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UNIVERSITY OF CALIFORNIA

Los Angeles

Students Becoming Mathematicians through Mathematical Modeling Learning Environments: A
Design-Based Study

A dissertation submitted in partial satisfaction of the requirements for the degree

Doctor of Philosophy in Education

by

Martin Christopher Romero

2015

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ABSTRACT OF THE DISSERTATION

Students Becoming Mathematicians through Mathematical Modeling Learning Environments: A Design-Based Study

by

Martin Christopher Romero

Doctor of Philosophy in Education

University of California, Los Angeles, 2015

Professor Concepción M. Valadez, Chair

Algebra has functioned as a gatekeeper for urban school students. Interventions have involved everything from doubling math time to new technologies. However, it is clear that change will not occur unless we address the content of mathematics, the pedagogical approach, attention to students and context together. In this Design-Based Research (Design Experiment) Study, I designed a learning environment that re-envisioned the urban school algebra classroom using the "science of patterns" (Devlin, 1996; Kneebone, 2001; Steen, 1988; Resnik, 1997; Mason, Burton, Stacey, 1982). Undergirding this learning environment is the engagement of students in mathematical modeling - iterative cycles of expressing, testing, and revising of their interpretations (Doerr & Lesh, 2011). The design experiment provided insights about students' learning trajectories as they used functions to construct their models as well as the pedagogy and tools that supported participation. It was found that when students engaged in the classroom tasks, that specific mediating process (interactions and activity) emerged that led to student learning. In this case, learning included: 1) students using their derived mathematical models to

answer/pose contextual questions about the quantities, and in the process of deriving models, 2) students developing an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations.

The dissertation of Martin Christopher Romero is approved.

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University of California, Los Angeles

2015

Dedication

This dissertation is dedicated to my wife, daughters, parents, and grandma. My wife has unendingly supported me through our lives together. Without Laura, this dissertation would not be possible; “we did it.” My daughters, Sylvia and Lauren are with me all the time; I am a better man because I have them in my life. This dissertation is something I want them to be proud of. My parents, Filbert and Sylvia have been there every step of the way; it’s a good feeling knowing I can count on you for anything. And my grandma, you have been my biggest fan since I can remember; I am so happy I can share this moment with you. Love you guys.

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Acknowledgements

There are so many people I am indebted to, so I apologize for not being able to acknowledge all of them in a lengthier fashion.

Profesora Concepción Valadez, my committee chair, thank you for taking in me and showing me what it means to be an academic. My process from beginning to end has been marked with your guidance; I would not be done without your “kicks in the butt”; but more importantly, your genuine confidence in my abilities.

Professor Megan Franke, thank you so much for the time and wisdom you allocated to me. I could not ask for a better mentor to model my future work with students.

Dean/Professor/Friend Jody Priselac, I would not be in graduate school if it were not for your commitment to me. Allowing me to be part of the UCLA Teacher Education Program made all of this possible. Please accept my genuine appreciation; I am so thankful to have a believer like you.

UCLA Teacher Education Program, meeting my students and faculty colleagues shaped my perspective on education and the undergirding ideas of equity in my dissertation. I am especially thankful to Annamarie Francois, Jaime Park, Imelda Nava, Kyndall Brown, Carolee Koehn, Jackie Honda, Yolanda Reyna, June Choi, and Jeff Share. I learned so much from you.

Bill Sandoval, thank you for letting me borrow your ideas to design my study, I hope I did your work justice.

Uri Treisman, your words and ideas are forever linked to how I think about mathematics education and the work I want to do.

Dr. Juan Lara and Dr. Julie Mendoza, if it weren't for our meeting at Roosevelt High School in 2008 and your opening of the door, I would never have imagined myself being a doctoral student and future holder of a doctorate of philosophy.

My 2010 Urban Schooling Cohort, it has been a distinct pleasure to get to know and learn from you, it is an honor to be a member of the group.

And lastly, I want to thank Chuck Vaughn, Domingo Torres-Rangel, Evelyn Torres-Rangel, Tom Walters, and Mario Cantu; teachers who helped changed my life trajectory.

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Chapter 1 – Introduction

Problem Statement

Urban school students (USS¹) face a multitude of unmatched in-and-out of school challenges that impact their future academic and life outcomes (Duncan & Murnane, 2011; Kozol, 1991). It is no surprise then that these realities also affect the mathematics teaching and learning these students experience. On the 2011 “nation’s report card” (National Assessment of Educational Progress-NAEP) for California, marked gaps along racial, ethnic, and socioeconomic lines were persistent. Only 17% of Latino and 19% of Black students scored at or above proficient on the 4th grade NAEP; while 57% and 64% of White and Asian students scored at or above proficient respectively. On the same measure, 18% of low socioeconomic status (SES) students scored at or above proficient and 56% for non-low SES students (California Department of Education, 2012). As a result, mathematics often plays a gatekeeping role for USS by limiting their access to advanced classes and by the simple fact that too many students cannot navigate their way through traditional school mathematics (Adelman, 2006; Stone, 1998; Walston & McCarroll, 2010). Amplifying these struggles is the fact that USS are amongst the fastest growing demographic groups in the nation; making success with urban schools and USS a national imperative (Scheurich, Goddard, Skrla, McKenzie, & Youngs, 2011).

The urgency to address this issue is linked to what past research says about the negative educational outcomes for students who are not successful in mathematics – especially for those who progress from middle school into high school without success in Algebra 1. It is no surprise then that Algebra 1 has been saddled with the moniker of “gatekeeper,” identified as a subject

¹ Note: In this paper I will use the term urban school students (USS) as proxy for urban city students from communities of color, low-income families, and families of immigrant status.

that hinders USS from gaining access to post secondary education. Having spent most of my career working with secondary USS, I can confirm that those students who often fail Algebra 1 are fast-tracked to dropping out of high school or at least not graduating in four years.

Consequently, the practical effect is that when these students have a strong negative experience in mathematics and in education more broadly; they come to believe that learning mathematics and success in school is beyond them.

Rationale for Study

Mathematics Success

Within the landscape of inequitable educational opportunities for urban school students (USS), the problem this study addresses is the lack of success USS have in secondary school mathematics, and more specifically, Algebra 1. This is critically important given the opportunities and benefits possibly accrued because one is successful in mathematics. For example, success could mean students become proficient mathematicians (National Research Council, 2001) as well as develop a "sense of efficacy (empowerment) in mathematics together with the desire and capability to learn more about mathematics when the opportunity arises." (Cobb & Hodge, 2010, p. 161). Simultaneously, success in secondary school mathematics is a gateway to advanced course sequences that prepares students for college entry and enrollment into highly regarded Science, Technology, Engineering, and Mathematics (STEM) based college majors. Cognitively, success in mathematics helps develop a students' mathematical thinking and reasoning abilities - which are most often beneficial when solving math problems, but may be transferred to other contexts where critical thinking and problem solving is involved. Lastly, coupling the intellectual status of mathematics in our society with the perception of its inaccessibility to the general population, success in mathematics provides USS with a form of social, academic, and cognitive capital to better position themselves in the

world.

In order to address the problem, this study examines an approach to teaching algebra that will help students see the relevance of mathematics, engage them in mathematical thinking and mathematical practices, and dispel the belief that mathematics is a rigidly contained system of facts, procedures, and algorithms where the teacher always holds the answer to the question (Stein, Grover, & Henningsen, 1996). This will be done by building a classroom learning environment hinged on the idea of mathematics as a "science of patterns" (Devlin, 1996; Kneebone, 2001; Steen, 1988; Resnik, 1997 ; Mason, Burton, Stacey, 1982). This characterization of mathematics links the learning of mathematics to a fundamental neuroscience view; "human brains operate...in terms of pattern recognition rather than logic"(Edelman , 2006, p.83). Further buttressing a motivation to learn mathematics from this perspective, Hawkins (2004) argues that, "patterns are all the brain knows about. They are pattern machines" (as cited by Laughbaum, 2008, p.589).

Beyond the cognitive rewards, there are advantages to viewing mathematics in this light. Immediately it gives credence to the argument that the learning of mathematics is a real-world activity. We can use mathematics to describe the patterns that exist all around us. For secondary school mathematics, these symbolic descriptions may come in the form of mathematical functions students learn early in their study of algebra. Most importantly, by expanding these real-world patterns to encompass all types of data found in our daily lives, we help answer the dreaded question commonly asked of mathematics teachers, "When will I ever use this in my life?"

USS' Struggles, Algebra for All, and Mathematics Equity

To further develop the study's rationale, it is important to clarify why USS struggle with mathematics, what has been done to address this issue, and what we can do moving forward to

help improve USS's mathematics success. To explain this, the following discussion is parsed into three sections, 1) an exploration of possible explanations for the low rates of success we have had with teaching mathematics to USS, 2) an examination of the "Algebra for All" movement meant to equalize opportunities for all students to a meaningful and rigorous mathematics education, and 3) a highlight of current calls for mathematics equity.

USS' Struggles

When reflecting on data showing tremendous disparities in mathematics achievement between USS and their non-urban-school peers, one first has to consider non-classroom issues to explain these gaps. This section briefly explains how poverty, segregation, and high-stakes testing impact USS. It ends focusing on mathematics classroom instruction.

Poverty and Segregation: Poverty's intractable impact on social, academic, and economic outcomes for USS is undeniable (Anyon, 2005). This is exacerbated by the fact that many USS go to schools lacking the resources needed to help support their success. Historically, it has been documented that low achievement statistically correlates to low socioeconomic status and with lack of home and school resources (Lacour & Tissington, 2011). Complicating the effect poverty has on students and learning are the findings that USS still attend highly segregated schools (Orfield, Kucsera, & Siegel-Hawley, 2012). "The consensus of nearly sixty years of social science research on the harms of school [racial] segregation is clear: separate remains extremely unequal" (Orfield et al., 2012, p.7). This segregation leads to inequities that pervade their school life, especially the quality of teaching these students experience (Haberman, December 1991; Kohn, 2000). With that said, it is important to state from an equity perspective that:

these findings do not suggest that poor students are of low intelligence; rather, the studies point to the power of the economy — and of economic hardship — to place extremely high hurdles to full development in front of children who are poor (Anyon, 2005, p. 76)

High-Stakes Testing: As a result of differential results on traditional measures of achievement, USS are more likely to attend schools which emphasize the use of standardized testing as a vehicle for reform. The rise of high-stakes testing impacts the quality and type of instruction these students receive. Darling-Hammond and Rustique-Forrester (2005) found that teaching to the test occurs with greater frequency in schools where stakes are attached to the test and where students are generally lower performing. Kohn (2011) observed that for poor children their curriculum consists of basic skills development, reliance on worksheets, more rote practice, more memorization, and a diminished emphasis on critical thinking. Kohn's major indictment of standardized testing is that it serves mostly to make dreadful forms of teaching appear successful. USS have suffered the most from the proliferation of high-stakes standardized assessments (FAIRTEST, 2011); with so much riding on their outcomes, meaningful instruction supporting higher-order thinking skills is stifled and often replaced with rote memorization techniques, "drill and kill" activities, and unashamed teaching to the test (Advancement Project, 2010). This brief synopsis is an appropriate foreshadowing of the types of mathematics instruction that lead to low success rates with USS.

Harmful Mathematics Instruction: Having spent significant time in schools during this test-frenzy atmosphere as a mathematics educator, I can personally attest to contributing in the detrimental practices reported by researchers and practitioners. Often times, we were not concerned about learning as much as we were about improving test scores on high-stakes exams. We spent numerous weeks training students to answer problems they were likely to encounter on standardized tests; essentially relegating their education into who can best eliminate three out of four bubbles (Hopkinson, 2011). This reductive version of mathematics instruction is actually in line with how many people view what mathematical learning is; even prior to the high-stakes accountability tests, studies have established that traditional school mathematics instruction in the United States primarily focused on routine procedures, rote exercises, and memorization (Stigler, Givvin, & Thompson, 2010).

As a result, the nature of traditional school mathematics instruction needs challenging. A premise of this study that the narrowly restricted view of mathematics is indicative of the impoverished teacher-centered experience students receive in school classrooms, the reification of mathematics as something completely disconnected from reality emerges early in a students' academic trajectory. Students' disaffection for learning mathematics directly impacts their motivation to want to do well in the subject as well as understand the content. When faced with this dilemma, USS do not seem to respond very well, as indicated by most local and national achievement results (Aud et al., 2011; EdSource, 2012)

Even more damaging, traditional school mathematics bears little likeness "to the mathematics of life or work...in which mathematicians engage" (Boaler, 2008). This denies students the opportunity to wrestle with challenging problems, acquire effective problem solving skills, and make sense of the mathematics they are learning (Schoenfeld, 2004). Learning mathematics through a traditional perspective has detrimental effects on how students use mathematical thinking and reasoning to productively solve problems and verify whether or not their answers to questions are reasonable (Stigler et al., 2010). Immediate consequences of this procedural fluency focus sets students up to see mathematics as a set of abstract disconnected facts that have little meaning, even to the point where they begin to see learning mathematics conceptually as "just wasting their time, time needed for practicing and memorizing" (Givvin, Stigler, & Thompson, 2011). The effect from this perspective is that students do not develop conceptual understanding and are unable to navigate problem situations that are not identical to the examples found in the book. This lack of success easily translates to decreased motivation to persevere through complex multi-step problems. Additionally, the critical thinking, problem solving, and analytical reasoning skills found in rich mathematical situations are the types of skills needed to be successful in post-secondary education (Conley, 2007).

In sum, there is no single explanation as to why many USS fail mathematics. As presented here, there are many issues that go beyond classroom instruction that need to be addressed to ensure more success for USS. Poverty, segregation, and high-stakes testing are all

out of teachers' hands. However, teachers in urban schools have a socially just responsibility to be attuned to their students' social and academic needs as well as to be aware of equitable instructional practices that may empower them. Currently, traditional school mathematics as conceived in most schools, continues to offer students only a shell of what mathematics is and how it is practiced. In fact, during the Iris M. Carl Equity address at the National Council of Teachers of Mathematics (NCTM) national conference, Treisman (2013) emphasizes that in his work with urban schools across the country, math classrooms are more interested in "filling in bubbles rather than connecting the dots" and are mostly "driven by a compliance mentality on tests that are neither worthy of the children nor worthy of the discipline they purport to reflect." With his in mind, it is easy to argue that this bankrupt version of mathematics functions as a gatekeeper for most students and will continue to do so, especially for USS.

Algebra for All

The acknowledgement of the successful completion of Algebra 1 in middle school as a gateway to advanced mathematics prompts educational reformers and equity activists to ask how to make this course available to everyone. The presumption being - especially for disadvantaged students- that giving them access to Algebra 1 earlier, can dramatically reduce the opportunity and achievement gap (Evan, Gray, & Olchefske, 2006). Many districts are now encouraging and even forcing more students to take Algebra 1 in middle school (Musen, 2010). The following section examines the effects of these implementations.

Tracing the "Algebra for All" call to its equity-driven roots, Silva, Moses, Rivers, and Johnson (1990) argued that the practice of offering Algebra 1 only to certain students has had detrimental effects on the mathematics education of urban school students. They further state that this "rationing of algebra" (Pg. 5) closes the door to a broad range of academic and professional pursuits while at the same time having a negative long-term effect on the well being of these students. In order to have a clear understanding of the impact of "Algebra for All" on

school and student achievement, we must take a thorough look at current literature on such reform efforts.

In an early analysis of the impact of high school algebra among students who differ in their math skills prior to entering high school, Gamoran and Hannigan (2000) describe the findings that helped initiate this all-inclusive push for all students taking algebra. It was found that students who completed Algebra 1 not only learn more math, but also pursued higher level courses of study. Gamoran and Hannigan's (2000) quantitative findings were consistent with the hypothesis that taking algebra earlier promotes academic achievement. All students regardless of prior math skills benefit from taking algebra in ninth grade. However, their study did suggest that these undifferentiated math classes may not benefit low achievers as much as high achievers, which could mean that students that need the most help do not get it. In addition, Gamoran and Hannigan's (2000) work does not address the negative effects of placing students in algebra who do not have the basic skills to be successful. They stated that the impact of an "Algebra for All" policy had still not been fully assessed.

Like previous studies, Allensworth, Nomi, Montgomery, and Lee (2009) quantitatively studied the effects of an "Algebra for All" implementation in the Chicago Public Schools (CPS) and found contrasting results to Gamoran and Hannigan (2000). Allensworth et al. (2009) reveal that other than gaining course credit, there are no observable benefits of enrolling in Algebra 1 instead of remedial math courses. In fact, among the students who took algebra instead of remedial math, their grades declined and math failure rates increased. Students were no more likely to proceed to advanced college-preparatory math courses because of the policy. Allensworth et al. (2009) address the issue of classroom instruction by remarking that with the change of classroom composition, it is imperative that substantial changes in pedagogy need to

be instituted for more students to be successful. They do not say what these changes should be and do not address what happens to those students who fail algebra because of lack of preparation. In a policy brief analyzing the work done by Allensworth et al. (2009), Mazzeo (2010) underscores the key premise that mandatory curriculum policies are suppose to offer greater equity in course taking which lead to improvements in students learning and educational outcomes. However, based on the results of CPS's "Algebra for All" push, he suggests that states and districts implementing mandatory curricula reforms "should focus attention on the quality of classroom instruction and the depth of tasks" (Pg.10) students are asked to complete.

In 2008 Loveless examined the National Assessment for Educational Progress (NAEP) math test results of the increased number of eighth-grade students enrolled in algebra as a result of the "Algebra for All" policy. He argues that although NAEP test scores had gone up for 8th graders since 2000, the scores of the top performing eighth graders dropped over that time. They found this was a result of more unprepared students being enrolled in algebra; upwards of 7.8 percent of eight grade students enrolled in algebra scored at the second grade level. Loveless (2008) found no evidence to suggest that having these students with extreme gaps in mathematical understanding be placed in an Algebra 1 classroom. In addition, Loveless (2008) discovered an increased instructional load placed on teachers to differentiate instruction for students with varying skills; so one possibility is that poor student performance was tied to teachers' instructional decisions. What this study did not include was any analysis of the effects of classroom instruction on student's performance and any follow up on what happens to students who fail Algebra 1 in the 8th grade.

In arguably the most comprehensive review of 8th grade Algebra 1 performance data, Williams et al. (2011) examined the readiness levels of 8th grade students (n=69,945) who took

the end of course 8th grade Algebra 1 California Standardized Test (CST) and compared them to their achievement results they obtained on the Algebra 1 CST. These students' readiness levels were determined by their achievement on the 7th grade end of course CST. Not surprisingly, they found that the most-prepared students typically took Algebra 1 in 8th grade and generally scored proficient or higher on the Algebra 1 CST. They also found that the least-prepared students, if placed in Algebra 1, generally did not even score at a basic level. Interestingly, the data indicated that schools serving mostly low-income students not only placed higher percentages of students into Algebra 1, but they placed higher percentages of students into Algebra 1 who were not proficient on the 7th grade CST. Thus, despite the well-intentioned efforts of expanding access to Algebra 1 to more students, "placing all 8th graders into Algebra 1, regardless of preparation, sets up many students to fail" (Pg.6). As a conclusion, Williams et al. (2011) state that there is sufficient reason for concern about too early placement into Algebra 1, particularly since there is little research on how 8th grade Algebra 1 failure affects students' future outcomes and their disposition towards mathematics as they progress into high school.

It is important to note that Algebra 1 can serve students beyond giving them access to the prerequisite march to Calculus, it can also provide as an "intellectual gateway" to abstract reasoning, critical thinking, and problem solving (Muller & Beatty, 2008). All of which have been identified as critical cognitive skills needed by students to be college ready (Conley, 2007). These additional cognitive benefits further heighten pressures to promote participation in Algebra 1 for USS. In fact, Evan et al. (2006) argue that Algebra 1 has functioned as central player in "maintaining and institutionalizing the achievement gap by systematically reducing the access of minority and low-income students to upper level mathematics... that is a precondition for college success" (pp. 13).

Having spent my entire career working in urban schools, I am immediately aware of the academic performance of students on end-of-year Algebra 1 assessments. In fact, despite the current climate of high stakes testing and California's Algebra for All initiative for 8th graders, eighty percent of California's students enrolled in Algebra 1 as 9th graders have historically scored below proficient on the state's standardized tests. Even more shocking, most urban schools have "non-proficiency" rates closer to ninety-five percent. This serves as a reminder that the current "Algebra for All" push has not done enough. Regardless of the findings that more students now take the course and are even passing the course at higher numbers, it is evident that not enough time has been spent on defining what it should look like when students learn Algebra 1. Liang, Heckman, and Abedi (2012) argue that "the algebra-for all policy did not appear to have encouraged a more compelling set of classroom and school-wide learning conditions that enhanced student understanding and learning of critical knowledge and skills of algebra" (p.340). This last finding is exactly why this current study can have important ramifications to what Algebra 1 curriculum and instruction looks like for more students.

Mathematics Equity

Since the publication of the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics, 1989, 2000) standards for school mathematics, equity concerns have garnered increased attention by mathematics researchers. NCTM (2008) states:

Excellence in mathematics education rests on equity—high expectations, respect, understanding, and strong support for all students. Policies, practices, attitudes, and beliefs related to mathematics teaching and learning must be assessed continually to ensure that all students have equal access to the resources with the greatest potential to promote learning. A culture of equity maximizes the learning potential of all students (p. 1).

This attention to equity is buoyed by the reality the schools produce unequal outcomes for USS, especially in regards to mathematical proficiency (Ball, 2002).

The equity perspective of this study is rooted in Cobb and Hodges's (2010) and Gutierrez' (2002; 2012) long established work and viewpoint on constitutes mathematics equity for students. For Cobb and Hodge (2010), equity "encompasses students' development of a sense of efficacy (empowerment) in mathematics together with the desire and capability to learn more about mathematics when the opportunity arises" (p.181). Gutierrez (2002) takes a critical stance, her version of equity states, "the inability to predict mathematics achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language" (p.19). Gutierrez (2012) elaborated on her notion to include four dimensions: access, achievement, identity and power. Access refers to the availability of "tangible resources that students have to participate in mathematics". Achievement relates to the conventional and non-traditional ways to measure student proficiency. Identity embraces more than students seeing themselves as proficient doers of math, but "incorporates the question of whether students find mathematics not just 'real world' as defined by textbooks or teachers, but also meaningful to their lives" (p.20). The power dimension extends the identity piece; students taking up "issues of social transformation" and opportunities for them "to use as an analytic tool to critique society" (p.19).

By addressing issues of power and social consciousness through mathematics, Gutierrez (2012) implicitly proposes a Critical Race Theory (CRT) turn to achieving mathematics equity for USS. Although not a direct focus of this study, this critical perspective is foundational in examining why USS are academically underperforming. Solórzano (2005) identified five tenets of CRT that should inform theory, research, pedagogy, curriculum and policy: (1) the intercentricity of race and racism; (2) the challenge to dominant ideology; (3) the commitment to social justice; (4) the centrality of experiential knowledge; and (5) the utilization of

interdisciplinary approaches. (p.274-275). In regards mathematics equity, Gutierrez' power dimension of equity addresses the second and third tenets of CRT.

Furthermore, the connection between CRT and obtaining equitable access and outcomes for USS lies with the idea that CRT is viewed as a "social justice project" (Yosso, 2005, p.74) . This perspective has led math educators to employ a pedagogy known as teaching mathematics for social justice (Bartell, 2010; Frankensein, 1983; Gutstein, 2003; Wager & Stinson, 2012). "Appealing to their sense of fairness, teaching mathematics with social justice issues can motivate students to learn the mathematical skills necessary to solve complex problems" (Gutierrez & Irving, 2012, p.24). Although research is still emerging about the benefits of this pedagogy, "the approach appears to be especially successful at engaging students who have lost interest in mathematics...by connecting mathematics to the world outside school" (Gutierrez & Irving, 2012, p.24) .

In sum, it is of utmost importance to extend the conversation about equity in the mathematics classroom beyond being solely about providing students the opportunity to learn and giving them access to gate keeping math courses. This extension is crucial to drive conversation and action towards addressing disparate achievement results for USS and their non-urban school peers.

Research Questions

With these circumstances in mind, here are the questions this study addressed:

1. What does teaching and learning look like in a secondary school mathematics classroom that approaches the teaching of algebra with problems set in daily life observations?
2. How does the use of graphing technology assist students' in mathematical thinking and the engagement of mathematical practices?
3. What is the trajectory of student's mathematics identity as they learn algebra from a "science of patterns" perspective?
4. What is the learning trajectory of student's use of functions to model data through a curve fitting process?

In order to answer these questions, I engaged students in the mathematical modeling of real-world data; this was done by creating "model-eliciting" activities related to the topic of data

modeling and curve fitting

A learning environment was based on a “science of patterns” perspective was envisioned using the following scenario. After classroom formalities, the lesson begins with the presentation of a potential pattern found in the world. For an algebra class, the patterns – which come in many forms - could be a table of values that hypothetically contain the stopping distance of a car for a given speed. The context of the presented data is discussed as a class after the teacher prompts the students to discuss everything they know about the data at this point. The discussion then takes a speculative turn as the teacher encourages the students to ask "I wonder" questions. As the questions come in, the teacher records each response. Using these "I wonder" inquiries as guide, the teacher elicits potential mathematical descriptions from the students to initiate the modeling process. The students then convert the data to a visualize representation by plotting the data using graphing technology. From here, students use their knowledge of functions to search for ways to mathematically describe or model the data with an algebraic representation. During the course of this modeling process (which is cyclical in nature), the teacher encourages students to convince, conjecture, and engage in mathematical practices by sharing their progress to the class or by collaborating in small groups. It is during this thought revealing sequence that students learn that multiple solutions are possible and that mathematics can be experimental in nature. As activity continues, they also learn from each other to "iteratively express, test, and revise their interpretations or conceptualizations of mathematical learning” (Doerr & Lesh, 2011, p.248) . In this context, students use graphing technology to test their models against the visual representation of the data. This process concludes when students are able to verify and justify that their model is a good fit to the original data as well solve posed problems from the teacher, solve problems that were co-constructed with their peers, or even

pose additional problems to be considered.

During this study I used a design experiment methodology to answer my research questions. I developed cycles of investigation as I engaged students in the modeling process of real-world data as described above. These investigations began with students hypothesizing about which functions they believe best models the pattern formed by the data; they then proceeded to test and revise their models via the feedback they receive from their peers, the teacher, and from their use of graphing technology. These cycles allowed me to revise the process I used to instruct students as well as “identify and account for successive patterns in student thinking by relating these patterns to the means by which their development was supported and organized” (Cobb et al., 2003, p.11). Since a primary goal of the methodology is to go through iterations of test and learn sequences to document and unpack the processes that lead to learning, each opportunity for students to develop mathematical models in the class was considered another iteration of my study.

This process of describing the world with mathematical models is a conduit for students to “focus on patterns, regularities, and other systematic characteristics of structurally significant systems” (Lesh and Lehrer, 2003, p.112). Furthermore, this purposeful design of having students engage in model-eliciting activities was in line with Schoenfeld’s (1992) goals for mathematics instruction- having students make sense of what mathematics is and how it is done. To that end, the methodology of employing model-eliciting activities had students developing new ways of thinking about how to make “symbolic descriptions” and “sense making systems” to describe the world (Lesh & Lehrer, 2003).

Chapter 2 – Literature Review and Theoretical Grounding

The mathematical fate for urban school students (USS) has been coupled with inequitable opportunities and academic underachievement. With USS often measured by standardized testing assessments, they have traditionally not performed well along those metrics. This underachievement, as indicated on most local and national achievement results (Aud et al., 2011; EdSource, 2012), has had detrimental effects on the future academic and life outcomes for these students. In order to provide a context for the study, the presentation of this chapter is done in two parts.

The purpose of **Part 1**, divided into 3 sections, is to provide a rationale for the design of the learning environment. 1) It explains how learning mathematics from a "science of patterns" perspective leads to students engaging in the practices of mathematicians. 2) It elaborates on what mathematical thinking and problem solving are as well as describe the benefits of engaging in those practices. 3) I conclude this part by examining research on modeling and "model-eliciting" activities. **Part 2**, delves into learning from both the cognitive and sociocultural perspective. I end by claiming that defining learning from a participatory framework provides an opportunity for mathematics educators to extend the definition of student achievement beyond those skills only measurable by multiple-choice exams.

Part 1: “Science of Patterns”, Mathematical Thinking/Problem Solving, and Mathematical Modeling

“Science of Patterns”

At the heart of the study was the mathematics I expected students to engage in. Using the depiction of mathematics as the “science of patterns” to guide my curriculum and instructional choices provided students with productive opportunities to engage in mathematical thinking. Supporting these choices is Tall’s (2008) proposal that a fundamental human attribute

essential for mathematical thinking and long-term learning is the sensory capacity to recognize patterns. This recognition of patterns provokes what Burton (1984) calls the “statement of a generalization”. He describes such statements as “the building blocks used by learners to create order and meaning out of an overwhelming quantity of sense data” (p.38). While adhering to similar sentiments, Schoenfeld (1992) extends these ideas by adding “mathematics is an inherently social activity in which a community of trained practitioners (mathematical scientists) engages...in systematic attempts, based on observation, study, and experimentation” (p. 335). These prevailing descriptions of mathematics help situate what a mathematician does and what disciplinary mathematical learning can look like in classrooms. It further allows us to reconceptualize what mathematics is worth being taught, what it means to be an expert (mathematically proficient), and what activities can be used to develop students’ emergent mathematics proficiency.

The Mathematical Method

Devlin (2012) proposes a “Mathematical Method” analogous to the Scientific Method used by scientists to engage in the practices of their discipline. Although listed in a sequential manner (Figure 2-1), a mathematician may move back and forth through the cycle when needed. The goal of the study was to support students’ mathematical growth by providing them with meaningful patterns to consider; then guiding students to study and develop an abstract notation for these patterns. From a modeling perspective, this meant students constructed or identified a mathematical structure that represents the pattern. Within this context, the mathematical method provides a means for students to engage in mathematical thinking and the practices of mathematicians.

- *Identify a particular pattern in the world*
- *Study it*
- *Develop a notation to describe it*
- *Use that notation to further the study*
- *Formulate basic assumptions (axioms) to capture the fundamental properties of the abstracted pattern*
- *Study the abstracted pattern, establishing truths by means of rigorous proofs from the axioms*
- *Develop procedures that you and others may use to apply the results of the study to the world*
- *Apply the results to the world*

Figure 2-1 The Mathematical Method (Devlin, 2012)

Recalling the classroom vignette introduced in chapter 1 that describes a “science of patterns” learning environment, which I have included below with special annotations, it is important to connect the “Mathematical Method” with the proposed classroom setting for the study.

(Identify a particular pattern in the world) After classroom formalities, the lesson begins with the presentation of a potential pattern found in the world around us. For an algebra class, the patterns- which come in many forms- could be a table of values that hypothetically contain the stopping distance of a car for a given speed.

(Study It) The context of the presented data is discussed as a class after the teacher prompts the students to discuss everything they know about the data at this point. The discussion then takes a speculative turn as the teacher encourages the students to ask "I wonder" questions. As the questions come in, the teacher records each response. Using these "I wonder" inquiries as guide, the teacher elicits potential mathematical descriptions from the students to initiate the modeling process.

(Develop a notation to describe it) The students then convert the data to a visualize representation - they do this by plotting the data using graphing technology. From here, students use their knowledge of functions to search for ways to mathematically describe or model the data with an algebraic representation.

(Use that notation to further the study) During the course of this modeling process (which is cyclical in nature), the teacher encourages students to convince, conjecture, and engage in mathematical practices by sharing their progress to the class or by collaborating in small groups. It is during this thought revealing sequence that students learn that multiple solutions paths are possible and that mathematics can be experimental in nature.

(Formulate basic assumptions (axioms) to capture the fundamental properties of the abstracted pattern) As activity continues, they also learn from

each other to "iteratively express, test, and revise their interpretations or conceptualizations of mathematical learning" (Loerr & Leah, 2011, p.248). In this context, students use graphing technology to test their models against the visual representation of the data.

(Study the abstracted pattern, establishing truths by means of rigorous proofs form the axioms)This process concludes when students are able to verify and justify that their model is a good fit to the original data as well solve posed problems from the teacher, solve problems that were co-constructed with their peers, or even pose additional problems to be considered. (Chapter 1 this paper)

By overlaying the "Mathematical Method" with the hypothetical classroom it is easy to see that the students learning in this environment participate as mathematicians- they enact the "Mathematical Method". Although the last two steps of the process are not included in the vignette, I argue that those two become enacted over a period of time when students continually engage in the learning environment. Through the development of their mathematical proficiency, students will be able to use the models they find and be able to at some point "develop procedures that you and others may use to apply the results of the study to the world" and ultimately to "apply the results to the world" (Devlin, 2012). This application of results brings us full circle. The "Mathematical Method" is directly linked to Burton's (1984) argument that the study of mathematics provides a particular means – specializing, conjecturing, generalizing, and convincing- to describe the world (Burton, 1984). Making this connection is vital; it ensures that the mathematics and the practices that students engage in within this classroom are akin to what mathematicians actually do as well as provide students with an answer to the often asked question, "When will we ever use math in our real lives?" Considering this study is focusing on students who are at a critical stage in their mathematical development, a tangible response to this question can give more students a reason to productively engage in mathematical activity.

Mathematical Thinking and Problem Solving

Mathematical thinking and problem solving are at the core of what mathematicians do. Because it is believed that mathematical thinking engages the same cognitive resources that are available for thinking in general (Tall, 2008), it is appropriate to look at two definitions of thinking respectively presented by cognitive scientists and a mathematics educator:

- thinking is the systematic transformation of mental representations of knowledge to characterize actual or possible states of the world, often in service of goals (Holyoak & Morris, 2012, Pg.XX) .
- thinking is the means used by humans to improve their understanding of, and exert some control over, their environment (Burton, 1984, p.36).

It is believed that mathematical thinking and the study of mathematics provides particular means to characterize the world and exert some control over it. Burton (1984) describes these means – the operations, processes, and dynamics of mathematical thinking - as encompassing four processes, specializing, conjecturing, generalizing, and convincing. An important aspect of these processes is the demarcation between them and the corpus of knowledge (rules and procedures) typically described as mathematics. This distinction is important because of the premise – one in which I believe- that “mathematical thinking is used when tackling appropriate problems in any context area, although questions of a mathematical nature might more readily expose such thinking” (Burton, 1984, p.36). This transfer of mathematical thinking to assorted domains is at the heart of the argument of why it is important for school mathematics to have a focus on a disciplinary learning in classrooms. As a result, it becomes important to give students the opportunity to actively engage in the dynamics of mathematical thinking. These dynamics include: “query assumptions, negotiate meanings, pose questions, make conjectures, search for justifying and falsifying arguments that convince, check, modify, alter, be self-critical, be aware of different approaches, be willing to shift, renegotiate, and change direction” (Burton, 1984, p.48).

With this in mind, Schoenfeld (1992) posits the case that thinking from a mathematical point of view means “seeing the world in the ways mathematicians do” (p.340). He further goes on to describe mathematical thinking as:

a mathematical point of view-valuing the processes of mathematization and abstraction and having the predilection to apply them, and developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structured mathematical sense-making (p. 335)

All these views of mathematical thinking bolster the argument that classroom mathematics needs to be experienced in a manner that accentuates the main features of what it means to think mathematically. This is done by helping students recognize that mathematics is not a rigidly contained system of facts, procedures, and algorithms where the teacher always holds the answer to the question (Stein, Grover, & Henningsen, 1996).

Some argue that engaging students in problem solving is a core goal of mathematics education (Wiggins, 2011). The National Council of Teachers of Mathematics (National Council of Teachers of Mathematics, 2000) explicitly states that “problem solving means engaging in a task for which the solution method is not known in advance” and “in order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings” (retrieved online on March 5, 2012). It is important to note that NCTM’s depiction of problem solving is similar to what others have said about this idea:

- Individuals responding to a problem that he or she does not know how to ‘comfortably’ with routine or familiar procedures(Carlson & Bloom, 2005)
- A thinking process in which a solver tries to make sense of a problem situation using mathematical knowledge she/he has and attempts to obtain new information about that situation till she/he can ‘resolve the tension or ambiguity’ (Nunokawa, 2005)
- A problem arises when a living creature has a goal but does not know how this goal is to be reached. Whenever one cannot go from the given situation to the desired situation simply by action, then there has to be recourse to thinking. (Bassok & Novick, 2012, as cited by Duncker, 1945)

I would further these ideas by distinguishing between “creative” and “noncreative” problem solving and “divergent” problems-many possible solutions- and “convergent” problems—one correct answer (Smith & Ward, 2012). The distinction between “creative” and “noncreative” is the concept of a problem being well-defined or ill-defined. A problem is well-defined if its beginning state and goal state are clearly specified as well as the operations needed to solve the problem. “Creative problems are considered ill-defined, primarily because multiple hypothetical solutions might satisfy the goals of the problem” (Smith & Ward, 2012, p.462). Regardless, in order for these problem-solving processes to surface in the classroom, an explicit effort must be made to have students experience solving problems in new and old contexts as well as developing a repertoire of problem solving heuristics they can use to manage themselves from a given state to a desired state. This can only occur through deliberate instructional practices developing a students’ mathematical disposition while engaging in a reconceptualized vision of what mathematics is. This reconceptualization is “based in part on...detailed understandings of the nature of thinking and learning and of problem solving strategies and metacognition; evolving conceptions of mathematics as the ‘science of patterns’ and of doing mathematics as an act of sense-making” (Schoenfeld, 1992). It has been argued that when students learn problem solving in the context of their math classroom, they “acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations” (National Council of Teachers of Mathematics, 2000) that could ultimately be helpful to them in their daily school and work life. This is especially true considering Bassok and Novick (2012) claim that “when people attempt to find or devise ways to reach their goals, they draw on a variety of cognitive resources and engage in a host of cognitive activities” (p.428). As a result, this is more evidence that a participatory and disciplinary approach to mathematics education can have a lasting and positive

impact on students' cognitive growth. In sum, it is appropriate to end this section with Schoenfeld's (1992) more than fitting description of the goals of mathematics instruction, especially as it relates to the goals of this study:

Mathematics instruction should provide students with a sense of the discipline—a sense of its scope, power, uses, and history. It should give them a sense of what mathematics is and how it is done, at a level appropriate for the students to experience and understand. As a result of their instructional experiences, students should learn to value mathematics and to feel confident in their ability to do mathematics. (p. 345)

Mathematical Modeling and “Model-Eliciting” Activities

The research completed on modeling and model-eliciting activities is done from models and modeling perspective (MMP). It is a multi-tiered methodology that examines students, teachers, and researchers as they “go through sequences of cycles in which they iteratively express, test and revise their interpretations” (Doerr & Lesh, 2011, p. 248). These cycles typically begin with students participating in a classroom modeling activity— usually called a model-eliciting activity (MEA) – which is designed to be both “model-eliciting” and “thought-revealing” (Doerr & Lesh, 2011). Lesh and Lehrer (2003) describe two major purposes of MEAs :

(a) identifying the mathematical understandings and abilities that are needed for success when “mathematical thinking” is needed beyond school in a technology-based age of information, and (b) identifying students who have extraordinary abilities that may not have been apparent based on past records of low performance on the narrow and shallow band of tasks emphasized in traditional textbooks and tests. (as cited in Lesh, 2001) (p.116)

With these explicit goals as a foundation for studying students engaging in a mathematical modeling process, MMP and MEA are ideal in assisting researchers to operationally define what is meant by “important mathematics, mathematical activity, and learning at the individual and social levels” (Lesh, Middleton, Caylor, & Gupta, 2008, pp. 115). Within its iterative cycle of researchers studying teachers, teachers studying students, and students creating and testing their mathematical models, MMP provides a framework to better understand what these ideas mean.

And considering the context of this study and my Design-Based Research (DBR) implementation; this can be especially beneficial for students who have traditionally struggled mathematically by determining how to best provide them with the learning experiences necessary to develop mathematical efficacy and proficiencies.

Combining the goals of MEAs with those of this study will bear fruit in two areas. First, it will help students become proficient mathematicians, and secondly but equally as important, it can democratize mathematical success for those USS who traditionally have not enjoyed such rewards.

Part 2: Learning Theories – Cognitive and Sociocultural

Having spent many years teaching in a mathematics classroom for urban schools, indelible debates between two steadfast factions of teachers defending their style of teaching was a common sight during department meetings. One side was for lecturing at the board and “drill and kill” assignments, while the other was a proponent of group work and the teacher working as a facilitator of learning. What was missing from these afternoon meetings was any discussion about the purpose of education or what learning theories had to say about what were doing in our classrooms. Early on, I found myself siding with the collaborative learning supporters. However, as I struggled to implement group work in the class and found students negotiating my cooperative tasks into teacher-directed lessons, I could readily understand the arguments for a more explicit model of instruction. Given those experiences, although reductive in description, I learned over time that learning in the classroom takes on many identities, and both the cognitive and sociocultural perspectives have important roles in helping students learn and develop identities as learners. It is from this mindset and related calls that educators “apply in the classroom what we know about humans as intelligent, learning, thinking creatures” (Bruer, 1993, p.1), that I present an overview of what knowledge is, where knowledge is found, and

what it means to learn within the cognitive and sociocultural perspectives. I will end my discussion with a portrayal algebra instruction that is line with the situative perspective studied by Greeno (2003).

Cognitive Perspective

Information Processing: As an alternative to behaviorism, the early cognitive perspective is a knowledge-centered theoretical framework that describes the human mind as a computing device that builds and executes production systems. These production systems – knowledge in this perspective- can be seen as a mental algorithm that we follow to complete a cognitive task. Additionally, the cognitive perspective emphasizes the major role that memory plays in helping us translate new information into a form that is meaningful so that we can retrieve it and be able to use it at a later time. Part of retrieving the information is through mental structures called schemas. These schemas provide a way for us to accumulate information but also influence what we notice, how we interpret and how we remember it (Derry, 1996; Greeno, Collins, & Resnick, 1996; Mayer, 1996). In sum, we can think of knowledge from this viewpoint as “a large collected database of information, organized into some logical order in the mind, which can be accessed and then described through human language” (Leonard, 2002, p.29).

Knowing what knowledge is from this perspective leads to a natural question, where can it be found? Inherent in the cognitive viewpoint is that knowledge is found in the mind and situated with individuals (Cobb, 1994). This individualist approach is prominent in schools where the goal of education is often times seen “as helping students develop the intellectual tools and learning strategies needed to acquire the knowledge that allows people to think productively” (Bransford, Brown, & Cocking, 1999, p.5) about the subjects being taught. When considering how students learn, the cognitive viewpoint posits that in order to acquire new

knowledge, our current schemas and prior knowledge help facilitate this new learning (Mayer, 1996). It is believed that prior knowledge affects how we interpret and process instruction and consequently affects what can be learned. Additionally, we can learn from new encounters only if we have explicit instruction about how to interpret these new mental representations (Bruer, 1993). Given this depiction of learning through the acquisition of mental representations and one's prior knowledge, it is appropriate to turn to a discussion on constructivism.

Constructivism: Considered as an alternative metaphor to describe cognitive development, constructivism is the perspective that individuals create knowledge and understanding through experiences and reflection, with knowledge resulting from the active mental processes of the learner as new experiences are interpreted through existing knowledge structures (Edwards, Esmonde, & Wagner, 2011). Another way to view constructivism is through a “learner-centric” lens, where learners have “some prior knowledge and experience as a basis from which to test out their hypotheses and build their own set of content to solve a particular set of problems posed by the instructor” (Leonard, 2002, p.37).

Drawing its tenets from Piaget's developmental learning theories (Packer & Goicoechea, 2000) and Vygotsky's emphasis on socially meaningful activities as an important influence on human cognition (Schunk, 2008); constructivism has had a major influence on educational settings. This influence, which has materialized in the shape of “reform instruction”, has a central focus on the individual acquiring knowledge or concepts through knowledge construction (Mayer, 1996) and “conceptual change” (diSessa, 2006). An appropriate summation of this knowledge construction and acquisition process is given by Sfard (1998):

Since the time of Piaget and Vygotsky, the growth of knowledge in the process of learning has been analyzed in terms of concept development. Concepts are to be understood as basic units of knowledge that can be accumulated, gradually refined, and combined to form ever richer cognitive structures. (p.5)

Sociocultural Perspective

The sociocultural perspective is the rejection of the view that the locus of knowledge is in the individual. Sociocultural theorists believe cognition is a property of individuals that is distributed among people, and their environments which include the tools, artifacts, and books that they use (Greeno et al., 1996; Mayer, 1996; Putnam & Borko, 2000) . As a result, knowledge or knowing is seen as:

both an attribute of groups that carry out cooperative activities and an attribute of individuals who participate in the communities of which they are members. A group or individual with knowledge is attuned to the regularities of activities, which include the constraints and affordances of social practices and of the material and technological systems of environments. (Greeno, 1996, p.17)

An important distinction of this conception is the move from "knowledge" to "knowing"- a more process-evoking term (Greeno, 1997)- which is a signifier for a participation framework that creates a foundational shift in how knowledge is viewed. Sfard (1998) views this emerging shift as "the permanence of hav[ing] giving way to the constant flux of doing"(p.6). For sociocultural theorists, this participation-turn towards making the "individual-in-social-action" as a unit of analysis places the explanation of psychological development at the center of sociocultural activity (Cobb, 1994). This sociocultural view holds that collaborative cognition is the product of group interactions over time (Putnam & Borko, 2000)

Consequently, since knowing is about individuals participating in a community, then learning is about the improving or strengthening of one's participatory abilities (Greeno et al., 1996; Sfard, 1998). Within this context, Wenger (1998) uses the term participation "to describe the social experience of living in the world in terms of membership in social communities and active involvement in social enterprises" (p.54). Wenger's idea of "communities of practice" situates learning as being developed along a trajectory of growth that includes one's identity and the enculturation into a community's ways of thinking and practicing (Greeno, 1997; Putnam &

Borko, 2000).

An essential aspect of this learning trajectory is the search for one's identity within the community; which powerfully results in both the person and community being transformed (Packer & Goicoechea, 2000). Identity development is important from a school perspective since classrooms can be considered complex social environments where students are always in search for their "learner" identity. For example, the development of one's mathematical identity in a mathematics classroom is considered a form of learning that goes beyond traditional claims of mathematical proficiency. From this perspective, once an individual begins to associate him or herself as a legitimate participant, they can argue that they are part of the community. Their increased participation including the use of the language, tools, and discourses of a mathematician, allows them to stake community membership. Within this context, I believe that you do not need content matter expertise or have a Ph.D. in mathematics to say you are mathematician, as long as you engage in the mathematical practices of a mathematician while developing math content knowledge, you are a mathematician.

To conclude this brief overview of the two perspectives, I recognize that the continuum that connects the early cognitive perspective and the more participatory frameworks of social cultural learning theories is complete with multiple perspectives and arguments. However, this move from a cognitive perspective to a participatory framework of learning has moved us from a "narrow view of learning as something people gain to a broader conception of learning that can include what people do" (Forman, 2003). More importantly, I agree with Sfard's (1998) thesis that this participatory turn replaces the image of learning as one's private possession to a democratizing view that can alter people's beliefs about learning and teaching; which has the potential to change how we view the purpose of education.

Chapter 3 - Methods

This chapter describes the main features of Design-Based Research (DBR) and why I chose this methodology. I then continue by using conjecture mapping (Sandoval, 2013) to explain what the features of my design do and how they work together to produce my hypothesized learning outcomes. The last part concludes with the curricular design of the intervention, and the logistics (instructional resources, participant and site context, intervention structure, data analysis) of the study.

Design-Based Research/Design Experiments

I enacted a Design-Based Research (DBR) (Barab, 2006; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Confrey, 2006; Schoenfeld, 2006) methodology to answer my posed research questions. Due to its focus on instructional design and classroom-level research (Cobb, Stephan, McClain, & Gravemeijer, 2001), it provided the best opportunity to study learning in the classroom from an inquiry-based perspective. This allowed for the potential to “create instructional theories at the level that holds the best possibilities to provide solutions to common ills in education” (Confrey, 2006, see location 6166 of Kindle Edition).

Supporting my methodological choice, Schoenfeld (2006) argues that the best way to examine the potential of ideas for educational implementations is to examine them in the classroom setting. DBR is especially advantageous in this regard since its foremost goal is to “produce data that enable those involved in the study to draw warranted conclusions about student learning and what contributes to it” (Schoenfeld, 2006, p. 196). Reinforcing this argument, Barab and Squire (2004) allude to the role context plays in learning and where DBR is best situated, “in the buzzing, blooming, confusion of real-life settings” (pg. 4, Table1). Furthermore, a byproduct of DBR is that not only do researchers and practitioners gain a better

understanding of learning in complex environments, it provides extended and meaningful learning opportunities for those students and teachers involved in the study (Barab, 2006).

Looking at DBR/DE from a broader view, Cobb et al. (2003) identifies five crosscutting features of DBR/DE studies:

- The development a class of theories about both the process of learning and the means that are designed to support that learning
- A highly interventionist nature which are test-beds for innovation.
- The creation of conditions for developing theories yet at the same time placing these theories in harm's way.
- Multiple iterations
- Producing theories that do real work; having potential for rapid pay-off be also speaking directly to the types of problems that practitioners address in the course of their work (Cobb et al., 2003, p.10)

In addition to these features, I highlight Sandoval's (2013) view that the main effort of design research is on "explicating casual processes" (p.17) that lead to desired outcomes within the proposed learning environment. With that said, the varied foci of each of these characteristics position DBR to be an appropriate methodology to study the mathematical learning and participation of students while engaging in mathematical and data modeling.

Conjecture Mapping

In response to questions about Design Based Research's (DBR) methodological rigor, Sandoval (2013) submits conjecture mapping (Figure 3-1) - a "technique for conceptualizing design research [as] a means to specify theoretically salient features of a learning environment design and map out how they are predicted to work together to produce desired outcomes"(p. 2).

He argues that:

Whatever the context, learning environment designs begin with some high level conjecture(s) about how to support the kind of learning we are interested in supporting in that context. That conjecture becomes reified within an embodiment of a specific design. That embodiment is expected to generate certain mediating processes that produce desired outcomes. The ideas a research team has about how embodied elements of the design generate mediating processes can be articulated as design conjectures. The ideas

they have about how those mediating processes produce desired outcomes are theoretical conjectures. (p. 4)

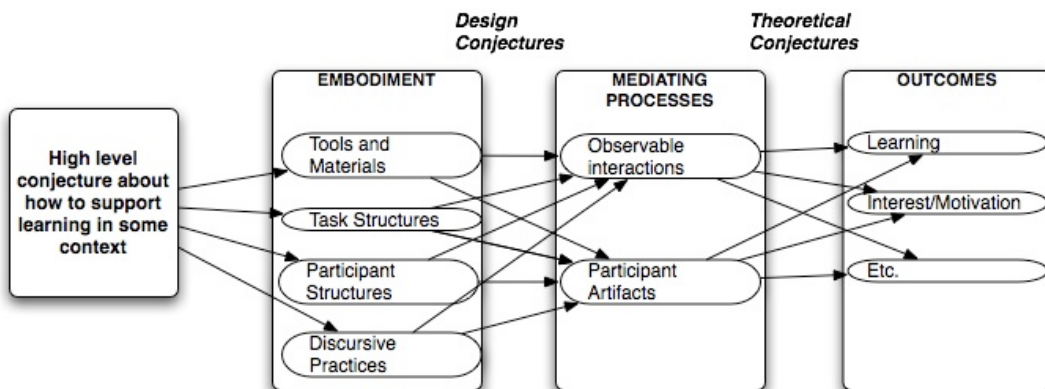


Figure 3-1 – Conjecture Map

Using Sandoval's conjecture mapping framework to hypothesize answers to my research questions and identify a means for testing them, Table 3-1 outlines the key components of my design. More importantly, by following the structure of the framework, I posited initial outcomes for students who learn in my proposed learning environment.

To begin, my high level conjecture is about how to support mathematical learning for students who have traditionally not been ready for 8th grade Algebra 1. The exact conjecture: *Engaging in a technology rich mathematical modeling learning environment helps students develop a frame of mind in which they begin to see how they can use mathematics to describe the world* results from my argument that learning mathematics from a “science of patterns” perspective allows students to see the act of doing mathematics as a real-world activity. As a consequence, they would be more motivated to engage in mathematical activity. My choice to employ data modeling and curve fitting in the learning environment was linked to the benefits found from the having students engage in “model-eliciting” activities. This includes extending the range of mathematical understandings and abilities being assessed, which would lead to a “broader range of students ...[emerging] as having exceptional potential” (Lesh & Lehrer, 2003, p.116).

A critical aspect of the embodied elements of the learning environment was the use of tools within the task and participant structures. Foremost was the need to have tasks helping students begin to think about what mathematics is, the benefit of productively engaging in mathematics, and what it means to do mathematics. Also, because studying mathematics from the data modeling and curve-fitting perspective are not common in traditional curricula, activities needed to be developed to engage students in the data modeling process. Additional tools included graphing technologies and interactive systems so that students got immediate feedback about their model representations as well dynamically sharing their progress with peers. A major reason to focus on such technologies is because of the extensive benefits afforded to students who have daily access to the technology (Roschelle & Singleton, 2008). Furthermore, graphing technologies were found to “facilitate communication and sharing of knowledge in both private and public settings, especially when the technology was treated as a partner or extension of self” (Goos, Galbraith, Renshaw, & Geiger, 2003, p.87). From a Vygotskian perspective, graphing technologies mediate learners abilities to “acquire mathematical process and concepts” (Rivera and Becker, 2004, p.82)

Given the task structures that had students engage in data modeling activities to study patterns and develop notations to describe these patterns, it was important to consider the mediating processes that would emerging as students used graphing technologies within these structures. My hypothesized mediating processes – certain kinds of activity and interactions resulting from the embodiment of my design - lead to my conjectured learning outcomes. It was my intent that the elements of the conjecture map lead to an initial hypothetical learning trajectory that will ultimately be refined as I enacted my research design.

Table 3-1

Conjecture Mapping Design

High Level Conjecture	Engaging in a technology rich mathematical modeling learning environment helps students develop a frame of mind in which they begin to see how they can use mathematics to describe the world
Embodiment	<p><u>Tools/Materials</u></p> <ul style="list-style-type: none"> • Graphing technology • Interactive display mechanism • Patterns Curriculum/Activities <p><u>Task Structures</u></p> <ul style="list-style-type: none"> • Pattern Activities • Graphing Stories / CBR • Identifying Functions in Images • Identifying Functions from Data/Scatterplots • Deriving Models in Images • Deriving Models from Data <p><u>Participant Structures</u></p> <ul style="list-style-type: none"> • Cooperative Learning
Mediating Processes	<ul style="list-style-type: none"> • Students recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs. • Students recognize the relevant quantities involved in the situation. • Students represent these quantities in tabular, graphical, algebraic forms. • Students hypothesize/explain how change in one quantity affects change in another quantity. • Students choose functions that are relevant to the situation. • Students test/revise whether their chosen function is an appropriate model
Outcomes	<ul style="list-style-type: none"> • Students use their derived models to answer/pose contextual questions about the quantities. • In the process of deriving models, students develop an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations

Description of Mediating Processes

As discussed previously, the mediating processes found in Table 3-1 are conjectured to lead to the hypothesized outcomes. The following is a more in depth description of the interaction and activity associated with the mediating processes.

- Students recognize the relevant quantities involved in the situation. (They do this by stating the relationships existing between the quantities):
- Students can represent these quantities in tabular, graphical, algebraic forms. (They take the values that represent the quantities and represent them in a mathematical form. This is seen when students make a scatter-plot from a table of values, and then find a mathematical model representing the plotted data):
- Students hypothesize/explain how change in one quantity affects change in another quantity: (In the process of making sense of a of quantities and representing them in mathematical forms, students hypothesize how the quantities are related to each other.)
- Students choose functions that are relevant to the situation. (This means they use a visual inspection of the graphical representation to choose a matching function. Or they use the contextual situation to argue why one function would be more appropriate than another (eg. population growth = exponential):
- Students test/revise whether their chosen function is an appropriate model, (They do this by plotting the function along with the data to gauge the fit of the function to the data. Students revise their model until it fits the data. Their revision can begin with guesses about how to adjust the function, but ultimately students learn how the coefficients of the function affect its fit):

Description of Hypothesized Outcomes

As Sandoval (2013) describes, the outcomes in Table 3-2 should be as a result of the mediating processes being generated from the embodiment of the design.

- Students use their derived models to answer/pose contextual questions about the quantities
- In the process of deriving models, students develop an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations

The first outcome is the application or real-world use of functions and the second outcome is connected to students understanding how the parameters of different functions effect the graphs of functions.

Curricular Content

The curricular content for the intervention/enrichment program consisted of concepts and skills found in the Common Core State Standards (CCSS) proposed for 8th graders enrolling in Algebra 1. It included activities that had students thinking about what mathematics is, why they would want to study mathematics, and what it means to do mathematics. The instructional

content was divided into six learning segments (Table 3-2). The learning segments were sequenced so that students learned the concepts and skills in a focused and coherent manner. This focus relates to isolating a smaller number of core mathematical ideas for a course and coherence is the idea that standards are "articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchal nature of the disciplinary content from which the subject matter derives"(Schmidt and Hoang, 2012, p.295, as cited in Schmidt et al., 2005). For this teaching experiment, the specific content included relationships between quantities, concept of function, linear functions and equations, and quadratic functions and equations. The original intent was to include exponential functions and equations, but due to time constraints the content was excluded. The reason these topics were included was because they represent the big ideas of Algebra 1 found in the CCSS. Additionally, the content is important given the academic background of the participants and their impending enrollment into Algebra 1 in 8th grade.

Table 3-2 (Appendix A) provides a description of the content, tasks, and activities for the classroom intervention. The mathematics content is in line with what I described above, however, due to the nature of design experiments, the amount of content students experienced differed based on the daily analysis and reflections from the classroom activity. It is important to remember that this study examined a way of teaching Algebra through mathematical modeling, so a keen eye was paid to how students engaged in the modeling process. This meant looking for ways to refine the modeling tasks to improve instructional design.

Table 3-2

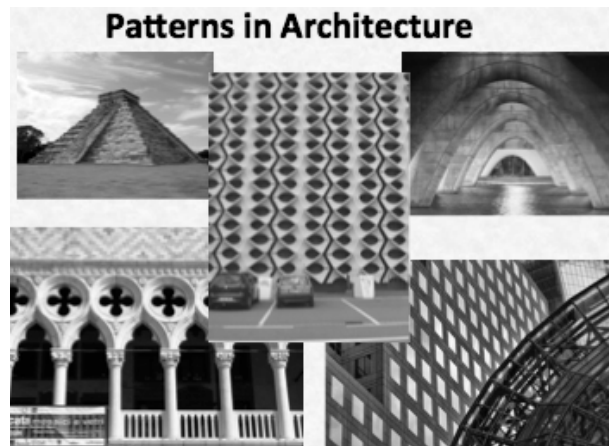
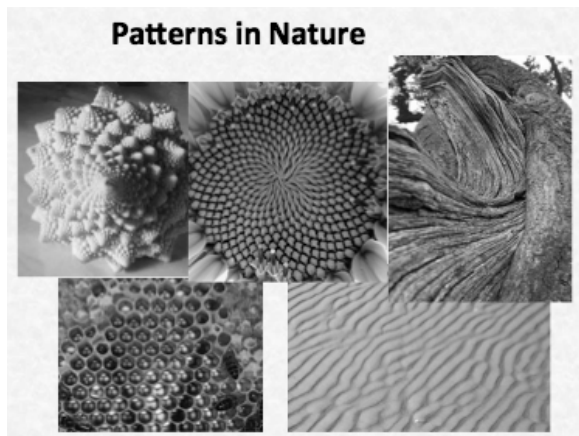
<i>Description of Content</i>		
Task	Activities	Math Content
Introductory activities around Calculator/Navigator use. Introduction to Patterns and	<ul style="list-style-type: none"> • Log-Into TI Navigator system • Introduction to calculator and its use to 	<ul style="list-style-type: none"> • Use of handheld technology • Numerical, Symbolic,

How Mathematics is a Real World Activity.	<ul style="list-style-type: none"> complete basic math operations • Have discussion of what mathematics is and what it means to engage in the study of mathematics • Pattern identification and creation activities. 	Graphical Patterns
Graphing Stories Tasks and Calculator-Based Ranger (CBR) Tasks	<ul style="list-style-type: none"> • GraphingStories.com graph creations • CBR motion detector activities 	<ul style="list-style-type: none"> • Functional relationships • Graphical representations of related quantities • Rate of change
Identifying Functions in Images Tasks	<ul style="list-style-type: none"> • Introduction to functions and their graphs • Cooperative Group Task • Calculator Driven Task • Pencil-Paper Task 	<ul style="list-style-type: none"> • Functions (Linear, Quadratic, Cubic, Exponential, Rational, Absolute Value, Square Root, Sine) • Identifying functions based on their graph
Identifying Functions in Data/Scatterplots Tasks	<ul style="list-style-type: none"> • Plotting data and identifying a function that possibly could represent the data • Creating a table of values, plotting data and identifying a function that possibly could represent the data • Collect data, creating a table of values from the data, plot data and identify a function that possibly could represent the data 	<ul style="list-style-type: none"> • Creating scatterplots with technology • Identifying patterns in plotted data • Identifying functions based on their graph
Modeling Tasks	<ul style="list-style-type: none"> • Deriving models from images • Deriving models from data • Deriving models from collected data 	<ul style="list-style-type: none"> • Linear equations ($y=mx+b$) • Quadratic equations (standard and vertex form) • Math Modeling (Hypothesizing and creating appropriate models based on the situation)

Tasks / Activities

Pattern Activities

During these tasks students were challenged to recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs. The goal for the initial task (Figure 3-2) was to create an opportunity for students to begin to think about what patterns are, how they can “arise from the world around us, from the depths of space or time, or from our inner workings of the human mind”, and how they connect to the idea of mathematics as a “science of patterns”.



Patterns in Numbers

O, T, T, F, F, __, __, __

1, 2, 4, __, __, __

1, 4, 9, __, __, __

1, 1, 2, 3, 5, __, __, __

Count to 25 starting at 1 by either 1's or 2's.

$$1^2 = 1 = 1$$

$$2^2 = 4 = 1 + 3$$

$$3^2 = 9 = 1 + 3 + 5$$

$$4^2 = 16 = 1 + 3 + 5 + 7$$

$$5^2 = 25 = 1 + 3 + 5 + 7 + 9$$

$$6^2 = 36 = 1 + 3 + 5 + 7 + 9 + 11$$

$$7^2 = 49 = 1 + 3 + 5 + 7 + 9 + 11 + 13$$

anuljoun

The sum of N odd integers starting at 1 equals N squared.

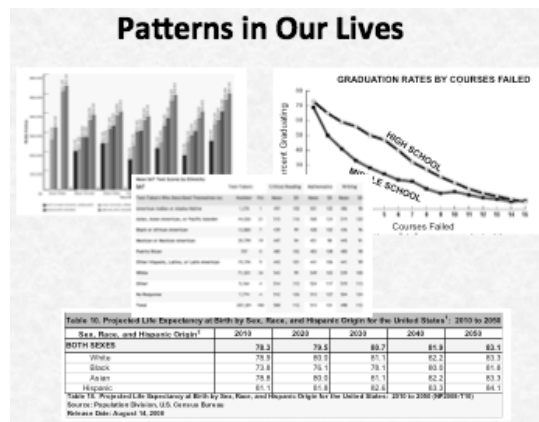


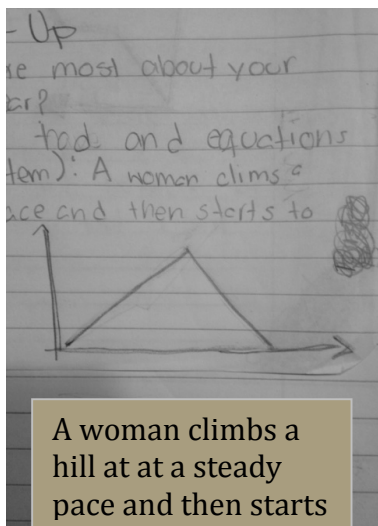
Figure 3-2- Initial Pattern TASK

CBR / Graphing Stories

The goal of the Graphing Stories.com tasks were to have students begin to make connections between physical contexts found in the real world, functional relationships existing within the physical context, and graphical representation of the functional relationship. The tasks originated from the GraphingStories.com website; they asked students to graphically represent a contextual situation that was provided to students in a fifteen second video clip (Figure 3-2 – Bottom 2 Images). These graphical representations become the basis for wanting to study those mathematical functions found in algebra courses.

The purpose of the calculator based ranger (CBR) activities were to have students actively participate in the creation of mathematics via physical movement. Their creations

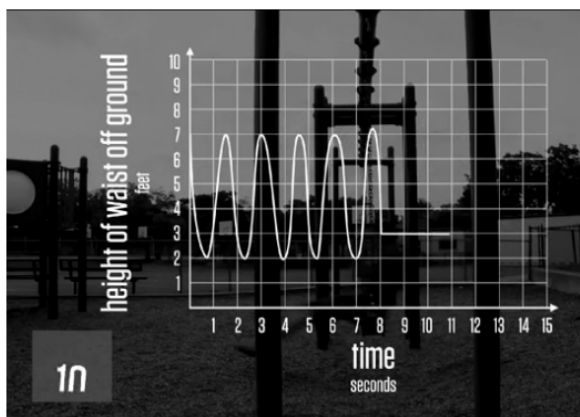
involved drawing mathematical representations of quantities, physically moving to produce graphical representations, and by explaining the relationship between quantities. The CBR is a motion detector that collects distance, velocity, and acceleration data while connected directly to a TI graphing calculator. These activities helped students recognize relevant quantities involved in a physical situation (Figure 3-3 – Top 2 Images).



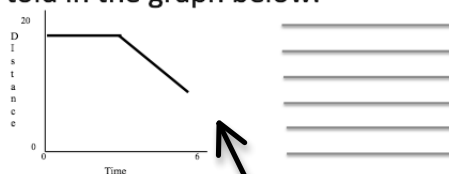
Height of Waist Off Ground

Author: Dan Meyer

Tags: parabolic, periodic, increasing, decreasing, piecewise, playground, swings



1. Journal Prompt: What story is being told in the graph below.



Possible words/phrases to use in your sentences: increase, slowed down, sped-up, stood still, forward, towards, for ___ about seconds, back, jumped, ran

First you stand still for 3 seconds far away then after 3 seconds you walk or run

Elevation

Author: Arianna Hoshino

Tags: piecewise, constant, increasing, decreasing, grass

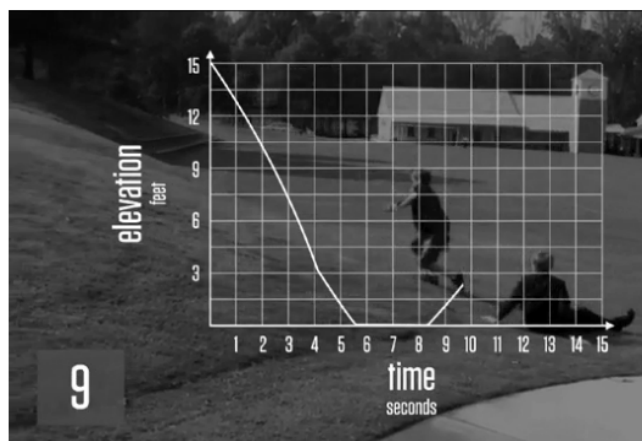


Figure 3-3- CBR and Graphing Stories Samples

Identifying Functions in Images

The purpose of these tasks (Figure 3-4) was to see if students could identify graphs of functions in real world images. Images were presented to students and they chose functions

whose graphs could be found in the images. I hypothesized that being able to do this would allow students to begin to think about how to choose functions in contextual situations. This task developed a student's ability to connect mathematical representations to contextual situations.



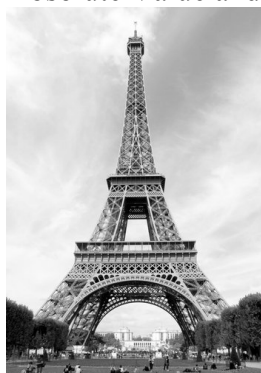
Cubic and Quadratic Functions



Absolute Value and Linear Functions



Sine Function



Rational and Quadratic Function

Figure 3-4 - Functions in Images

Identifying Functions in Data

The tasks of identifying functions in data/scatterplots (Figure 3-5) during the class included three specific scenarios. The first was for a student to be presented with plotted data from some contextual or “real-world” situation, and then asked to identify and choose a function whose graphical representation matches that of the plotted data. A second scenario involved students taking data from a table of values and representing this data in a graphical form. From there students used their knowledge of functions and their graphs to choose a function relevant to the situation. This task helped students begin to see data as being represented in a graphical

form. A final scenario involves students identifying a function in data during the initial steps of the modeling process.

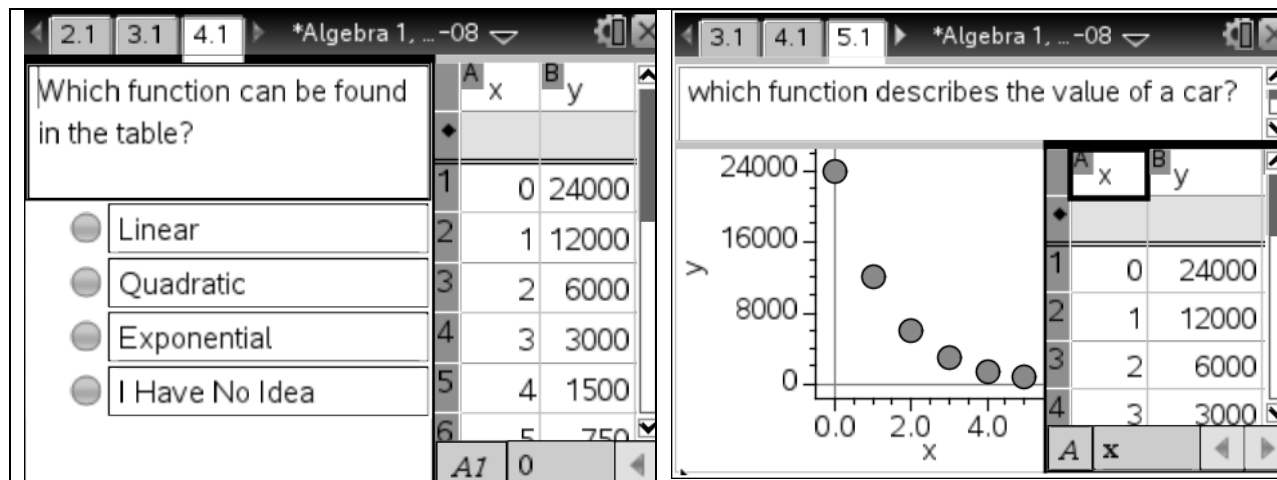


Figure 3-5 – Functions in Data

Deriving Models Tasks

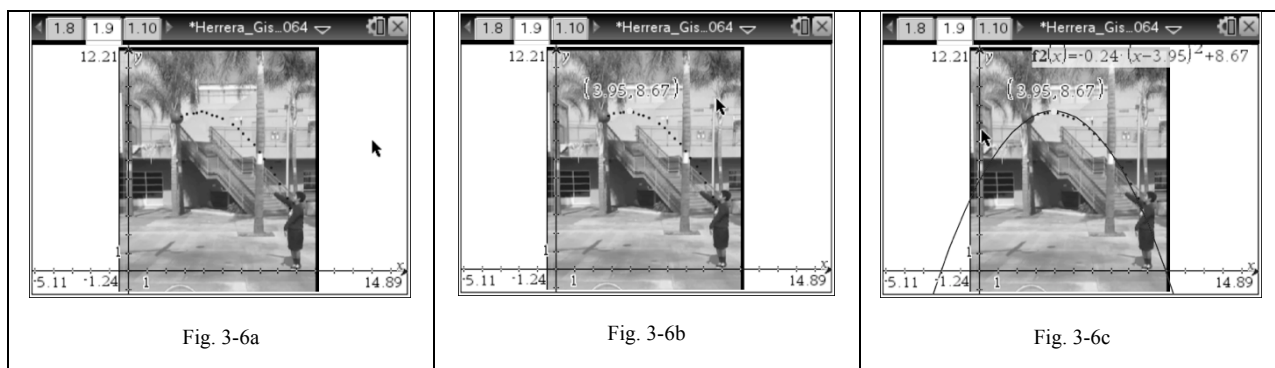
The specific tasks include deriving models of functions in images, deriving models from data, and deriving models from collected data. The intention of the latter two tasks and to some degree the first task was for students to use their models to answer/pose contextual questions about the quantities involved in the situation and to develop an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations.

Instructional Design

Mathematical Modeling/ Curve Fitting and Classroom Practices

In order to engage the participants in increased opportunities to learn and to document their learning and mathematical practices, I created “model-eliciting” activities around the topic of data modeling and curve fitting. To guide the creation of these activities, I chose data that can be approximated by those functions found in the Algebra 1 curriculum. An example of using curve fitting to represent or model patterns in the real world can be found in Figure 3-6abc. By using current handheld graphing technologies, the path of a shot basketball (Figure 3-6a.) is first

modeled by transposing the Cartesian plane on top of the image of the basketball (Figure 3-6b.). This produces data points that represent the path of the ball; a polynomial function is then obtained through the iterative process of expressing, testing, and revising until an appropriate model is determined. As discussed previously, this iterative cycle follows Devlin's (2012) proposed Mathematical Method where students identify a pattern, study it, try to develop a notation to describe it, and then use that notation to further the study. In our specific data modeling situation, this means students encounter a contextual situation where they produce a set of values that can be abstracted by a mathematical function. They will use graphing technology to plot these values and study the graphical representation or pattern formed from the data. From here, students express a representation of the data by trying to identify a mathematical model (function) that fits the data. To test whether this function is an appropriate model, they plot the function along with the data to gauge the fit of the function to the data. Depending on the fit – the graph goes through the points - of the function to the data, the students then revise their model based on the visual feedback of the graph along with the data and their knowledge of the function. With this revision, the students are then engaged in the iterative cycle of expressing, testing, and revising in order to find an appropriate model for the data. Once a model is generated, students then answer posed questions regarding the context of the situation or even pose their own questions to answer.



Cartesian Plane on top of image	Values approximating the path	Math model describing the path
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Figure 3-6 – Physical Situations (Basketball Shot)

Another way to engage students in the modeling process is through real-world data. In the case below (Figure 3-7), the tabular data represents the predicted life expectancy for four demographic categories. As can be seen from the scatterplots that are formed using graphing technology, a distinctive linear pattern is evident. From here, students studied the linear pattern in the act of producing an abstract notation to represent the data. In this scenario, students start with the slope-intercept equation ($f(x)=mx+b$) to begin their iterative process of modeling the data. In sum, these are some of the initial methods students will employ to model data in this learning environment. Additional methods or choices of topic were considered as a result of the “analyze and revise” portion of the design research methodology.

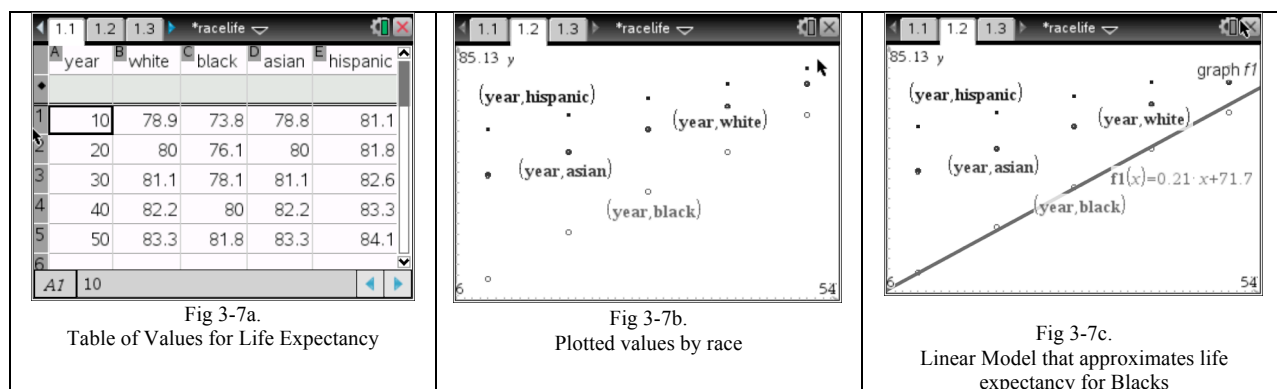


Figure 3-7 – Life Expectancy Data

Classroom Pedagogies

In addition to engaging struggling math students through data modeling and curve fitting, the instructional practices in the classroom attempted to follow what Boaler (2002, 2008) describes as positive characteristics of mathematics classrooms. These characteristics include “reasoning about applied problems, discussing mathematical ideas, and actively engaging in mathematical learning” (Boaler, 2012, p.3). In order to achieve these goals, students were arranged in collaborative working groups during the instruction of the study. Because working in groups has shown to have great potential to support student learning in the mathematics

classroom (Webb et al., 2009), opportunities were provided for students to work together while developing their mathematical models and conceptual understanding of Algebra topics.

Classroom Technology

This study utilized the Texas Instruments Nspire-CAS (Nspire-CAS) handheld device and the Texas Instruments Navigator Wireless Network (TI-Navigator) in conjunction with the data modeling and model-eliciting activities. The Nspire-CAS is a multi-function graphing calculator with applications allowing the user to create spread sheets, do statistical analysis on data, and compute solutions to algebraic equations via the Computer Algebra System. The ease of use and visual features of the Nspire-CAS makes graphing, curve fitting, and data analysis a fluid process allowing students to participate in the express, test, and revise process of mathematical modeling. The TI-Navigator system creates a connection between students and teacher wirelessly networking each student's graphing calculator to the classroom computer. The TI-Navigator is designed so that teachers can track the progress of individual students or the class in real time, view student coursework, check problem solving techniques, guide performance, and use instant feedback to create a dynamic learning environment (Texas Instruments, 2012)

Given the context of the study, it was my hope that the potential utility of these technologic systems to the mathematical modeling process would help facilitate mathematical thinking, conceptual understanding, and procedural fluency for the participants. Additionally, the prospective usefulness of these advanced tools in the classroom coheres to Lesh et al.'s (2008) position that "we must find ways to use technology to provide all children with democratic access to powerful ideas" (p. 114). Specifically, all participants in this study had access to a TI-Nspire CAS in class as well as being able to connect to the TI-Navigator.

School and Participant Context

I used an independent Pre-K – 12 charter school in the greater Los Angeles area as the study site. The school population is approximately 75% Latino and 24% African American, with the remaining demographics made up of various groups. The total number of students in the entire span is approximately 1200 with about 25% considered English Language Learners. intervention/enrichment program. These students had just completed 7th grade mathematics and were enrolling in Algebra 1 in the proceeding academic year.

The participants for the study were recruited from the school's matriculating 8th grade cohort of students. With the help a school site administrator and the school site counselor, students and parents were informed a summer math class that would help prepare 8th grade students for Algebra 1. Twenty-four students were allowed to enroll in the course based on their desire to participate or by suggestion that they could benefit from the course. These students were consulted about the study and the potential of the program to help them successfully be prepared to engage in Algebra 1.

Program / Class Structure

I was both the primary researcher for the project and teacher of the mathematical modeling Algebra 1 course. There were also two pre-service student teachers in the classroom to observe me teach and assist with data collection. With the course spanning five weeks (June 24-July 26, no school July 4,5), I worked with the participants on a block schedule format (Figure 3-8). This format takes the following form: I met with them Monday, Wednesday, and Friday from 10:00am to 12:30pm. These meeting times allowed me to see them for seven and half hours a week for a total of thirty-five hours during the five-week course. It is important to note that my intervention course is not the only academic instruction the participants receive. During the time they were not with me, they engaged in other academic and enrichment activities.

Additionally, I gained access to this site because of my past history with the school. I was one of the founding faculty members of the high school portion of the pre-K-12 campus. I previously served as faculty member and administrator for the school for four years. None of the participants was a former student.

Week														
1			2			3			4			5		
M	W	F	M	W	F	M	W	F	M	W	F	M	W	F
2.5	2.5	2.5	2.5hrs.	2.5	No	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
hrs.	hrs.	hrs.		hrs.	School	hrs.	hrs.	hr.	hrs.	hrs.	hrs.	hrs.	hrs.	hrs.

Figure 3-8 – Class Schedule

Data Sources and Projected Analysis

Multiple sources of data were collected and analyzed in order to understand how the task and participant structures led to the emergence of the mediating processes in the design.

Important in this process are the observable interactions and artifacts evidenced in the class.

This evidence was also used to demonstrate the mediating process led to desired outcomes.

Table 3-3 outlines the timing of the data collection activities – see Appendix B for a complete description of the rationale for each collection.

Table 3-3

Data Sources

Timing of Collection	In-Experiment
Date(s)	June 24- July 26, 2013
Activity	<ul style="list-style-type: none"> *UCLA MDPT Free-Response Algebra 1 Readiness Task (First Class Session) *Participant Calculator Responses (Daily) *Participant Written/Math Responses (Daily) *Student Discourse (Video – 5 Times in Total (Once a week)) *UCLA MDPT Free-Response Algebra 1 Readiness Task (First Class

Session)

*Reflections on individual and group learning trajectories. Includes
Field notes / Student daily Observation Form (see Appendix C)

*Teacher Reflections on Instruction (Daily) (Appendix D)

Limitations

Although the purpose of this study was to better understand what teaching and learning would look like in an Algebra 1 classroom engaging in mathematical modeling, due to time constraints of the summer course, results of the this study did not include students interacting with exponential functions. This critical component of the Algebra 1 curriculum, especially as it relates to the Common Core State Standards, was not studied. Exponential functions are applicable to many aspects of the real world, so my belief is that the inclusion of them would have further supported students in connecting mathematics to their lives.

Chapter 4 - Data Analysis and Findings

The presentation of data and findings is organized in two parts. Part 1 shows how and where the mediating processes emerge from within the tasks; students' interaction and artifacts are examined. Part 2 maps out how the existence of the mediating processes function to produce the learning outcomes.

Part 1 – The Emergence of Mediating Processes

Introduction

The presentation of the analysis and findings for Part 1 focuses on the task structures students engaged in within the learning environment. For each task, I display the evidence of mediating processes emerging from within the task structures, and also discuss how the use of graphing technology along with my adjustments to instruction supported student learning (the emergence of mediating processes and outcomes) within my learning environment. The task structures analyzed include: Pattern Activities, Calculator Based Ranger (CBR)/Graphing Stories, Identifying Functions in Images, Identifying Functions in Data/Scatterplots, and Deriving Models.

Each section heading for Part 1 is titled with a name of a task structure. Within these sections is a description of the various tasks students engaged in, a discussion of what it looks like for students to work and participate while completing these tasks, and how the mediating processes emerged. Additional content includes the role technology and instruction played with student learning in this environment. It is important to note that for the first tasks (Pattern Activities / CBR and Graphing Stories), I only summarize the tasks and interactions in order to focus on the more complex tasks related to student learning. My in-depth findings and analysis

are for the following tasks: Identifying Functions in Images, Identifying Functions in Data/Scatterplots, and Deriving Models.

Developing a Mindset: Pattern Activities Tasks and CBR / Graphing Stories Tasks

The Pattern Activities and CBR/Graphing Stories tasks occurred at the beginning of the course. The goal for these first two tasks was for students to see mathematics being used to describe the world and that the use of mathematical functions is one way to do that.

Task Structures

Pattern Activities

Starting the course with pattern activities created an opportunity for students to begin to think about what patterns are and how they could be described mathematically. Figure 4-1abcd provides examples of the types of questions prompted to students. The questions ranged from numerical and letter patterns to graphical and geometric patterns. These activities were integrated into the first two class sessions and then cycled in throughout the course during opening warm-up prompts. During these tasks students recognized, described, and made generalizations about regularities in patterns of numbers, symbols, and graphs.

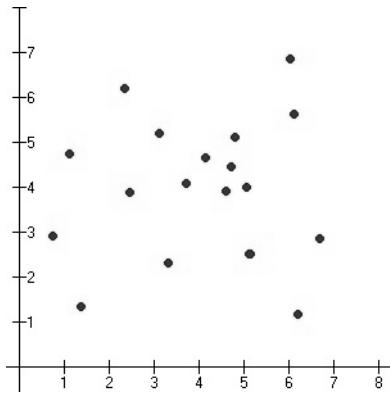
5, 1, -3, ____, -11, -15, ____, ____, -27, ____

O, T, T, F, F, S, ____, ____, ____

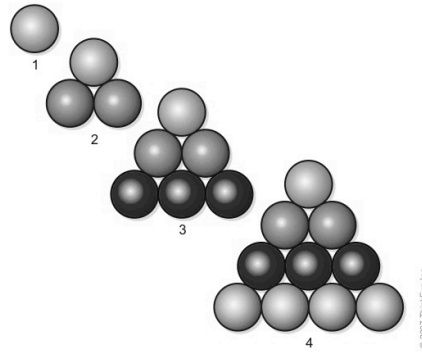
2, -4, 8, -16, ____, ____, 128, ____

4-1c.

4-1b



4-1c.



4-1d.

Figure 4-1 – Pattern Activities

CBR / Graphing Stories

Graphing Stories

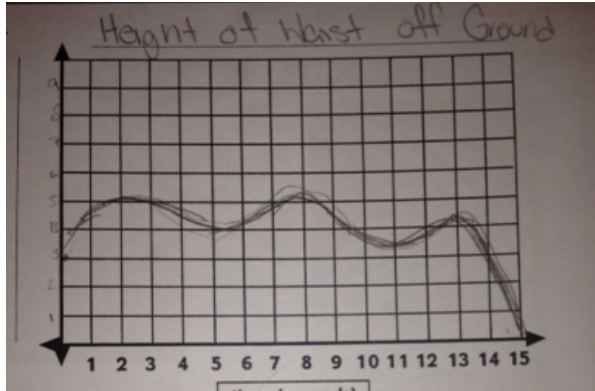
The goal of Graphing Stories.com tasks was to have students make connections between physical contexts found in the real world and the functional relationships existing within the physical context. These graphical representations of the functions became the basis for wanting to study those mathematical functions found in Algebra courses. In the process of completing graphing stories (Figure 4-2) students linked the action in the video to mathematical representations. To represent the situation students identified a pattern of regularities in their graph and the action playing out in the video. This type of interaction led students to have to recognize the relevant quantities involved in the situations being presented. Although this only involved time (0 to 15 seconds) as the independent variable, students recognized how another quantity (dependent variable) was affected as time elapsed. Furthermore, they figured out how to represent the relationships between the quantities in graphical forms.

Calculator Based Ranger (CBR)

The purpose of the calculator based ranger (CBR) activities was to have students actively participate in the creation of mathematics via physical movement. Their creations involved drawing mathematical representations of quantities, physically moving to produce graphical

representations, and by explaining the relationship between quantities. Using a CBR motion detector to track distance versus time data, students were asked to walk in a manner that would produce specific graphs (See Figure 4-2c. for a sample graph). Students were also asked to draw the graph of a specific situation (“A woman climbs a hill at a steady pace and then runs down the other side”, Figure 4-2d).

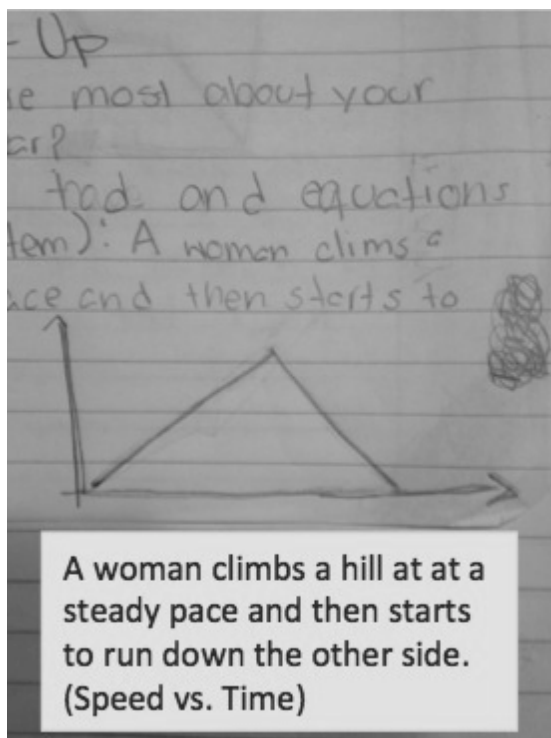
These activities helped students recognize relevant quantities involved in a physical situation. This was seen when they physically walked in front of a motion detector to produce a graph of a real-world context. In these cases the quantities were time and distance from the motion detector. Important with these tasks is the opportunity for students to see that physical contexts found in the real world can be described using mathematics; they begin to make sense of how to model real-world situations.



4-2a



4-2b



4-2c

Figure 4-2 Student Work

Identifying Functions in Images Tasks

Task Structure Description

The purpose of these tasks was to see if students could identify graphs of functions in real world images. I hypothesized that being able to do this would allow students to begin to think about how to choose functions in contextual situations. Finding a graph of a function in an image is a simplified proxy for finding a mathematical model to represent real-world contextual situation; this task develops a student's ability to connect mathematical representations to contextual situations. They begin to make sense of what it means to do mathematics: identify a pattern, describe with mathematical notations, and then use the notation for something in the world.

An example of these tasks can be found in Figure 4-3. Images were presented to students and they were expected to choose a function whose graph could be found in the image. It is

1. Journal Prompt: What story is being told in the graph below.

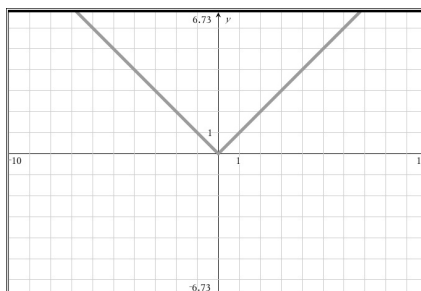


Possible words/phrases to use in your sentences: increase, slowed down, sped-up, stood still, forward, towards, for __ about seconds, back, jumped, ran

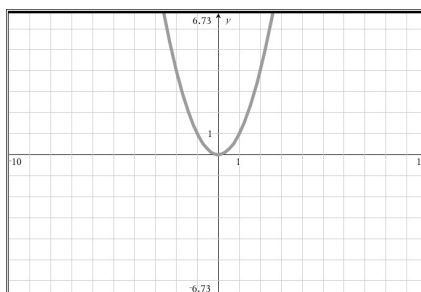
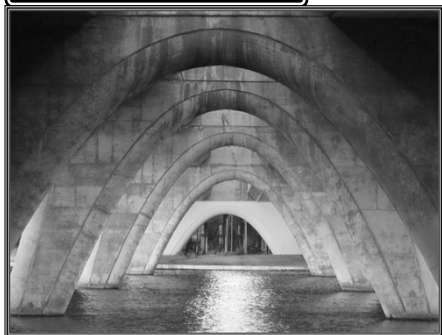
First you stand still for 3 seconds far away then after 3 seconds you walk or run

4-2d

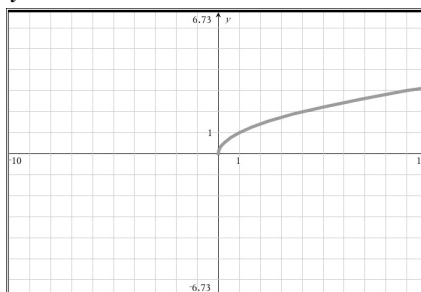
important to note that the representations found in the images may not appear identical to that of the actual graph. This means students began to implicitly apply transformations of functions to identify them in the images.



Absolute Value Function
 $y = |x|$



Quadratic Function
 $y = x^2$



Square Root Function
 $y = \sqrt{x}$

Figure 4-3 – Sample Images with Functions

In order to support students' ability to identify functions in images, students were given a resource, a tool containing the name of a function, its algebraic representation, and its graph. See Figure 4-4 for an example of one function given to students. See Appendix E for the entire

document. When provided with images, students could then use their resource to choose relevant functions based on its graphical representation. This means students would analyze the graphical shape of functions and see if they can find in an image.

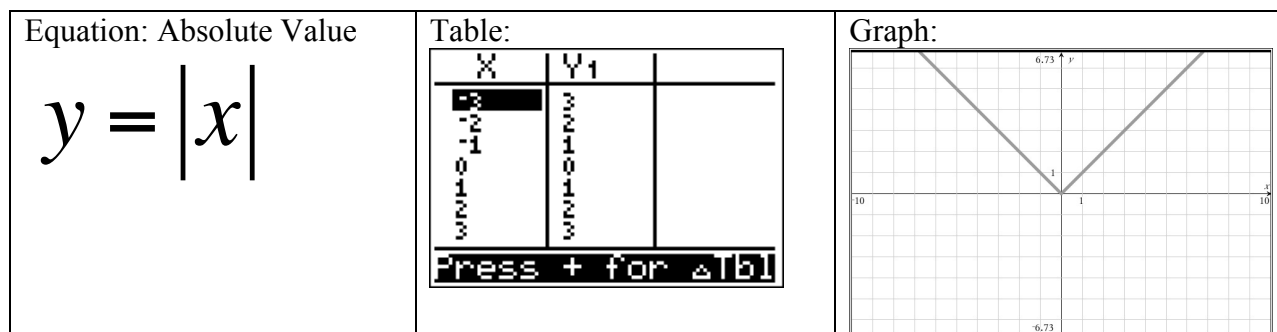


Figure 4-4 – Function Resource for Students

Three main tasks were given to students to identify the graphs of functions in images, a cooperative group task, a technology task, and the last was done with pencil-paper. The cooperative group task presented students with an image on the screen and they were asked to decide in their working groups which graph could be found in the image. They chose their functions by sending a group member to the front of the class holding a large graph of the function. The technology version included students being electronically sent images to their handheld graphing calculator and then identifying them by submitting responses from their handheld to my computer. The pencil-paper version included printed images where students identified graphs of functions by naming them and drawing over the image.

Task Findings: Student Work and Participation

The analysis presented here looks at the choices students made for functions they see in images and also analyzes classroom discourse around the choices they are made. It is not until the next set of tasks (identifying functions in data/graphs) that students were asked to write why they made the choices they did. I chose not to have them write a reason for their choice with these tasks since this was the first time they were trying to connect functions (graph of complex

functions) to real world images. The intent was to see if they would gradually develop a mindset that mathematics can be used to describe the world. So having them write reasons for the choices this early could possibly deter them fully participating at time when it is important to dive into the tasks without any caution of failure.

For these specific identifying functions in images tasks, two specific mediating processes emerged. One, choose functions relevant to a contextual situation; meaning students examined an image of a real-world representation, then chose a function based on the shape of its graph they believed could be found in the image. Two, students recognized and described patterns of regularity in graphs. This occurred when students analyzed an image, looked for patterns in the image, and then described those patterns in multiple manners.

Cooperative Group Task

Students showed that they could choose the appropriate function when working together. In making decisions about the image containing a parabola (Figure 4-5), all the groups chose the graph of the quadratic function as their answer. Students could be seen shuffling through their stack of functions and matching them up to the projected image. A snapshot of their activity is found in Figure 4-12. One of the students in Group 1 is explaining in Spanish to the other student by pointing to the image on the screen while comparing it to the their stack of functions on the table. Three of the students in Group 2 have their eyes pointed to the screen while examining a function they have in hand. The fourth student in Group 2 is going through the stack functions to find a match. The students in Group 3 and Group 4 can be seen discussing their options before deciding which function matches the image on the screen (Figure 4-5). Mathematical discussion leads to the development of mathematical practices; making sense of problems, arguing and critiquing others, and communicating their mathematical ideas. By

choosing the quadratic function as being represented in the image, students in their groups recognized the parabolic pattern in the image and connected that to the quadratic equation.

The types of activity and interaction described for the image in Figure 4-5 continued for the remaining images (Figures 4-6 to 4-11). Students compared, contrasted, and explained their choices about which functions were appropriate. This was evident in the way they talked to each other during the choice of functions. “I know this one” and “Sine, Sine, Sine” when they first saw the image in Figure 4-6. When finding a function for Figure 4-7, different students could be heard saying, “It could probably be this one”, “It can’t be this one”, “Could it be this one?” In some cases, students defended and described their choices by saying, “It curves more”, “I think it is this one, because if you flip it over, it is the same thing as that [exponential function], “you really don't see the bottom part”, “You should flip it, flip it to the right.” These exchanges show students recognizing and describing regularities of patterns in images.

Although student discussion did not always include the names of the functions, their comments indicated an ability to recognize a function in an image. I cite further evidence of while they discussed images found in Figures 4-9 to 4-11. “This one, this one” (a student showing a function to another student), “Maybe its that car, the car has this on top” (as a student points to a function on a sheet a paper), “Look the top of the car” (a student seeing a function on the top of the car), “I know I know” (a student recognizing immediately a function in the image), “Her nose, the top of the nose”, “Look at the car” (students trying to explain to another student where a function can be found in the image). In an exchange between two students, one student asked the question, “Where do you see cubic?” The other student responded with, “the car it goes like this and that, remember he showed us the pattern?” (This discussion occurred outside

the view of the camera, but I imagine that the student was using hand gestures to enhance his argument).

Quadratic Function

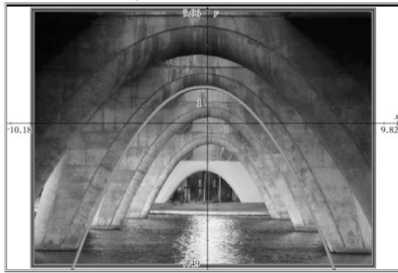


Figure 4-5 – Quadratic Function

Cubic Function

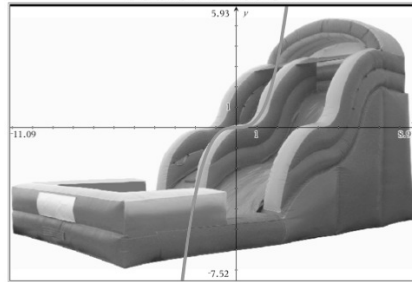


Figure 4-6 – Cubic Function

Square Root Function



Figure 4-7 – Square Root Function

Rational Function



Figure 4-8 – Rational Function

Absolute Function



Figure 4-9 – Absolute Value Function

Sine Function



Figure 4-10 – Sine Function

Exponential Function



Figure 4-11 – Exponential Function



Figure 4-12 – Student Activity During Cooperative Group Task

In addition to their conversations about functions, students communicated their ideas via pointing and hand gestures (Figures 4-13 to 4-19). A conversation (Figure 4-19) shows one student explaining where the exponential function can be found in the image (Figure 4-11), he uses his hand to demonstrate to a Spanish-only speaking student and then says “mira de lado”, which means “look on the side.” Highlighted in Figures 4-13 to 4-15 is similar communication, students pointing to graphs on their table or screen while comparing and contrasting and making choices about which functions are appropriate. These instances show students fully participating in the process of choosing functions appropriate for the situation. It also shows them engaging in important mathematical practices.

Constructing viable arguments – an important mathematical practice- is demonstrated by students in Figure 4-16 and Figure 4-17. Students used their hands and fingers to make an argument for their choice while trying to decide on a function that is represented in the image. Figure 4-18 shows a student asking to come to the board; “Can I show you?” was his question, as he wanted to defend to his choice. He proceeded to come to the board and rotate the graph he

had in hand so that it mapped on to the image on the board. This prompted another student to say, “exact too”, meaning that the rotated graph fit perfectly onto the portion of the image he was referring to. On a mathematical content note, this indicates an intuitive application of the transformation of a function, a concept not learned until a more advanced algebra or pre-calculus class.



Figure 4-13



Figure 4-14



Figure 4-15



Figure 4-16 - Student using hand gestures to demonstrate the shape he sees in the image displayed on the board and how it matches up to what is on paper.

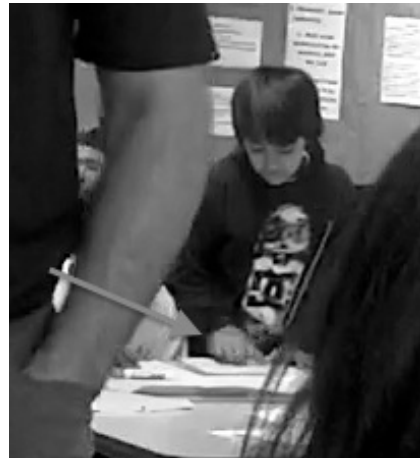


Figure 4-17 - Student uses his finger to point at the graph on paper as he explains to another student how it is represented in the image.

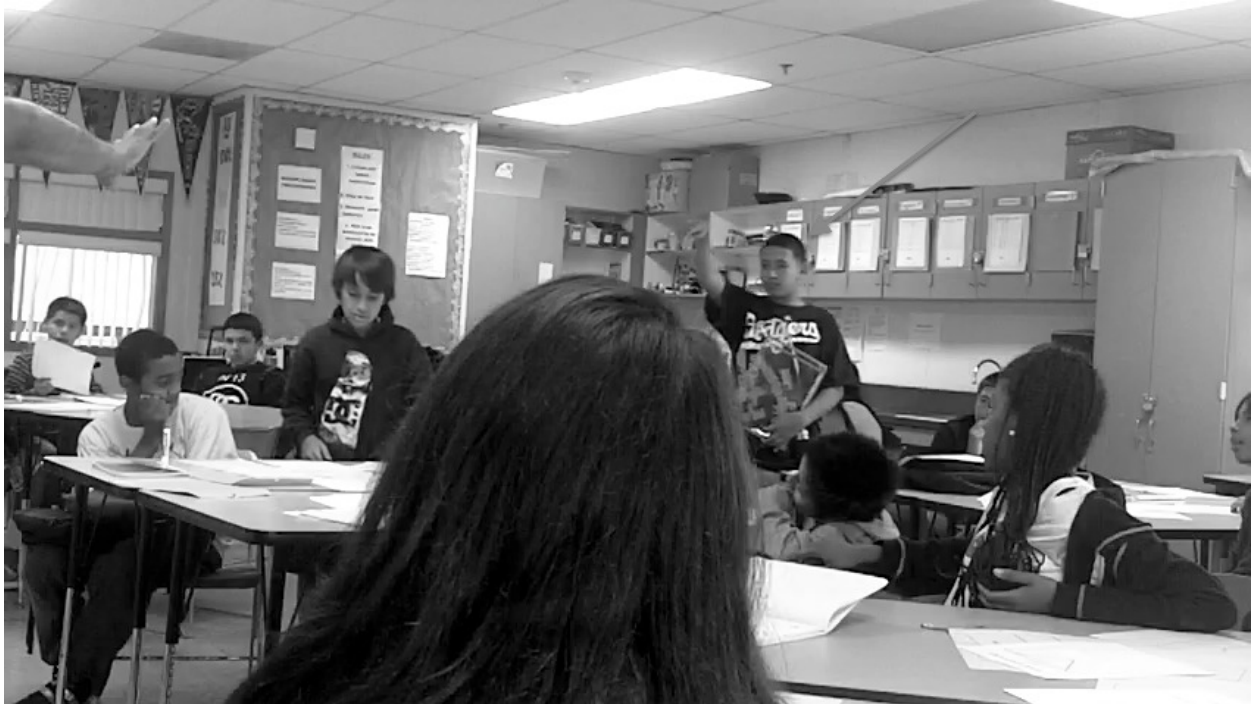


Figure 4-18 - Student raising his hand and leaning forward saying, “Can I show you?”; he wants to argue is point about his choice of function.

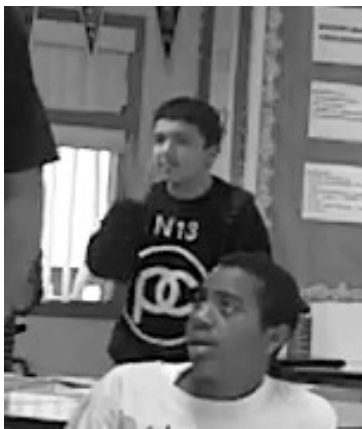


Figure 4-19 - Student demonstrating with his hand and speaking in Spanish to another student to show where he sees an exponential function, “mira de lado” [look at the side].

Group analysis shows that when students are working together they can choose appropriate functions relevant to the situation. Table 4-1 shows the results for the groups’ choices. Important in these interactions is not that they chose the correct function, but that they used the graphical patterns in the image to identify a function representing that pattern. For this task, students associated the graphs of functions to real world objects; which meant they are

recognized a visual pattern and then found a way to represent that pattern in a mathematical manner.

These conversations and interactions are examples of students recognizing and describing patterns of regularity in graphs. I use the term graphs to mean the images that are being represented on a coordinate axes system. As students work collaboratively, each interaction showed students either recognizing a pattern in the image or describing to others where the pattern can be found. What can be inferred from the type of language used is students are still early on in the ability to use formal mathematical language to describe the patterns. Only a few students use the name of functions (“Sine, Sine, Sine”, “Where do you see cubic?”) to describe the pattern at this stage, while most either identify a location in the image (“the car has this on top”, “Her nose, the top of the nose”) or beginning to use some math terminology (“remember he showed us the pattern?”, “It curves more”). As will be seen in another task, students get better at describing these patterns by the using mathematical terminology to name the patterns. I argue that this initial activity of recognition and description are mediators for students to choosing functions relevant to a contextual situation, which will then be a mediator for other mathematical activity.

Table 4-1

<i>Group Results – Cooperative Task</i>	
<i>Image – Function(s) Found In Image</i>	<i>Group Responses</i>
Figure 5 (Quadratic)	All groups identified the graph of a Quadratic function.
Figure 6 (Cubic, Sine)	Five groups identified a Sine curve in the image and one group identified a cubic function.
Figure 7 (Quadratic, Square Root, Exponential)	Five groups identified an Exponential function in the image and one group identified a quadratic function.
Figure 8 (Rational, Linear)	Four groups identified a rational function, one group identified a linear function, and one group did not respond.

Figure 9 (Absolute Value)	Five groups responded Absolute Value function and one group did not respond.
Figure 10 (Sine, Linear)	Five groups chose a Sine function and 1 group chose a Cubic function.
Figure 11 (Exponential, Linear)	All groups chose Exponential Function.

Technology-Driven Task

Technology as a tool was used to support students in recognizing and choosing functions found in images. The initial tasks are multiple response items; students are given functions to choose from when making their decisions. The purpose for the multiple choices is in line with the idea that since these are initial tasks regarding these ideas, I wanted to provide students with the best opportunities to participate. The results for the images are presented in chronological manner, meaning they are presented in the order students encountered them in the class.

Table 4-2 summarizes the images presented to students and their choices for functions found in the images. The results for these three images occurred during the first day students were asked to identify functions in an image. From the first one to the last one, students showed an improved ability to look images and find graphical representations of functions in the images. The number of students who could correctly identified an appropriate function in the image was up to 95% by the last one for the day. In addition, the participation rate of students, at minimum, 18 of the 22 students submitted an answer when prompted about choosing a function in an image. This meant that students engaged with the idea that functions can used to describe the world.

Table 4-2

Number of Participating Students	Image and Function Choices	Student Responses
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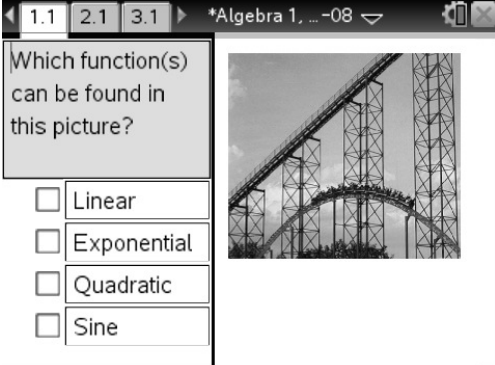
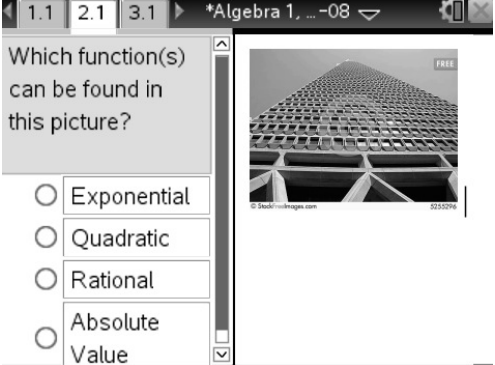
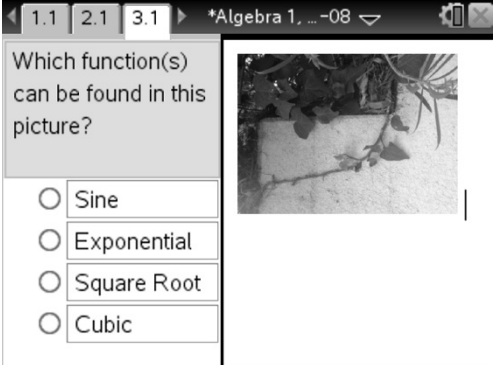
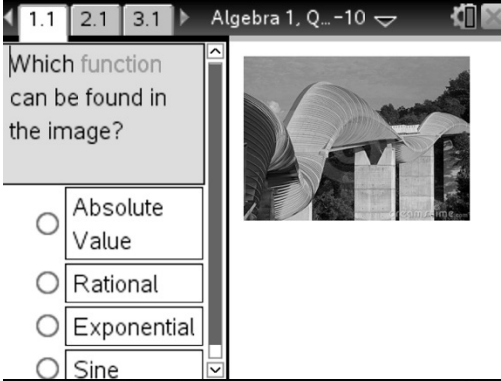
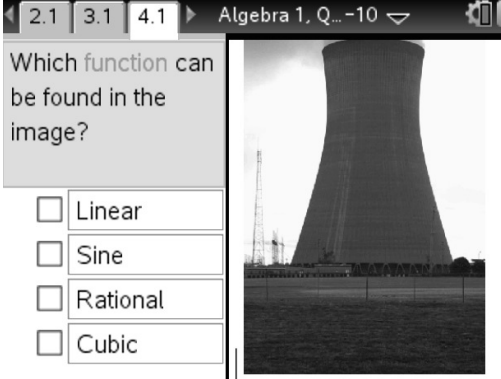
N=22		<p>22 different students responded with a total of 32 choices of functions. Of these selections, 13 chose a linear function, 9 chose an exponential function, 6 chose a quadratic function, and 4 chose a sine function.</p>
N=18		<p>Each student only chose one answer although they were able to choose multiple responses. The overwhelming choice by students (14 of 18, 78%) was the absolute value function. As for the other students, 1 chose an exponential function, 1 chose a quadratic function, and 2 chose a rational function. 19 (95%) of the students said they saw an exponential function in the image. There was a lone student who chose the sine function; I am not sure why they made this choice. However, this student did choose appropriate functions with the two previous images.</p>
N=20		

Table 4-3 summarizes results from a second day of having students identifying functions in images. The results for the first two images show students analyzing an image and choosing a function - based on visual inspection - that is relevant to the situation. Similar to the results from the last image of the previous day, all but one student chose the function that was the most evident in the image of the bridge. Second image was included since I believed it would cause students pause when deciding which function was represented. The ambiguity in the picture forced to students to compare the graphs of functions to the peculiarities of the image. As a result

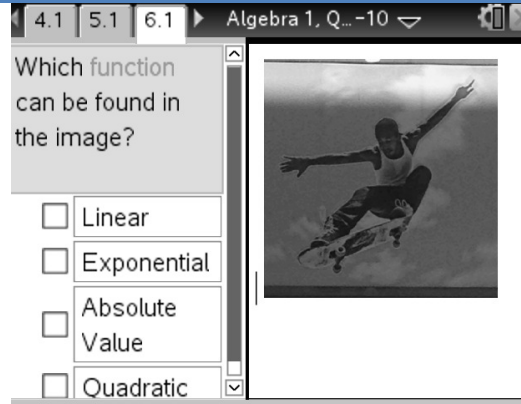
students chose multiple functions to represent the situation. For me, the rational function would have been my choice considering the symmetric nature of the rational function and the symmetric shape in the image. More students choosing the rational function meant that they picked up on the symmetry and shape between the image and the graph. This type of recognition is essential in identifying a potential model from plotted data. As of the choice of a cubic function, I believe the curved shape found in the image could lead to this conclusion. Using that as a measuring stick, this shows students thinking mathematically. For the last image (skater), students identified multiple functions; each of these could easily be argued for as appropriate, especially given the various patterns in the image.

Table 4-3

Summary Results

Number of Participating Students	Image and Function Choices	Student Responses
N=17		Sixteen of seventeen students identified a sine graph in the image of the sinusoidal shaped bridge. One student chose the exponential function;
N=22		Sixteen of twenty-two students chose a rational function, eleven of twenty-two chose a cubic function, three students chose the linear function, and three chose the sine function.

N=22



The linear function was chosen by fourteen of the twenty-two students and the exponential function was chosen by ten of the twenty-two students. The choices by four of the students for the absolute value function and the choice by two of the students of the quadratic function are reasonable with respect to the image.

Pencil-Paper Tasks

The pencil-paper tasks, completed after the cooperative group and technology tasks, asked students to identify functions in images without the support of multiple choices. This meant they had to first individually analyze the image (Figure 4-20) and then find a function from their toolkit (See Appendix E) that they believed was represented in the image.



Figure 4-20 – Pencil-Paper Task Image

The results of the students' choices can be found in Table 4-4. The data represents the functions chosen by students and the number of students who chose that function; students were able to choose multiple functions. For example, of the twenty students who participated in the activity, eight of them identified the graph of a linear function in the image. Additionally, all

students chose a variety of functions they found in the image (Figure 4-20). This indicates that students recognized patterns in the image and chose a function to describe the visual pattern. At this stage it is important to note that whether or not the actual functions are in the image is not important at this point. For example, the arch in at the bottom of the tower is more than likely not a parabola, but that does not make the choice of a quadratic function incorrect in this case. Students are made a connection between the real world aspect of the image and a possible representation with algebraic notation and language. This is a mediating process for them being able to produce a formal mathematical model from data.

Table 4-4

Student Results Functions in Images

Function Found in Image	Number of Students Identifying Function
Linear	8
Quadratic	11
Cubic	11
Exponential	7
Square Root	8
Absolute Value	12
Rational	2
Sine	1

A second pencil-paper task, a culminating activity for these types of tasks, presented students with 11 images (APPENDIX F), they were to choose one function whose graph was represented in the image. The data is reported by the number of images for which students identified an appropriate function (Table 4-5). Eighteen students completed this task. Four students correctly identify a function in each image but thirteen students correctly identified at least 8 functions in the images. One student performed poorly, however, I believe it was not due to their ability, but due to them just not completing the task.

Overall, students developed the ability to see functions in the images by comparing the shapes of the graphs to objects in the images. The marking on the image (Figure 4-21) outlines

the graph the student identified; this ability is evidence that the mediating process of choosing appropriate functions relevant to a contextual situation emerges. This helped students make sense of what it means to do mathematics: identify a pattern, describe with mathematical notations, and then use the notation in a productive manner.

Table 4-5

Culminating Paper-Pencil Task Results

Number of images identified	Number of Students
11	4
10	9
9	3
8	1
4	1

1. Quadratic Function

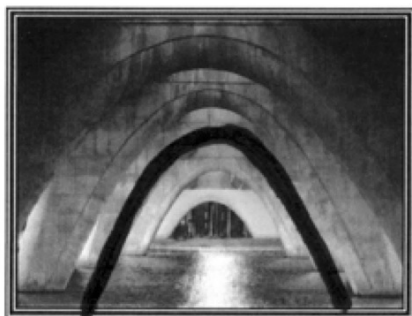


Figure 4-21 - Student Identifying Function in Image

Instructional Design

The plan for instruction for these tasks centered on providing students with a resource (APPENDIX E) to support their choice of functions found in images. This resource should be seen as a tool for them to use to begin to think about how mathematics can be used to describe the world. Observed during these tasks were students using this resource in productive ways to

choose an appropriate function. They were seen looking at the images and referring to the resource to choose a function. The resource became a centerpiece between multiple students as they discussed possible functions that could represent the situation.

My initial plan prior to engaging students in these tasks was to have images appear on their handhelds, and then they would interactively respond and submit their choice of functions to my computer via the handheld. However, I made adjustments to my instruction based on how students engaged with the content prior to these tasks. Looking at my observation notes during the study “I learned that I need to make sure students are actively engaged in the lesson and need to alter the types of activities they engage in.” As a response, the collaborative activity analyzed above was my attempt to have students participate in mathematical activity. Based on the results, not only did mediating processes emerge, but students used mathematical language in a give-and-take manner to discuss the naming of functions found in images. After this collaborative task, I proceeded with sending images to students’ handhelds so they choose the function and submit it to me electronically.

One thing I learned from the transition of initial tasks (Pattern Activities and CBR/Graphing Stories Tasks) is the need to approach classroom instruction like I was a classroom teacher. Meaning that I could not just expect the students just to be well-behaved and fully engaged into the content because I was conducting a study. I needed to conceive of sound instructional strategies that took into account students’ prior experiences and knowledge, as well as strategies that help students have positive learning experiences. These ideas led me to implement the collaborative learning activity as a way to introduce students to functions as a representation of the world. By the end of these tasks, students were able to identify a function inside a picture.

Technology as a Mediator

Since students have access to a handheld graphing calculator and I was using the TI-Navigator computer system (the TI-Navigator system creates a connection between students and teacher wirelessly networking each student's graphing calculator to the classroom computer), the use of technology becomes an important tool to support the emergence of mediating processes and ultimately the desired learning outcomes. As a result, it is important to discuss where in the participation of these tasks did the technology support such learning goals. Up to this point, students were using the handheld to log-in to the TI-Navigator system and to respond to polling questions using the handheld to submit their answers to me. They had also been introduced to the basic features of the handheld via Cross-Number computational activity (Appendix G).

The technology used during these tasks included students choosing functions they find in images that were sent to their handhelds. This personalization was to allow students a safe space to process and think about their choices. They would choose the function, submit it via the handheld, and their response anonymously displayed on my computer. In turn, I would project for the class to see. The purpose of the public display was to encourage conversation about the choices and have students defend their choices when they would see alternative answers. As seen from the results section, most students were able to choose appropriate functions, and classroom discussion showed evidence of students thinking and reasoning about mathematics as well as the beginning use of correct academic language.

Identifying Functions in Data/Scatterplots Tasks

These sets of tasks followed immediately after the identifying functions in images tasks. Three versions of tasks prompted students to identify functions in data or scatterplots. Each task structure is described prior to the presentation of results and findings of students engaging in these tasks.

Task Structure Description

Task Type 1 – Plotted-Data: The first was for a student to be presented with plotted data from some contextual or “real-world” situation, and then asked to identify and choose a function whose graphical representation matches that of the plotted data. For example, in Figure 4-22, data from a contextual situation is plotted (scatterplot) and students are then asked to access what they know about the graphical forms of functions to choose one that best represents the data.

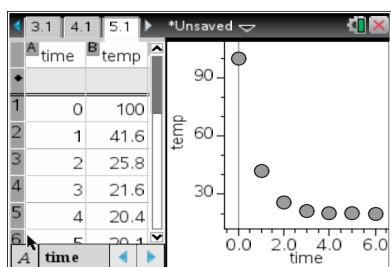


Figure 4-22 – Plotted Data

Task Type 2 – Tabular Data: A second task involves students taking data from a table of values and representing this data in a graphical form. Students used their knowledge of functions and graphs to choose a function relevant to the situation. By transforming the table of values into a scatterplot, students were then able to describe the data with a mathematical notation. Figure 4-23 shows an example of a table of values from a contextual situation.

The “Science of Patterns” - Real Life Situations

<p>D. Height of golf shot. A golfer hits a ball.</p> <p>x = the time that has elapsed in seconds. y = the height of the ball in meters.</p>	<p>Table:</p> <table border="1"><thead><tr><th>time</th><th>height</th></tr><tr><th>x</th><th>y</th></tr></thead><tbody><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>25</td></tr><tr><td>2</td><td>40</td></tr><tr><td>3</td><td>45</td></tr><tr><td>4</td><td>40</td></tr><tr><td>5</td><td>25</td></tr><tr><td>6</td><td>0</td></tr></tbody></table>	time	height	x	y	0	0	1	25	2	40	3	45	4	40	5	25	6	0	<p>_____ Function</p> <p>Sketch and Label the Graph</p> <div style="border: 1px solid black; height: 150px; width: 100%;"></div>
time	height																			
x	y																			
0	0																			
1	25																			
2	40																			
3	45																			
4	40																			
5	25																			
6	0																			

Figure 4-23 – Identifying Functions in Data/Scatterplots Task 2

Task Type 3 – Initial Modeling: A third task involved students identifying a function in data during the initial steps of the modeling process. Figure 4-24 shows a scatterplot of data that has been collected by students. The process of finding a mathematical model included students interacting with real-world data, representing it in graphical form, and identifying a pattern (asking them which function could represent this data). This process mediated their ability to find a formal mathematical model.

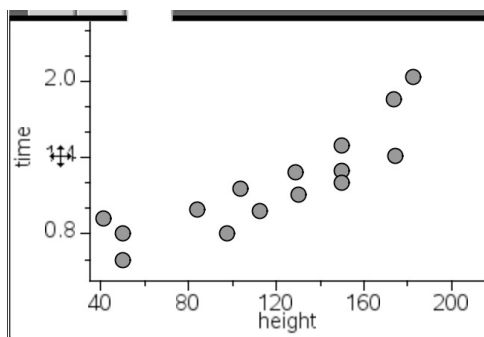


Figure 4-24 – “Real-World “Modeling Data

Student Work and Participation

The following analysis shows how three mediating processes emerged while students participated in tasks where they identified functions in data and scatterplots. The mediating processes were; students recognized and described regularities in patterns in graphs, represented quantities in tabular and graphical forms, and choosing a function relevant to the situation. Both group and individual level findings will be discussed.

Plotted-Data - Data Analysis

Table 4-6 shows the results of three successive scatter-plots presented to students on their handheld devices. For each, students individually responded by submitting their answer to my computer. In each case, the majority of students chose an appropriate function to represent the

plotted-data. For the first plot, thirteen of fifteen students chose a relevant function; for the second one, seventeen of twenty chose a relevant function; and for the last one, twenty-one of twenty-two chose the cubic function. These results indicate that students recognized that pattern in the plotted data, connected the basic graphical form of a function to that pattern, and then chose an appropriate function.

For those students who did not choose an appropriate function, there were only a few instances where students chose an unrelated function. For the first plot, only the absolute value function was unreasonable because of its “V” shaped graph; and only one student chose this function. In the second plot (curved), three students chose the linear function and none chose the sine function. Since no explanation accompanied student choices, it is difficult to know exactly why three students chose a linear function to represent the curved data. Important to note is that by not choosing the sine function, it shows students can identify functions that are not relevant to the situation; an ability on a larger scale that is an essential for effective mathematical thinking.

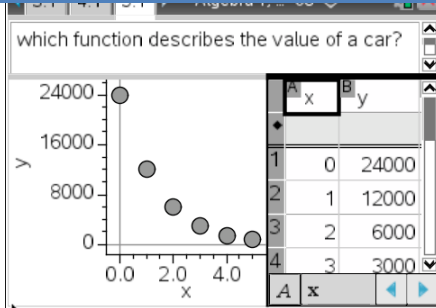
For the final prompt of this task type, one student chose a rational function. The student who chose the rational function previously had no trouble identifying a relevant function for the other tasks, so it is difficult to know exactly why they did not choose the cubic function like all the other students. A thought is that the plotted data demonstrates similar behavior to that of the y-values of the rational function; the y-values are unbounded (increase without stopping) for both functions. Assuming that this may be a reason why the student chose the rational function as the representation, it shows that the student used the patterns in the graph to make a choice.

Table 4-6

Plotted-Data Student Results

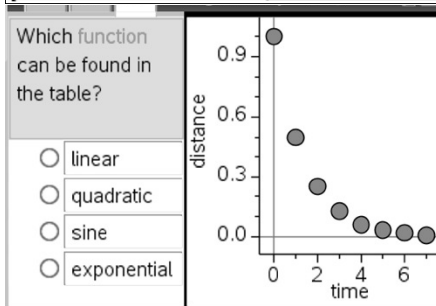
Number of Participating Students	Plotted-Data	Student Responses
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N=15



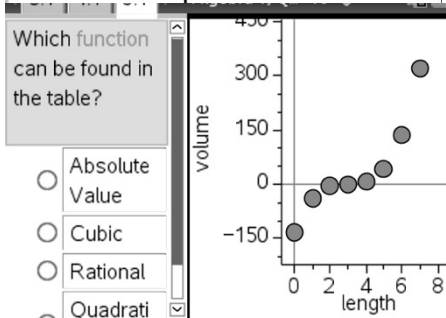
Of the fifteen participating students, seven chose exponential function, four chose rational function, two chose $y=2^x$ (algebraic form for an exponential function), and one chose absolute value.

N=20



Of the twenty participating students, fourteen students identified the data as an exponential function, three as quadratic, and three as linear.

N=22



Of the twenty-two students participating, twenty-one chose the cubic function and one student chose the rational function as the representation.

Tabular Data - Data Analysis

For these task types, students represented a table of values in graphical form and identified a function that represented the plotted data. Figure 4-25 provides an example of what sixteen of twenty-one students produced in this task. These students represented the quantities in a table, plotted the data by hand and using technology, and identified the graphical pattern as a linear function. One of the sixteen students not only chose a linear function to represent the situation, but the student found a specific function ($y=x+1$) to describe the situation (Figure 4-26). Whether the student used the description in the context, the values of the quantities, or the plotted to data to derive the equation is unknown, but this attempt forms beginning steps to deriving a mathematical model.

Three students produced a correct table but did not identify a function to describe the data; the reason for this is unknown. More interestingly, two students created the table, plotted the data, and then described the data with an incorrect function; one student chose absolute value and the other chose exponential function. Its possible the student who chose the absolute value was thinking of the “V” shape graph produced by the absolute value function in terms of pieces. Thinking of absolute value graph in piece-wise sections – two linear parts – makes the choice of the absolute value function acceptable. The student in this case not only recognized a pattern after representing the quantities in graphical form, but also chose a function that is relevant to this situation.

2. Complete the table of values. Then plot them using your calculator to determine which function describes the pattern.

Tu Daddy's Pizza (Price Menu)
 Large Cheese Pizza \$5.00
 \$1.00 for each topping

<i>Toppings</i>	<i>Cost</i>
x	y
0	5.00
1	6.00
2	7.00
3	8.00
4	9.00
5	10.00
6	11.00

The pattern that is formed by the table of values is a(n) Linear function.

Figure 4-25 – Student Written Work #1

2. Complete the table of values. Then plot them using your calculator to determine which function describes the pattern.

Tu Daddy's Pizza (Price Menu)
 Large Cheese Pizza \$5.00
 \$1.00 for each topping

<i>Toppings</i>	<i>Cost</i>
<i>x</i>	<i>y</i>
0	5.00
1	6.00
2	7.00
3	8.00
4	9.00
5	10.00
6	11.00

The pattern that is formed by the table of values is a(n) $y = x + 1$ function.

Figure 4-26 – Student Written Work #2

A second task (Appendix H), completed after the plotting-data tasks, used technology to study graphical patterns and functions represented in multiple forms (graphical, tabular, algebraic). The analysis highlights common responses made by most students, and unique answers that are incorrect, but demonstrate the emergence of mediating processes.

The first two responses (Figures 4-27,4-28) are representative of how fourteen of twenty students responded to the prompts. Although the hand plots are not scaled correctly (probably due to the translation from the hand-held to paper), they show students representing the data in multiple forms, recognizing patterns, and choosing functions relevant to the situation. At this time point in the class, students had not been asked to formally name the patterns with algebraic representations. However, naming the pattern with a function name was a primer for finding a model for the data.

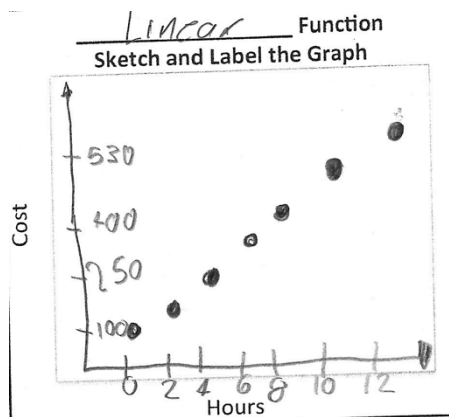


Figure 4-27 Linear Function Sample

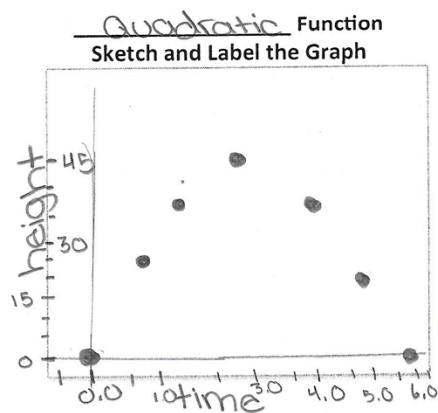


Figure 4-28 Quadratic Function Sample

The two examples in Figure 4-29 and Figure 4-30 are of interest because of the incorrect mathematics displayed. However, these students still demonstrated they could choose a function relevant to the situation. They do this by matching what they know about graphs of functions to the data they plotted. The response in Figure 4-29 is for the same data found Figure 4-28, however, the student inverted the columns in the table and plotted the quantities time versus height. Unfortunately, the inversion is evidence that the student did not reason about relationship between the quantities. Despite these shortcomings, the student still identified a quadratic function as the best representation for the data.

Due to the lack of labels (Figure 4-30) on the axis it appears the student plotted the points based on looking at a graph on a calculator. The student chose the cubic function as the representation for data although the context of the data was periodic in nature (sinusoidal). Given the initial stages of growth for these students in regards to functions representing the real world, the choice of a cubic function makes sense based on the curved shape of the data. Both the cubic function and the sine curve have points of inflection (point where graph changes concavity), which makes these graphs look similar. Based on these characteristics, the student chose a function relevant to the situation.

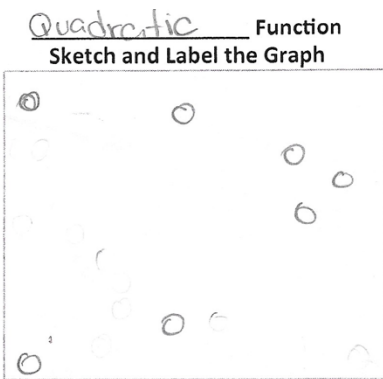


Figure 4-29 Quadratic Function Sample 2

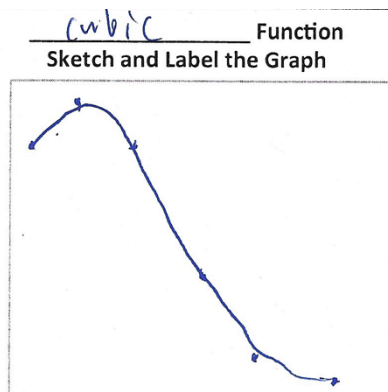


Figure 4-30 Cubic Function Sample

Initial Modeling - Data Analysis

This task type had students identify a function from data they have collected. The data and analysis come from the initial stages of students going through the modeling process. In later tasks, students used derived models from this process to predict an outcome for an event of interest. In this first case (Appendix I), students predicted the time it would take for a miniature parachute to fall a distance of 3 stories (965 cm). In order to do so, they collected data for the drops of the miniature parachute at various heights under 200 cm. For a second task students collected and plotted for a situation where students found a mathematical model representing a physical a bungee jumping situation. They did this using rubber bands and Barbie dolls to replicate a bungee jump using various lengths of bungee. Figure 4-31 and Figure 4-32 display sample student work for each task.

Starting Height (x)	Fall Time Trial 1	Fall Time Trial 2	Fall Time Trial 3	Average Fall Time (y)	Ordered Pair (x,y)
50	0.8	0.9	0.6	0.8	(50, 0.8)
100	0.9	0.9	1.0	≈ 0.9	(100, 0.9)
130	1.5	1.0	1.4	1.3	(130, 1.3)
150	1.6	1.4	1.2	1.4	(150, 1.4)
200	1.3	2.1	1.9	≈ 1.8	(200, 1.8)

Figure 4-31 Sample Student Work Barbie

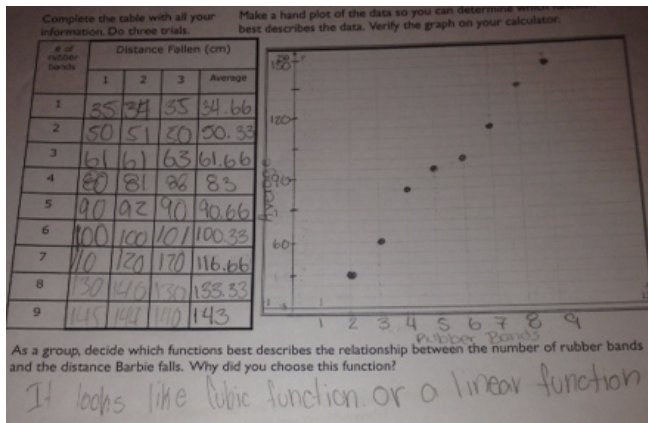
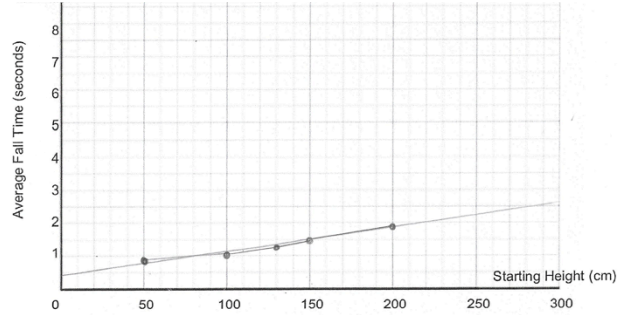


Figure 4-32 Sample Student Work Barbie

Across both tasks, although at different rates of participation, students collected and represented data in tabular and graphical forms (89% for task 1 and 65% task 2, Table 4-7). They also showed the ability to analyze various features of a graph then choose a relevant function to represent the data. More telling than their choice of function was the explanation for their choice. Comments like “it looks like exponential because it looks curv[y]” and “linear because I see the diagonal lines going a positive slope” show a level of mathematical sophistication and understanding by students to eventually derive a mathematical model to represent this situation and make predictions. Evident in these responses is the students’ ability to choose a function by comparing the pattern of the plotted data to that of graphs of known functions. Table 4-7 contains more evidence, “Linear, I think [it] is linear because the line is

going straight and because there isn't any other function that looks like this." This student went through their database of functions to choose a function that matches the data.

A unique example (Figure 4-32) provides more evidence of mediating processes emerging. A student's response for their choice of a function, "it looks like [a] cubic function or a linear function" is based on the shape of the plotted data. From a global perspective, the overall trend of the data appears to be linear, but upon further inspection, the center of the data shows a change in concavity. This change in concavity gives the data a cubic look; a prominent feature for the graph of the basic cubic function. The student developed ability to not only to represent data in multiple forms (tabular and graphical), but to analyze various features of a graph and then choose a relevant function to represent the data.

This written work and explanation of why they chose a particular function indicates how students thought about graphical patterns. A student describing why they choose a function based on how the data looks, led to them choosing an appropriate function whose graphical representation resembles the plotted data. Students interacting with real-world data by taking the data, representing it in graphical form, and identifying a pattern (asking them which function could represent this data) mediates their ability to find a math model to represent the data. This further buttresses the evidence that the mediating processes (representing quantities in tabular, graphical, algebraic forms, recognizing and describing regularities in patterns of graphs, and choosing functions that are relevant to a situation) emerged within these tasks.

Table 4-7

Task	Functions Chosen	Student Written Reasoning
Parachute Activity (N=19)	Seven students responded that the data showed a linear pattern, six students identified the exponential function as a possible	<p>"It looks like exponential because it looks curv[y] and because the points"</p> <p>"And it looks like linear because it is kinda straight"</p>

Bungee Jumping Activity (N=17)	function, three stated the square root function, and one said the cubic function fit their data, two students did not respond	“Linear because I see the diagonal lines going a positive slope”
		“Exponential because you can see the lines curve a little bit”
		“Its exponential or linear function because it has a curve and it also looks a diagonal line”
		“Linear because it looks like a line”
		“The function is linear because it almost looks like a line”
		“It looks like a cubic function or a linear function”
	Eleven of the seventeen students stated the data looked linear and/or cubic. The rest of the students either plotted and did not choose a function (n=5) or simply did not plot the data (n=2).	“liner por ninguna funcion se paresca a la linear”
		“Each time you add a rubberband to a barbie doll, the barbie doll bungee jumps lower than before (it makes a positive slope going diagonally - linear function)”
		“because is looks like a line”
		“Linear I think is linear because the line is going strait and because there isnt any other function that looks like this”

Instructional Design

Identifying functions from data and scatter plots is a more complex task than identifying a function in an image. Looking at a real-world image immediately contextualizes the situation for the students; while examining and plotting numerical data adds a layer of abstraction to the process of choosing an appropriate function to represent the situation. Without explaining to students the numerical characteristics of different functions, students have to first plot the data before identifying a function that could be a representation for the data. This means a procedural

step needs to be completed prior to students being able to recognize and describe the pattern; this step is to plot the data. During this course, students were shown how to do this to do using their handheld graphing technology. I will discuss in the next section how this supported student learning. Given this background, I learned that it was important to create alternative learning experiences and to contextualize the data otherwise some students would lose interest in the task.

These specific tasks occurred about half way through the course, so by this time I had become aware of classroom instruction that facilitated increased student engagement. For instance, I utilized more collaborative activities where students distributed the work in their groups to accomplish specific goals. In one task, students were provided with eight tables of values representing real-world situations (Appendix J) they were to identify an appropriate function for the data. Instead of having students complete all the tables of values, students were each assigned two of the situations to work on. They then met with another student in the class to explain and verify their findings. Once they were confident with their work, they returned to their original group to explain why they chose the function they did.

Another thing I did was to begin classes analyzing data from the real world or data that was collected by them. I did this to contextualize the mathematics they were doing; examples included showing videos of situations and presenting data about those situations and having them physically collect data in order to answer a question. Videos I used were cars skidding to stops to analyze stopping distances, a CNN video of the Asiana Airlines plane crash in San Francisco (Summer, 2013) to discuss the data on landing a plane, and videos students made while shooting a basketball to determine the path of the ball. Collected data included their shoe size and height, distance they live from school and the time it takes to get to school, and identifying the horizontal distance and height of a ball as it was tossed in front of a whiteboard. Many of these

activities were added to my daily plans in order to better connect the mathematics they were learning to real-world situations.

Technology as a Mediator

In addition to role the handheld graphing calculator and TI Navigator played in the previous tasks - interactive communication and feedback between students and myself – the handheld was used to assist students with the process of graphically representing data. Within these tasks students were expected to choose an appropriate function that represents the data found in a table of values. To do this, students first had to make a scatterplot. However, to assist with the tedium of hand plotting or those students who need additional support with the procedures to correctly produce a plot, students were allowed to use the handheld to produce the scatterplot. The flexible use of the handheld to quickly produce these plots allowed students to examine the patterns in the distributions. Being able to see the data plotted in a correct and efficient manner mediates their ability to then choose a function that could be relevant to the situation.

Figure 4-33 is a sample of written and calculator work produced by a student. The student begins by creating a table of values based on the given situation. Using a graphical approach to analyze the data, the student then enters the data into a list on the handheld. Once into the calculator, a scatterplot is quickly produced. With the data now plotted, the student is then able to make an educated decision about which function represents this data. In this case, the student identifies the linear function as the appropriate choice. Since there is no explanation, we can only assume because the data forms a straight line that this is why the student chose the linear function. It is important to note that I could have first instructed to students to examine the rate of change in the table as a way to identify an appropriate function. However, because only the linear function produces a somewhat trivial result when considering the rate of change or first

differences, I thought it was a conversation worth having only after students approached these situations from a graphical perspective.

For these tasks, having students engage in identifying functions in data and scatterplots with the use of technology led to the emergence of students being able represent quantities in tabular and graphical forms, recognize and describe regularities in patterns of graphs, and choosing functions that are relevant to a situation. In sum, the results show what it looks like for students to participate and produce mathematics as well as what it looks like for the mediating processes to emerge.

2. Complete the table of values. Then plot them using your calculator to determine which function describes the pattern.

Tu Daddy's Pizza (Price Menu)
 Large Cheese Pizza \$5.00
 \$1.00 for each topping

<i>Toppings</i>	<i>Cost</i>
x	y
0	5.00
1	6.00
2	7.00
3	8.00
4	9.00
5	10.00
6	11.00

The pattern that is formed by the table of values is a(n) Linear function.

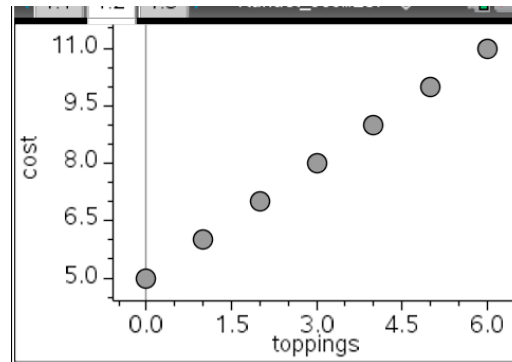
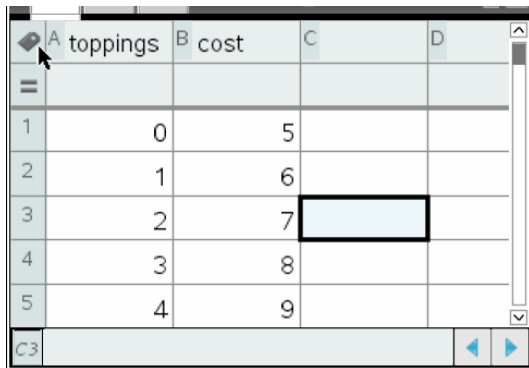


Figure 4-33 Student Written and Calculator Work

Deriving Models Tasks

The deriving models tasks are at the heart of what it means to do mathematics from a “science of patterns” perspective. The creation of models to describe a pattern follows the early steps of Devlin’s *Mathematical Method* (2012):

- *Identify a particular pattern in the world*
- *Study it*
- *Develop a notation to describe it*
- *Use that notation to further the study*

This section will begin with a brief description of what the tasks looked like.

Task Structures Description

The specific tasks include deriving models of functions in images, deriving models from data, and deriving models from collected data. The intention of the latter two tasks and to some degree the first task was for students to use their models to answer/pose contextual questions about the quantities involved in the situation and to develop an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations. An example of each can be found in Appendix K. It is important to keep in mind that at the core of the modeling process is the use of technology to engage in the iterative process of expressing, testing, and revising until an appropriate model is determined.

Student Work and Participation

The analysis and findings for these tasks will come from a snapshot of the specific tasks students engaged in during the class. I analyze the work from the perspective of identifying how students engaged in the iterative process of finding a model to fit the data in the given situation. The focal mediating processes emerging from these modeling tasks are students representing quantities in multiple forms (tabular, graphical, and algebraic), students choosing functions relevant to a situation, and students testing and revising to determine whether their chosen function is an appropriate model for the situation. Students recognizing regularities in patterns

Table 4-8

Modeling Process (Street Task)

Number of Students (N=19)	Modeling Process
n=2	This group obtained a linear model for the streets mainly by altering both the slope and the y-intercept in an iterative process. They then checked to see if the model matched the street in the image.
n=13	It appears this group derived their model by first calculating a correct slope for their model and then altering the y-intercept to check to see if an appropriate fit was made, meaning they found a linear model that matched the street in the image.
n=4	This group either did not do anything significant in regards to the assignment or simply did not document the work they did to obtain the models for the streets.

The first group of students found models to represent the streets in the image. The results show the work for one of the streets outlined in the assignment. Although the complete task asked students to find models for five streets, I limit my analysis to the first assignment prompt – finding a mathematical model representing the street that goes through the two points $(-4,0)$ and $(-2,4)$, Broadway Street (See arrow on Figure 4-34). The written work for the two students is found in Figure 4-35.

The work for both students shows a sequence of iterative attempts to obtain a model for the Broadway Street. They began their modeling process starting with the equation $y=x$; this shows students knew they wanted to use a linear equation to model Broadway Street. Explanations of how the model is altered to find a better fit is also seen in the student work. In the first case (Figure 4-35), the student begins altering the original model by recognizing that the $y=x$ model does not match or fit Broadway Street, the student says “make it higher” to acknowledge the $y=x$ needs to be moved up to fit the street. The second student then proceeded in a similar manner; choosing to revise their model each time by changing the y-intercept. The

student stated, “a lot”, “higher”, “higher”, “lower”, “perfect”, as they described the alterations needed to adjust their model to fit Broadway Street. The trajectory of alterations, $y=x$, $y=x+2$, $y=x+4$, $y=x+6$, $y=2x+8$, shows the student tried to find a model by examining the graph of the chosen model and comparing it to the street on the image.

Another implication of this work is the connection between the changes students suggested and how that altered their choices for the parameters in their models. For example, the first student chose $y=x+4$ as the next model after $y=x$ and recognized that this was not good; they suggested to “make it align” to fix the problem. The next moves were also to the y -intercept, $y=x+2$, $y=x+6$; this moved the linear model down and then up. Following the $y=x+6$ model, the student said they needed to make the their model a “little lower”. Based on the suggested adjustments by both students, their models were altered appropriately until they obtained a good-fit model. A full trajectory of the student’s models can be found in Appendix L. Each picture displays the graph and equation for the model being tested.

A limitation of this analysis is not being fully aware of any mental or undocumented modeling attempts by the students in the modeling process. If existing, knowing the models could shed further light on their thinking and how the meditating processes emerged within the tasks. With that said, each student showed evidence that they chose a function relevant to the situation - a linear model to represent the street; and that they used their knowledge of linear functions along with the graphing technology to test and revise their models to obtain a line of best fit.

Broadway $f_1(x)=mx+b$	
Equation	What Adjustment are you going to make?
$y=x$	make it higher
$y=x+4$	make it a ligh
$y=x+2$	higher...
$y=x+6$	little lower
$y=5x+6$	higher.
$y=2x$	I need a plus.
$y=2x+8$	I got it.

Broadway $f_1(x)=mx+b$	
Equation	What Adjustment are you going to make?
$y=x$	alot
$y=x+2$	higher
$y=x+4$	higher
$y=x+6$	lower
$y=2x+8$	perfect

Figure 4-35 Student Models

The second group consisted of the students ($n=13$) who derived their model by first identifying a linear function as a possible fit, then calculating a slope for their model using the given points on the image, and then altering the y-intercept to check to see if an appropriate fit was made. My evidence that they calculated the slope first rather than using a trial and learn method is based on how they recorded the slope in their equation. They either recorded the slope in fraction form ($4/2$), meaning they used the classic “rise over run” method ($m = \frac{y_2 - y_1}{x_2 - x_1}$), or they recorded the correct slope of 2 for their initial model, so I am hypothesizing that they also calculated the slope first before doing any trial and learning process to obtain a model that represents Broadway Street (Figure 4-34).

Figure 4-36 shows student responses for four students who calculated the slope prior to engaging in their search for a model to fit Broadway Street. It is evident in their iterative attempts that they knew how to choose an appropriate function and alter it to find a function that goes through the street. Their documented adjustments also show they knew how to change parameters in the model to find their model. Their comments for their adjustment included “I just need to find the y-int”, “change the y-int” “move up to 4”, “slope is good, move to -4”. These statements indicate that these students believed their slope is correct. The student’s comment, “keep the slope” indicated that they were aware that a model with a slope of 2 is a good start. This good start is probably related to the fact that their first attempt, $y=2x+3$, is parallel to Broadway Street.

These examples show student recognizing the linear pattern in the street, choosing an appropriate function to begin the modeling process, and obtaining a final model via a test and revise strategy. Their trail of equations is a good way to see the students thought process until the graph of the model overlaps the street they were representing.

Broadway $f(x)=mx+b$	
Equation	What Adjustment are you going to make?
$y=x$	go higher
$y=2x+3$	Keep the slope
$y=2x+6$	
$y=2x+7$	change y intercept into 8
$y=2x+8$	

Broadway $f(x)=mx+b$	
Equation	What Adjustment are you going to make?
$y=4/2x+4$	I just need to find the y-intercept.
$y=4/2x+8$	I just add 4 more to the y-intercept and I get it.

Broadway $f(x)=mx+b$	
Equation	What Adjustment are you going to make?
$\frac{4}{2}x+8$	change the y-int
$\frac{4}{2}x+7$	change the slope
$\frac{4}{2}x+7$	change the y-inter
$\frac{4}{2}x+8$	right

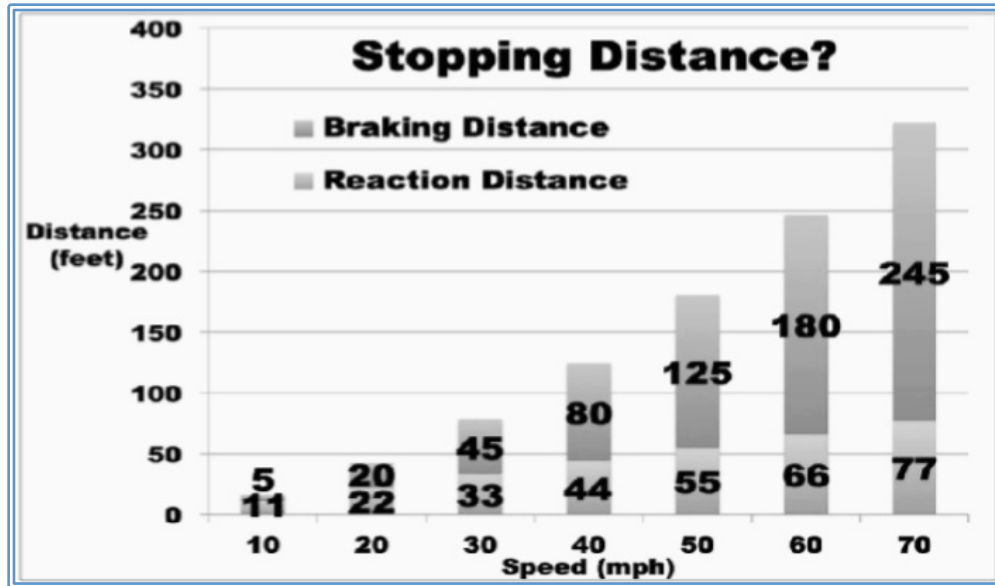
Broadway $f(x)=mx+b$	
Equation	What Adjustment are you going to make?
$y=2x$	slope is good so through -4
$y=2x-4$	move up to 4
$y=2x+4$	move up
$y=2x+8$	up
$y=2x+6.93$	up
$y=2x+8$	got it.

Figure 4-36 Student Models

Deriving Models from Data/Collected Data

The task analyzed here prompted students with the following graphic, scenario, and questions (Figure 4-37).

At many automobile crime scene investigations, investigators often measure the skid marks left behind by those cars involved in the accident. By measuring and recording the length of the skids marks, crime scene investigators can determine the speed of the car. We will use the data below to determine how they do this.



1) What is the stopping distance for a car that is traveling 35 mph and 100 mph? 2) When driving in front of a school, the posted speed is 25 mph. One day you are driving in front of a school and have to come to a quick stop. The neighborhood officer sees this and measures the skid mark left behind your car. He measures the skid mark; it turns out to be 50 feet. He then writes you a ticket for speeding, was he right or wrong to do so?

Figure 4-37 Stopping Distance Prompt

Those students who successfully addressed the prompted questions made sense of the quantities involved, created a table of values from the numbers in the graphic, made a scatter plot using the values in the table, recognized a pattern in the plotted data, identified a function to represent the pattern, and then determined a mathematical model that fit the data. These skills, which were my hypothesized mediating processes, emerged (students showed the ability to do these things) while students engaged in the given tasks.

By the end of this activity, which was one of the last tasks students completed in the course, work samples of the eighteen students ($n=18$) who turned in their work and had attempted some aspects of the task (Table 4-9) were collected. Without discounting those students who completed little, this analysis focuses on the first two groups of students. Figures 4-38 to 4-43 show the written work for six of the eight students who documented their process for finding a model to fit the Stopping Distance data. From this group, two categories of students emerged in this activity; one group identified a linear function as an appropriate function to model the data, and the other group recognized that the linear function might not be the most appropriate model and altered their plan of attack by trying a quadratic function.

Table 4-9

Student Results Stopping Distance

Number of Students (N=18)	Stopping Distance Modeling Process
n=8	eight students documented their modeling process with written and technological documentation,
n=6	six students completed the work mostly on the calculator, either with a test and revise process or with the transformation tool
n=4	four students completed little or nothing.

To start, the two students who used only a linear function for their model both approached their changes in a similar manner. Each chose to alter the value of the slope during the process. Indicating this are their explanations for adjustments, “add to the slope” and “put it more up” (Figure 4-38, Figure 4-39). The second statement accompanied successive changes in the equation for the model, “ $y=x$ ”, “ $y=5x$ ”, “ $y=4$ ”, “ $y=2x$ ”, “ $y=2.3x$ ”, “ $y=4.0x$ ”. These adjustments showed that the student tried to fit the graphs to the data points by altering the “ m ” (value of the slope) parameter in the $y=mx+b$ equation. This meant they were aware of how the change in parameters of the linear function impacts how the graph appears.

Attempt	Equation	What Adjustment are you going to make?
1	$2x+2$	make both numbre grente
2	$4x+5$	add 0.6 to the slope
3	$5x-59.8$	add .0.5 to the slope
4	$5.56x-64.6$	
5	$5.15x+38$	

- 1: Make both numbre grente
 - 2: add to the slope
 - 3: add 0.5 to the slope
- Figure 4-38 Student Models**

1	$y = x$	Put it more up
2	$y = 5x$	
3	$y = 4x$	
4	$y = 2x$	
5	$x = 2.3x$	
6	$y = 4.0x$	

- 1: Put it up more

Figure 4-39 Student Models

The second group of students who documented their work started the model seeking process by choosing a linear function (Figures 4-40 to 4-43). Two of them started with “ $y=x$ ”, the others started with “ $y=3.91x$ ” and “ $y=4.7x-39$ ”. As I mentioned previously, it was not documented why they chose those starting functions, all we know is that their initial choice was a linear function. The item differentiating this group is their choice to change their model type from a linear to a quadratic function. Students articulated some of their thought process, “Now I think it is quadratic”, “Not even close to a line”, “Try a different function”, “try a Quadratic” (Figures 4-40 to 4-43). These statements indicate a conscious recognition on the students’ behalf that their initial plan of attack to use a linear function to model the data was appropriate in this case.

From this group, Figures 4-41 and 4-43 show the students’ understanding of how the parameters in the models need to be changed in order for the model to graphically fit the data. The two students used the following phrases when describing the adjustments they made: “add a slope and y-int”, “change the vertex”, “change the a”, “I just need to find a slope”, “found my y-intercept but just need my slope”, “I’m really close now”, “I’m just trying to find the x-value”. The students are making an explicit connection between the graph and how altering certain

values in the equation influence how the graph looks and fit the data. Figure 4-44 shows the graphical trajectory of the models from Figure 4-43.

Attempt	Equation	What Adjustment are you going to make?
1	$y = 3.91x$	Not even close to the line
2	$y = 0.15(x-20.5)^2 + 40.5$	So close
3	$0.062(x-1.19)^2 + 9.65$	Yay????
4		↑ ↑
5		↑ ↑
6		The Correct one
7		one

- 1: Not even close to the line
 - 2: So close
 - 3: Yay???? The Correct One
- Figure 4-40 Student Models**

Attempt	Equation	What Adjustment are you going to make?
1	$x = 4.7x - 39$	Try different function
2	$x = 0.07(x+18)^2 + 5$	IDK
3		

- 1: try a different function
 - 2: IDK
- Figure 4-42 Student Models**

Attempt	Equation	What Adjustment are you going to make?
1	$y = x$	I just need to find a slope.
2	$y = x + 10$	found my y-intercept but just need my slope.
3	$y = 0.7(x-1.8)^2$	I'm really close. Now I think it is quadratic.
4	$y = 0.7(x-1.8)^2 + 7.9$	I'm just trying to find the x-value.

- 1: I just need to find the slope
 - 2: found my y-intercept but just need my slope
 - 3: I'm really close. Now I think it is quadratic
 - 4: I'm just trying to find the XXXXX
- Figure 4-41 Student Models**

Attempt	Equation	What Adjustment are you going to make?
1	$y = x$	add a slope and y-int
2	$\frac{1}{2}x + 10$	try the Quadratic
3	$1(x-5)^2 + 4$	change the vertex
4	$1(x-10)^2 + 11$	change the a
5	$0.094(x-7.65)^2 + 22.6$	it was close
6	$0.094(x-0)^2 + 0$	
7	$0.081(x-0)^2 + 6$	

- 1: add a slope and y-int
 - 2: try the Quadratic
 - 3: change the vertex
 - 4: change the a
 - 5: it was close
- Figure 4-43 Student Models**

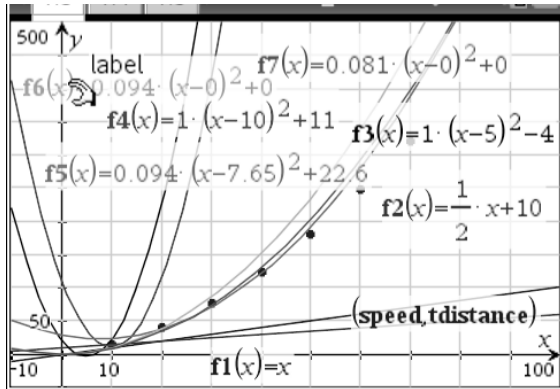
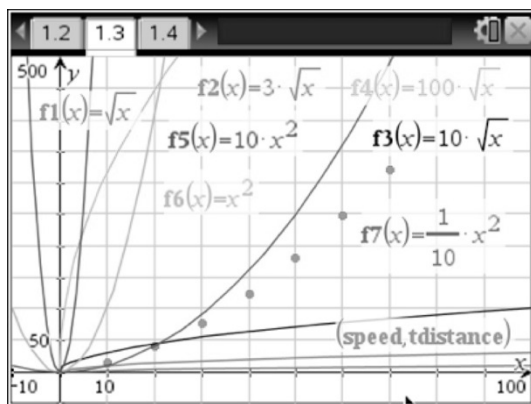


Figure 4-44 Student Models

This analysis is for those students who did not document in writing their modeling process but instead found their model on the handheld either by a test and revise process or with a transformation shortcut. The evidence for first case was found in the trail of functions left on the students handheld. Evidence for the second case was based on my interactions with students while they were obtaining a model during the lesson. It is important to note that the transformation shortcut is a process where students manually fit a graph over the data. While using this method, the equation dynamically changes with respect to the transformations being made to the graph. This results in students finding a model without altering the parameters in the equation. Although this was not the original intent of the task, I allowed some students to use this process. For me this was still evidence of them choosing an appropriate function for the situation and them using tools strategically to engage in mathematics. I viewed this as a way for all students to participate in the modeling process, especially those who had produced little work up to this point or were struggling with the mechanics of finding a model. For both cases, I collected their work from their handheld.

I highlight the work of one of three students who completed their test and revise method for finding a model using the calculator without documenting the process in writing. Figure 4-45

shows the seven functions and the graphs of those functions. Based on the pattern of the data points, the student chose the square root function ($y = \sqrt{x}$) to model the data. This was a reasonable choice since the graph of the square root function has the appearance of a quadratic function; the square root graph is mirror reflection of the quadratic function. Seen in the student's function trajectory is their attempt to fit the data by a test and revise process. After a few attempts by altering the parameters of square root function, the student makes a change of direction and tries a quadratic function for their model. The student altered the leading parameter to fit the graph to the data. They do this by changing the values from 10, to 1, and then to 1/10. This indicates their understanding of how the parameters in the equation impact the shape of a graph. Although their final model does not go through the data points, the student showed an ability represent the data in multiple forms, to choose a function relevant to the situation, and then use a test a revise method to find a model for the situation.



$$\begin{aligned}
 f1(x) &= \sqrt{x} \\
 f2(x) &= 3\sqrt{x} \\
 f3(x) &= 10\sqrt{x} \\
 f4(x) &= 100\sqrt{x} \\
 f5(x) &= 10x^2 \\
 f6(x) &= x^2 \\
 f7(x) &= \frac{1}{10}x^2
 \end{aligned}$$

Figure 4-45 Student Models

This final analysis is for students who did not document their modeling finding process and used the transformation tool on the calculator to find their model. Given the interactive nature of the process- students using the click and drag feature to manipulate the graph of the function – it was difficult to demonstrate what this looks like when a student engages in that

process. Figure 4-46 has the end result for two students who used the process. I recall these students were sitting next to each other and told me they used this process to find the quadratic and linear model. The students tried three different models, and based off the order of the labeled functions, they started with the square root function ($f_1(x)$) and progressed to the quadratic ($f_2(x)$) and linear ($f_3(x)$) functions. Two items indicating their use of the transformation tool are the level of precision for the parameters in the equations ($f_2(x)$ and $f_3(x)$) and the fact that they have different equations although working together during the process. As a subtext, these two students were often unproductive and displayed some resistance to the content during the course. So although the use of the tool was not intended learning outcome, its use supported students in finding a model to describe the data. Despite the fact they did not revise their models by manually altering the parameters in the equations, the students represented the quantities in multiple forms (tabular, graphical, and algebraic), recognized a pattern, chose a function that could represent that pattern, and then used a transformation tool appropriately to find an appropriate model for the situation.

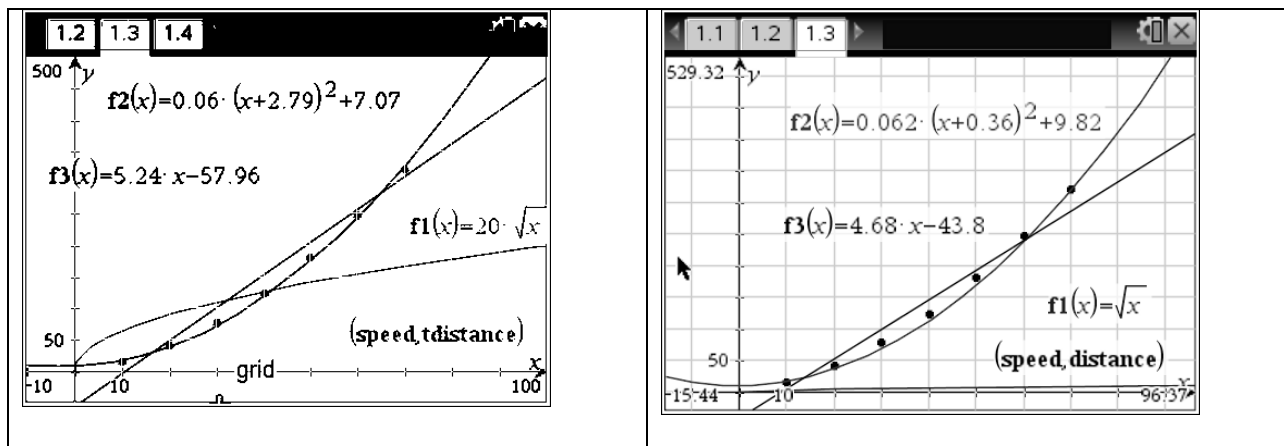


Figure 4-46 Student Models

Instructional Design

Given there was a time limit for the course (5-weeks), a major challenge was gauging how long it would take for students to productively engage in the various tasks. So engaging students in the derivation of models tasks – which encompassed more steps than previous activities – took longer to complete than I anticipated. This was also because students were participating at different levels based on their motivation, behavior, and prior knowledge. Another thing to consider was the progression of the tasks, the deriving models tasks mostly happened later in the course; being able to give all students adequate time to fully develop their models was not always possible. As happens in regular classes, these challenges resulted in students progressing at different paces and with my having to find ways to engage students in the content. At the same time, I wanted to make sure I was not being too prescriptive with my instruction; telling students exactly what to do in order to move through content. Student activity had to result from genuine interaction with data, patterns, and the modeling process.

A specific thing I did to help students connect mathematical modeling to the real world was to provide students with physical contexts. These contexts included collecting data to model parachuting, bungee-jumping, and the path of a basketball being shot. These situations provided a context to pose and answer questions regarding the developed models. The use of the parachute and bungee jumping were part of my original plan, but modeling the path of the basketball was not. Additionally, I included the use of a video analysis tool to capture students from the class shooting the basketball. I traced the path of the ball so students could find a model to represent the situation (Figure 4-47).



Figure 4-47 – Traced Path of Basketball

Technology as a Mediator

The students used handheld technology while deriving models from images and data.

The use included entering the data into a list, having the technology plot the data, and then using the graphing feature to test and revise their models until they found a good fit. In terms of our classroom discussions, a good fit meant that the graph of their function resembled the way the plotted data looked, or the graph went through the points. A sample of what this looks like for the students can be found in Figure 4-48. Emerging from this activity were mediating processes leading to students using their models to answer posed questions about the data. Specific things students did were represent quantities in tabular, graphical, and algebraic forms, choose functions relevant to the situation, and test and revise whether their chosen function is an appropriate model.

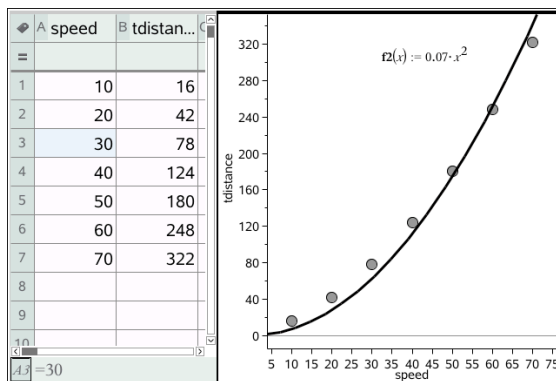


Figure 4-48– Fitted Model

The latter mediating process – deriving a mathematical model using a test and revise approach – was a new activity for students. The technology was integral to this process because students were able to get immediate feedback on whether or not their chosen model fit the data. Once students entered in the data and created a plot, they instantly saw the graph of the model as soon as they entered the equation into the calculator. This instant response by the calculator prompted the students to reflect on whether or not their choice of functions was appropriate, and if the model was appropriate, if the model needed to be revised. Important in these interactions with technology is students are able to do something that would be time-consuming and tedious to complete using pencil and paper. The procedure of fitting a potential model to data takes seconds using the handheld technology; fitting the same model by hand could take upwards of 5-10 minutes. This difference in productivity, especially for students with a negative disposition towards mathematics, gives them entrée into heady mathematical activity otherwise not accessible to them without technology.

An additional activity emerged while students derived models to fit data. Some students discovered alternative to come up with their model. Without any guidance, some students figured out that when they entered in a model to determine a fit, they could alter their initial

model by “clicking” on its graph, and then dragging and moving the graph until it fit the data – the graph went through the data points. When students transformed their model using this process, the equation for the model would also change accordingly, which would provide students with an equation that represents the plotted data. Two things tipped me off to students using this method to obtain their equation. When sharing models and graphs with the class, the parameters in the models appeared to be too precise to have come from a trial and learn process; an equation of $5.24x - 57.96$ for a linear model compared to $5x - 57$ would be cause pause. A second hint had to do with the specific students who were presenting these precise models. By this time in the course, the pace at which students were obtaining their models varied. This varied because of interest in the course, behavior issues, or some students just needed more time to do the procedural aspects of finding a model. It turned out that some students were obtaining models faster than normal, so I inquired about their process, and they shared the transformation tool on the calculator. Although the overall intent was for them to derive their models by a test and revise process where they alter the parameters in the models, this transformation tool gave certain students access to deriving a mathematical model that could be used in answering and posing questions about the relevant quantities. This was especially important for students who would not have engaged in the process of altering parameters. In a way, these students used a revise and test method while they were first choosing a model and then transforming the graph to find a fit. In the end, the technology provided students with a way to engage in the modeling process and develop a mindset that mathematics can be used to describe the world.

Summary

The preceding sections showed how the mediating processes emerged from within the embodiment of the learning environment. This summary consolidates the findings on mediating processes and summarizes the findings of technology as a mediator in the learning environment.

The Emergence of Mediating Processes

Important in the design of the study is the embodiment of the learning environment leading to students participating and interacting in ways that ultimately produce the hypothesized learning outcomes. I have showed how and where the emergence of the mediating processes occurred within the task structures students encountered. Below is a set of tables summarizing the findings (Tables 4-10 to 4-13). The first column lists the specific tasks in the learning environment, the second column contains the mediating processes emergent in the tasks, and the third column restates the evidence from above.

Table 4-10

Developing a Mindset (Initial Tasks)

Tasks	Mediating Processes	Sample Evidence from Text
Pattern Activities	Students recognized, described, and made generalizations about regularities in patterns of numbers, symbols, and graphs. Students recognized the relevant quantities involved in the situation. Students hypothesize and explain how change in one quantity affects change in another quantity.	This was evident when students extended sequences of numbers and symbols based on their own reasoning, and generated their own patterns and sequences that demonstrated regularities when prompted to do so.
CBR/ Graphing Stories	Students can recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs. Students are able to recognize the relevant quantities involved in the situation. Students hypothesize/explain how change in one quantity affects change in another quantity.	In the process of completing graphing stories (Figure 2) students made a connection between the action in the video and how the action is represented in mathematical ways. These activities helped students recognize relevant quantities involved in a physical situation. This was seen when they physically walked in front of a motion detector to produce a graph of a real-world context.

Table 4-11

Identifying Functions in Images

Tasks	Mediating Processes	Sample Evidence from Text
Cooperative Group Tasks	Students can recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs. Students choose functions that are relevant to the situation.	The evidence of students' recognition is seen when they interacted with other students in the process of choosing a function; this process included them describing the patterns to other students. For example, seen in Table 3 is student conversation, they recognized a graphical pattern in the image and then described to others what the pattern was or where the pattern could be found in the image.
Technology Task	Students can recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs. Students choose functions that are relevant to the situation.	Choosing an appropriate function relevant a contextual situation evolved by students first recognizing that some pattern or regularity exists, then finding a way to represent that pattern in mathematical manner. With these tasks this meant students found a pattern in an image and then identified a function whose graph represents that pattern. The evidence of students engaging in this activity can be found with the results of students successfully naming a function whose graph can be found in an image.
Pencil-Paper	Students can recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs. Students choose functions that are relevant to the situation.	The marking on the image outlines the graph the student saw in the image; this ability is evidence that the mediating process of choosing appropriate functions relevant to a contextual situation emerges.

Table 4-12

Identifying Functions in Data/Scatter-Plots

Task	Mediating Process	Sample Evidence from Text
Plotted Data	Students recognized and described regularities in patterns in graphs Students choose functions that are relevant to the situation.	Students take the basic graphical form of a function and connect to this to plotted data. This is evident in that nine students chose the exponential

		function to represent the scatterplot, which is correct, but four more students chose the rational function. This choice is reasonable given that the right side of the rational function $y=1/x$ closely resembles the plotted data.
Tabular Data	Students recognized and described regularities in patterns in graphs Students can represent these quantities in tabular, graphical, algebraic forms. Students choose functions that are relevant to the situation.	Figure 33 provides an example of what sixteen of twenty-one students produced in this task. These students represented the quantities in a table, plotted the data by hand and using technology, and identified the graphical pattern as a linear function. One of the sixteen students not only chose a linear function to represent the situation, but the student found a specific function ($y=x+1$) to describe the situation.
Initial Modeling	Students recognized and described regularities in patterns in graphs Students can represent these quantities in tabular, graphical, algebraic forms. Students choose functions that are relevant to the situation.	They also showed the ability to analyze various features of a graph then choose a relevant function to represent the data. More telling than their choice of function was the explanation for their choice. Comments like “it looks like exponential because it looks curv[y]” and “linear because I see the diagonal lines going a positive slope” show a level of mathematical sophistication and understanding by students to eventually derive a mathematical model to represent this situation and make predictions.

Table 4-13

Deriving Models

Tasks	Mediating Processes	Sample Evidence from Text
From Images	Students choose functions that are relevant to the situation. Students test/revise whether their chosen function is an appropriate model	The student begins the attempt at finding a model by starting with the equation $y=x$. The significance of this starting equation is that it shows students know they want to use a linear equation to model Explanations

**From Data /
Collected
Data**

Students are able to recognize the relevant quantities involved in the situation.
Students can represent these quantities in tabular, graphical, algebraic forms.
Students hypothesize/explain how change in one quantity affects change in another quantity.
Students choose functions that are relevant to the situation.
Students test/revise whether their chosen function is an appropriate model

of how the model is altered to find a better fit is also seen in the student work. In the first case, the student begins altering the original model by recognizing that the $y=x$ model does not match or fit Broadway Street, the student says “make it higher” to acknowledge the $y=x$ needs to be moved up to fit the street.

Two of them started with “ $y=x$ ”, the others started with “ $y=3.91x$ ” and “ $y=4.7x-39$ ”. As I mentioned previously, it was not documented why they chose those starting functions, all we know is that their initial choice was a linear function. The item differentiating this group is their choice to change their model type from a linear to a quadratic function. Students articulated some of their thought process, “Now I think it is quadratic”, “Not even close to a line”, “Try a different function”, “try a Quadratic” (Figures 40-43). These statements indicate a conscious recognition on the students’ behalf that their initial plan of attack to use a linear function to model the data was appropriate in this case.

The Role of Technology

Technology was integral to the design of the learning environment. Specifically, handheld technology was to support the emergence of the mediating processes that would lead to desired learning outcomes. With that said, it is clear technology played a role in supporting students in the learning environment. Although more is to learn about how to successively use this technology to further students mathematical thinking, problem solving, and modeling. I am confident saying that we could not have gotten to some of the mediating processes without the

use of technology. In no way could students have engaged in the ways they did without the ability to flexibly use the handheld to engage in the tasks.

Within this learning environment, graphing technology supported students in representing data in multiple forms – tabular, graphical and symbolic. The ease at which students moved from a table of values to a graphical form allowed them to then quickly recognize a pattern in the data and then name the pattern. The naming of the pattern involved them going through the process of finding a function whose graph would go through or best-fit the data. By removing the cognitive load or the time it would take to hand plot and graph various models, the graphing technology gave students easy access to the type of thinking needed to fit a model to the data. Students identified the pattern with a function name and used an algebraic notation to see if they named it correctly. Additionally, in the process of checking if they chose the appropriate function, students' flexible use of the technology allowed them to learn how the adjustment of parameters in a function influence the shape of the graph. This trial and learn process, mediated by the technology, developed a students' understanding of how the algebraic representations are related to its graphical form. The technology also gave students a private place to experiment with their ideas before having to share or communicate their thoughts. This is important for students who previously have not had success in mathematics. This was evident when students used the dynamic graphing feature to find a model to fit data. Compared to the others students who were manually changing parameters to fit the function to the data, these students used a trial and learn process of moving a function to fit the data and allowed the technology to derive the equation for them. Although a different process than thinking about how to change parameters to make the graph move into place, this dynamic method provided these students with access to

finding a model and eventually asking and posing questions about the situation using their derived model.

Part 2 – Mediating Processes to Learning Outcomes

Introduction

The findings discussed in this section focus on how the mediating processes function to produce desired learning outcomes. Important in this chapter (and with Design Experiments) is to produce a level of analysis so that it is possible to backtrack from learning outcome to mediating process (Sandoval, 2013). The learning outcomes were: 1) students used their derived models to answer/pose contextual questions about the quantities, and in the process of deriving models, 2) students developed an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations.

This section presents the work of three students (Student A, Student B, Student C) for three different tasks where students had to find a model for a given situation and then use their model to answer a real world question. These modeling tasks were culminating activities following students engaging in the tasks previously discussed. Three different student's work will be examined to lay out the learning that emerged; one student's work will serve as a reference for each of the three tasks. The student work is shared to highlight the role of the mediating processes in learning and to provide an in depth analyses of what students were learning. Additionally, a summary of how well the groups performed in regards to these modeling tasks is presented.

Student Learning on Task 1: Stopping Distance

A video of various cars speeding and skidding to stops was demonstrated for students as an introduction to the Stopping Distance task. This task began with the prompt and data in Figure 4-49.

At many automobile crime scene investigations, investigators often measure the skid marks left behind by those cars involved in the accident. By measuring and recording the length of the skids marks, crime scene investigators can determine the speed of the car. We will use the data below to determine how they do this.

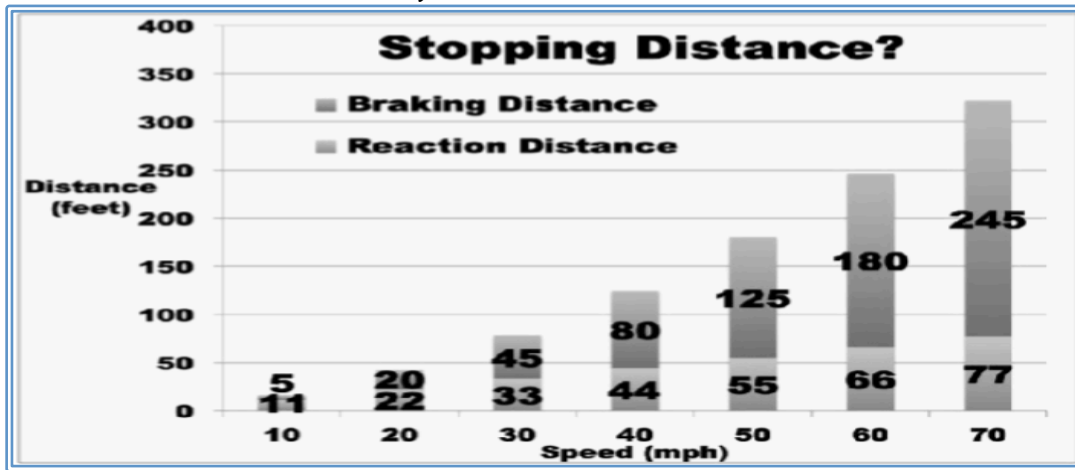


Figure 4-49 - Stopping Distance Task

Individual Analysis (Student A)

In the process of engaging in this modeling task (Appendix K), Student A used their derived models to answer posed questions and developed an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations. Figure 4-50 shows a portion of Student A’s response (to making sense of the Stopping Distance data. The work first shows the emergence of mediating processes; she recognized both quantities increased as the speed increased and that the braking distance had a greater rate of change than the reaction distance (Figure 4-50a). In Figures 4-50bc Student A represented the quantities – by summing the bars - in tabular and graphical forms. It is important to note that she tried to make further sense of the data by adding a zero in the speed column; it is possible that she was hypothesizing about what the total stopping distance should be for this speed.

From here (Figure 4-50d), the student conjectured which function could possibly represent the graphed data. Neither choice mathematically correct, but Student A is used their knowledge of what the functions looked like to make the connection the graphed data. I

considered each response as a reasonable choice, the exponential has the same shape, a “little curve”, and the square root is possible given that stopping distance data is quadratic in nature.

What do you know for sure?	What do you wonder?
It increase each time for both	Why is the Braking distance increases more than the Reaction Distance

[Left Column] “It increase each time for both”

[Right Column]”Why is [is] the Braking distance increases more then the Reaction Distance”

Figure 4-50a

Speed	Total Stopping Distance
10	16
20	42
30	78
40	124
50	180
60	246
70	322

Figure 4-50b

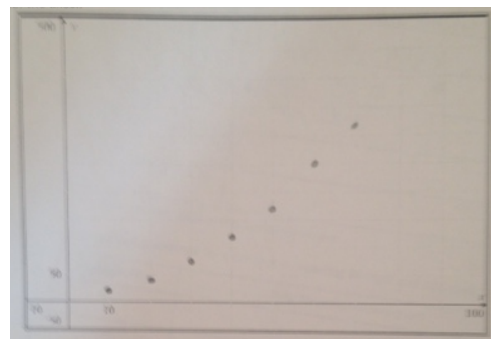


Figure 4-50c

Square Root because it has a little curve or exponential

“Square root because it has a little curve or exponential”

Figure 4-50d

Figure 4-50 – Mediating Processes Emerging (Student A)

Figure 4-51 shows evidence of Student A testing and revising whether her chosen function is an appropriate model. It is in this process that the mediating processes lead to a desired learning outcome. While attempting to fit their chosen model to the data, the student changed the parameters of the function in response to how well the graph of the function fit the data. Student A developed an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations. This was done using handheld technology with the

student recording each successive equation and adjustment in the table. However, we do not know why Student A made her choices. We can only infer from the adjustments why they made them.

Attempt	Equation	What Adjustment are you going to make?
1	$y=x$	add a slope and y-int
2	$\frac{1}{2}x+10$	try the Quadratic
3	$1(x-5)^2-4$	change the vertex
4	$1(x-10)^2+11$	change the a
5	$0.094(x-7.65)^2+22.6$	it was close
6	$0.094(x-0)^2+0$	
7	$0.081(x-0)^2+0$	

- 1: $y=x$
- 2: $\frac{1}{2}x+10$
- 3: $1(x-5)^2-4$
- 4: $1(x-10)^2+11$
- 5: $0.094(x-7.65)^2+22.6$
- 6: $0.094(x-0)^2+0$
- 7: $0.081(x-0)^2+0$

Figure 4-51 – Modeling with Test and Revise Process (Student A)

As modeling process progressed, more evidence of student learning is seen in the way Student A adjusted the quadratic function appropriately by altering the parameters for the vertex form ($y=a(x-h)^2+k$) of the quadratic function. Student A began with the equation of $y=x$ (Figure 4-52a), then transitioned from the second $\frac{1}{2}x+10$ to third $1(x-5)^2-4$ (Figure 4-52b) equation by shifting directions from a linear to a quadratic model. Although she originally stated “square root” or “exponential” as a possible model for the data, Student A ends up with final model that fits the data (Figure 4-52c).

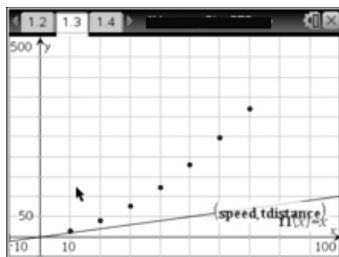


Figure 4-52a $f_1(x)=x$

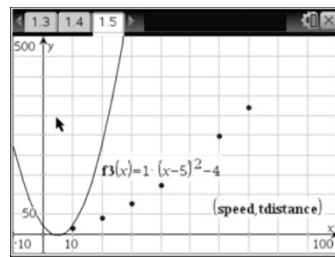


Figure 4-52b $f_3(x)=1(x-5)^2-4$

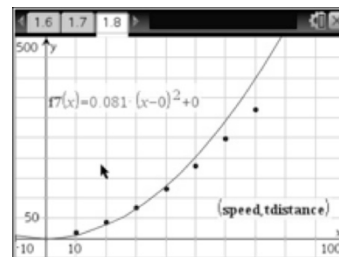


Figure 4-52c $f_7(x)=0.081(x-0)^2+0$

Figure 4-52 - Stopping Distance Models (Student A)

Student A ends this task using her derived model to answer contextual questions about the quantities involved in the situation (Figure 4-53). The student answered by substituting values into the equation. The process of substitution in this case shows that the student understands how the quantities in this situation are related to their derived model.

In order to answer the more difficult question of how fast was the car going if its skid mark is 50 feet, the student used a test and check strategy to determine the speed of the car. Up to this point in the class, students had not been instructed to solve a quadratic equation, so the student had limited methods to choose from to answer the question. One aspect of the outcome that is not observed is the student posing a question in regards to the model and the situation. This occurred because I did not give students an opportunity to do so. Students were still in their initial stages of deriving models and using their models to answer questions, so in the design of the activities, I did not include a prompt or task where students were asked to pose questions that could be answered using their models.

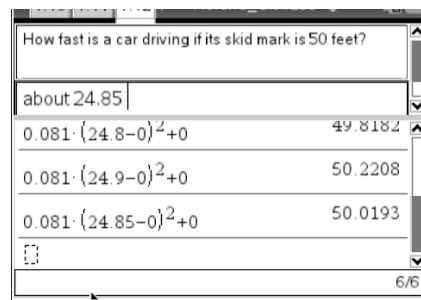
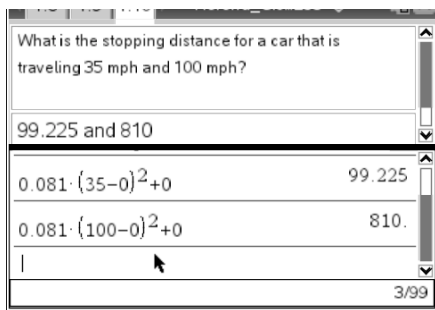


Figure 4-53 – Using Derived Model to Answer Posed Questions (Student A)

Class Analysis

I used individual students' work to show what the learning outcomes look like and how they surfaced within the class. In order to buttress the evidence that learning resulted for more

than one student, it is important to get a larger sense of the number of students who did or did not perform similarly to the highlighted student above.

For the Stopping Distance task, eighteen students ($n=18$) submitted written work. Of those eighteen students, eight students ($n=8$) completed the task similarly to the student above. Common features included the use of a test and revise process for obtaining their model of best fit and the use of a substitution process to answer the posed questions regarding the scenario. Of the eighteen students, six students ($n=6$) did not document their work for finding a model but recorded some model to fit the data. It is unclear if these students used a test and revise approach or the transformation tool to find their model. The remaining students ($n=4$), did little or no work. As previously discussed, the Stopping Distance task was one of the last tasks completed by students during the course. There was a time pressure to complete content and make sure students experienced the entirety of the course. As a result, some students did not have adequate time to find their model through a test and revise approach or even with the technology.

Student Learning on Task 2: Barbie Bungee Jumping

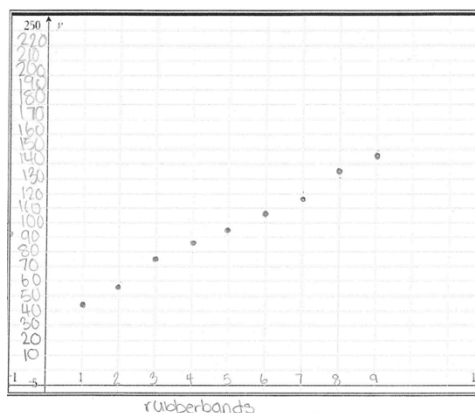
The Barbie Bungee Jumping was an activity where students created different sized bungee cords using large rubber bands to simulate a bungee jump by tying a Barbie doll to the cords and then dropping Barbie from different heights. The purpose of the activity was to have students determine how many rubber bands are needed to construct a bungee cord that will get Barbie as close to the ground as possible when dropped from a pre-determined height of 965 cm.

Individual Analysis (Student B)

Figure 4-54 shows a portion of a typical student response (Student B) to making sense of the Barbie Bungee Jumping task. The mediating processes: recognizing relevant quantities in a situation, representing these quantities in multiple forms, explaining how change in one quantity

affects change in another quantity, recognizing patterns in graphs, and choosing a function relevant to the situation are all evident in the work.

# of rubber bands	Distance Fallen (cm)			
	1	2	3	Average
1	43	44	45	$\frac{144}{3}$
2	54	60	60	$\frac{152}{3}$
3	73	72	76	74
4	80	83	90	84
5	99	102	104	102
6	110	111	116	112
7	120	128	128	125
8	135	139	140	138
9	143	149	150	147



As a group, decide which functions best describes the relationship between the number of rubber bands and the distance Barbie falls. Why did you choose this function?

Each time you add a rubberband to a barbie doll, the barbie doll bungie jumps lower than before. (It makes a positive slope going diagonally) - (Linear Function)
 Now try and find the equation of the function that best fits the data

“Each time you add a rubberband to a Barbie doll, the Barbie doll bungie jumps lower than before. (It makes a positive slope going diagonally) – (Linear Function)”

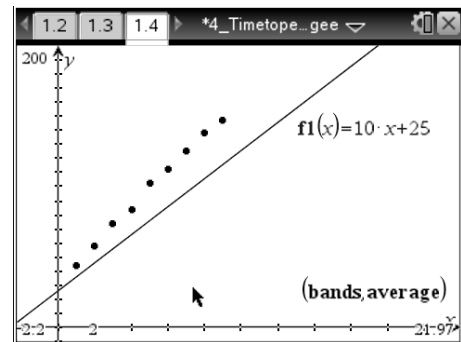
Figure 4-54 – Multiple Representations of Functions (Student B)

In engaging in this task there is evidence that the student was able to make sense of the data in a way that led to deriving a model for the data and then using the model to answer posed questions. The student completes the table and then makes sense of the quantities to produce a scatterplot of the table. She makes a statement regarding the pattern in the graph and the relationship between the quantities. The student then named a function that is relevant to the situation based on the graphical appearance of the plotted data.

Figure 4-55 shows five documented models Student B obtained through the test and revise process to fit their function to the data. Each successive change demonstrates their understanding of how to manipulate the parameters of the function to fit the graph to the data as well as a developing understanding of the underlying structure of functions as it relates to its symbolic and graphical representations. With the third model ($y=15x+25$), the student

comments, “almost through”, referencing the fit of the line to the data. Continued adjustments include altering the value of the slope and the y-intercept. These changes made by the student nestle the graph of the linear model into place – through the points. These small adjustments particularly show thoughtful student engaging in the test and revise process with knowledge of how the changing parameters influence the graph. Pictured in Figure 8 is Student B’s initial and final model.

Equation	What Adjustment are you going to make?
$y = 10x + 25$	Make it steeper & slope higher.
$y = 25x + 25$	Make it a little lower than before.
$y = 15x + 25$	Almost through.
$y = 13.5x + 35$	just a little bit more.
$y = 12.7x + 35$	Yay me!!! ☺



$y = 10x + 25$ | Make it steeper & slope higher
 $y = 25x + 25$ | Make it a little lower than before
 $y = 15x + 25$ | Almost through
 $y = 13.5x + 35$ | just a little bit more
 $y = 12.7x + 35$ | yay me!!!

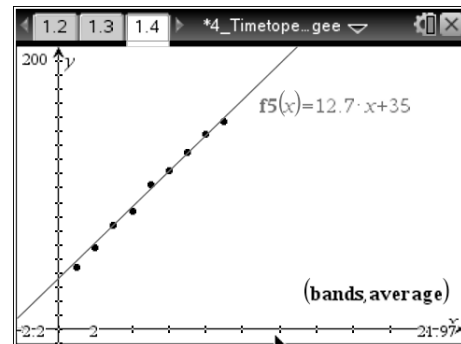


Figure 4-55 – Modeling Process (Student B)

The second learning outcome – using a derived model to answer a contextual question – emerges when the Student B determined the number of rubber bands needed to get their Barbie as close to the ground as possible. Figure 4-56 shows the derived model being used to solve an equation that produces the number of rubber bands. The displayed answer, 71, is determined after the Student B rounds the original answer up to 72. She recognized that 72 rubber bands

was an over estimate, meaning that her Barbie would hit the ground if they use 72. The final result, highlighted in a square, is 71 rubber bands.

It is important to note in this process of using her model, the task guided the student to the procedure of setting the equation of their model equal to 965. This was my instructional decision to support students through the idea of how we can use models to answer contextual question related to the quantities involved in the model. This decision was also based on the time constraints places on us during the course.

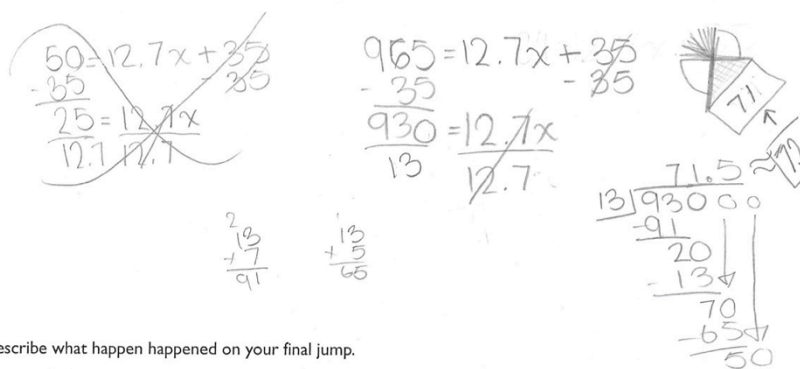


Figure 4-56 – Solving the Barbie Problem (Student B)

Class Analysis

In total, twenty students (n=20) returned written work and/or technology documentation for this activity. Of the twenty students, eleven students (n=11) completed the task similar to the student whose work was discussed in the above section. This meant the students responded to all prompts, used a test and revise method to obtain their model, and then used their model to determine the number of rubber bands needed for Barbie to get as close to the ground as possible. Five students (n=5) recorded some work, but did not document any process for obtaining their model. As a result, there was no work showing them using a model to answer the posed question. It is possible that because this was a collaborative task, groups collected data together, that these students did not document work because others had already completed the task. The remaining students (n=4), documented little or no work. For the same reasons, these

students possibly did nothing because others in their groups had already completed the task. An alternative reason could be some of them were just not interested in the activity. This is reasonable given some of the resistance demonstrated by students during the course.

Student Learning on Task 3: Modeling with Quadratics

The purpose of the Modeling with Quadratics task was to give students a first experience at using quadratics to model a physical situation. I also chose this task so that I could involve students in the creation of the data. The task was for students to determine if shot basketball would go into the hoop, they would do this finding a model for the data and visually seeing if the model goes through the basket.

Individual Analysis (Student C)

Student C's work shows the mediating process of testing and revising their model to fit the data. It also shows evidence of that mediating process supporting his understanding of the underlying structure of functions as it relates to its symbolic and graphical representations. To start, Student C used the basic quadratic function (Figure 4-57) to fit the data and then proceeded with altering the parameters to move the graph around the coordinate axes to go through the points. This altering of the parameters to fit the graph to the data is an example of a learning outcome. Moving from the second to the third equations, the student changed the leading parameter from positive to negative in order to change the orientation of the graph.

The student also stated "wider" as an adjustment after the seventh equation, as a response they change the parameter from (-1) to -0.11, which produced a fairly good fit. Not known is how the student knew to use -0.11. My best guess is they played with the number without recording their changes. A second thing could have been the use of the transformation tool, but because of the previous models and their lack of precision to more than 1 decimal place, I doubt

this student used that method. This interplay by Student C further established his understanding of the graphical and symbolic representations of functions.



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Task: Determine the equation of a quadratic function that models the path of a basketball.



Fitted Model

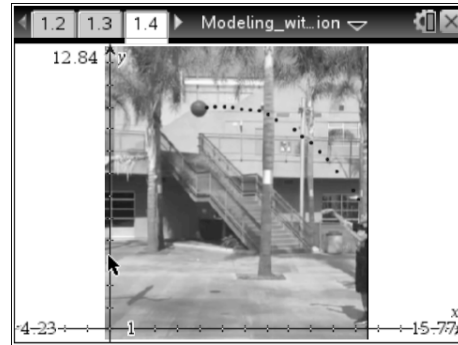
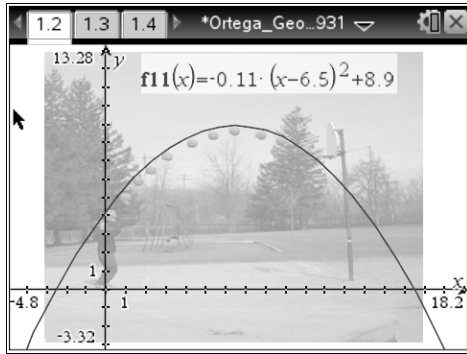
Figure 4-57 – Modeling Process (Student C)

Equation	What adjustments are you going to make?
x^2	make it side ways
$2(x-0)^2+3$	
$-1(x-0)^2+9$	lower
$-1(x-0)^2+8.5$	move to the right
$-1(x-7)^2+8.5$	went off
$-1(x-6.5)^2+8.5$	more up
$-1(x-6.5)^2+8.9$	wider
$-0.11(x-6.5)^2+8.9$	good

- 1: x^2 | make it side ways
- 2: $2(x-0)^2+3$
- 3: $-1(x-0)^2+9$ | lower
- 4: $-1(x-0)^2+8.5$ | to the right
- 5: $-1(x-7)^2+8.5$ | went off
- 6: $-1(x-6.5)^2+8.5$ | more up
- 7: $-1(x-6.5)^2+8.9$ | wider
- 8: $-0.11(x-6.5)^2+8.9$ | good

In the process deriving and making sense of the model and the context, Student C demonstrated what learning look like by using his derived model to answer questions related to the situation. Figure 4-58 shows a brief response where, the answer 8.91 refers to the maximum height the ball travels based on what the model predicts. The answer also shows Student C understood the meaning of the parameter's value for their model. The final model $(-0.11(x-6.5)^2+8.9)$ is in vertex form, so the 8.91 represents the y-value of the vertex. The vertex in this case is the highest point on the graph. Student C knew the y-value represented the height of the ball. Another example shows him responding in a similar fashion in manner. However, his

model is in vertex form, so it is reasonable to assume he identified the vertex using the handheld and then substituted those values into the equation. The last answer indicates Student C used his model to answer questions about the contextual situation. He knows that the 10.1 in the equation represents the maximum height of the ball.



In this activity, the horizontal distance traveled by the basketball is the independent variable. What is the dependent variable?

height

Determine the maximum height of the basketball in meters. Explain your reasoning.

$$h = 8.91$$

...this year season...

$$0.12(x - 52)^2 + 10.1$$

9. How high does Daniel's shot go?

10.1

Figure 4-58 – Using Derived Model to Answer Posed Question (Student C)

Class Analysis

In total, eighteen students (n=18) returned written work and/or technology documentation for this activity. There were nine students (n=9) who completed models for the three situations, and used their models to answer posed questions. During the modeling process, students used a test and revise process, the transformation tool, or a combination of both. Seven students (n=7) partially completed the work and submitted at least one model for the situations. There was some attempt at answering posed questions, but not all of them were addressed. Finally, there were two students (n=2) who did little or no work. Their lack of work could be explained by their lack of interest in the activity or their inability to derive a model.

Across the three modeling tasks, the two tasks that involved students actively participating in the data collection (Barbie Bungee and Modeling with Quadratics) had more students demonstrating full or partial learning outcomes (16 out of 20, 80% and 16 out of 18, 89% compared to 14 out of 18, 78%)(Table 4-14). There was a consistent pattern amongst the students in the level of work they produced; which corresponded to amount of learning demonstrated. One interesting finding was in the Modeling Quadratics task, two students who had produced little or now work in the previous tasks, engaged in the modeling process to determine if the shot went into the basket. I believe this was the case because I video recorded these two students shooting a basketball, and embedded the video and images from their shots in the task. Table 17 shows that for this task, only two students did little or no work instead of four.

Table 4-14– Class Analysis Summary

Student Learning	Task		
	Stopping Distance	Barbie Bungee	Modeling with Quadratics
Number of students producing work similar to presented Student (A,B, or C)	N=8	N=11	N=9
Number of students producing partial work compared to presented Student (A, B, C)	N=6	N=5	N=7
Number of students producing little or no work	N=4	N=4	N=2

Summary

Part 2 of this chapter provided evidence of how my hypothesized mediating processes emerged and led to the desired learning outcomes when students actively participated in the

discussed tasks. This summary reviews the main findings regarding mediating processes leading to learning outcomes and also discusses other findings from the study.

Main Findings (Mediating Processes Leading to Outcomes)

Figure 4-59 contains an overall summary of mediating processes appearing within categories of tasks. For example, in the column labeled *Patterns*, the “YES” indicates that the mediating process of “Students recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs” emerged in the Patterns tasks. The subsequent “YES”s indicate the same thing for the identified mediating processes within the specified tasks, all of these were outlined in the Findings Chapter.

Of particular interest in Figure 4-59 are the “**” in the Identifying Functions in Data tasks. I purposely marked these mediating processes because I did not have evidence of them emerging within this category of tasks. I originally thought students would recognize the relevant quantities involved in the situation and hypothesize/explain how change in one quantity affects change in another quantity when engaging in these tasks, but I had no evidence of this happening. Students being able to interact with the quantities involved in situations seem like an important process that would lead to the desired learning outcomes. Despite this not happening, it did not prevent students from using their derived models to answer/pose contextual questions about the quantities. Regardless, this ability to understand how these related quantities interact with each other provided students with a deeper understanding of how their choice of functions to represent the quantities might depend upon how the change of one quantity affects the other.

Further analysis of the tasks leads me to believe that I would have to alter the tasks and possibly the ways students interacted so they would have opportunities to discuss how the quantities are related and how that relationship is depicted in the scatterplot of the data. This would then lead to further discussions about function characteristics, such as the linear functions

display a constant rate of change relationship, or that exponential functions can be described in terms of growth or decay based on a multiplicative relationship between the variables. This altering of tasks is in line with how I approached each day's instruction, using what I learned from each lesson guided my choices of how I would proceed the next day. These discussions can be found in the Instructional Design sections of the Findings Chapter.

Mediating Processes	TASKS					Outcomes
	Pattern	CBR / Graphing Stories	Identifying Functions in Images	Identifying Functions in Data	Deriving Models	
Students recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs.	Yes	Yes	Yes	Yes	Yes	Students are able to use their derived models to answer/pose contextual questions about the quantities. In the process of deriving models, students develop an understanding of the underlying structure of functions as it relates to its symbolic and graphical representations
Students recognize the relevant quantities involved in the situation.		Yes		**	yes*	
Students represent these quantities in tabular, graphical, algebraic forms.				Yes	Yes	
Students hypothesize/explain how change in one quantity affects change in another quantity.		Yes		**	yes*	
Students choose functions that are relevant to the situation.			Yes	Yes	Yes	
Students test/revise whether their chosen function is an appropriate model					Yes	

Figure 4-59- Summary of Mediating Processes and Outcomes

Another column of interest is the Deriving Models task. Based on the data, all the mediating processes emerged, but there were sparse examples of students recognizing relevant quantities involved in a situation and hypothesizing/explaining how change in one quantity affects change in another quantity. Like the previous discussion, having students engage in these processes could benefit them while deriving models and using them to answer/pose

contextual questions about the quantities. To help this development, the task would need to be altered in manners previously discussed.

In addition to the altering of the tasks and participant structures to produce mediating process that lead to desired learning outcomes, is to consider the forms instrumentation used to measure and identify mediating processes and outcomes. In the spirit of design experiments, more iterations of this path of study would lead to better in forms of engaging students and developing ways to document their learning. In this study, I could find effective ways to have students pose and answer contextual questions requiring the use of their derived models to do so. I could also design alternative ways through artifact collection and interaction documentation to uncover the important functions that lead to outcomes.

Other Findings

With an emphasis on showing how mediating processes lead to the desired learning outcomes, it is important to consider that the actual existence of the mediating processes emerging within the tasks is a finding itself. Knowing certain tasks support my hypothesized student interaction and activity, is an important take-away from the study. The fact the mediating processes manifest when students engage in the tasks provide evidence that the learning environment is conducive to student learning. Even with some students not producing evidence of the learning outcomes, the strength of a design experiment is that we can go back and make adjustments to best support student learning. This finding helps us understand how these types of tasks lead to productive mathematical activity.

Another finding – related to the preceding discussion – is the tweaking of tasks during the course to get to the mediating processes. As structured in the design of the study, I constantly reflected on the tasks and how they led to productive student interactions and activity. My reflections guided my daily instructional decisions. I found more students interacting with the

mathematics in productive ways after thoughtful modifications to tasks were implemented on a daily basis. All this is in line with the crosscutting features of DBR/DE studies: The development a class of theories about both the process of learning and the means that are designed to support that learning, a highly interventionist nature which are test-beds for innovation, the creation of conditions for developing theories yet at the same time placing these theories in harm's way, multiple iterations, and producing theories that do real work; having potential for rapid pay-off but also speaking directly to the types of problems that practitioners address in the course of their work (Cobb et al., 2003, p.10).

Chapter 5 – Discussion, Contributions, Summary Conclusions

Introduction

The genesis for this study is rooted in my experiences as a mathematics educator in urban schools. I began this study to better understand ways of teaching and learning algebra in order that more students might participate in rigorous mathematics content while engaging in the productive practices of mathematicians. My intent was to build a classroom learning environment based on the description of mathematics as the "science of patterns" (Devlin, 1996; Kneebone, 2001; Steen, 1988; Resnik, 1997; Mason, Burton, Stacey, 1982) and to study this learning environment using Design-Based Research (DBR) (Barab, 2006; Cobb et al., 2003; Confrey, 2006; Schoenfeld, 2006).

The goal of this study was not to produce a perfect curriculum showing results where all students learned, but to study specific tasks that shaped the types of activities and interactions students had in the learning process. I learned that students, who often are not successful in algebra, are fully capable of using mathematics as a tool to describe the world. This study showed types of tasks that can be used to inform teachers, especially within the context of the new Common Core State Standards (2011) and its focus on mathematical practices. This chapter ends with a discussion of the results as related to the study's research questions, a summary of how these results connect to important classroom issues, and the contributions of this study to the literature on mathematics education.

Research Questions

What does teaching and learning look like in a secondary school mathematics classroom that approaches the teaching of algebra with problems set in daily life observations?

My experiences as a mathematics educator in urban schools were not only the impetus for me to conduct this study, but these experiences formed the foundation of what I believed students were mathematically capable of when provided with the conditions to see math as a

useful, worthwhile, and doable endeavor. I combined my on-the-ground work in schools with budding ideas on learning to shape a classroom environment leading to what I called learning within a mathematics classroom describing mathematics as a “science of patterns”. This learning is directly connected to my original high-level conjecture that engaging in a technology rich mathematical modeling learning environment helps students develop a frame of mind in which they begin to see how they can use mathematics to describe the world.

However, it would be misleading you to say that during the study all students interacted and persevered with mathematics in ways that I had predicted or hoped for. Despite sixteen years of classroom experience, but perhaps because I had been out of the classroom for four years, I naively expected that when students engaged in the specific tasks I aligned with the “science of patterns” perspective, they would immediately adopt the practices of mathematicians. I fantasized their going from students with little or no interest in mathematics to ones who now saw how math could be applied to their life and begin to actually enjoy the process of doing mathematics. I believed they would begin to identify themselves as mathematicians- those who does math because they enjoy it, because they know they are good at it, and because they know how to persevere when the task became challenging.

So what really happened? What did teaching and learning look like in the classroom? On the surface, it may have looked like any other urban school classroom. There was hustle and bustle of students coming into class; students interacting with each other and me as they found their seats and materials. Students asking the typical questions, “What are we going to do today?” “Can we do something fun?” During the class, there was student resistance and even misbehavior. This summer school experience offered them no academic credit, it was offered to them as Algebra 1 enrichment course to prepare and expose them to Algebra 1. The students

were eighth graders of differing abilities and mathematics success in seventh grade. Some students were not always interested in doing mathematics. Therefore, I had to quickly pull out my teacher bag-of-tricks in order to support and help students. This meant that I had to sometimes put aside my “researcher hat” and begin to think like the urban schoolteacher I had been for sixteen years. Although this aspect is not central to the study, it is discussed in instructional design sections of the findings and analysis chapter. I had to account for student motivation and it shaped my instructional choices to better engage students in the tasks. These aspects did have an impact on my data collection, I had to find ways for students to feel comfortable responding to journal prompts, confidently discussing math ideas with each other, and actively participating in the practices of mathematicians.

Given this somewhat challenging environment, and as displayed in the findings and analysis chapter, student learning did take place. The mediating processes emerged while students engaged in the tasks, and students displayed evidence of the hypothesized learning outcomes. This is especially significant given the real-world context of the study, done in the “in the buzzing, blooming, confusion of real-life settings” (Barab and Squire, 2004, pg. 4, Table 1) where student success in mathematics is often tied to their in-class learning opportunities (Treisman, 2013).

Viewing the classroom with a more formal lens and appropriating the language of mediating processes and outcomes, students had the opportunity to recognize, describe, and make generalizations about regularities in patterns of numbers, symbols, and graphs. Describing the patterns in a mathematical manner was one way learning looked for students in the class. Further, students had the opportunity to begin to think about their world in a mathematical context. This meant that students were provided with real-world situations where they had to

recognize relevant quantities in the situation. Students hypothesizing and explaining how change in one quantity affected change in another quantity was a tangible example of them learning in this context. Teaching is providing students with opportunities to use math to describe their world and learning is students going through iterative process of finding a suitable mathematical function to describe it.

Important in this process is the appropriate use of technology. This meant students used technology to represent patterns in multiple forms. These forms – numerical, tabular, graphical – then lead to an algebraic description or mathematical model. Ultimately, students used their derived models to answer/pose contextual questions about the situation.

A final aspect of learning in a “science of patterns” classroom connects to a participatory framework (Sfard, 1998). From this perspective, increased participation using the language, tools, discourses, and methods of mathematicians allows students to claim community membership. Although I was unable to fully study this focus, it is important to recognize that as the mediating processes led to learning outcomes, the students’ learning trajectory included the development of their identity as mathematicians.

How does the use of graphing technology assist students’ in mathematical thinking and the engagement of mathematical practices?

Graphing technology supported students in representing data in multiple forms – tabular, graphical and symbolic. The ease at which students moved from tables of values to graphical forms allowed them to then quickly recognize a pattern in the data, and subsequently name the pattern. The naming of the pattern involved going through the process of finding a function whose graph would go through or best-fit the data. By removing the cognitive load or the time it would take to hand plot and graph various models, the graphing technology gave students easy access to the type of thinking needed to fit a model to the data. This was in line with McCulloch

(2011) , who described that use of technology lessened students' frustration in the problem solving process and that a tool like the graphing calculator could help convert frustration into perseverance since they had an “instrument with which they can act in response to feelings of frustration” (p.177).

More students were willing to participate in the tasks because the technology gave students a safe place to test their ideas before having to share or communicate their thoughts. This was important for students who previously had not had success in mathematics. Additionally, the development of their mathematical thinking flourished. As research has shown, students worked collaboratively more often when technology was available (Burrill et al, 2002); they were willing to share “the products of [their] problem solving strategies” (Robutti , 2009). This was evident when students used the dynamic graphing feature to find a model to fit data. Compared to the others students who were manually changing parameters to fit the function to the data, these students used a “trial and learn” process of moving a function to fit the data and allowed the technology to derive the equation for them. Although this is a different process than that of thinking about how to change parameters to make the graph move into place, this dynamic method provided these students with access to finding a model and eventually asking and posing questions about the situation using their derived model. More importantly, it provided students with common language to communicate their ideas to more expert students; all students were able to be members of the mathematicians' community.

Although there is much more to be learned about technology and how it can support student thinking and participation, there is no doubt that technology supported the development of mathematical thinking and the engagement of mathematical practices. In sum, graphing technology helped students effectively represent mathematical objects in multiple ways, it

provided access to mathematical modeling and productive ways of mathematical thinking, and was a vehicle for fruitful discussions about the characteristics of functions and mathematics in general.

What is the trajectory of students' mathematics identity as they learn algebra from a "science of patterns" perspective?

Going into the study with a design experiment approach allowed me to think about how I could create and enact a learning environment where students would learn mathematics in ways not often seen in urban schools. I believe that a fundamental goal of mathematics education is to "produce students who are willing to engage in challenging mathematics and see the value in doing so; in other words, to develop students with positive mathematics identities" To study this, I proposed traditional ways of measuring identity within the study (questionnaires, journal prompts). However, as I proceeded, what emerged was related to understanding how the mediating processes led to desired learning outcomes. While I know I observed instances of productive mathematics identities being developed, it did not turn out to be my focus of analyses.

Although I was unable to answer my posed question, in no way should the study of mathematics identity be overlooked when compared to traditional manners of measuring student performance. With mathematics identities being associated to many related affective factors influencing mathematics performance (Boaler, 2002; Boaler & Greeno, 2000), including persistence in the field, identity is a powerful factor determining mathematics learning (Bishop, 2012). I believe learning mathematics from a "science of patterns" perspective and engaging in ways presented in this study can positively affect how students sees themselves with respect to the field and practices of mathematics.

What is the learning trajectory of student's use of functions to model data through a curve fitting process?

A student's learning trajectory begins with the development of mindset – the idea that mathematical functions can be used to describe the world. This mindset was developed through the graphical representation of related quantities found in the world and the graphs of functions. Students then developed an understanding of how to represent functions in multiple forms. This back-and-forth allowed students to interact with real-world data by taking the data, representing it in graphical form, and making a connection between the plotted points of paired quantities and the graphs of functions.

Students then hypothesized which function best represented the data and then checked whether or not their conjecture was correct. Their verification process included naming the graphed pattern with an algebraic notation (function), and then graphing the function to see how well it fit the data. The goodness-of-fit process was based purely on how well they graphed function went through the plotted points. Additionally, in the process of checking if they chose an appropriate function (mathematical model), students flexibly used technology to learn how the adjustment of parameters in a function influenced the shape of the graph. This trial and learn procedure, mediated by the technology, developed students' understanding of the underlying structure of how changing parameters in a function changes the functions graphical representation. The learning trajectory ended with students using their chosen and adjusted function to answer and pose contextual questions about the related quantities. Overall, students learned that functions are flexible objects used to describe and model data found in the world.

Contributions and Future Research

Important in all studies, is the answer to central question, "What does this all mean?" My posed research questions focused on what teaching and learning looked like in a re-envisioned algebra classroom, how students participated and interacted with each other, how technology

assisted students in doing mathematics, and could this learning environment help students begin to see the relevance of mathematics to their lives.

Contributions of the Study

The results of the study yield the following contributions: Firstly, study presents a potential way to support students in mathematics who have traditionally not been successful in mathematics. The approach demonstrated in this research is specifically related to types of tasks and student use of technology in the process of modeling and participating in mathematical practices. Having students successful in mathematics and beginning algebra (algebra 1) will lead to better educational outcomes for those students.

A second contribution revolves around finding ways to have mathematics be more meaningful and relevant for students. This relevance is two-fold; one is related to engaging students in mathematical activity couched in real-world contexts, and more importantly, contexts immediately related to their lives. Approaching mathematics from a “science of patterns” perspective opens the door to use mathematical modeling as a tool to describe the world.

A third contribution, one interconnected with definitions of mathematical proficiency, is the development of *procedural fluency and conceptual understanding*. These two concepts, which are at the heart of any instructional program, developed in this study when students showed fluency in both areas. Understanding the concept of using functions to model the world developed their procedural fluency of representing data in multiple forms; this included their ability to alter a function’s parameters to have its graph fit data. I believe given more time, it might be possible to develop other procedural skills when engaging students in a learning environment of the sort demonstrated in this study.

In the end, one overarching significance of the study, is its contribution to mathematics equity. Having more students value mathematics while at the same time emerging as

mathematically proficient, gets to a main motivation and responsibility of mathematics teachers.

It is also in line with Schoenfeld's (1992) thoughts on mathematics instruction:

Mathematics instruction should provide students with a sense of the discipline—a sense of its scope, power, uses, and history. It should give them a sense of what mathematics is and how it is done, at a level appropriate for the students to experience and understand. As a result of their instructional experiences, students should learn to value mathematics and to feel confident in their ability to do mathematics. (p. 345)

Ultimately, I see the findings of the study beginning to point (if even ever so slightly) in a direction where learning mathematics from the perspective shown here can empower students with a sense of agency and belief that they can exert control of the world. This happens by them being given opportunities to legitimately participate in a community of mathematicians while engaging in the mathematical practices of a mathematician. This extension of mathematics to students, is a democratizing process where more students can, want, and do demonstrate their mathematical abilities.

Future Research

Future research in this area should involve a longer period of study of students engaged in learning mathematics from the “science of patterns” perspective. It should be done with Design-Based Research approach - so much could be learned about the daily happenings of a real-world classroom – and from alternative methodologies able to examine other factors of student identity and disposition.

A specific research agenda that comes to mind is student identity. By the end of this study, I was not sure how students saw themselves in terms of being a doer of mathematics. Although there were engaging in the practices, developed conceptual and procedural fluency, I am not sure the students were conscious of the high level mathematical thinking they were doing. I say this because I did not see or at least effectively measure if students were cognizant of the

type of mathematics they were doing. I was hoping that when they realized what they were doing that it would inspire students to want to do more.

Another area ripe for study has to do with having students participate in real-world mathematics. Mathematical modeling is a catalyst for having students do mathematics in the pursuit of answering real-world questions. With that said, I believe a question exists about what is real-world and what relates to students' lives. This is where I think studying the context of the modeling situations would be fruitful for mathematics educators. With future work, I would like to see how having social justice themed contexts impacts the ideas examined in this study.

Last and definitely not least, future research must run along the lines of equity and opportunity. A key question to ask and seek answers to is "what can teachers do to equalize learning opportunities for students?" My inquiry is motivated by my findings, Treisman's (2013) testimony that "opportunity to learn is something we need to work on as math educators", and NCTM's (2008) statement that "excellence in mathematics education rests on equity" (pg.1).

Conclusion

When I first completed my data collection for this study, my thoughts were shrouded with insecurities and doubts. I wondered out loud to more than one empathetic ear that maybe I would need to do this all over again. "The students didn't learn anything", "What did I actually learn?", "They [my committee] trusted me to be out in the real world conducting research?", and "Who actually cares about what I just did?" were just some of the utterances I made in my post-study gloom. It wasn't until talking to more-knowing individuals (advisor and committee members) that I was put to ease. A turning point occurred, after one of my self-doubting ramblings, when a committee member asked me to respond to a question without thinking deeply about it. The simple but insightful question was, "would I do the study again?" My instant

reaction was “yes, yes I would”. Once I thought about it in this manner, I was ready to move forward.

Of course, the enactment of my design experiment had its issues. Some students were resistant, I did not have enough instructional time, outside the class factors were coming into play. I could not collect the data intended and at the same time I collected the data I intended. This was all overwhelming, too much data and not enough data, all at the same time. My instinctive response was buoyed by what I saw in students as they engaged in the tasks. I saw students participate with mathematics in ways that traditional mathematics did not give them access to. The things (mediating processes and outcomes) students were doing and the way they were thinking, even if they were not aware of it, could have lasting benefits if they further experience mathematics from this perspective.

Appendices

Appendix A – Curriculum/Instructional Content

Task	Activities	Math Content
Introductory activities around Calculator/Navigator use. Introduction to Patterns and How Mathematics is a Real World Activity.	<ul style="list-style-type: none"> • Log-Into TI Navigator system • Introduction to calculator and its use to complete basic math operations • Have discussion of what mathematics is and what it means to engage in the study of mathematics • Pattern identification and creation activities. 	<ul style="list-style-type: none"> • Use of handheld technology • Numerical, Symbolic, Graphical Patterns
Graphing Stories Tasks and Calculator-Based Ranger (CBR) Tasks	<ul style="list-style-type: none"> • GraphingStories.com graph creations • CBR motion detector activities 	<ul style="list-style-type: none"> • Functional relationships • Graphical representations of related quantities • Rate of change
Identifying Functions in Images Tasks	<ul style="list-style-type: none"> • Introduction to functions and their graphs • Cooperative Group Task • Calculator Driven Task • Pencil-Paper Task 	<ul style="list-style-type: none"> • Functions (Linear, Quadratic, Cubic, Exponential, Rational, Absolute Value, Square Root, Sine) • Identifying functions based on their graph
Identifying Functions in Data/Scatterplots Tasks	<ul style="list-style-type: none"> • Plotting data and identifying a function that possibly could represent the data • Creating a table of values, plotting data and identifying a function that possibly could represent the data • Collect data, creating a table of values from the data, plot data and identify a function that possibly could represent the data 	<ul style="list-style-type: none"> • Creating scatterplots with technology • Identifying patterns in plotted data • Identifying functions based on their graph
Modeling Tasks	<ul style="list-style-type: none"> • Deriving models from images • Deriving models from data • Deriving models from collected data 	<ul style="list-style-type: none"> • Linear equations ($y=mx+b$) • Quadratic equations (standard and vertex form) • Math Modeling (Hypothesizing and creating appropriate models based on the situation)

Appendix B - Summary of Data Collection

Type / Time	Source	Method/Instrument	Rationale/Projected Analysis
This data was collected during the first and last class meeting	Individual Free Response Math Task	UCLA MDPT Free-Response Algebra 1 Readiness Task	The purpose of this data was to provide an in-class pre/post baseline of students' modeling skills.
This data was collected on a daily basis	Participant Calculator Responses	Collected through QuickPoll response mechanism from the TI Navigator Software. I will be able to itemize and digitally store participants mathematical responses	The purpose of data was to track student progress as they develop their mathematical modeling skills, mathematical thinking, mathematical practices, and concept of function as they learn through the science of patterns perspective. This data is used to identify how the graphing technology supports their development in mathematical modeling skills, mathematical thinking, mathematical practices, and concept of function.
This data was collected on a daily basis	Participant Written/Math Responses	The responses were collected on a daily basis. This included homework assignments and in class tasks. Journals were given to students in order keep track of their work.	The purpose of data was to track student progress as they develop their mathematical modeling skills, mathematical thinking, mathematical practices, and concept of function as they learn through the science of patterns perspective.
This data was collected every class session	Student Discourse	This data was collected via video recordings. I recorded the entire class discussion engaged in a model-eliciting task. We also video recorded individual discussions during this process. This was done as the assistants or I	The purpose of this data was monitor student discourse as they engaged in various mathematical practices while mathematical modeling. This data was cross-referenced with other forms of data.

		supported students during class.	
Group Debriefing / End of each class session	Reflections on individual and group learning trajectories. Field notes / Student daily Observation Rubric (see Appendix D)	Debriefing with teacher and classroom research assistants. We recorded their engagement of mathematical practices using a Mathematical Practices rubric (See Appendix D for rubric) We discussed our ratings for various students and discussed what aspects of the instruction, curriculum, and graphing technology allowed them to engage or not engage in mathematical practices. This lead us to discussing how instruction would be revised for the next lesson.	The data was essential to track individual and group trajectories in regards to the their engagement in mathematical practices as their experience the curriculum and instruction. These observations were triangulated with video recordings of class sessions as well as student work (written and calculator).
Reflections/ Collected on a Daily basis	Teacher Reflections	My reflections were documented at the end of each class session. Reflections questions (See Appendix E) were used to guide the analysis. Additionally, the field-notes taken during the debriefings with research assistants were also used to guide the content of the reflections. Further analysis of student written and calculator work were used to inform the reflections.	These reflections guided the changes and revisions needed during the class. This was important given the iterative nature of the teaching experiment.

Appendix C - Student Daily Observation Form

Observer:

Date:

There is evidence that the student...	TU T	MU T	ST	M T	TT	Examples (Events of Interest) / Pedagogical and Content Notes
Students are able to recognize the relevant quantities involved in the situation.						
Students can represent these quantities in tabular and graphical form.						
Students hypothesize/explain how change in one quantity affects change in another quantity.						
Students choose functions that are relevant to the situation.						
Students test/revise whether their chosen function is an appropriate model.						
Students use their model to answer/pose contextual questions about the quantities.						
Notes:						

Appendix D - Lesson Reflection Form

When you consider the content learning of your students and the development of their mathematical thinking and practices, what do you think explains the learning/development or differences in learning/development that you observed during the learning segment?

Based on your experience teaching this learning segment, what did you learn about your students as mathematics learners (e.g., easy/difficult concepts and skills, easy/difficult learning tasks, easy/difficult features of academic language, common misunderstandings)?

What are the next steps given what your understanding of students engaging in mathematical practices and mathematical thinking today?

Appendix E – Student Function Tool

The "Science of Patterns" - Notes

Equation: Linear $y = x$	Table: 	Graph:
Equation: Quadratic $y = x^2$	Table: 	Graph:
Equation: Absolute Value $y = x $	Table: 	Graph:
Equation: Square Root $y = \sqrt{x}$	Table: 	Graph:

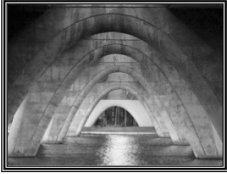
The "Science of Patterns" - Notes

Equation: Exponential $y = 2^x$	Table: 	Graph:
Equation: Rational $y = \frac{1}{x}$	Table: 	Graph:
Equation: Cubic $y = x^3$	Table: 	Graph:
Equation: Sine $y = \sin(x)$	Table: 	Graph:

Appendix F – Pencil Paper Task

The "Science of Patterns" - Picture/Function Match

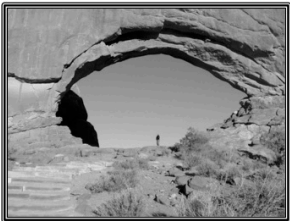
1. _____ Function 2. _____ Function



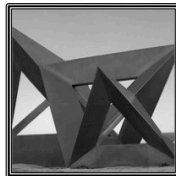
3. _____ Function 4. _____ Function



5. _____ Function

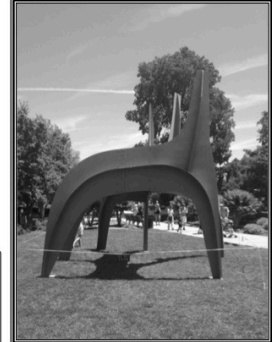


6. _____ Function



The "Science of Patterns" - Picture/Function Match

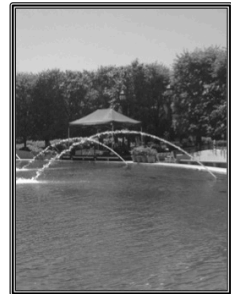
7. _____ Function 8. _____ Function



9. _____ Function



10. _____ Function



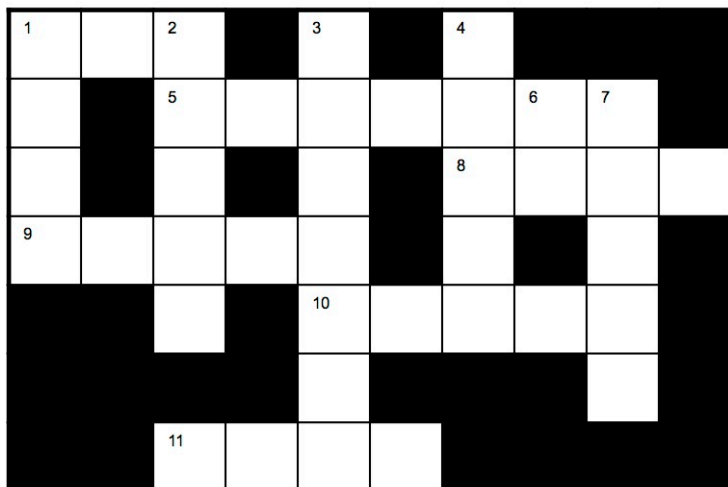
11. _____ Function



Appendix G – Cross Number Activity

CROSS-NUMBER PUZZLE

Complete the cross number puzzle below by using your Nspire to evaluate each expression. Each digit, decimal point, and negative sign will occupy each square. Round all answers to the nearest hundredth. If your answer does not fit the squares provided then check your calculation.



Across

Down

1) $12+11 \cdot 10$

1) $\frac{12+3}{7+5}$

5) $6543(132+329)$

2) $\frac{463}{94} \cdot 47$

8) $45^2 + 6^2$

3) -320^2

9) $\left(\frac{1}{2}\right)^4 + 3.7^3$

4) $-\sqrt{500(17852+1993)}$

10) $42000 \cdot 25^0$

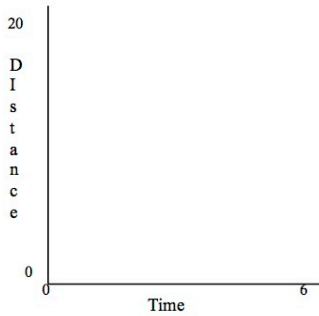
6) $\sqrt{337+504}$

11) $6\sqrt{11} + \sqrt[3]{4.4} - 1.83^5$

7) $\frac{9710}{15(17)}$

Appendix H – Science of Patterns Task
The “Science of Patterns” – Learn Check

1.



Johnny starts 20 feet from the motion detector and runs towards the detector for about 3 seconds, he then stops at about 10 feet and stands still.

DRAW THE GRAPH THAT REPRESENTS THE SITUATION

2. Complete the table of values. Then plot them using your calculator to determine which function describes the pattern.

Tu Daddy’s Pizza (Price Menu)
 Large Cheese Pizza \$5.00
 \$1.00 for each topping

<i>Toppings</i>	<i>Cost</i>
x	y
0	5.00
1	
2	
3	
4	
5	
6	

The pattern that is formed by the table of values is a(n) _____ function.

3. Identify 3 functions that can be found in the picture. Outline where the functions can be found.

List 3 functions you found:



Appendix I – Falling Circles Task

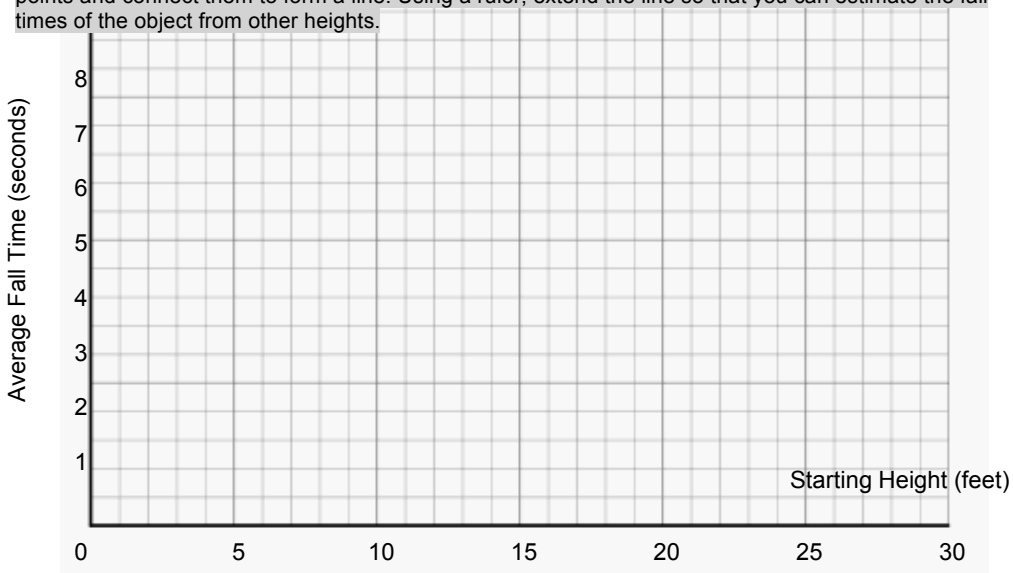
Falling Objects Activity

Objective: Use your skills in data collection, graphing linear equations, and finding the equation of a line in slope-intercept form to predict the time it will take for an object to fall from any height.

- 1) **Make Observations:** Make sure to listen to the teacher for instructions. Describe the object that you will use in this activity (sports ball, paper, coffee filter, etc.)
- 2) **Collect Data:** Make a table of values with the starting height in the first column, the time trials in the next three columns, the average time in the fourth column, and the ordered pairs.

Starting Height (x)	Fall Time Trial 1	Fall Time Trial 2	Fall Time Trial 3	Average Fall Time (y)	Ordered Pair (x,y)

- 3) **Find the Pattern:** Use this coordinate plane to make a careful graph of the data. Plot the (x,y) points and connect them to form a line. Using a ruler, extend the line so that you can estimate the fall times of the object from other heights.



Appendix J – Real-World Activity

The “Science of Patterns” - Real Life Situations

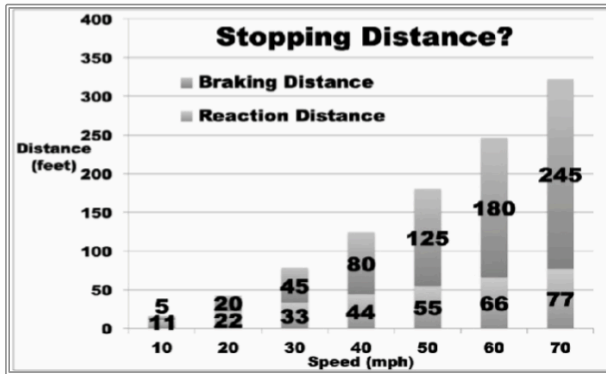
<p>D. Height of golf shot. A golfer hits a ball.</p> <p>x = the time that has elapsed in seconds. y = the height of the ball in meters.</p>	<p>Table:</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="border: none;">time</th> <th style="border: none;">height</th> </tr> <tr> <th style="border: none;">x</th> <th style="border: none;">y</th> </tr> </thead> <tbody> <tr><td style="border: none;">0</td><td style="border: none;">0</td></tr> <tr><td style="border: none;">1</td><td style="border: none;">25</td></tr> <tr><td style="border: none;">2</td><td style="border: none;">40</td></tr> <tr><td style="border: none;">3</td><td style="border: none;">45</td></tr> <tr><td style="border: none;">4</td><td style="border: none;">40</td></tr> <tr><td style="border: none;">5</td><td style="border: none;">25</td></tr> <tr><td style="border: none;">6</td><td style="border: none;">0</td></tr> </tbody> </table>	time	height	x	y	0	0	1	25	2	40	3	45	4	40	5	25	6	0	<p style="text-align: right;">Function</p> <p style="text-align: center;">Sketch and Label the Graph</p> <div style="border: 1px solid black; height: 100px; width: 100%;"></div>
time	height																			
x	y																			
0	0																			
1	25																			
2	40																			
3	45																			
4	40																			
5	25																			
6	0																			
<p>E. Test drive A car drives along a 200 meter test track.</p> <p>x = the average speed of the car in meters per second. y = the time it takes to travel the length of the track in seconds.</p>	<p>Table:</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="border: none;">speed</th> <th style="border: none;">time</th> </tr> <tr> <th style="border: none;">x</th> <th style="border: none;">y</th> </tr> </thead> <tbody> <tr><td style="border: none;">1</td><td style="border: none;">200</td></tr> <tr><td style="border: none;">2</td><td style="border: none;">100</td></tr> <tr><td style="border: none;">4</td><td style="border: none;">50</td></tr> <tr><td style="border: none;">5</td><td style="border: none;">40</td></tr> <tr><td style="border: none;">8</td><td style="border: none;">25</td></tr> <tr><td style="border: none;">10</td><td style="border: none;">20</td></tr> <tr><td style="border: none;">11</td><td style="border: none;">12.5</td></tr> </tbody> </table>	speed	time	x	y	1	200	2	100	4	50	5	40	8	25	10	20	11	12.5	<p style="text-align: right;">Function</p> <p style="text-align: center;">Sketch and Label the Graph</p> <div style="border: 1px solid black; height: 100px; width: 100%;"></div>
speed	time																			
x	y																			
1	200																			
2	100																			
4	50																			
5	40																			
8	25																			
10	20																			
11	12.5																			
<p>F. Balloon A man blows up a balloon.</p> <p>x = volume of air he has blown in cubic inches. y = diameter of the balloon in inches.</p>	<p>Table:</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="border: none;">volume</th> <th style="border: none;">diameter</th> </tr> <tr> <th style="border: none;">x</th> <th style="border: none;">y</th> </tr> </thead> <tbody> <tr><td style="border: none;">0</td><td style="border: none;">0</td></tr> <tr><td style="border: none;">1</td><td style="border: none;">1.25</td></tr> <tr><td style="border: none;">2</td><td style="border: none;">1.8</td></tr> <tr><td style="border: none;">3</td><td style="border: none;">2.2</td></tr> <tr><td style="border: none;">4</td><td style="border: none;">2.5</td></tr> <tr><td style="border: none;">5</td><td style="border: none;">2.8</td></tr> <tr><td style="border: none;">6</td><td style="border: none;">3.1</td></tr> </tbody> </table>	volume	diameter	x	y	0	0	1	1.25	2	1.8	3	2.2	4	2.5	5	2.8	6	3.1	<p style="text-align: right;">Function</p> <p style="text-align: center;">Sketch and Label the Graph</p> <div style="border: 1px solid black; height: 100px; width: 100%;"></div>
volume	diameter																			
x	y																			
0	0																			
1	1.25																			
2	1.8																			
3	2.2																			
4	2.5																			
5	2.8																			
6	3.1																			

Appendix K - Modeling Activities Task 2

Stopping Distance

The "Science of Patterns" – Stopping Distances

At many automobile crime scene investigations, investigators often measure the skid marks left behind by those cars involved in the accident. By measuring and recording the length of the skids marks, crime scene investigators can determine the speed of the car. We will use the data below to determine how they do this.

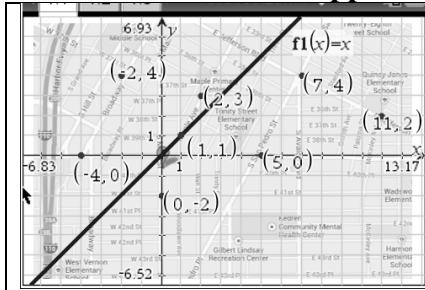


What do you know for sure?	What do you wonder?

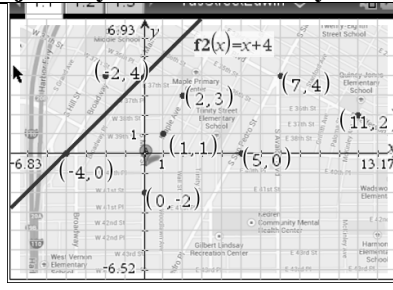
Using the table above, create a new table by identifying the important quantities (variables) involved in the stopping distance situation.

Speed	Total Stopping Distance
10	
20	
30	
40	
50	
60	
70	

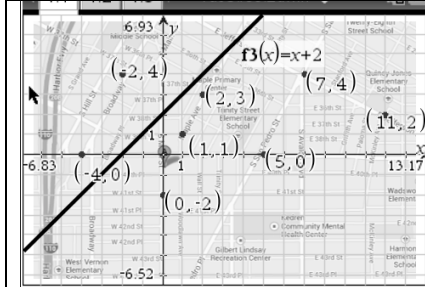
Appendix L – Student Trajectory Streets Activity



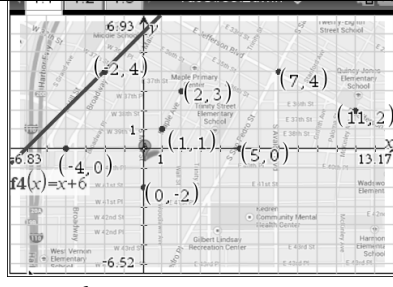
$y=x$



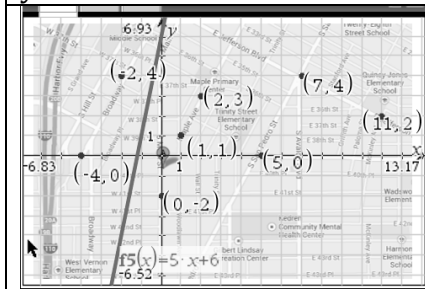
$y=x+4$



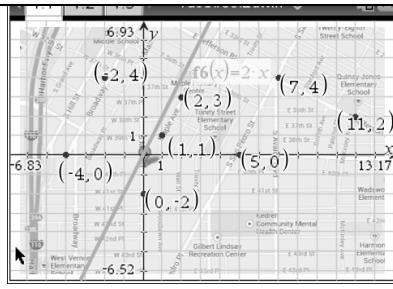
$y=x+2$



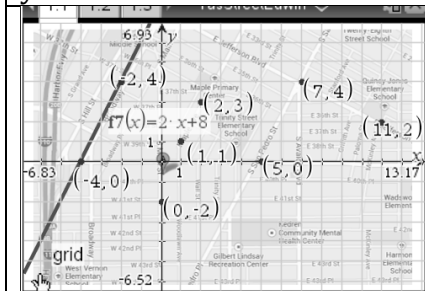
$y=x+6$



$y=5x+6$



$y=2x$



$y=2x+8$; model of best fit

Appendix M – Bungee Jumping Activity

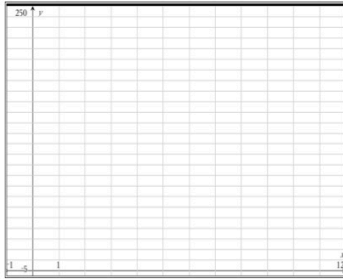
Algebra I
Name: _____

Barbie Bungee Jumping

Complete the table with all your information. Do three trials.

Make a hand plot of the data so you can determine which function best describes the data. Verify the graph on your calculator.

# of rubber bands	Distance Fallen (cm)			
	1	2	3	Average
1				
2				
3				
4				
5				
6				
7				
8				
9				



As a group, decide which functions best describes the relationship between the number of rubber bands and the distance Barbie falls. Why did you choose this function?

Now try and find the equation of the function that best fits the data.

Equation	What Adjustment are you going to make?

Algebra I
Name: _____

Barbie Bungee Jumping

Find out how many rubber bands you will need to get Barbie as close to the ground as possible. You will do this by setting your equations equal to the distance Barbie will have to fall. You will then solve for x and that answer will be the number of rubber bands you need. Show your work or explain how you got your answer.

Describe what happen happened on your final jump.

Conclusions: What problems did you encounter? How did the jump go? If you were to revise your jump, what would mathematics would you do differently?

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