UC Santa Barbara
Ted Bergstrom Papers

Title

A Test for Efficiency in the Supply of Public Goods

Permalink

https://escholarship.org/uc/item/0gp06544

Authors

Bergstrom, Ted
Roberts, Judith
Rubinfeld, Daniel
et al.

Publication Date

1988-03-18

A Test for Efficiency in the Supply of Public Education

by

Theodore Bergstrom, Judith Roberts, Daniel Rubinfeld, and Perry Shapiro

Current version: April 21, 1995

Abstract. This paper devises and applies a statistical test for efficient provision of local public education. The test is based on the "Samuelson condition" of equality between the sum of marginal rates of substitution and marginal cost. The econometric method is a micro-based approach to the estimation of the marginal rate of substitution function. This method accounts for possible "Tiebout bias" caused by the fact that individuals may choose their school districts in accordance with their tastes for education.

A Test for Efficiency in the Supply of Public Education

Theodore Bergstrom, Judith Roberts, Daniel Rubinfeld, and Perry Shapiro

1. Introduction

The question of whether governments spend too much or too little has been the subject of much debate but little econometric testing. This paper conducts an empirical test of whether local governments spend more or less than a Pareto optimal amount on local public goods. Our procedure is very simple in principle. We check whether the "Samuelson first order conditions" (Samuelson, 1954) for efficient provision of public goods are satisfied. The Samuelson conditions require that the sum of individual marginal rates of substitution between a public good and the private numeraire equals the marginal cost of the public good. This condition is necessary for an interior Pareto optimum and in a well-behaved convex economy is also sufficient.

Theoretical arguments have been made which suggest that the amount of local public goods provided may be nearly efficient. Some of these arguments depend on the effectiveness of majority voting. Bowen (1943) proposed a model in which the amount spent on public goods is the median of the quantities desired by voter-taxpayers, where each voter realizes that in return for the benefits of additional public expenditure, he or she will have to bear a predetermined share of the extra cost. Bowen showed that if marginal rates of substitution are symmetrically distributed and public goods are paid for by a uniform "head tax", then majority rule leads to a Pareto efficient supply of public goods. Bergstrom (1979) extended the Bowen efficiency theorem to include some cases where wealth is not symmetrically distributed and where there is a proportional wealth tax. Ledyard (1983) presented a model in which voters make rational decisions about whether to vote, candidates choose their positions strategically and political equilibrium is efficient. A different theoretical case for the efficiency of local public goods supply was inspired by the work of Tiebout (1956). Tiebout suggested that the competitive forces engendered by people "voting with their feet" might lead to approximate efficiency. This literature is ably summarized and criticized by Bewley (1982) and Stiglitz (1982).

There is an interesting literature that argues that local public goods are undersupplied. Barlow (1970) suggested that the Bowen conditions are typically not satisfied and offered evidence that in the case of local school expenditures in Michigan, the median quantity demanded is less than the Pareto optimal amount. Some economists believe that voters systematically underestimate the benefits of public goods. Galbraith (1958) attributed this to the effect of private advertising. Downs (1960) argued that because information is costly and a single voter has a negligible effect on public outcomes, it is rational for voters to be less than fully informed about the effect of public goods. This, Downs argues, leads to a systematic underestimation of benefits, which are poorly understood, relative to tax costs which are easily understood. Some writers have suggested that local governments supply too little public goods because there are "spillovers" in benefits from one city to

another. These effects were analyzed by Brazer (1961), Weisbrod (1964), and Williams (1966).

Others have made appealing arguments that "too much" is spent. Romer and Rosenthal (1979), Brennan and Buchanan (1971), Denzau and Mackay (1982), argue that bureaucrats may manipulate the choices offered to voters in such a way as to lead to greater expenditure than the median of the most preferred amounts of voters. Shapiro and Sonstelie (1982) found evidence to support the hypothesis of bureaucratic manipulation. Stiglitz (1982) argued that renters' incentives in elections are inconsistent with efficiency. Courant, Gramlich and Rubinfeld (1979) suggested that public employee market power might also lead to inefficient levels of public provision.

Our test is designed to detect undersupply of the kind described by Barlow or oversupply due to bureaucratic manipulation. Since we deal with consumers' reported preferences about expenditures in their own cities, we will not be able to detect undersupply or oversupply that occurs because people don't know what is good for them. We will also be unable to discover whether there is undersupply because of unrewarded spillovers from one city to another. Furthermore, our results can tell us nothing about whether efficiency would require a different assignment of people to cities. A test of the type we suggest can at best only determine whether the existing population of a city could make a Pareto improvement for its members by increasing or decreasing its public expenditures.

2. Methodology

Suppose that we observe a number of communities (school districts in our case) each of which supplies a local public good. Consumer i who lives in community j has a marginal rate of substitution between the public and private goods that is a function of the form

$$m_i = m(A_j, Y_i, H_i, Z_j) \qquad (1)$$

where A_j is the amount of the public good provided in community j, Y_i is i's disposable income (consumption of private goods), H_i is a vector of personal characteristics of person i (such as age, sex, family status, etc.) and Z_j is a vector of characteristics of community j (such as its population, climate, proximity to other cities, etc.).

To make a manageable task of estimating the sum of marginal rates of substitution, we will have to make some restrictive assumptions about the functional form of $m(A_j, Y_i, H_i, Z_j)$. In particular, we assume that individual marginal rate of substitution functions are of the form:

$$m(A_j, Y_i, H_i, Z_j) = \beta_0 + \beta_1 \ln A_j + \beta_2 \ln Y_i + \sum_k \beta_{3k} \ln Z_{jk} + \sum_k \beta_{4k} H_{ik} + \epsilon_i. \qquad (2)$$

Let X_i denote the column vector $(1, \ln A_j, \ln Y_i, \ln Z_j, H_i)$ of right-hand variables of the mrs equation. Equation 2 can then be written simply as:

$$m_i = \beta' X_i + \epsilon_i. \qquad (3)$$

2

We estimate the marginal rate of substitution function, $m(Y_i, A_j, H_i, Z_j)$, for expenditures on public primary and secondary education in Michigan using a 1978 survey of Michigan households (see Courant, Gramlich and Rubinfeld (1979)). Once we have estimates of this function, based on this statewide sample, we can try to predict the sum of marginal rates of substitution in individual school districts. We can then compare this sum with the marginal cost of public goods to the school district. If we find that the predicted sum of marginal rates of substitution in this school district is greater than our estimate of the marginal cost of schooling to the district, it may be that the district is spending too little on public goods from the standpoint of efficiency. It might also be the school district is different in some way that we have not measured from other districts in the state. But if there is a systematic tendency to undersupply (oversupply) local public education, then when we compare the predicted sums of marginal rates of substitution to marginal costs for a large number of school districts we should expect to find that the predicted sums of marginal rates of substitution on average tend to exceed (fall short of) the estimated marginal costs of public goods.

From the Courant, Gramlich, Rubinfeld survey, we can calculate respondent i's tax price, t_i. If her home community provided the same amount that she would choose given her tax price, then each respondent's tax price would be equal to her marginal rate of substitution, so that $m_i = t_i$. We could then estimate the parameters of the marginal rate of substitution function simply by running a regression in which the dependent variable is the tax price, t_i.

While people may tend to move to communities where the provision of public goods is in accordance with their tastes, there is no reason to expect unanimous agreement within communities about expenditure levels. It is true that if people sorted themselves so that residents of the same community had nearly identical tastes and incomes, then with an equitable tax structure and a reasonably responsive government, we would expect all residents to be getting approximately the amount of public goods that they would choose for themselves given the tax structure. But there are strong economic forces that lead to diversity within communities. People with different occupational skills find it advantageous to work together and to live in close proximity. The housing stock may be highly variable in age, quality, size of units, and attractiveness of location. It is not surprising therefore that within communities there is a wide range in the incomes, education levels, ages, and family sizes of residents. While taxes of different types of consumers could possibly be adjusted to achieve near unanimity about quantities, we cannot expect on *a priori* grounds that communities will be in Lindahl equilibrium. And indeed empirically there is no such near-unanimity. In the Courant, Gramlich, Rubinfeld survey about 25 per cent of the respondents wanted higher and about 17 percent wanted lower expenditures on public goods than were currently supplied in their communities.

The fact that community choice is voluntary leads to a potentially important statistical problem. This problem of "Tiebout bias" (Goldstein and Pauly (1981)) can be described as follows. Define the variable η_i to be the difference between household i's marginal rate of substitution and its tax price. Thus we have:

$$t_i = m(A_j, Y_i, H_i, Z_j) + \eta_i \qquad (4)$$

3

and therefore from (3) it follows that

$$t_i = \beta' X_i + \epsilon_i + \eta_i. \tag{5}$$

If η_i were uncorrelated with variables X_i, then the mismatch between t_i and m_i would cause no econometric problems. Then if ϵ_i is also uncorrelated with X_i, unbiased estimators of the parameters of equation (5) could be found by ordinary least squares. (One instance where this would be the case is where η_i is zero for all i.) We want to take account of the possibility that η_i is correlated with the right hand variables, X_i. For this reason we will have to use a more elaborate econometric method which is described in the remainder of this section.

We focus on a survey question in which respondents were asked whether they would prefer school spending to increase, decrease or remain the same, if they knew that their taxes would change to reflect these expenditure changes. Because the responses take the form of discrete rather than continuous variables, we must use a qualitative response model. We postulate that people only say that they are dissatisfied with the current state of affairs if their tax price is sufficiently different from their marginal rate of substitution, If the difference is not sufficiently large, they respond that they want local school expenditures to remain the same (S). If their marginal rate of substitution is sufficiently larger than their tax price, they respond that they want more expenditures (M) and if it is sufficiently smaller they respond that they want less (L). The concept of sufficient difference is formalized by a parameter δ such that the response is (M) if $m_i > t_i + \delta$; (L) if $m_i < t_i - \delta$; and (S) if $t_i - \delta < m_i < t_i + \delta$.

Recalling equation 3, we see that the probabilities of individual i's responses conditional on a tax rate t_i and vector of characteristics X_i are:

$$P(M_i|t_i, X_i) = P(\epsilon_i > t_i - \beta'X + \delta)$$

$$P(L_i|t_i, X_i) = P(\epsilon_i < t_i - \beta'X - \delta)$$

$$P(S_i|t_i, X_i) = P(t_i - \beta'X - \delta \leq \epsilon \leq t_i - \beta'X + \delta). \tag{6}$$

Assuming that ϵ has a standard normal distribution, the probabilities can be expressed in terms of the standard normal cumulative distribution as follows:

$$P(M_i|t_i, X_i) = 1 - F\left(\frac{t_i - \beta'X + \delta - E(\epsilon|t_i, X_i)}{\sigma(\epsilon|t_i, X_i)}\right)$$

$$P(L_i|t_i, X_i) = F\left(\frac{t_i - \beta'X - \delta - E(\epsilon|t_i, X_i)}{\sigma(\epsilon|t_i, X_i)}\right)$$

$$P(S_i|t_i, X_i) = F\left(\frac{t_i - \beta'X + \delta - E(\epsilon|t_i, X_i)}{\sigma(\epsilon|t_i, X_i)}\right) - F\left(\frac{t_i - \beta'X - \delta - E(\epsilon|t_i, X_i)}{\sigma(\epsilon|t_i, X_i)}\right). \tag{7}$$

If ϵ were distributed independently of (t_i, X_i), it would be the case that $E(\epsilon|t_i, X_i) = 0$ and standard, ordered probit estimation would yield consistent and efficient estimators

4

of the parameters β and δ. But we suspect that this is not the case. As we argued in connection with equation (5), even if we are willing to assume that ϵ_i is uncorrelated with X_i by itself, the correlation between ϵ_i and t_i *conditional on* X_i will generally be nonzero. Similarly, since the vector X_i includes as one of its components actual school spending levels in i's community ($\ln A_j$), the correlation between ϵ_i and X_i *given* t_i is likely to be nonzero. Failure to recognize these possible correlations can result in what we call *Tiebout bias*, or bias that occurs because people take into account their preferences for public goods when they decide where to live.

To deal with possible correlation between ϵ and (t_i, X_i), we introduce a set of instrumental variables which plausibly have a negligible influence on the marginal rate of substitution functions but which may affect either t_i or A_i. In the next section we will describe these instrumental variables, which are denoted by the vector W_i. Let us define $\tilde{X}_i$ to be the vector obtained by dropping the component $\ln A_i$ from the vector X_i. Our approach is to add two equations to the model, which may themselves be simultaneously determined. The reduced forms are follows:

$$t_i = \theta_{11}\tilde{X}_i + \theta_{12}W_i + \omega_{1i} \tag{8}$$

$$\ln A_i = \theta_{21}\tilde{X}_i + \theta_{22}W_i + \omega_{2i}. \tag{9}$$

where ω_1 and ω_2 are random errors assumed to be uncorrelated with W and $\tilde{X}$. We estimate the system of equations, (7)-(9), using the method of full information maximum likelihood. The relevant likelihood function is described in the appendix. A detailed discussion of the approach can be found in Rubinfeld, Shapiro and Roberts (1987).

3. Predicting Individual Marginal Rates of Substitution

Our source of data is a sample of 1093 Michigan homeowners from a survey of Michigan voters residing in many different school districts. The fact that the sample includes voters from different school districts is important since we want to estimate the effects of characteristics of the school districts in which a respondent lives as well as the respondent's own characteristics on her willingness to pay for an additional unit of local public education. The individual characteristics of the respondents were recorded in the survey. The characteristics of the school systems were obtained from the Michigan Department of Education, while other community characteristics were taken from the 1970 U.S. Census First Count and Fourth Count School District data tapes. The definitions of all variables used in the estimation procedure are given in Table 1.

Measurement of Independent Variables for the Prediction Equations.

The way in which we measure quantity and price variables requires some discussion. The quantity of local public education that a respondent experiences is measured by *per student* expenditures in the local school district. Per student expenditures by itself is the

appropriate measure if there are constant returns to scale in the production of local education and if the costs of educational inputs are the same across districts. To allow for the possibility that there may be increasing or decreasing returns to scale, we included variables for total enrollment in the school district and average enrollment per school in the district. If there are increasing (decreasing) returns to scale, then providing an extra unit of education would be cheaper (more expensive) in larger school districts.

To allow for differences in costs due to differences in the wages paid to teachers, we included a variable measuring the average teachers' salary in the county where the school district was located. We used the average teachers' salary for the county in which the school district was located rather than the average teachers' salary in the district itself since differences in the latter might be strongly influenced by differences in the quality and experience of teachers while differences in the former might more closely reflect the market conditions facing the school district. Not only might the prices of school inputs differ from district to district, but so might the price of private goods. The average wage rate of non-teachers is used as a surrogate for a local price index for private goods.[1] We also included as a variable the mean *per capita* income in the respondents' home county.

Since the commodity of interest is per student school expenditures, the tax price t_i paid by family i is the cost to i of increasing the expenditure *per student* in the school district where the family resides by one dollar. In the survey, each respondent reported the assessed value of his home. The tax share of respondent i was taken as the ratio of the assessed value of i's home to the total assessed value of property in i's home district. This tax share multiplied by the number of students in the district is the tax price, t_i.

We do not know much about the tax shares perceived by renters. Therefore we did not estimate separate marginal rate of substitution functions for renters. Instead we assumed that renter preferences for public goods were the same as the preferences of homeowners (though of course their tax prices might be very different). When we later construct estimates of sums of marginal rates of substitution in the community, we must add in the estimated sum over renters as well as homeowners.

The respondent's disposable income was recorded from the survey. We included several other variables that describe individual characteristics which might influence demand. One variable of interest is whether a respondent has children in the local public schools. We included separate dummy variables for whether a respondent had children of school age and for whether a respondent had preschool children. We also include a variable for whether the respondent has children in private school. Other variables describe the respondent's race, sex, educational level, and whether the respondent is over 65 years of age, retired, unemployed, or receiving welfare payments.

We use four instrumental variables which could reasonably be expected to influence a taxpayer's actual tax price or the local school district's expenditure level but would not

---

[1] Differences in wages due simply to the occupational mix are excluded since we were able to find wages by occupation and compute a weighted average using the same weights for each occupation across communities.

have a direct effect on willingnesses to pay for local public education. The first is the fraction of households in i's community with income within 30% of the community median income. The more homogeneous a population, the more agreement there should be about the correct level of spending. Furthermore, a person in a homogeneous community is not as likely as one in a heterogeneous community to have extremely high or low tax shares, since property values are likely to be more nearly equal. The second instrument is the percent change in educational expenditures between the fiscal year 1977-78 and 1978-79. Because moving is costly, households will often choose to remain in a community even though local spending changes at a different rate than household demand.

The third and fourth instruments are intended to measure the ease of "voting with one's feet". In urban areas there are many school districts within commuting distance of one's job. In an isolated community or a rural area there will typically be only one school district. If the workplace requires diversity of tastes and income, one would expect more sorting into communities of relatively homogeneous tastes and income in the suburbs than in isolated communites. We therefore include a dummy variable for whether a respondent resides in a central city and another dummy for whether the individual resides in an SMSA.

The Prediction Equations for Individuals

We write the equation for predicting the marginal rate of substitution of individual i as:

$$\hat{m}_i = \hat{\beta}' X_i + \hat{\epsilon}_i(t_i, X_i, W_i). \tag{10}$$

where $\hat{\epsilon}_i(t_i, X_i, W_i)$ is the estimated value of ϵ_i conditional on t_i, X_i and W_i. As we show in the appendix, the expected value of the error term can be written:

$$E(\epsilon_i | t_i, X_i, W_i) = \gamma_t t_i + \gamma_X' X_i + \gamma_W' W_i \tag{11}$$

Our estimating procedure provides us with unbiased estimators of the γ's so that

$$\hat{\epsilon}_i(t_i, X_i, W_i) = \hat{\gamma}_t t_i + \hat{\gamma}_x' X_i + \hat{\gamma}_w' W_i \tag{12}.$$

Equation (10) is therefore equivalent to:

$$\hat{m}_i = \hat{\gamma}_t t_i + (\hat{\beta}' + \hat{\gamma}_x') X_i + \hat{\gamma}_w' W_i. \tag{13}$$

The full information maximum likelihood estimates of the parameters of equation (13) are given in Table 2. Estimates of β are in column 1 and estimates of γ are in column 2.

While the primary purpose of this effort is to provide prediction equations to be used for other purposes, it is of interest to notice the magnitudes of price and income elasticities that are implicit in the parameter estimates that we use. With convex preferences, we should expect the marginal rate of substitution between education and other goods to vary inversely with the amount of public education and directly with disposable income

(since the latter measures expenditures on "other goods"). As can be seen from Table 2, the estimated effect of household income on marginal rate of substitution was significantly positive and the estimated effect of expenditure on marginal rate of substitution was significantly negative. The implied income elasticity of demand based on these estimates is .23. The implied price elasticity of demand is -.87.[2]

The reader is free to explore and interpret other coefficients that appear in the table. The estimates of γ_t and γ_A (where γ_A is the element in the vector γ_X that is associated with $\ln A$) tell us about the correlation between ϵ and (t, X), which is to say, the importance of Tiebout sorting. As we demonstrate in the appendix, if there were no correlation between ϵ and (t, X), then γ_t and γ_A would both be zero. We find, using asymptotic t-tests, that both of these coefficients are significantly different from zero at the 1% level.

In Table 3 we present the maximum likelihood estimates of the marginal rate of substitution function where no correction for Tiebout sorting is made. These estimates constrain ϵ to be uncorrelated with ω_1 and ω_2, the unobserved determinants of tax price and actual expenditures (that is, γ_t and γ_X are constrained to zero). Almost all the coefficients of the constrained model are substantially larger than those of the general model. The likelihood ratio test between the constrained and unconstrained models yields a Chi-square(2df) of 8.10. As a result, the hypothesis of zero covariance, or the exogeneity of both expenditures and tax price, can be rejected at the 1% level.

4. Testing for Efficiency of Community Expenditures

Predicting Community Sums of Marginal Rates of Substitution

Now that we have prediction equations for individual marginal rates of substitution, we can use these equations, together with information available from the Census about the distributions of economic and demographic characteristics in school districts, to predict the sum of marginal rates of substitution in any school district in Michigan. The way we proceed is as follows. Let S_j be the set of households in community j and let $m^j = \sum_{i \in S_j} m_i$. Since Equation (13) is linear in t, X, and W, we can estimate the sum of marginal rates of substitution as follows:

$$\hat{m}^j = \sum_{i \in S_j} \hat{m}_i = \hat{\gamma}_t t^j + (\hat{\beta}' + \hat{\gamma}'_x)X^j + \hat{\gamma}'_w W^j \tag{14}$$

where $t^j = \sum_{i \in S_j} t_i$, $X^j = \sum_{i \in S_j} X_i$ and $W^j = \sum_{i \in S_j} W_i$.

---

[2] These estimates are obtained by solving the equation $m = \beta_1 \ln A + \beta_2 \ln Y +$ constants for $\ln A$ as a function of the other variables. When a consumer is getting the desired amount of public goods, m is equal to the price p that he pays for them. Then we have $\ln A = (p - \beta_2 \ln Y + \text{constants})/\beta_1$. The income elasticity is therefore just $-\beta_2/\beta_1$ and the price elasticity evaluated at the mean price is just $\bar{p}/\beta_1$. Using our estimates β_1 and β_2, we find .23 and -.87 respectively.

If we sum the expression in Equation (3) over all consumers in community j, we have

$$m^j = \sum_{i \in S_j} m_i = \beta' X^j + \epsilon^j \tag{15}$$

where $\epsilon^j = \sum_{i \in S_j} \epsilon_i$. From adding the expressions in Equation 11, it follows that

$$E(\epsilon^j | t^j, X^j, W^j) = \gamma_t t^j + \gamma'_X X^j + \gamma'_W W^j. \tag{16}$$

From Equations (14), (15), and (16) it follows that:

$$\hat{m}^j - m^j = (\hat{\beta}' - \beta' + \hat{\gamma}'_x - \gamma'_x) X^j + (\hat{\gamma}_t - \gamma_t) t^j + (\hat{\gamma}'_w - \gamma'_w) W^j - \epsilon^j + E(\epsilon^j | t^j, X^j, W^j). \tag{17}$$

For each community, we must find the aggregate vectors, X^j, and W^j. Most of the components of these vectors are aggregates that are readily available in statistics published by the U.S. Census or the Michigan Department of Education. For example the census records the number of households in each community, the number of persons over 65 years of age and the number of persons with a college education.

Calculating the sum of $\ln Y$ in each community is slightly more difficult. Here the problem is twofold: the Census reports information on income rather than log of income, and the form of the information differs between families and unrelated individuals. The approach we have taken is to estimate the mean of $\ln Y$ for families from the information available on family income and the mean of $\ln Y$ for individuals from the information on individual income. Weighting these by the number of families and the number of unrelated individuals, respectively, we obtain the district-wide mean of $\ln Y$. The distribution of families in a school district was given for fifteen income brackets, allowing us to estimate the median family income (we assume a uniform distribution within each bracket). Then assuming income is distributed log-normally, we have, for families,

$$mean(\ln Y) = median(\ln Y) = \ln median(Y).$$

For unrelated individuals, the only data available were aggregate income and the number of individuals. For them we took $\ln mean(Y)$ as an approximation for $mean(\ln Y)$.

Efficiency in the Interest of Society as a Whole

Let c_j be the marginal cost of providing the local public good to community j. Then the Samuelson efficiency condition for community j is expressed by the equation

$$c_j = m^j = \sum_{i \in S_j} m_i = \beta' X^j + \epsilon^j. \tag{18}$$

In our application the public good is *per student* expenditures in the local public schools. The cost to a school district of increasing expenditures per student by one dollar is equal

to the number of students in the district. Therefore c_j is just the number of students enrolled in school district j. If the only benefits from local schools accrue to people living in the school district, and if district j spends the efficient amount, then we should have $m^j/c_j = 1$. For the 497 Michigan school districts, the average ratio of $\hat{m}^j$ to c_j is 0.748. The distribution of the computed values of the ratio of community mrs to marginal cost is shown in Figure 1. More than 80% of the school districts have $\hat{m}^j/c_j < 1$.

It is possible to use our estimates to identify school districts for which there is an especially large divergence between the sum of marginal rates of substitution and marginal cost. But it is important to remember that for a particular community, such a divergence is not necessarily an indication of inefficiency. It might be that unobserved differences in tastes for education or unobserved cost differences account for the discrepancy. In our notation, this would mean that ϵ^j differs from its expected value.

Although $\epsilon^j - E(\epsilon^j|t^j, X^j, W^j)$ will vary across individual districts, we can expect that the average value of this difference over the 497 school districts in Michigan will be very close to zero. To test for a systematic tendency for overspending or underspending in Michigan, we therefore want to study the average over all Michigan school districts of the ratio $\hat{m}^j/c_j$. Under the null hypothesis that there is no systematic bias toward overspending or underspending, we would have $\frac{1}{n}\sum_{j=1}^{n}\frac{\hat{m}^j}{c_j} = 1$. From Equation (17) it follows that

$$\frac{1}{n}\sum_{j=1}^{n}\frac{\hat{m}^j}{c_j} - 1 = (\hat{\beta}' - \beta' + \hat{\gamma}'_x - \gamma'_x)\bar{X} + (\hat{\gamma}_t - \gamma_t)\bar{t} + (\hat{\gamma}'_w - \hat{\gamma}_w)\bar{W} - \bar{\epsilon} \qquad (19)$$

where $\bar{X} = \frac{1}{n}\sum_{j=1}^{n}\frac{X^j}{c_j}$, $\bar{t} = \frac{1}{n}\sum_{j=1}^{n}\frac{t^j}{c_j}$, $\bar{W} = \sum_{j=1}^{n}\frac{W^j}{c_j}$ and $\bar{\epsilon} = \frac{1}{n}\sum_j \frac{\epsilon^j - E(\epsilon_j|t_i, X_i, W_i)}{c_j}$.

The variance of the expression in Equation (19) comes from two sources. One source is the variance of the $\hat{\beta}$'s and $\hat{\gamma}$'s around the "true" β's and γ's. The second source of variance is the random variable $\bar{\epsilon}$. Since $\bar{\epsilon}$ is a mean of statistically independent community specific random variables taken from 497 communities, the latter source of variance can reasonably be neglected. As we see from Equation (19), the difference $\frac{1}{n}\sum_{j=1}^{n}\frac{\hat{m}^j}{c_j} - 1$ is a linear combination of the difference between our parameter estimates for the β's and γ's and their expected value. Since we have estimates of the variance-covariance matrix for these estimates, we can readily calculate an estimate of the standard deviation of the expression in Equation (19). According to our computations, the random variable $\frac{1}{n}\sum_{j=1}^{n}\frac{\hat{m}^j}{c_j}$ takes the value .748 and the estimated standard error of this random variable is .09. Therefore we must reject the hypothesis of efficiency at the 5% significance level.

These results suggest that the sum of marginal rates of substitution between local public education and other goods in Michigan communities tends to be less than the marginal cost of public goods. If the only beneficiaries of local public education are the residents of the community in which the education is provided, this would mean that most communities spend too much on local public education from the standpoint of social efficiency.

10

Efficiency in the Interest of the Localities.

There is another way in which the efficiency of local governments could be measured. Instead of comparing the sum of marginal rates of substitution with the marginal cost of local public education, we might compare this sum with the fraction of marginal costs that are borne by the voters in the school district. This would give us a test of whether school districts are acting efficiently in the interest of their own residents.

In Michigan, at the time of this survey, marginal increments to local revenue came from the local property tax. In most school districts a large fraction of the property tax base is non-residential property and, of course, much of this property is either not locally owned or concentrated in the hands of a very small number of voters. (For the included communities non-residential property is 35% of the property tax base.) If people believe that taxes assessed on non-residential property are fully exported and if the fraction of the local property tax base that is residential property is s^j, then efficiency from a purely local point of view requires that $m^j/c_j = s^j$.

For 454 of the 497 communities that we observed, $\frac{m^j}{c_j} - s^j$ is positive and for the remaining communities $\frac{m^j}{c_j} - s^j$ is negative. Under the null hypothesis that, on average, $\frac{m^j}{c_j} = s^j$, Equation (19) would be replaced by

$$\frac{1}{n} \sum_{j=1}^{n} \frac{\hat{m}^j}{c_j} - \bar{s} = (\hat{\beta}' - \beta' + \hat{\gamma}'_x - \gamma'_x)\bar{X} + (\hat{\gamma}_t - \gamma_t)\bar{t} + (\hat{\gamma}'_w - \hat{\gamma}_w)\bar{W} + \bar{\epsilon} \qquad (20)$$

where $\bar{s} = \frac{1}{n} \sum_{j=1}^{n} s^j$.

As it turns out, $\bar{s} = .649$ and $\frac{1}{n} \sum_{j=1}^{n} \frac{\hat{m}^j}{c_j} = .748$. The estimated standard error of the random variable $\frac{1}{n} \sum_{j=1}^{n} \frac{\hat{m}^j}{c_j} - \bar{s}$ is .09. Therefore we are not able to reject the hypothesis that, on average, $\frac{m^j}{c_j} = s^j$.

5. Concluding Remarks.

This is, as far as we know, the first attempt in the literature to empirically test the Samuelson conditions for efficient provision of public goods. Accordingly, we urge the readers to interpret our results cautiously. Other data sources and other methods of approach may lead to very different conclusions. To illustrate this point, we must point out that our corrections for Tiebout bias had a very strong effect on our results. If we estimate the marginal rate of substitution functions, making no correction for Tiebout bias, but otherwise pursuing the same methods, we find that the average value of the ratio of summed marginal rates of substitution to marginal cost in Michigan school districts is 1.62 while with the correction, the average value is 0.748. Thus the uncorrected estimates would suggest substantial underspending and the corrected estimates suggest a tendency toward overspending.

The choice of which variables to treat as endogenous also makes a difference, albeit a smaller difference, to the efficiency calculation. When the endogeneity correction is applied only to expenditures, and not to price, the average ratio of summed mrs to marginal cost is about .94. However, when the endogenity correction is applied to tax price alone, the resulting efficiency calculation is .80, much closer to the result of applying the endogeneity correction to both variables.

These facts suggest that if someone wants to prove either that too much or too little is being spent, he can find the desired result by fiddling with the specification of the model. We think that the correction we have chosen is preferable on theoretical grounds to the uncorrected estimates. Other data sets and other choices of instrumental variables may lead to different conclusions. But if there is a strong tendency in one direction or another, we expect that it would be confirmed in repeated studies. We hope that others will try to investigate this question empirically.

With these qualifications in mind, suppose we take our empirical results as roughly correct, what conclusions could we draw? On average it appears that the sum of marginal rates of substitution for local public education is lower than marginal cost. If there are no spillovers of benefits from one school district to others, this would mean that local school districts are overspending from the standpoint of social efficiency. But, on average, the sum of the residents' marginal rates of substitution is about equal to the share of the costs that are borne by local taxpayers. This means that local school districts tend to act quite efficiently in promoting the interests of the voting population which does not bear the entire cost of local education. Whether this is a socially efficient arrangement depends on the size of the externalities generated by local education for the rest of the economy.

So, even if our empirical results were unquestioned, we wouldn't have ended the debate on whether too much or too little is being spent. Whether or not the expenditure level is efficient from the viewpoint of society as a whole it would be of considerable interest if our conclusion that local school districts tend to act efficiently in the interests of local voters is corroborated by other research.

Appendix

The system of equations 3, 8, and 9 in the text is assumed to have a vector of random disturbances $(\epsilon, \omega_1, \omega_2)$ with a joint normal distribution with mean zero and with off-diagonal covariances that are not necessarily zero. It is the fact that these off-diagonal terms, $\sigma_{\omega_1 \epsilon}$, $\sigma_{\omega_2 \epsilon}$ and $\sigma_{\omega_1 \omega_2}$ are non-zero that causes Tiebout bias to be a problem.

A full information maximum likelihood procedure is used to obtain consistent parameter estimates. Letting R represent an individual's more/same/less response, the likelihood function that is maximized represents the joint probability of observing the given set of $(R, t, \ln A)$ vectors. For example, given X_0 and W_0, the likelihood of observing a "more" response, tax price t_0, and expenditure level, A_0 is:

$$pr(more, t_0, \ln A_0) = \int_{\frac{t_0 + \delta - \beta' X_0}{\sigma_\epsilon}}^{\infty} f(\epsilon, \omega_{10}, \omega_{20}) d\epsilon$$

where $\omega_{10} = t_0 - \theta_{11}\tilde{X}_0 - \theta_{12}W_0$ and $\omega_{20} = \ln A_0 - \theta_{21}\tilde{X}_0 - \theta_{22}W_0$. Using Bayes' law,

$$f(\epsilon, \omega_1, \omega_2) = f(\epsilon|\omega_1, \omega_2)f(\omega_1, \omega_2) = f(\epsilon|\omega_1, \omega_2)f(\omega_1|\omega_2)f(\omega_2).$$

And, from standard results for conditional distributions of multivariate normal variables, $E(\epsilon|\omega_{10}, \omega_{20}) = \lambda_1 \omega_{10} + \lambda_2 \omega_{20}$ and $E(\omega_{10}|\omega_{20}) = \lambda_3 \omega_{20}$ where

$$\lambda_1 = \frac{\sigma_{\omega_1}^2 \sigma_{\epsilon \omega_1} - \sigma_{\epsilon \omega_2} \sigma_{\omega_1 \omega_2}}{\sigma_{\omega_1}^2 \sigma_{\omega_2}^2 - \sigma_{\omega_1 \omega_2}^2}$$

$$\lambda_2 = \frac{\sigma_{\omega_1}^2 \sigma_{\epsilon \omega_2} - \sigma_{\epsilon \omega_1} \sigma_{\omega_1 \omega_2}}{\sigma_{\omega_1}^2 \sigma_{\omega_2}^2 - \sigma_{\omega_1 \omega_2}^2}$$

and

$$\lambda_3 = \frac{\sigma_{\omega_1 \omega_2}}{\sigma_{\omega_2}^2}.$$

Then the term in the likelihood function corresponding to this observation is

$$\ln\left(1 - F\left(\frac{t_0 - \beta' X_0 + \delta - \lambda_1 \omega_{10} - \lambda_2 \omega_{20}}{\sigma_{\epsilon|\omega_1, \omega_2}}\right)\right) - \frac{1}{2}(\ln 2\pi\sigma_{\omega_1|\omega_2}^2 + \ln 2\pi\sigma_{\omega_2}^2)$$

$$-\frac{(\omega_{10} - \lambda_3 \omega_{20})^2}{2\sigma_{\omega_1|\omega_2}^2} - \frac{\omega_{20}^2}{2\sigma_{\omega_2}^2}$$

In a similar fashion, the likelihood function for the entire sample becomes

$$L = \sum_{i \in Less} \ln F \left(\frac{t_i - \beta' X_i - \delta - \lambda_1 \omega_{1i} - \lambda_2 \omega_{2i}}{\sigma_{\epsilon | \omega_1, \omega_2}} \right)$$

$$+ \sum_{i \in Same} \ln \left(F \left(\frac{t_i - \beta' X_i + \delta - \lambda_1 \omega_{1i} - \lambda_2 \omega_{2i}}{\sigma_{\epsilon | \omega_1, \omega_2}} \right) - F \left(\frac{t_i - \beta' X_i - \delta - \lambda_1 \omega_{1i} - \lambda_2 \omega_{2i}}{\sigma_{\epsilon | \omega_1, \omega_2}} \right) \right)$$

$$+ \sum_{i \in More} \ln \left(1 - F \left(\frac{t_i - \beta' X_i + \delta - \lambda_1 \omega_{1i} - \lambda_2 \omega_{2i}}{\sigma_{\epsilon | \omega_1 \omega_2}} \right) \right)$$

$$- \frac{n}{2} (\ln 2\pi \sigma^2_{\omega_1 | \omega_2} + \ln 2\pi \sigma^2_{\omega_2}) - \sum \frac{(\omega_{1i} - \lambda_3 \omega_{2i})^2}{2\sigma^2_{\omega_1 \omega_2}} - \frac{\sum \omega^2_{2i}}{2\sigma^2_{\omega_2}}.$$

Substituting $\omega_{1i} = t_i - \theta_{11} \tilde{X}_i - \theta_{12} W_i$ and $\omega_{2i} = \ln A_i - \theta_{21} \tilde{X}_i) - \theta_{22} W_i$, we maximize this function with respect to β, δ, θ, λ_1, λ_2, λ_3, $\sigma_{\epsilon | \omega_1, \omega_2}, \omega_2, \omega_1$, and σ_{ω_2}.

We are concerned here, however, with predicting marginal rates of substitution, rather than with estimates of the structural model. That is, we are concerned with

$$m = \beta' X + \lambda_1 (t - \theta_{11} \tilde{X} - \theta_{12} W) + \lambda_2 (\ln A - \theta_{21} X - \theta_{22} W).$$

Gathering terms and making obvious substitutions, we can write

$$m = \beta' X + \gamma_t t + \gamma'_x X + \gamma'_W W.$$

These are the reduced form parameter estimates β and γ that are reported in Table 2. Notice that $\gamma_t = \lambda_1$ and $\gamma_A = \lambda_2$. Some of the other parameter estimates are reported in Table A1.

References

Barlow, R. (1970) "Efficiency Aspects of Local School Finance," *Journal of Political Economy*, **78**, 1028-1039.

Bergstrom, T. (1979) "When Does Majority Rule Supply Public Goods Efficiently?," *Scandinavian Journal of Economics*, **8**, 216-227.

Bergstrom, T., D. Rubinfeld and P. Shapiro (1982) "Micro-Based Estimates of Demand Functions for Local School Expenditures," *Econometrica*, **50**, 1183-1205.

Bewley, T. (1981) "A Critique of Tiebout's Theory of Local Public Expenditures," *Econometrica*, **49**, 713-737.

Bowen, H. (1943) "The Interpretation of voting in the Allocation of Economic Resources," *Quarterly Journal of Economics*, **58**, 27-48.

Brazer, H. (1961) "Some Fiscal Implications of Metropolitanism," in *Metropolitan Issues: Social, Governmental, Fiscal*, ed. G. Birkhead. Syracuse, New York: Maxwell Graduate School of Public Affairs, Syracuse University.

Brennan, G. and Buchanan, J. (1980) *The Power to Tax: Analytical Foundations of a Fiscal Constitution.* : .

Courant, P., Gramlich and D. Rubinfeld (1979) "Public Employee Market Power and the Level of Government Spending," *American Economic Review*, **69**, 806-817.

Denzau, A., R. MacKay and C. Weaver (1981) "On the Initiative Referendum Option and the Control of Monopoly Government," in *Tax and Expenditure Limitation*, ed. H. F. Ladd and T. N. Tideman. Washington, D.C.: Urban Institute, 191-222.

Downs, A. (1960) "Why the Government Budget is Too Small in a Democracy," *World Politics*, **12**, 541-563.

Galbraith, J. K. (1958) *The Affluent Society.* Cambridge, Mass.: Houghton-Miflin.

Goldstein, G. S. and M. V. Pauly (1981) "Tiebout Bias and the Demand for Local Public Goods," *Journal of Public Economics*, **16**, 536-550.

Gramlich, E. and D. Rubinfeld (1982) "Using Micro Data to Estimate Public Spending Demand Functions and Test the Tiebout and Median Voter Hypothesis," *Journal of Political Economy*, **90**, 535-560.

Heckman, J. (1979) "Sample Selection Bias as a Specification Error," *Econometrica*, **47**, 153-161.

Inman, R. (1982) "Wages, Pensions, and Employment in the Local Public Sector," in *Public Sector Labor Markets*, ed. P. Mieszkowski and G. Peterson. Washington, D.C.: Urban Institute, 110-118.

Ladd, H. (1975) "Local Education Expenditures, Fiscal Capacity, and the Composition of Property Tax Base," *National Tax Journal*, **28**, 145-150.

Ledyard, John (1983) "The Pure Theory of Large Two Candidate Elections," Northwestern University working paper.

Romer, T. and Rosenthal, H. (1978) "Political Resource Allocation, Controlled Agencies and the Status Quo," *Public Choice*, **33**, 4, 27-43.

Rubinfeld, D., P. Shapiro and J. Roberts (1987) "Tiebout Bias and the Demand for Public Goods," *Review of Economics and Statistics*, **69**, 3, 426-437.

Samuelson, P. A. (1954) "The Pure Theory of Public Expenditure," *Review of Economics and Statistics*, **26**, 387-398.

Shapiro, P., and Sonstelie, J. (1982) "Did Proposition 13 Slay Leviathan?," *American Economic Review*, **72**, 184-190.

Stiglitz, J. (1982) "The Theory of Local Public Goods Twenty-five Years after Tiebout: A Perspective," NBER Working Paper no.954.

Tiebout, C (1956) "A Pure Theory of Local Expenditures," *Journal of Political Economy*, **64**, 416-424.

Weisbrod, B. (1964) *External Benefits of Public Education: An Economic Analysis*. Princeton, N.J.: Princeton University, Industrial Relations Section.

Williams, A. (1966) "The Optimal Provision of Public Goods in a System of Local Government," *Journal of Political Economy*, **74**, 10-33.