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# Diagnosticity: Some theoretical and empirical progress

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## Abstract

We present progress towards a novel theoretical approach for understanding Tversky's famous 'diagnosticity' effect in similarity judgments, and an initial empirical validation. Our approach uses a model for similarity judgments based on quantum probability theory. The model predicts a diagnosticity effect under certain conditions only. Our model also predicts that changes to the set of stimuli to be compared can cause the diagnosticity effect to break down or reverse. In one experiment, we test and confirm one of our key predictions.

**Keywords:** similarity; diagnosticity; quantum probability

## Introduction

Our ability to perceive senses of similarity between objects is fundamental for cognitive processes such as categorisation, perception, language and decision making. Yet developing a successful model of our similarity intuitions has proven a major challenge. A natural approach has been to employ multi-dimensional psychological spaces, in which stimuli are represented as points and similarity between stimuli depends on distance. Such distance-based similarity models have proven popular, both because of their practical utility in cognitive modelling (e.g., categorization models; Nosofsky, 1984) and the theory they have enabled (e.g., Shepard, 1987). However distance-based similarity models are constrained in a number of ways. Some constraints arise from the fact that distance functions obey metric axioms; the most obviously intuitive of these is symmetry, implying  $\text{Sim}(A, B) = \text{Sim}(B, A)$ , where  $\text{Sim}$  denotes similarity. However there is also a more subtle constraint, which is that similarity should be a function only of the features of the stimuli being compared, and should be independent of the set from which the stimuli are taken. We call this constraint non-contextuality.

In a seminal study, Tversky (1977) provided an extensive empirical examination of distance-based similarity models. He obtained empirical evidence against all three metric axioms and so concluded that distance-based similarity models ought to be abandoned. Moreover, he reported a particular type of context effect in similarity judgments, such that the similarity between two stimuli can be affected by the presence or not of other available objects (broadly relevant to the similarity task). This context effect, called the diagnosticity effect, is also clearly beyond simple distance-based models of similarity. The diagnosticity effect is a fascinating demonstration of why similarity cannot be understood as a pairwise relation. Indeed, ever since Murphy and Medin's (1985)

highly influential work, cognitive psychologists have realised that there is more to similarity than just an objective (i.e., unmalleable by context or other factors) pairwise comparison.

The diagnosticity effect is central to the question of the adequacy of distance-based similarity models. This is because it is in fact possible to accommodate violations of the metric axioms within such models. Consider, for example, asymmetries; one needs to modify an expression like  $\text{Sim}(A, B) = f(d_{AB})$ , with  $f$  an arbitrary function, and  $d_{AB}$  the distance between objects  $A$  and  $B$  in psychological space, to something like  $\text{Sim}(A, B) = f(p_{AB} \cdot d_{AB})$ , where  $p_{AB}$  is a directionality parameter (Nosofsky, 1991). As long as  $p_{AB} \neq p_{BA}$ , asymmetries can emerge. By contrast, when considering the diagnosticity effect, there are no simple ways to modify standard distance-based similarity models to include context effects (cf. Goldstone & Son, 2005). What then is the most appropriate way to incorporate context in similarity judgments?

Tversky's (1977) diagnosticity demonstration involved a simple task, where participants had to select the country most similar to Austria, the target, from a set of three alternatives. When the alternatives were Sweden, Poland, and Hungary, most participants selected Sweden (49%), implying that  $\text{Sim}(\text{Sweden}, \text{Austria})$  was highest. When the alternatives were Sweden, Norway, and Hungary, Hungary was selected most frequently (60%), not Sweden (14%). Thus, changing the range of available alternatives can apparently radically change the similarity between the same two alternatives. Tversky's (1977) explanation for this result was that the choice of alternatives led to the emergence of different diagnostic features (either 'Eastern European' or 'Scandinavian' countries), which in turn impacted on the similarity judgment. Note, in Tversky's account, the relevant cognitive processes are sequenced as above, i.e., first there is a process of grouping, then an emergence of diagnostic features. As Tversky (1977, p. 342) notes "A change of clusters, in turn, is expected to increase the diagnostic value of features on which the new clusters are based, and therefore, the similarity of objects that share these features." Tversky (1977) employed 20 pairs of four countries and one further demonstration of the diagnosticity principle, based on schematic faces.

While Tversky's (1977) account is compelling, it relies on a notion of similarity and grouping between options which is underspecified or even circular (grouping depends on similarity, but, according to Tversky's account, similarity depends

on grouping; cf. Pothos et al., 2011). Further, the notion of the emergence of diagnostic features seems attractive for stimuli with many features, but this explanation must break down for stimuli with a small number of features, where the choice of which features to compare when making a similarity judgment is essentially fixed. In particular we report below the results of an experiment involving stimuli with only a single varying feature (their size), where any context effects cannot be accounted for by the emergence of diagnostic features.

The diagnosticity effect has eluded compelling replications (but see Evers, 2014). However we may find some evidence for the effect by looking not in the similarity literature, but in the decision making one. We note first that the fundamental cognitive principles guiding decision making may not be that different from those relevant in similarity judgments. Indeed, according to some authors there are deep commonalities between the two kinds of processes (e.g., Pothos, Busemeyer, & Trueblood, 2013; Shafir, Smith, & Osherson, 1990; Sloman, 1993). Significantly for our discussion of the diagnosticity effect, the decision making literature contains extensive replications of the so-called ‘similarity’ effect, which is analogous to the diagnosticity one. Consider a task whereby participants are asked to choose the option they prefer, amongst two alternatives *A* and *B*. A robust finding is that, when introducing a third option *C*, similar to *B*, then preference for *A* increases (see Trueblood et al., 2014, for a review).

There is a further lesson to be learned from the decision making literature; together with the similarity effect, researchers frequently also observe an attraction effect (and also a compromise one, of less relevance when using simple single feature stimuli.) The attraction effect is that, in the *A, B* choice as above, when introducing an option *C* clearly dominated by *A*, preference for *A* is enhanced. The attraction effect in decision making competes with the similarity effect. Is there an attraction effect in similarity judgments too? As far as we are aware, this has not so far been explored. But, if attraction effects can emerge in similarity judgments, then perhaps the interplay between the similarity/diagnosticity effect and the attraction effect is what explains the (apparent) brittleness of the diagnosticity effect. The main theoretical contribution of the present work is to propose a mathematical framework which produces predictions for both the diagnosticity and the attraction effect in similarity judgments. Our model reveals that the conditions for obtaining a diagnosticity effect are indeed highly constrained. We will present some preliminary empirical evidence to suggest that there is both a diagnosticity and an attraction effect in similarity judgments, in a way that is well captured by our model.

### **Previous modelling work**

Although this is not the place for a review of the existing similarity literature, we wish to mention a few important models of similarity that can account for the diagnosticity effect. For reasons of space, our focus will be restricted to Tversky’s (1977) findings. Of course, the empirical similarity litera-

ture has greatly progressed since then, nevertheless Tversky’s (1977) findings still provide a gold standard of attainment for novel similarity proposals.

Krumhansl’s (1978) distance-density model is a powerful way to extend standard distance-based similarity models. The main idea is that similarity depends on a modified distance function, which increases if the local density around the compared items increases. The distance-density model is consistent with the diagnosticity effect since, given options *A, B*, and target *T*, adding an option *C* similar to *A* increases the density around *A* (and *C*), and thus the modified distance between *A* and *T* increases, so the similarity between *A* and *T* decreases. However, the distance-density model has difficulty with other key Tversky (1977) results.

Ashby and Perrin’s (1988) General Recognition Theory (GRT) is a powerful similarity model, based on the idea that stimuli give rise to distributions of ‘perceptual effects’ in psychological space. With many stimuli present, psychological space is divided into response regions, such that each one is optimal for responding to a given stimulus. Similarity between two stimuli then depends on the overlap of the distribution of perceptual effects for the first stimulus, with the optimal response region for the second. The GRT deals well with the diagnosticity effect, since changing the range of alternatives in a similarity task changes the way psychological space is divided into optimal response regions. However, the GRT runs into problems accounting for symmetry violations, requiring additional assumptions to reproduce the observed asymmetries.

Nosofsky’s (1984) Generalized Context Model (GCM) is an extensively employed model of how categorization decisions are guided by similarity relations. The GCM can, in principle, capture the diagnosticity effect, because changing the range of available stimuli changes the overall similarity structure of the stimuli. This will change the dimensional attentional weights, which in turn impacts on the similarity ratings. No doubt these ideas are powerful and, verifiably, afford considerable explanatory power in supervised categorization. But their applicability in unsupervised situations, as in Tversky’s (1977) diagnosticity paradigm, is less clear: attentional weight change in the GCM is driven by the requirement to learn particular categorizations. Such requirements would be absent in e.g. the diagnosticity paradigm.

### **The quantum similarity model (QSM)**

#### **The original model of Pothos et al.**

Pothos et al. (2013), building on earlier work by Busemeyer et al. (2011), proposed a quantum similarity model (QSM), with a view to provide a comprehensive account of Tversky’s (1977) main findings. The QSM was developed using what we call quantum theory (QT), that is, the mathematics for how to assign probabilities to events from quantum mechanics, without any of the physics. QT can be employed as a theory of probability, beyond physics, in any area where there is a need to formalize uncertainty. In psychology, there

have been some arguments that the key characteristics of QT, e.g. interference, context and order effects resonate well with cognitive processes, at least in some cases (e.g., Bruza et al., 2009; Pothos & Busemeyer, 2009; Trueblood & Busemeyer, 2011; Wang & Busemeyer, 2013; for overviews see Busemeyer & Bruza, 2011; Pothos & Busemeyer, 2013).

Regarding the QSM, asymmetries can arise fairly naturally from the model, as well as violations of the triangle inequality, as observed by Tversky (1977). Violations of minimality can be accommodated in a generic way. Thus any explanation of the diagnosticity effect from the QSM will go hand in hand with satisfactory coverage of the other key findings from Tversky (1977), regarding violations of the metric axioms.

Below we introduce the original model of Pothos et al. and explain how it predicts a diagnosticity effect. In the next subsection we present an extension of the QSM which improves the treatment of context, and gives more detailed predictions.

The basic ingredient in the QSM is a complex vector space  $H$  (strictly a Hilbert space), representing the space of possible thoughts, which may be partitioned into subspaces, each representing a particular concept/stimulus. The subspace corresponding to concept  $A$  is associated with a projection operator  $P_A$ . The set of subspaces relevant to a particular set of similarity judgments need not be disjoint or complete, thus a particular thought may be associated with more than one concept. Although realistic psychological spaces may have high dimensionality, the important features can often be captured by a model with a much smaller dimensionality.

The dimensionality of the subspace corresponding to a concept is determined by the degree of knowledge we have about that concept. For example, if participants can be assumed to know more about China than Korea (cf. Tversky, 1977), then the dimensionality of the China subspace will be greater than that of the Korea one. These differences in dimensionality are a key feature of the QSM and give rise to predictions about asymmetries in similarity judgment. Equally, where we expect no systematic differences in degree of knowledge a convenient simplification is to assume that the subspaces are one dimensional, that is, just rays.

In the original QSM the knowledge state is given by a vector,  $|\psi\rangle$  in  $H$ . It corresponds, broadly speaking, to whatever a person is thinking at a particular time. For example,  $|\psi\rangle$  could be determined by the experimental instructions.

In the absence of any context elements the similarity between two concepts  $A$  and  $T$  in the QSM is essentially the sequential probability of thinking about  $A$  and then  $T$ <sup>1</sup>

$$\text{Sim}(A, T) = |P_T P_A |\psi\rangle|^2. \quad (1)$$

This expression for similarity can be thought of as a conjunctive probability that  $|\psi\rangle$  is consistent with possibilities  $A$  and  $T$  and, indeed, the QSM was developed as an extension of Busemeyer et al.'s (2011) model for the conjunction fallacy. In both these models, a projection step was

<sup>1</sup>To save space we ignore the normalisation of these expressions; in the experiments we discuss it turns out to be a constant.

assumed to be a step of thinking about the corresponding possibility/concept. For example,  $P_T P_A |\psi\rangle$  indicates thinking about possibility  $A$  first and then  $T$ . So, the way to incorporate context in the original QSM is as additional projections. For example, suppose the similarity between  $A$  and  $T$  is assessed in the context of a third option  $C$  (this could be an alternative in a forced choice task). Then, similarity would be computed as  $\text{Sim}(A, T; C) = |P_B P_A P_C |\psi\rangle|^2$ . In the case of Tversky's (1977) demonstration, the similarity expression was e.g.  $\text{Sim}(\text{Sweden}, \text{Austria}) = |P_{\text{Austria}} P_{\text{Sweden}} P_{\text{Hungary}} P_{\text{Poland}} |\psi\rangle|^2$  (the order of the projections for Hungary, Poland would need to be counterbalanced). Crucially, this similarity depends on the grouping of the context elements (it is higher when there is greater grouping), so it provided a natural way to accommodate Tversky's key finding and insight. But, the introduction of context in the QSM was somewhat heuristic and, moreover, it is not clear what new predictions are made. Our objective is to extend the QSM, with a view to address both these problems.

### An improved model of context

The first extension concerns employing a density matrix,  $\rho$  for the cognitive state, instead of a pure state,  $|\psi\rangle$ , as in Pothos et al. (2013). Essentially, a density matrix reflects classical uncertainty of which is the true pure state (e.g., for each participant), and it is a more general way of representing states in QT. The analogue of Eq.(1) in this case is,

$$\text{Sim}(A, T) = \text{Tr}(P_T P_A \rho P_A), \quad (2)$$

where  $\text{Tr}$  denotes the trace of a matrix. If the knowledge state is pure  $\rho = |\psi\rangle \langle\psi|$  and Eq.(2) reduces to Eq.(1).

As in Pothos et al. (2013), we include context effects by prior projections (i.e. between the initial state and the stimulus  $A$ ) onto the context elements. If the task is to select which of  $\{A, B, C\}$  is most similar to a target  $T$ , then when judging the similarity between  $A$  and  $T$  the context elements are  $B$  and  $C$ . The key technical extension we introduce concerns the idea that there are many possible sets of prior projections we could consider. We define the similarity between  $A$  and  $T$  in the context of  $B$  and  $C$ , denoted  $\text{Sim}(A, T; B, C)$ , as,

$$\text{Sim}(A, T; B, C) = \sum_i a_i^2 \text{Tr}(P_T P_A \Gamma_i \rho \Gamma_i^\dagger P_A) \quad (3)$$

where  $\Gamma_i$  is one of the strings of projectors given by,

$$\Gamma_i \in \{1, P_B, P_C, P_B P_C, P_C P_B, P_B P_C P_B, P_C P_B P_C, \dots\} \quad (4)$$

and  $a_i$  is a pre-factor which we will discuss in a moment. Thus in the extended QSM we do not include only a single possible string of prior projectors, but rather a whole infinite family<sup>2</sup>. This solves the problem of the arbitrariness of the original proposal, regarding context elements.

<sup>2</sup>This is easily extended to larger sets of context elements. The general form is

$\Gamma_i \in \{1, \text{all context elements, all strings of two context elements, all strings of three context elements, } \dots\}$

The  $a_i^2$  are the relative probabilities participants will follow the particular prior train of thought given by  $\Gamma_i$ . We suggest setting  $a_i = \alpha^{N/2}$ , with  $N$  the number of projectors in  $\Gamma_i$ , and  $\alpha < 1$  is some non-negative number. This has the advantage that there is only a single parameter, and that longer strings of projectors (i.e. larger numbers of prior thoughts) have lower relative probabilities of occurring in the model.

A final issue concerns how to set  $\rho$ . In the original QSM the initial state was taken to be a pure state with equal prior probability to be consistent with each of the stimuli. While it is reasonable to set it in a neutral way when only two objects/concepts are involved (as in the original model), when multiple objects are involved, this is not straightforward (e.g., it often precludes a representation of low dimensionality) and, furthermore, may be psychologically unreasonable. So, with multiple objects, we suggest that  $\rho \sim 1$  (the identity matrix)<sup>3</sup>.

Compared to Pothos et al (2013) our improved model comes at the price of extra complexity. It is worth pausing to see if we can give a conceptual overview of the way our model works. The model adds to the value of the similarity in the absence of context elements a number of terms which involve prior thoughts about the context stimuli (in this case  $B, C$ ), before the comparison between the two relevant stimuli (in the case  $A, T$ ) is made. Trains of thought involving multiple stimuli have higher amplitudes when these stimuli are close together in psychological space (a basic feature of QT), thus the value of the perceived similarity depends on the way that the context stimuli are grouped. Although the details depend on the values of the  $\{\alpha_i\}$ , if  $B, C$  are grouped but far from the  $A$  and  $T$ , this will lead to higher values of perceived similarity than if  $B, C$  are ungrouped, or are close to  $A$  or  $T$ . This is essentially the diagnosticity effect.

### The predictions of the extended QSM

In general it might be complicated to extract analytical predictions for set of stimuli that have large number of features, since the corresponding Hilbert space will be very large dimensional. However our model simplifies when we have a set of stimuli which vary in only a single feature, e.g. size. In this case we can embed the stimuli in a simple 2D Hilbert space. The projection operators are then onto rays, and the states of the stimuli are encoded in the angles between the rays. Assume we have a target  $T$  plus stimuli  $\{A, B, C\}$ . Taking the angle of the ray representing  $T$  to be 0 (this angle is arbitrary), and the angles between  $T$  and the other stimuli to be  $\theta_A, \theta_B, \theta_C$  respectively, the model predicts that the observed similarity between  $A$  and  $T$ , in the context of  $B$  and  $C$ , will be (Yearsley et al, in preparation),

$$\text{Sim}(A, T; B, C) = \cos^2(\theta_A) \times \left[ 1 + \frac{\alpha(\cos^2(\theta_A - \theta_B) + \cos^2(\theta_A - \theta_C))}{1 - \alpha\cos^2(\theta_B - \theta_C)} \right] \quad (5)$$

<sup>3</sup>If concepts are represented by subspaces of equal dimension  $\rho \sim 1$  has equal prior probability of being consistent with each concept, as in the original QSM. More generally,  $\rho \sim 1$  is a state neutral with respect to *features*, rather than *concepts*.

By looking at these expressions we can see three things,

- For  $\alpha = 0$  we recover the similarity expression when ignoring context.
- For small values of  $\alpha$ , the similarity between  $A$  and  $T$  is enhanced by grouping  $A$  with  $B$  and/or  $C$ .
- For larger values of  $\alpha$ , the similarity between  $A$  and  $T$  is enhanced if  $B$  and  $C$  are grouped.

A more detailed analysis (Yearsley et al, in preparation) shows that these features lead to the following predictions for the effects of context in the case of stimuli with a single relevant feature; first, all else being equal, smaller values of  $\alpha$  will lead to an attraction effect, larger values to a diagnosticity effect. Second the type of context effect observed depends on the angle  $\theta_C$  when  $\alpha$  is non zero. In the special case where  $\theta_A = -\theta_B > 0$  (cf Sweden and Hungary equally similar to Austria, as in Tversky's 1977 demonstration), we have that for  $\theta_B \lesssim \theta_C \lesssim \theta_A$ ,  $C$  is judged most similar to  $T$ . For  $\theta_C \gtrsim \theta_A$ , there is initially a diagnosticity effect, where  $B$  appears most similar to  $T$ ; as  $\theta_C$  gets larger this becomes an attraction effect, with  $A$  appearing most similar to  $T$ . Likewise, for  $\theta_C \lesssim \theta_B$ , there is at first a diagnosticity effect, with  $A$  most similar to  $T$ ; as  $\theta_C$  gets smaller this becomes an attraction effect, and  $B$  appears most similar to  $T$ .

We think these predictions are significant and at the heart of the debate about whether the diagnosticity effect is real or not. The extended QSM shows the brittleness of the diagnosticity effect. It depends on both  $\alpha$  and  $\theta_C$  and, therefore, unless the stimuli are carefully controlled, it seems likely that no diagnosticity effect will be observed! We believe this in part explains the paucity of replications in the literature (see Evers & Lakens, 2014, and Medin et al., 1995, for exceptions.)

Before exploring the novel predictions of the extended QSM regarding the interplay between diagnosticity and attraction, we first check that the extended QSM can capture Tversky's (1977) result (as the original QSM can). Recall the comparison concerned the similarity between a target Austria and two countries Hungary and Sweden, in the presence of one of two context items, Poland or Norway. When the context item is Poland, Sweden is preferred, when the context item is Norway, Hungary is preferred.

Our set up will be similar to that employed by Pothos et al. We set the angles between Austria and Hungary and between Austria and Poland to be equal to some base angle, plus some small random angles between  $\pm 5^\circ$ . We then set the angle between Austria and Sweden to be minus the same base angle, plus again a small random angle between  $\pm 5^\circ$ . The result is that Hungary and Sweden are roughly equidistant from Austria, while Hungary and Poland are closely grouped. We then compute the similarities between Austria and each of the other countries to determine which would be selected as most similar to Austria, averaging the results over  $10^4$  choices of the random angles. The results are displayed in Fig.1.

It is clear from this graph that the extended QSM reproduces the key result of Tversky (1977) for larger values of  $\alpha$ .

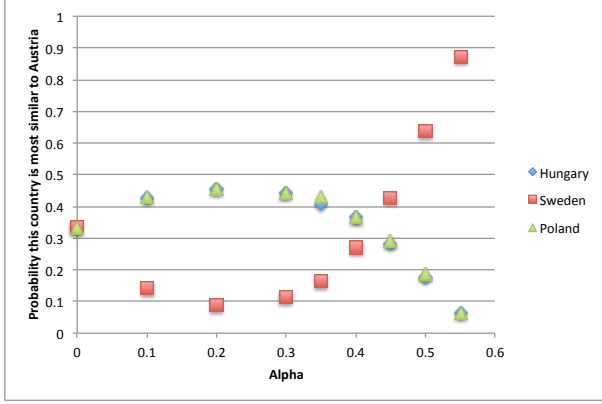


Figure 1: Simulated Probabilities of choosing each country as most similar to Austria. The base angle here is  $\sim 18^\circ$ .

For smaller values of  $\alpha$  our model also displays an attraction effect. That the crossover happens for values of  $\alpha$  between 0.4 and 0.5 may not be immediately meaningful, but it can be shown that this is equivalent to an average number of prior context thoughts between one and two. This seems appropriate considering the demands of Tversky's (1977) experiment (i.e. participants do consider the context elements, but long strings of prior thoughts are unlikely.)

That this set up does not quite reproduce the findings in Tversky (1977), since there the context country, Poland, was always rated as less similar to Austria than either of Hungary or Sweden. This can be achieved in our model by choosing a slightly larger base angle between Austria and Poland. However the intention of this demonstration was rather to show that a diagnosticity prediction can emerge in the extended QSM, in the way observed by Tversky, depending on the grouping between the context elements.

### An Empirical Demonstration

We now report an initial set of experimental results examining the key prediction of the extended QSM regarding the interplay between the diagnosticity and attraction effects.

**Participants** 200 experimentally naïve US residents were recruited via Amazon Turk, and were paid \$0.50 for their time.

**Stimuli** We employed as stimuli 17 spirals. The size of each spiral was given by the formula,

$$S_n = S_0(1.1)^n, \quad (6)$$

where  $S_0$  was the size of the initial target spiral, chosen to be  $7\text{cm}^4$ , and  $n$  runs from  $-8$  to  $8$ . According to Weber's law, Ps should rate the similarity between spirals as a function only of the difference in  $n$ , so,

$$\text{Sim}(S_n, S_m) = \text{Sim}(S_{n+k}, S_{m+k}). \quad (7)$$

<sup>4</sup>The sizes of the spirals depend on the screen on which the experiment is taken. However the relative sizes do not.

Thus the similarity between neighbouring spirals is constant, which simplifies the analysis. A pilot study of 100 participants confirmed the perceived similarity between  $S_0$  and the other spirals varied broadly in accordance with Weber's law.

**Procedure** Participants were given a series of trials where they were shown a target spiral  $T$  with size  $S_0$  and below this three other spirals  $A, B, C$ , of sizes  $S_A, S_{-A}, S_C$ , and were asked to indicate which of  $A, B, C$  they judged most similar to  $T$ .

The experimental trials were designed so that  $A$  took values from 1 to 8, and  $C$  took values from  $-8$  to  $8$ , but excluding 0. Participants were split into two groups. One group saw values of  $C$  from 1 to 8, and the other from  $-1$  to  $-8$ .

The presentation of the spirals  $\{A, B, C\}$  on the screen in each trial was partly randomized, the spirals were presented horizontally across the screen, with  $C$  in the centre and either  $A$  on the right and  $B$  on the left or vice versa. The order of presentation of trials within each group was randomized.

**Discussion** As space restrictions prevent us from reporting the full set of data, we focus on the similarity judgments corresponding to  $A = 4$ . This means we have a target spiral  $T$  of size  $7\text{cm}$ , spirals  $A$  and  $B$  of sizes  $10.2\text{cm}$  and  $4.8\text{cm}$  respectively, and then a spiral  $C$  whose size varied from  $3.3\text{cm}$  to  $15.0\text{cm}$ .

It is helpful to organize the data in terms of which other spiral is closer, and which one is further away from  $C$ . Diagnosticity means participants should prefer the spiral furthest from  $C$ , while attraction means participants should prefer the spiral closer to  $C$ , provided it is more similar to the target than  $C$ . The data is presented in Fig.2.

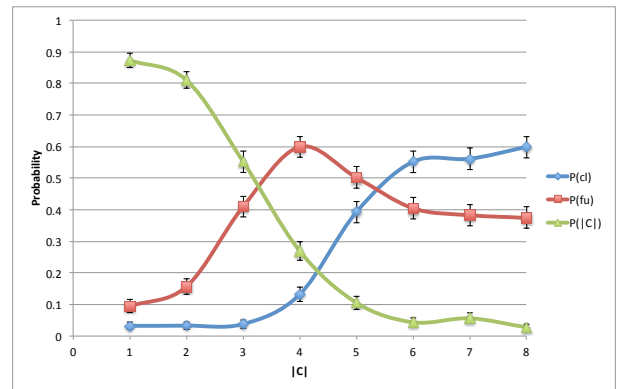


Figure 2: Probabilities for choosing spiral closest to  $C$  (cl), spiral furthest from  $C$  (fu) or  $C$  itself ( $C$ ), as most similar to  $T$  as function of  $|C|$ .  $A = 4$  in all cases. Error bars show SE.

The data show both a diagnosticity ( $|C| = 4, 5$ ) and an attraction effect ( $|C| \geq 6$ ). In addition there is a preference for the ungrouped over the grouped stimuli even for  $|C| \leq 3$  which, though not strictly a diagnosticity effect (since the context item is preferred over  $A$  and  $B$ ), still represents a context effect. The transition between the two effects is in qualitative agreement with the predictions of our model. For a

suitable choice of model parameters (an additional parameter is needed to convert similarity ratings (1-9 scale) into values between 0-1) we can reproduce rank order of preferences for all values of  $|C|$ .

Overall, our data clearly demonstrate both the existence of a diagnosticity effect for these single feature stimuli and that this effect can break down in a way predicted by the extended QSM. We reiterate the key point, the blue and red curves in Figure 2 correspond to spirals equidistant (verifiably so, from our pilot) to the target spiral  $T$ . We can see how the probability for selecting one spiral vs. the other can vary dramatically as the context stimulus,  $C$  (the green curve), changes. Clearly, further work is needed to validate more comprehensively the extended QSM, but we are encouraged by these results.

## Conclusions

We presented a new model of similarity under the influence of context, based on QT, building on earlier work by Pothos et al (2013). The model provides a formal, rigorous way to predict similarity judgments, based not only on the stimuli compared, but also on other broadly relevant stimuli, which constitute a contextual influence. We showed this model can account for Tversky's (1977) diagnosticity findings. Importantly, we also showed the model predicts a diagnosticity vs. attraction effect, based on the grouping of available stimuli, and provided an initial experimental validation of this prediction.

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