lottery). The standard deviations were not revealed to the participants, but the lottery with the higher SD had 4 written to the right of reward bar, which indicated reward reduction value with each hint. Similarly 8 was the constant reduction rate for the other reward. No reward was received on wrong guess. Participants were allowed to draw hints until the reward bar went to zero, in which case the trial would terminate with zero reward. The fixation box inside which the participants needed to foveate for a minimum of 1000ms was a rectangle with 1/4 the height and width of the screen.

Design Each participant went through a block of validation trials for learning to fixate on the displays to generate samples. In these trials, they had to make 90% successful fixations out of 30 total attempts in order to proceed to the main experiment, as detailed below. All participants successfully completed the validation block. They then did 10 practice trials of selecting between visuomotor lotteries before doing 50 main ones. The left-right position of the lotteries was randomized on each trial.

Procedure The participants were shown the instructions and a play through of how the experiment would look like, and calibrated the eye-tracker. They then had to go through validation trials in which they had to get used to the fixation process. For 30 times, a box appeared on one side of the split screen and the participants had to bring their gaze inside that side and fixate inside the box for 1 second in a limit of 5 seconds in total. If they were successful, the box turned green, otherwise it turned red. The box appeared in orange color to inform the participants that they have to 'bring' their gaze inside that side of the screen or in other words they needed to look outside the area of the corresponding lottery to turn the box black again and enabling another hint to be drawn. The 'bring the gaze inside' step in our procedure made sure that the fixations they make while estimating the location of the target don't result in drawing an indiscriminately large number of hints accidentally.

After validation, practice and main trials began, with participants free to sample hints for either of the targets at any time. If they drew a hint for a given target, the corresponding box turned orange, indicating the need to bring their gaze inside for drawing another hint. Participants were told to use the left mouse button to guess the position of the circle whenever they wanted to. On a correct guess, the circle appeared in green color and red otherwise. The points won for the particular trial were indicated afterwards, and the next trial began.

Before the beginning of the validation block and before every main trial, participants were given the option to recalibrate the eye-tracker. This was done so that they had an option to take a rest by dismounting their head between any two trials.

Results

Baseline Task Performance

As our first analysis, we check whether participants' performance matched baseline expectations of having been sensitive to information-theoretic task characteristics. In particular, we expected participants to have a lower success rate for the harder lottery, to have drawn more samples for the harder lottery, and to have seen greater success in hitting the target after having drawn more samples.

Two of these three baseline expectations hold true in our participants. The success rate, measured by the number of rewarded lotteries over selected lotteries, for easy lotteries was significantly greater than for hard lotteries (two sample t-test $t(9) = 4.29, p < 0.001$). Also, as shown in Figure 3, the success rate for both lotteries increases significantly with increasing number of samples with the slope of the best linear fit significantly positive, ($p < 0.001$) for both conditions across all participants. These results suggest that participants' responses were sensitive to the information processing demands of the task. However, the number of samples drawn for harder lotteries was not significantly different from the number of samples drawn for easier lotteries (two sample t-test $t(9) = 1.44, p = 0.16$), suggesting that differences in mental effort applied were small. We examine this discrepancy further below.

Preference for easier

Figure 4 shows that most participants had a preference for the easy lottery, represented by the green part of the bar. Across all participants, this preference was significant with one sample proportion $z(49) = 2.06, p = 0.039$. This revealed preference, prima facie, can be interpreted as aversion to effort, along the lines of Kool et al. (2010) and Westbrook et al. (2013).

Effort aversion is also evident in participants' behavior across time in our experiment. Across all participants, the correlation between trial number and total samples in that trial was significantly negative, $r(49) = -0.46, p < 0.001$. This negative correlation means that participants drew fewer samples on later trials. Concomitantly, success rate also significantly decreased with number of trials $r(49) = -0.74, p < 0.001$.

We also see a significant correlation between the number of samples drawn and fraction of easy lotteries chosen measured across blocks of 5 trials over the course of each participant's data $r(99) = -0.65, p = 0.03$. In other words, drawing fewer samples weakly predisposes participants to select the easier lottery more frequently. We next examine the pattern of sampling effort seen in the task to better understand participants' management of effort.

Mental effort in sampling lotteries

In Figure 5, red bars represent the average number of samples taken for the harder lottery and green bars show average samples taken for the easy lottery by all participants. The green and red vertical dotted lines show the optimal number of samples for both lotteries respectively. We find significant oversampling for the easy lottery ($z(9) = 2.97, p = 0.002$) as seen in Juni et al. (2011), but significant under-sampling for the harder lottery ($z(9) = -2.39, p = 0.01$) in comparison with their respective theoretical norms. Thus, we clearly see evidence of sub-optimal effort allocation for the harder lottery, supporting an effort-related interpretation of the revealed preference result reported above.

We also note that the difference in expected gain between the hard and the easy lotteries was significantly negative at the time the consequential choice was made, across all under-sampled harder lotteries (two sample t-test $t(398) = -7.4, p < 0.001$). Therefore, undersampling clearly was suboptimal in terms of rewards, indicating that people were willing to give up rewards to avoid effort.

We also tested if participants were responsive to success and failure, by adjusting their sampling effort. For this we measure the average change in the number of choice-samples following a success and failure. Our findings are illustrated in Figure 6. For the easy lottery, participants sampled 0.67 fewer samples for the same option when they experienced success on the previous trial($z(109) = -2.29, p = 0.02$) and 0.31 more same choice samples after failing on the previous trial ($z(126) = 2.02, p < 0.04$). This suggests that, at least at the cohort level, participants responded resource-rationally to the task design for the easier lottery, calibrating effort towards the minimum value needed to produce success. However, such adaptive calibration did not happen in the harder lottery (see Figure 6), where the change in same choice sampling was not significantly different from zero for either success or failure. Given the intrinsic difficulty of the harder lottery, and the systematic under-sampling seen for it, it is unsurprising that participants were not able to calibrate effort for it.

Attention and Preference

Eye-tracking studies of multi-option choice frequently use a drift-diffusion framework for modelling the process of attending to different options and accumulating information about the value of options by sampling(Krajbich & Rangel, 2011). In this formalism, while it is easy to explain why people select options they attend to more, the choice of which item people will attend to as a function of prior preferences is not yet characterised (Tavares, Perona, & Rangel, 2017). It is possible that reward-based attention capture causes observers to keep bringing their attention back to the option that they feel is more valuable.

Given the high visibility of the interaction between attention and preference afforded by our task, test the reward-based attention capture hypothesis estimating participants' probability of fixating on recently rewarded options.

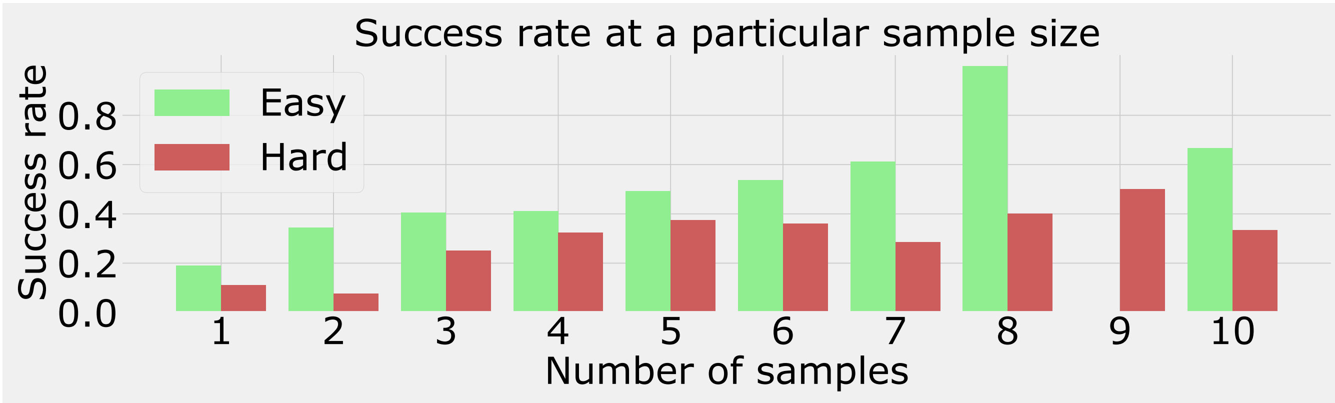

Figure 3: Increasing success with number of samples

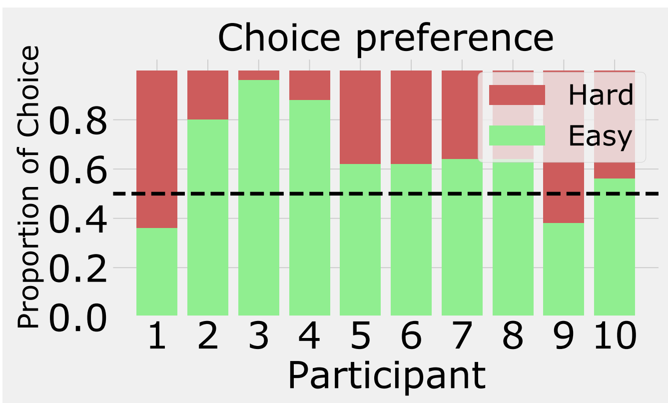

Figure 4: Choice Proportion

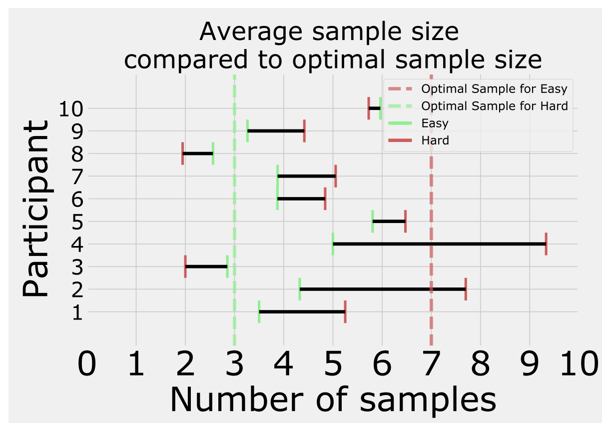

Figure 5: Comparison to optimal sampling

To this end, we fit a variation of the strategy selection model first proposed by (Otto, Markman, Gureckis, & Love, 2010), which tries to model strategy use in repeated choice settings like ours.

The model predicts the probability of sampling i as,

$$P(s_i,t|c_i,t-1) = P_{repeat} + (1 - P_{repeat}) \times Softmax(i)$$

,

This equation describes the probability of sampling item 'i' more (s_i), when the same item 'i' was chosen and rewarded at time t-1(c_i) as a combination of P_{repeat} or probability of repeating the choice, and a Softmax function which is:

$$Softmax(i,t) = \frac{e^{\gamma Q_{i,t}}}{e^{\gamma Q_{i,t}} + e^{\gamma Q_{j,t}}},$$

where γ is the exploitation parameter, with higher values leading to the better quality option being chosen more frequently. The quality of the option itself is defined as,

$$Q_{i,t} = Q_{i,t-1} + \alpha[r_t - Q_{i,t-1}],$$

where α is the learning rate and r_t is the reward received. Quality basically summarizes which option has been historically more rewarding for a participant up to the current trial.

If the last rewarded item is not sampled more, the model incorporates this information as,

$$P(a_j,t|a_i,t-1) = 1 - P(a_i,t|a_i,t-1).$$

In case the previous choice is not rewarded the sampling prediction is simply,

$$P(s_i,t|c_i,t-1) = Softmax(i)$$

,

To fit the model to data, we estimated parameters that maximized the likelihood for a participant,

$$L = \prod_{trials} P$$

Figure 7 shows that the probability of fixating more on the rewarding option which is significantly lower than 0.5 (one sample proportion $z(49) = -6.99, p < 0.001.$) at the cohort level, but demonstrates interesting individual differences.

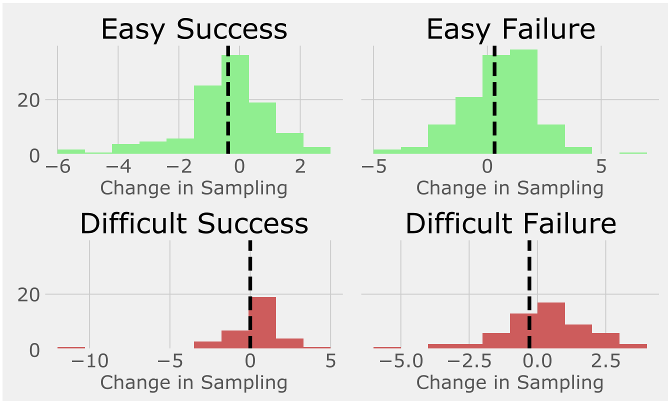

Figure 6: Histogram of change in sampling behavior as a function of success or failure on the previous trial for both easy and hard lotteries

Most participants appear to switch away from the rewarding option when deciding which one to focus on in the next trial, with the exception of two, whose attention appears strongly captured by the previous trial's reward. In the absence of a larger set of participants, the current findings suggest that, whereas reward-based attention capture appears to not dominate the decision of what to attend to next in our task, it could explain some participants' behavior quite well.

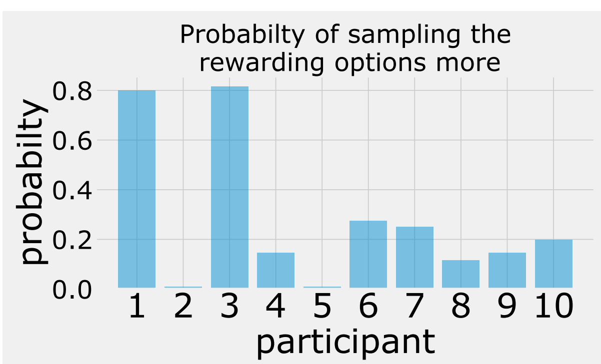

Figure 7: Model-based estimate of P_{repeat}, the probability of attending more to a that was rewarding on the previous trial

Discussion

Existing methods for measuring mental effort have serious limitations (Kool & Botvinick, 2018). While response time measures confound the cost of effort with the actual effort allocated (Shenhav et al., 2017), revealed preference-based measures confound the cost of effort with peoples' cognitive abilities and motivation level (Kool & Botvinick, 2018).

In this paper, we have presented a visuomotor selection task with attention-based evidence sampling, which offers mental effort measurements while significantly reducing confounds affecting existing measurements of mental effort. While response times could be greater for participants performing a task desultorily, individuals must pay attention for a specific period of time in order to obtain one sample in our task, thus coupling overt visual attention with each observed unit of effort. Unlike cognitive abilities loaded in other mental effort paradigms, such as working memory capacity (Westbrook et al., 2013), or task-switching ability (Kool et al., 2010), the ability to focus visually for about 1000ms is not as sensitive to individual differences (Shiffrin & Schneider, 1977), thus permitting units of mental effort in our paradigm to be comparable within and across individuals.

Consistent with earlier findings (Kool et al., 2010; Westbrook et al., 2013; Westbrook & Braver, 2015), our experimental findings showed mental effort to be aversive, with people willing to give up larger rewards in order to sample harder lotteries less, and preferring easier lotteries over harder ones as a consequence. A novel finding revealed in our experiment is that, at least for for easy tasks, people do adaptively adjust the magnitude of effort they must put forth between trials, consistent with theoretical resource-rationality expectations (Lieder & Griffiths, 2020). This observation, while requiring corroboration by a larger sample study, suggests that resource-rationality assumptions may have greater ontic significance than as pure modelling devices (Rahnev, 2020).

Undersampling of risky choices has previously been reported in numerical risky decisions-from-experience (Hertwig, Barron, Weber, & Erev, 2004; Hertwig & Pleskac, 2010). We note that it is unlikely that our participants under sampled the harder lottery for any of the possible reasons listed by Hertwig and Pleskac (2010). Our task had no memory demand, was not low stakes as wrong guesses resulted in zero reward and perceived difference in the outcomes of both the choices should not make a difference, expected reward for harder lotteries was consistently lower than that of easy one, at the time decisions were made. Thus, evidence for effort conservation in visuomotor -based decisions-from-experience offers another possible explanation for the under-sampling seen in numerical decisions-from-experience (Hertwig & Pleskac, 2010).

The major conceptual limitation in our task is that we do not account for the possibility of withdrawal of covert attention from the task. That is, someone may attend to the task overtly via eye movements while being mentally disengaged. While this possibility does not appear phenomenologically salient in the task in our experience with it, it is certainly theoretically realistic (Hunt & Kingstone, 2003). Future work may empirically measure covert attention shifts in our task by examining on-task microsaccades (Hafed & Clark, 2002).

Multiple experimental directions also present themselves immediately for future investigation. For instance, combining our task with the economic indifference approach of Westbrook and Braver (2015) should help estimate effort-cost curves using direct effort measurements. We anticipate such efforts will lead to greater clarity about the nature of mental effort.

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