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ON A TRIGONOMETRIC EXPRESSION FOR THE SINGLE-EDGE-NOTCH CASE

by

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The value of having closed form solutions for stress-intensity relationships should not be underestimated. These are particularly useful when synthesizing crack growth rates obtained under fatigue or stress-corrosion conditions into failure times. Of course, the accuracy of the stress intensity factor should not be sacrificed for the sake of simplicity. Considering the centrally-notched plate, Fedderson[1] has shown that the secant relationship approximates Isida's [2] solution to within about 0.3 percent for $2a/_W$ less than 0.8. The stress intensity is given by

$$K = \sigma[\pi a \sec \frac{\pi a}{W}]^{1/2} \qquad (1)$$

where σ is the gross applied stress, a is the half-crack and W is the plate width. In his discussion, Fedderson[1] pointed out the importance of surveying numerical solutions of other configurations for equally concise relationships.

Since we were currently performing some stress-corrosion cracking experiments with single-edge-notch specimens, it was of some value to have a closed form solution for this configuration. The current expression proposed for this specimen is given by Brown and Srawley[3] to be

$$K = Y \frac{P}{BW} (a)^{1/2} \qquad (2)$$

$$Y = 1.99 - 0.41(a/W) + 18.70(a/W)^2 - 38.48(a/W)^3 + 53.85(a/W)^4$$

where a is the crack length, P is the applied load and B is the specimen thickness. Surprisingly enough, a trigonometric approximation to the polynomial was found in a very short time. This was accomplished in the following manner. First, it is known that the stress-intensity for single-edge-notch specimens rises much more rapidly with increasing crack length than a symetrically-notched specimen. Thus, only squared trigonometric terms were considered. Secondly, the trigonometric function in this case would involve $\pi a/2W$. Also, it was decided to use 2 as the leading term instead of 1.99. Taking three typical values of 0.2, 0.4 and 0.6 for the aspect ratio, a table of sine, tangent, cotangent and cosine was formulated. A cursory examination showed that $2[1 + \sin^2 + \tan^2]$ gave a good approximation to equation (2). An even simpler expression uses $\sin^2 + \sec^2$ so that Y in equation (2) may be replaced by

$$f(\theta) = 2\left[\sin^2\left(\frac{\pi a}{2W}\right) + \sec^2\left(\frac{\pi a}{2W}\right)\right] \qquad (3)$$

Values for Y and $f(\theta)$ were calculated at 0.05 intervals of the aspect ratio from 0 to 0.70. In Table 1, it is seen that the two expressions generally agree within about one percent for aspect ratios up to 0.65. More important, for the range of a/W from 0.3 to 0.6 where most experiments are evaluated, the two expressions coincide within 0.77 percent. As the deviation of the polynomial from the collocation solution may be as large as 0.4 percent,[3] a 0.77 percent difference does not seem much larger. Also, the differences between equations (2) and (3) are seen to fluctuate about plus and minus as indicated in Table 1. Thus, it is possible that equation (3) might represent the single-edge-notch case as well as equation (2). Considering that a two-term trigonometric function is simpler than a five-term polynomial, it is suggested that the stress-intensity expression might be simplified to

$$K = \sigma(a)^{1/2} \cdot 2\left[\sin^2\left(\frac{\pi a}{W}\right) + \sec^2\left(\frac{\pi a}{W}\right)\right] \qquad (4)$$

ACKNOWLEDGMENTS

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REFERENCES

1. C. E. Fedderson, discussion to Ref. 3, p. 77.
2. M. Isida, Proceedings, Fourth U.S. Cong. of Appl. Mech., 1962.
3. W. F. Brown, Jr. and J. E. Srawley, Plane Strain Crack Toughness Testing of High Strength Metallic Materials, ASTM STP 410, Philadelphia, 1966.

Table 1. Stress Intensity Coefficients for Various Aspect Ratios

a/w	$Y^{(a)}$	$\frac{\pi a}{w}$	$\sin^2(\frac{\pi a}{2w})$	$\sec^2(\frac{\pi a}{2w})$	$f(\theta)^{(b)}$	$\Delta^{(c)}$	Percent Difference
0	1.9900	0	0	1.0000	2.0000	-0.0100	-0.5%
0.05	2.0117	.07854	.00796	1.0081	2.0320	-0.0213	-1.05%
0.10	2.1029	.15708	.02446	1.0251	2.0990	+0.0039	+0.19%
0.15	2.2466	.23562	.05304	1.0560	2.2182	+0.0284	+1.26%
0.20	2.4344	.31416	.09548	1.1054	2.4018	+0.0326	+1.34%
0.25	2.6654	.39270	.14666	1.1718	2.6370	+0.0283	+1.06%
0.30	2.9472	.47124	.20590	1.2591	2.9300	+0.0172	+0.58%
0.35	3.2955	.54978	.27319	1.3756	3.2976	-0.0021	-0.06%
0.40	3.7338	.62832	.34517	1.5269	3.7441	-0.0103	-0.28%
0.45	4.2938	.70686	.42191	1.7298	4.3033	-0.0095	-0.22%
0.50	5.0156	.78540	.49959	1.9982	4.9955	+0.0201	+0.40%
0.55	5.9467	.86394	.57826	2.3710	5.8984	+0.0460	+0.64%
0.60	7.1432	.94248	.65451	2.8941	7.0972	+0.0545	+0.77%
0.65	8.6700	1.0210	.72693	3.6634	8.7806	-0.1106	-1.26%
0.70	10.596	1.0996	.79391	4.8514	11.291	-0.605	-5.36%

(a) Equation (2)

(b) Equation (3)

(c) $\Delta = (a) - (b)$

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