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Low-Energy Transfer to Transport Swarms of CubeSats to Lunar Orbit

THESIS

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by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

EL_i	=	ith Lagrange Point of the Sun-Earth system
LL_i	=	ith Lagrange Point of the Earth-Moon system
$CRTBP$	=	Circular Restricted Three-Body Problem
SOI	=	Sphere of Influence
LEO	=	Low Earth Orbit
ESO	=	Earth Staging Orbit
LSO	=	Lunar Staging Orbit
X	=	State vector
C	=	Jacobi constant
Δv	=	Impulse
μ	=	Three-body constant
ϵ	=	Perturbation
$\Phi(t_i, t_0)$	=	State transition matrix
Δv_{MI}	=	Maneuver Insertion

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ABSTRACT OF THE THESIS

Low-Energy Transfer to Transport Swarms of CubeSats to Lunar Orbit

By

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Master of Science in Mechanical and Aerospace Engineering

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Professor Kenneth D. Mease, Chair

The use of CubeSats as space observation missions is now a reality. CubeSats are satellites in the category of small satellites (around 10 cm x 10 cm x 10 cm) that started as simply educational projects for students. But they soon spread out to scientific investigation and exploration.

In this work, a low-energy transfer is designed and studied to transport these small satellites from low Earth orbit to orbit about the Moon. The theory behind low-energy transfers, as well as the computational methods, are described. A low-energy transfer is a transfer that exploits natural pathways in position-velocity space created by the forces of the Sun, Earth, and Moon acting on the CubeSat to reach the final target. The three-body problem in a rotating frame shows equilibrium points. Periodic orbits exist in the neighborhood of these equilibrium points. The low-energy transfer takes advantage of these periodic orbits and their stability properties using them as staging orbits. With precise maneuvers, a vehicle can be placed on a stable manifold of target periodic orbit such that it naturally travels to the target periodic orbits with little use of propellant. The transfers reduce the CubeSat propulsion requirements at the expense of transfer time. These transfers normally take 4 to 6 months to travel from a low orbit at the Earth to an orbit about the Moon.

Following the procedures to design the transfer, this work develops, analyzes and compares a low-energy transfer with other transfer options. A case study is also shown to discuss the values obtained. The low-energy transfer is shown to reduce the propulsion requirements significantly in comparison to conventional direct transfers.

Chapter 1

Introduction

Space-based observation has always been a point of interest when talking about space missions. Space-based observation has helped to understand the Earth and its environment as well as the solar system and the universe. However, to observe space bodies that are far away, there is a need for space-based telescopes. Detection of exoplanets requires measuring truly low frequencies, even frequencies below 10 MHz.

When talking about space-based telescopes, a constellation of CubeSats serves as a space-based radio telescope capable of detecting frequencies below the mentioned 10 MHz. CubeSats are small satellites, ranging in mass between 500 kg to 0.1 kg. They were originally developed as training projects to expose students to the challenges of engineering practices and system design. But their use has soon spread to the international industry. There is a need for a constellation of these small satellites to detect low frequencies (big amplitudes). A swarm of CubeSats, working together, will act as a bigger antenna with an effective diameter equal to the distance between the satellites.

The reduced size of these CubeSats leads to a limited propulsive capability. It creates a need for efficient transfers from near-Earth orbit to lunar or libration point orbit. Using phase space manifolds converging to the target orbit provides the most fuel-efficient transfers in terms of fuel consumption at the cost of longer transfer times as will be explained in chapters 3 and 4.

Dr. Michael Freilich, Director of Earth Science at NASA Headquarters said: "Earth-observing mission portfolio will benefit greatly from the ability to launch small satellites into optimal orbits, when and where we want them". But what is the deal with these small satellites? Let's put some numbers on the table. The Cassini Mission, for example, is the mission that gathered the most information about Saturn to date. The cost of it raised to $3 billion. It cannot take the risk of coming too close to Saturn's rings because particles can damage it. CubeSats cost several orders of magnitude less than that. Around $50,000. Their reduced size implies more versatility in missions. They could be sent to Saturn's rings because they can orbit with the particles that could damage Cassini. CubeSats are small enough to avoid impacts. But their reduced size is not the main benefit. The real benefit is cost.

CubeSats are small payloads in a space mission. Due to their size and mass, there is no need to spend millions of dollars to place them in orbit. A single launch vehicle can deploy a constellation of these small satellites as explained before, to serve as a bigger receiver antenna. At the cost of transfer time, a mission following a low-energy transfer can place swarms of CubeSats in orbit with a very little economic cost.

The reduced economic cost decreases the pressure of failure of the mission. For the same reason, if an individual CubeSat is lost, the rest of the constellation can keep working together.

The low-energy transfer is constructed by using the gravity of massive bodies in the system considered. Periodic orbits are used as staging orbits. The periodic orbits that are used have stable and unstable manifolds. Stable manifolds can be viewed as sets of potential trajectories that a spacecraft could follow to reach the periodic orbit without the need for propulsive force. Similarly, unstable manifolds can serve as trajectories to departure the periodic orbit. A low-energy transfer exploits these natural dynamics to reduce propulsion requirements, usually at the expense of longer transfer time.

This work describes the process to create a low-energy transfer from the Earth to the Moon. The transfer is designed to transport a vehicle with little economic cost and it is later compared to other transfer options. Details are provided to shape the low-energy transfer from a low Earth orbit to a libration orbit around the Moon.

This work summarizes and reproduces the work by Parker and Anderson.[2] Later, the values obtained are compared with Parker and Anderson's results. In their book, they develop a method to design low-energy transfers from the Earth to the Moon using manifolds of periodic orbits. This work uses the same strategy to compute the low-energy transfer and it later compares the values of energy needed in the transfer with other transfer options found in Ref. [2].

Chapter 2 of this work describes the process to model low-energy transfers and also includes the model to reproduce the motion of the bodies and the assumptions made. In chapter 3, the method described in chapter 2 is applied to compute a low-energy transfer. The same chapter briefly describes what a low-energy transfer is and highlights the differences between the low-energy transfer and the conventional direct transfer. In chapter 4, a case study is evaluated. The results are presented and discussed in chapter 5.

Chapter 2

Modeling for Low-Energy Transfers

This chapter describes the procedure to construct a low-energy transfer from a low Earth orbit to a libration orbit around the Moon. The method to describe the motion of the system is explained and it is later used to develop the staging orbits and the trajectories in between them.

The circular restricted three-body problem (CRTBP) is the simplest model to describe the motion of a considered massless spacecraft under the effect of two massive bodies. The model reveals the manifold structure used in low-energy transfers. This work considers either the Earth and the Moon or the Earth and the Sun as the two primary bodies. The system composed by the Earth and the Moon is used when the sphere of influence of it dominates over the system composed by the Earth and the Sun. The two primaries are in 2-body motion unaffected by the spacecraft.

The CRTBP uses a rotating frame centered in the barycenter of the system composed by 2 massive bodies and the space vehicle, considered massless. The x-axis points towards the smaller of the bodies and the z-axis points towards the normal of the plane defined by the orbit of the bodies. Finally, the y-axis completes the frame. In this frame 5 equilibrium points appear, named Lagrange points due to the discovery of Joseph-Louis Lagrange.

2.1 Equations of Motion

The CRTBP model uses the rotating reference frame centered at the barycenter of the two massive bodies. The normalized equations of motion in this reference frame are given by:

$$\dot{X} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \\ 2\dot{y} + x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} \\ -2\dot{x} + y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \\ -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3} \end{pmatrix} \tag{2.1}$$

Where μ is defined as the three-body constant and it is simply the ratio between the smaller mass and the total mass of the system. For the system composed by the Moon and the Earth, $\mu = 0.01215$ and for the system with the Earth and the Sun $\mu = 3 \cdot 10^{-6}$. The parameters r_1 and r_2 are computed as follows.

$$r_1^2 = (x+\mu)^2 + y^2 + z^2 \tag{2.2}$$

$$r_2^2 = (x-1+\mu)^2 + y^2 + z^2 \tag{2.3}$$

2.2 Lagrange Points

A mechanical system with three objects: 2 massive ones and one considered massless, constitutes a restricted three-body problem. It was discovered that there were five special points in this frame where a gravitational equilibrium could be maintained. In other words, an object placed at any one of these five points would stay there, when viewed from the rotating frame, with the effective forces with respect to this frame canceling. These five points were named Lagrange points and were numbered from L1 to L5. While L1, L2, and L3 constitute unstable equilibrium points, L4 and L5 constitute stable equilibrium points.

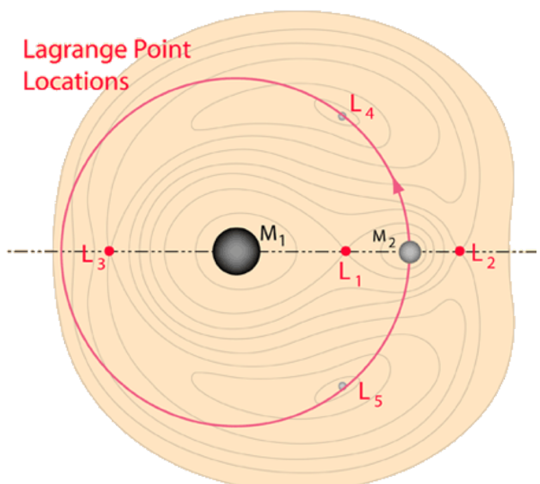

Figure 2.1: Lagrange points in the Earth-Moon system[3]

For this work, two different three-body systems need to be considered. The first one uses the L_2 point of the Sun-Earth-spacecraft system named EL_2. The second one uses either the L_1 or the L_2 points of the Earth-Moon-spacecraft system named LL_i. Lagrange points are important because in their neighborhoods there are families of periodic orbits. These periodic orbits are used as staging orbits during the low-energy transfer.

2.3 Periodic Orbits

The existence of the Lagrange points introduces indeed a set of solutions in the system of simple periodic symmetric orbits. Periodic orbits are closed-orbits that a spacecraft would follow from an initial point on it to the same point repeatedly in a fixed amount of time, the orbit period.

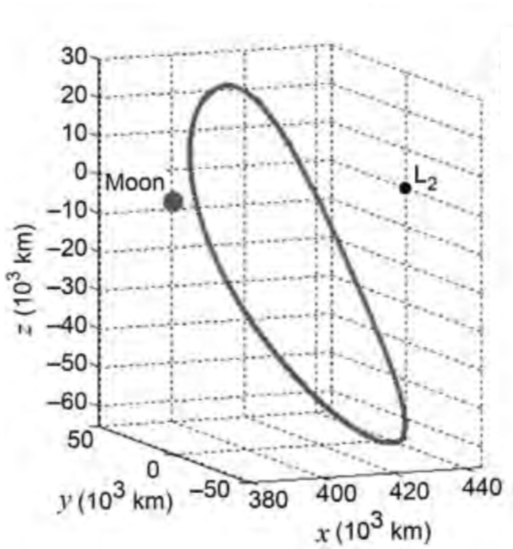

Figure 2.2: Example of a periodic orbit in the Earth-Moon system.[2]

Lyapunov and halo orbits are good examples of such orbits. The main idea of the low-energy transfer is taking advantage of the existence of periodic orbits and their associated manifolds. Once the vehicle is placed in one of the stable manifolds with the right state, it is going to follow a natural path towards the periodic orbit. The process has two main benefits – if the vehicle approaches the orbit on a tangential trajectory, it will not require any force to set it in place, which means no impulse is needed. The other benefit is the fact of using the periodic orbit as a parking orbit. It provides the full transfer a longer time frame for executing the launch.

2.3.1 Modeling the Periodic Orbits

The motion of a spacecraft in the Solar System can be modeled using different analysis. For this work, the Circular Restricted Three-Body Problem (CRTBP) has been used. This model is a remarkably good approximation of the motion of a considered massless object in the presence of two massive bodies whose relative motion is known. For example, a spacecraft in the Sun-Earth and Earth-Moon systems.

After determining how the spacecraft will move in the region dominated by the two massive bodies it is necessary to describe the motion when captured in the periodic orbits of interest. The Lagrange points have been mentioned before. These periodic orbits around them (in particular the L_1, L_2, and L_3) are easy to characterize by first replacing the origin of the frame at the particular Lagrange point.

$$\begin{aligned} x' &= x - (1 - \mu + \gamma) \\ y' &= y \\ z' &= z \end{aligned} \tag{2.4}$$

And linearizing the equations of motion after applying the previous displacement yields

$$\begin{aligned} \ddot{x}' - 2\dot{y}' - (1 + 2c)x' &= 0 \\ \ddot{y}' + 2\dot{x}' + (c - 1)y' &= 0 \\ \ddot{z}' + cz' &= 0 \end{aligned} \tag{2.5}$$

2.3.2 Jacobi Constant

The Jacobi constant or Jacobi energy is a useful parameter to characterize orbits. All trajectories happening under the gravitational attraction of the two massive bodies in the

CRTBP have a constant Jacobi energy. As long as no other perturbation occurs, the value of C will remain constant and it is defined as

$$C = (x^2 + y^2) + \frac{2 - 2\mu}{r_1} + \frac{2\mu}{r_2} - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 \tag{2.6}$$

2.4 Invariant Manifolds

As mentioned in section 2.3, periodic orbits have associated manifolds. Many of the periodic orbits in the Earth-Moon system are unstable meaning that their monodromy matrix has at least one unstable eigenvalue. A vehicle traveling on an unstable orbit will escape from it after experiencing a small perturbation. The escaping direction is defined by the eigenvectors of the unstable eigenvalues. These eigenvectors are tangent to the unstable manifold. Similarly, the periodic orbit has a stable manifold. If a spacecraft is precisely transferred onto the stable manifold, it will proceed to the periodic orbit under gravitational forces[1].

To compute the invariant manifolds of the periodic orbits, it necessary to evaluate the Jacobian of their states. Applying the CRTBP equations to the Jacobian of the states one obtains

$$J = \frac{\partial \dot{X}}{\partial X} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\partial \ddot{x}}{\partial x} & \frac{\partial \ddot{x}}{\partial y} & \frac{\partial \ddot{x}}{\partial z} & 0 & 2 & 0 \\ \frac{\partial \ddot{y}}{\partial x} & \frac{\partial \ddot{y}}{\partial y} & \frac{\partial \ddot{y}}{\partial z} & -2 & 0 & 0 \\ \frac{\partial \ddot{z}}{\partial x} & \frac{\partial \ddot{z}}{\partial y} & \frac{\partial \ddot{z}}{\partial z} & 0 & 0 & 0 \end{bmatrix} \tag{2.7}$$

[1]Small trajectory corrections will be required along the trajectory since the stable manifold is itself unstable

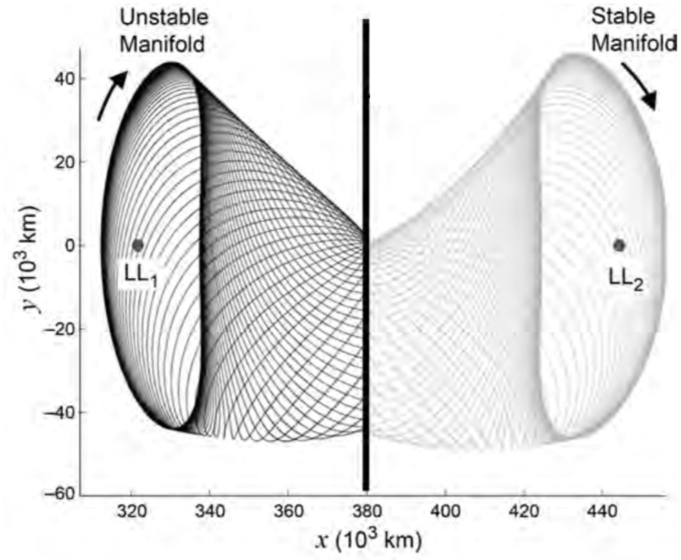

Figure 2.3: Representation of stable and unstable invariant manifolds in the Earth-Moon-spacecraft system with unstable periodic orbits.[2]

The eigenvalues of the Jacobian illustrate the unstable and stable directions to leave or approach the unstable periodic orbits respectively. The eigenvector associated with the larger real eigenvalue indicates the unstable direction v^U. With such direction, the unstable manifold can be produced by integrating the state forward in time $X^U = X \pm \epsilon\, v^U$. Similarly, the eigenvector associated with the other real eigenvalue indicates the stable direction v^S. With this direction, the stable manifold can be produced by integrating the state backward in time $X^S = X \pm \epsilon\, v^S$.

Tables 2.1 and 2.2 next summarize the eigenvalues of interest with the corresponding eigenvectors of the Lagrange points of interest.

Table 2.1: Summary of the eigenvalues of the Lagrange Points of interest

Lagrange Point	EL_1	EL_2	LL_1	LL_2	LL_3
λ_1	2.5179807	2.4696698	2.9320561	2.1586744	0.1778754
λ_2	-2.5179807	-2.4696698	-2.9320561	-2.1586744	-0.1778754

Table 2.2: Summary of the eigenvectors of the Lagrange Points of interest

Lagrange Point	EL_1	EL_2	LL_1	LL_2	LL_3
x	-0.7384946	-0.7380995	-0.7412247	-0.7350305	-0.6689538
y	0.3971436	0.4049124	-0.3410576	0.4632473	5.6219138
z	0	0	0	0	0
u	-1.8595151	-1.822862	-2.1733124	-1.5866915	-0.1189904
v	1	1	1	1	1
w	0	0	0	0	0
x	-0.7384946	-0.7380995	-0.7412247	-0.7350305	-0.6689538
y	-0.3971436	-0.4049124	-0.3410576	-0.4632473	-5.6219138
z	0	0	0	0	0
u	1.8595151	1.822862	2.1733124	1.5866915	0.1189904
v	1	1	1	1	1
w	0	0	0	0	0

Summarizing the above, each Lagrange point can be treated as a saddle point. After a perturbation, the space vehicle will obtain the unstable manifold direction when propagating forward in time. Similarly, it will follow the stable manifold when propagating backward in time.

2.4.1 Invariant Manifolds of Periodic Orbits

The previous section describes the invariant manifolds of the Lagrange points of the systems. But the Lagrange points are simply points. In other words, they are zero-dimensional structures. The objective of this work is to place the spacecraft into invariant manifolds that will lead it towards periodic orbits, or one-dimensional structures. Therefore, invariant manifolds are two-dimensional structures.

Ideally, to evaluate the stability of the trajectory, it would be necessary to consider each point along the orbit as an initial point. But the computation of the eigenvalues and eigenvectors of the Jacobian of all the states along the orbit is infeasible. A solution is to use the state transition matrix around the orbit. The state transition matrix is used to propagate the stable and unstable directions along the orbit from t_0 to $t_0 + T$ producing what is called the monodromy matrix. Consequently, the unstable and stable directions at the time t_i are given by $v_i = \Phi(t_i, t_0)v$.

At the point on the orbit where the perturbation is applied, the necessary equations are

$$X_i^S = X_i \pm \epsilon \frac{v_i^S}{|v_i^S|} \tag{2.8}$$

$$X_i^U = X_i \pm \epsilon \frac{v_i^U}{|v_i^U|} \tag{2.9}$$

Where ϵ is described as the perturbation applied as mentioned before. The magnitude of the perturbation is an important value to consider. A small perturbation gives a close approximation when drawing the manifold that follows. However, the spacecraft will require more time to depart from the orbit.

Chapter 3

Low-Energy Transfer Computational Method

This chapter summarizes the computational method to shape a low-energy transfer. The chapter also describes some other transfer options that will later be compared to the low-energy transfer designed.

3.1 Low-Energy Transfer Strategy

Putting the theory about periodic orbits and invariant manifolds together one may construct the low-energy transfer by following the recipe step by step. First of all, a periodic orbit in the Sun-Earth system is needed. This orbit, named as Earth Staging Orbit, is going to serve as the intermediate staging orbit after the departure from the Earth and before traveling towards the Moon. This orbit must fulfill some requirements – it must be unstable, its stable manifold has to intersect the Low Earth Orbit (LEO) where the spacecraft is parked and its unstable manifold has to intersect the final target orbit. By meeting these requirements two main points can be observed. Since the stable manifold intersects the LEO there is no need of an additional maneuver to transport the vehicle from the LEO to the manifold. This reduces to 0 any potential impulse required between the maneuver insertion and the LEO. Another major benefit is that the target orbit is reached at a much lower relative velocity than the direct transfer. This implies a notorious reduction in the Δv and in most cases a total elimination of it since the vehicle is naturally captured by the target orbit.

Second, a target orbit is needed. This orbit is going to be named Lunar Staging Orbit. And for this work, it is going to be a periodic orbit around the L_1 or L_2 Lagrange points in the Earth-Moon system. If the mission requires it, it is possible to transport the spacecraft from this orbit to a low Moon orbit or even the lunar surface. For the same reason as with the Earth Staging Orbit, this orbit must be unstable and its stable manifold has to intersect the unstable manifold of the ESO.

Lastly, the starting parking orbit is needed. The starting parking orbit is a LEO that will serve as the starting point from where the spacecraft will perform the maneuver insertion. The ESO is defined, so the state there is known. By backward integration, one can reproduce the stable manifold until the LEO is intersected. Similarly, from the ESO again but with forward integration, one can compute the unstable manifold until the LSO is targeted. Figures 3.1 and 3.2 summarize the process.

To leave the starting parking orbit, impulsive thrust is assumed. That is, the position of the spacecraft remains the same before and after the maneuver is applied and the velocity changes instantaneously. This is a good approximation when carrying high-thrust engine. The thrust time in this type of engine will be short enough that the position will not change much.

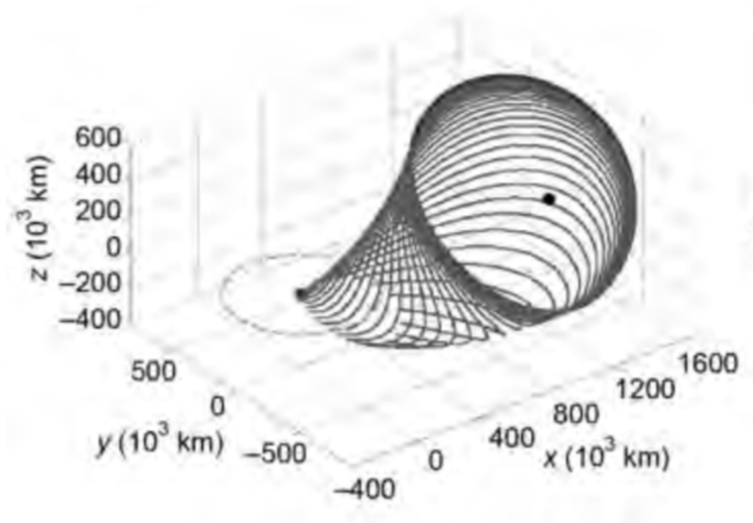

Figure 3.1: Stable manifold about the EL_2 Lagrange point in the Sun-Earth-Moon system. One can observe how the LEO orbit is intersected near the Earth.[2]

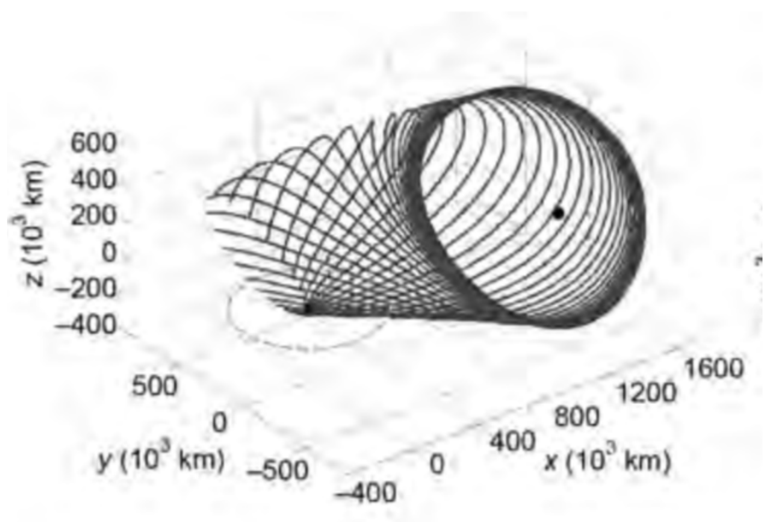

Figure 3.2: Unstable manifold about the EL_2 Lagrange point in the Sun-Earth-Moon system.[2]

3.2 Summary of Low-Energy Transfers and Other Transfer Options

There are many ways to transport a spacecraft through space. This chapter compares one of the most used trajectories in the past, the direct transfer, with a low-energy transfer.

3.2.1 Direct Transfer

When traveling from the Earth to the Moon everyone can think about the Apollo missions. The Apollo missions are quick, direct transfers. They reached the Moon 3 days after leaving the Earth.

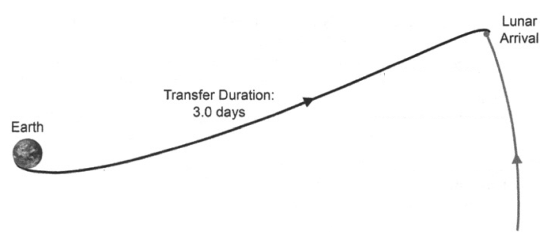

Figure 3.3: Example of a direct transfer from the Earth to the Moon.[2]

This trajectory requires only the gravitational force of the Earth and the Moon. The vehicle starts from a low altitude orbit at the Earth. After a maneuver, the vehicle is placed in a cruise orbit that targets some orbit about the Moon after a second maneuver. Direct transfers have durations between hours and weeks depending on the amount of propellant willing to spend. Typically, an efficient direct transfer sends the spacecraft in about 4 days. But they can be used either to send the vehicle to the surface of the Moon or to orbit around it.

3.2.2 Low-Energy Transfer

In comparison to the direct transfers, the low-energy transfers use gravity to reduce the amount of fuel consumption. The Sun's gravity takes the vehicle beyond the orbit of the Moon by slowly increasing the periapse altitude of the orbit. When the vehicle returns toward the Earth, it encounters the Moon on a nearly-asymptotical trajectory. As a result, the Moon naturally captures the vehicle saving the necessary propellant to reduce the energy at that point of the trajectory.

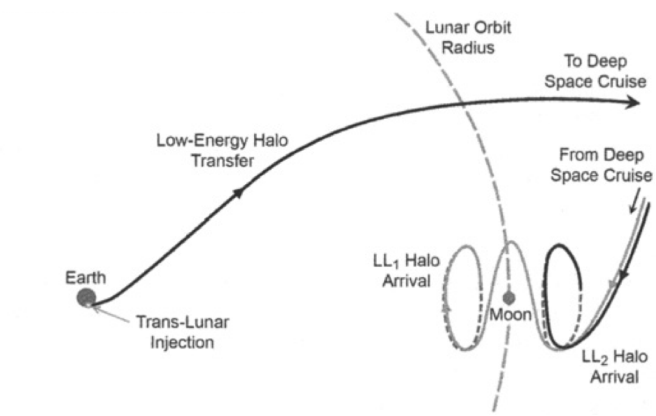

Figure 3.4: Example of a low-energy transfer from the Earth to a halo orbit at the L_1 or L_2 points at the Moon.[2]

As mentioned before, low-energy transfers save propellant at the cost of longer transfer times. These trajectories need around 3 months to reach their final destination. As seen in Figure 3.4, the orbit crosses the orbit of the Moon, implying that a lunar flyby is possible to reduce even more the required energy to complete the trajectory. Similarly to the direct transfers, low-energy transfers can be used to either land the spacecraft on the surface of the Moon or place it in an orbit around it – lunar libration orbits or lunar orbits.

The main benefits of the low-energy transfer are the following. First of all, the increased savings in energy, something that can be translated into economic savings. The gravity does most of the work in shaping the trajectory so that the spacecraft does not need to apply force.

A second major benefit is the flexibility of the launch. One of the problems of direct transfers is the reduced window of time to perform the launch. It is a quick trajectory so the relative position between the Earth and the Moon has to be precise. On the other hand, the low-energy transfer provides a larger window of time. Its long duration allows for adjustments and small corrections to reach the desired target.

Implicitly from the second benefit above, the flexibility of the low-energy transfers provides a wider range of potential target orbits on a given date.

Direct transfers have short periods of time when they require a maneuver. Low-energy transfers, however, can wait some days before performing a maneuver. This provides the operations team more time to prepare the spacecraft before needing a maneuver.

Another benefit is that low-energy transfers can be used to place several vehicles into different orbits with a single launch vehicle. That is an important benefit when talking about CubeSats. As mentioned before, the telescope consists of swarms of small CubeSats interacting together at different locations. Using direct transfers, placing the CubeSats one by one would require an extensive amount of propellant.

Lastly, low-energy transfers can be used to target any location on the Earth while returning from the Moon. This goes back to the flexibility of these transfers. When doing the reverse route, from the Moon to the Earth, using relatively short quantities of propellant the spacecraft can travel directly to any location on the Earth due to the large duration of low-energy transfers.

3.2.3 Low-Energy Transfer vs. Direct Transfers

There are other ways to transport a spacecraft from the Earth to the Moon other than just the direct transfer and the low-energy transfer. For example, the direct transfer can extend its launch window if staging is performed in between the trajectory. Another example is known as the low-thrust trajectory. This trajectory is similar to the low-energy transfer but requires a longer time. An example of the low-thrust transfer is the SMART-1, it reached the Moon after a 2-year trajectory. But let's compare numbers between the low-energy transfer and the conventional direct transfer.

The direct transfer typically takes between 3 to 6 days to reach the final destination. In some cases, it is possible to perform the trajectory in hours, but the transfer is far from optimal. On the other hand, a low-energy transfer takes between 2.5 and 4 months. The difference between taking 2 or 4 months is mainly the parking time at the periodic orbits.

The previous statement is what makes low-energy transfers more flexible in terms of launch periods. The fact of being able to park in intermediate periodic orbits provides the spacecraft with a large window of time to wait for the desired relative position between the Earth and the Moon.

But the main purpose of the low-energy transfers is what gives them their name, the reduced energy that they take. In comparison to an optimized direct transfer, a low-energy transfer can save more than 400 m/s of Δv in transfers to lunar-libration orbits. And more than 120 m/s of Δv in transfers to low-altitude lunar orbits. All these savings are in comparison to optimized direct transfers. When the direct transfer has not been optimized the difference in Δv is even more remarkable.

3.2.4 Other Transfer Options

A variation of the direct transfers are the fast direct transfers. These types of transfers are direct transfers via interior manifolds. They follow the same procedure as direct transfers but using manifolds within the plane that describes the Moon around the Earth.

Direct transfers with bridge are another variation of the conventional direct transfers. They are simply direct transfers in which a third maneuver is required to intersect the manifold. The manifold used does not intersect the original parking orbit so there is need of a transfer from the LEO to the manifold.

Another group of transfers to consider is the named complex transfers group. They are characterized by high initial altitude and relatively low Δv budget.

Transfers in the efficient transfers group are characterized by low initial altitude and low Δv budget. Trajectories belonging to this group have a low Δv budget without paying too much transfer time.

The last group of transfers to consider is the group of long duration transfers. These transfers momentarily travel away from the Earth-Moon system without escaping from it. They may include a lunar flyby.

Chapter 4

Case Study: CubeSat Transfer from Earth Orbit to Lunar Orbit

In this section, a low energy transfer is computed using Matlab following the procedure described in the previous chapters. The values of the energy obtained after computing the transfer are analyzed and compared to those for other transfer options found in Ref. [2].

For this work, a 185 km altitude LEO has been used as the initial parking orbit. Secondly, one must define a Lunar Staging Orbit. The LSO is described in the previous chapter as the final target orbit that may be used to transport the spacecraft to lunar orbits or to the lunar surface later on. For this work, a halo orbit about the Earth-Moon L_2 point has been selected with a Jacobi constant value of 3.05.

The intermediate parking orbit defined as the Earth Staging Orbit is also a required input. A halo orbit about the Sun-Earth L_2 point with a Jacobi constant value of 3.00077207 has been used. Figure 4.1 next shows the top view of the Earth Staging Orbit. For simplicity, it is assumed that the plane of rotation of the Moon around the Earth is in the ecliptic plane.

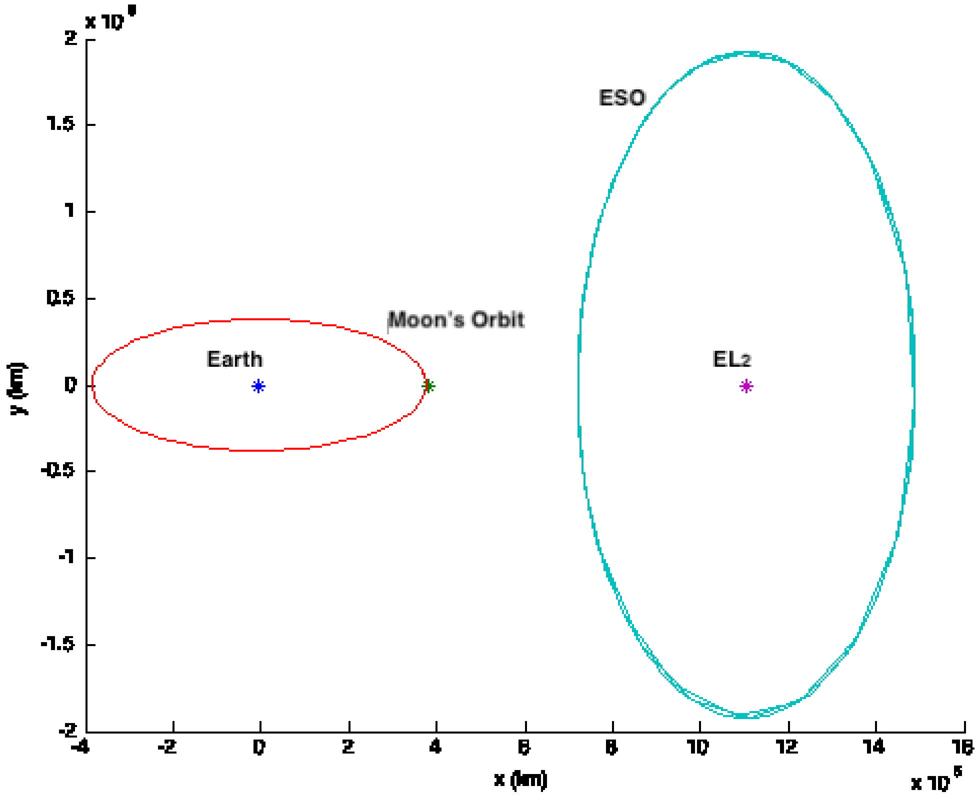

Figure 4.1: Planar view of the Earth Staging Orbit in the Earth-Moon plane of rotation.

The following step is to pick a point on the ESO of where the trajectory is desired to intersect it. The point chosen at the ESO determines the magnitude of the required perturbation. How much does this affect the final value? The state of the vehicle at the intersection with the ESO is different at each fiber of the stable manifold. This means that the larger the required velocity is at Earth departure, the larger the required impulse at departure.

The Matlab code developed in this work plots fibers of the stable manifold shown in Figure 4.2. Fibers are families of 1-dimensional smooth submanifolds (curves) inside the manifold itself.[11] From high to low in the intersection with the ESO, in Figure 4.2 the velocities at the LEO departure in normalized units are 19.6, 15.4, 14.9, 17, and 18.6. This shows a minimum in the third trajectory when the chosen point is at the south of the ESO.

Figure 4.3 shows the plot of the first segment of the trajectory considering the previously mentioned minimum. However, taking a look into the transfer time in normalized units the same trajectories show values of 0.2785, 0.3785, 0.4185, 0.3445, and 0.2745. Exchanging again transfer time for cost the trajectory considered, although having the minimum energy, it has the largest transfer time.

What is necessary next is to perturb the state at the chosen point to leave the periodic orbit. As explained in the previous chapter the magnitude of the perturbation is an important factor to consider. Here the initial value is set as the formula in Eq. 4.1 and it is then adjusted to reach the final destination.

$$\epsilon = \frac{100}{\sqrt{u^2 + v^2 + w^2}} \tag{4.1}$$

The first integration results in the first part of the trajectory, after leaving the low Earth orbit and until arriving at the ESO. Figure 4.2 shows fibers of the stable manifold.

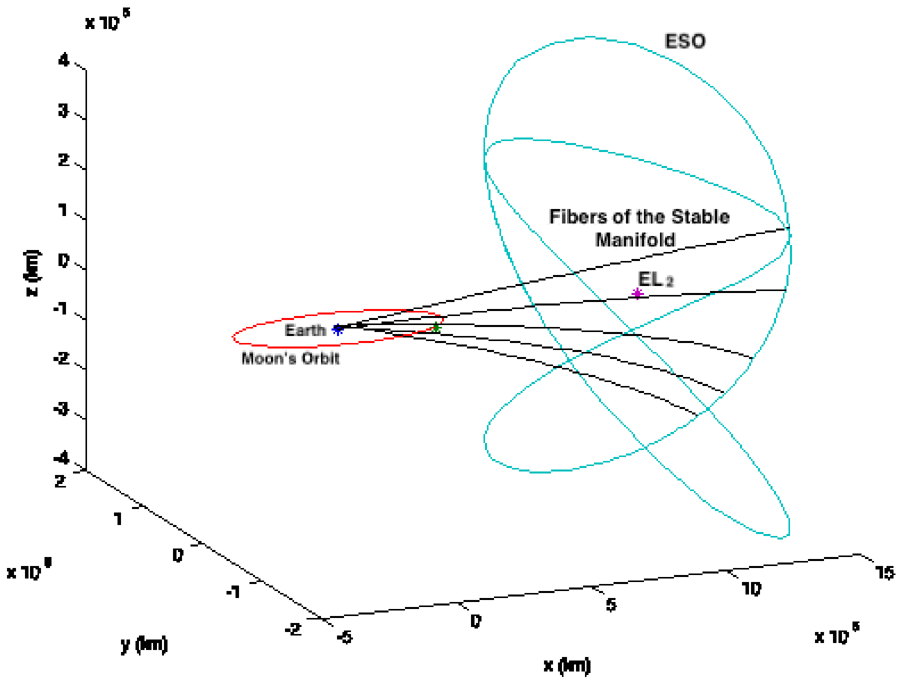

Figure 4.2: View of fibers of the stable manifold.

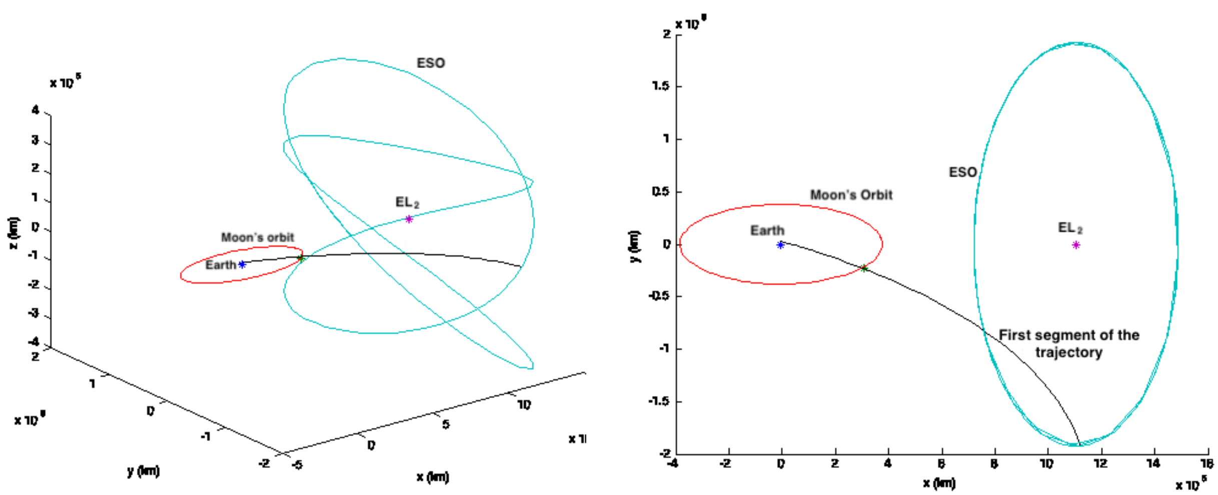

Figure 4.3: View of the first segment of the trajectory. 3D view (left) and planar view (right).

Second, perturbing the state of the vehicle at the point where it arrives at the ESO, the unstable manifold is integrated. Figure 4.4 shows the plot of one component of the unstable manifold of the trajectory. Also, Figure 4.5 shows the plot of the completed trajectory.

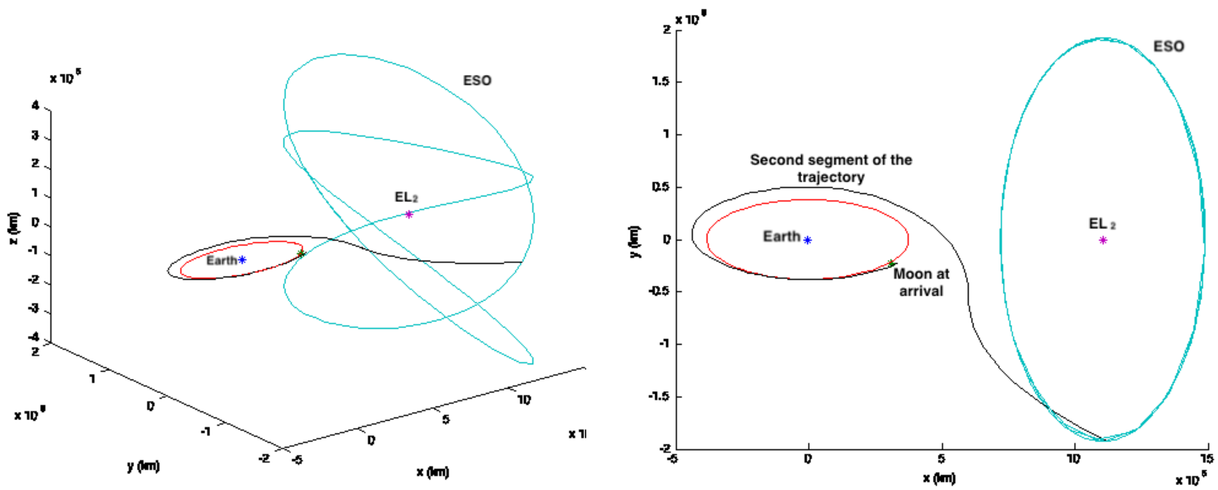

Figure 4.4: View of the second segment of the trajectory. 3D view (left) and planar view (right).

Once the trajectory is computed, the two main values of interest are known – the time and the total impulse. The total impulse is divided into two different applied forces. The first one occurs when the spacecraft leaves the LEO parking orbit, and it is inserted onto the stable manifold towards the ESO. The stable manifold intersects the LEO. The LEO departure state is known and the state at the beginning of the manifold is also known so the difference in velocity is the required external energy to apply. The second impulse occurs at the arrival into the ESO, the spacecraft needs to leave the stable manifold and join the path of the unstable manifold. Again, both states are known so the difference in velocities is the required impulse to be given. It is necessary to remember the unstable manifold reaches the final destination asymptotically so that no impulse is required – the spacecraft is naturally attracted by the periodic orbit at the Moon. Table 4.1 shows the values obtained and Table 4.2 compares them with other trajectories.

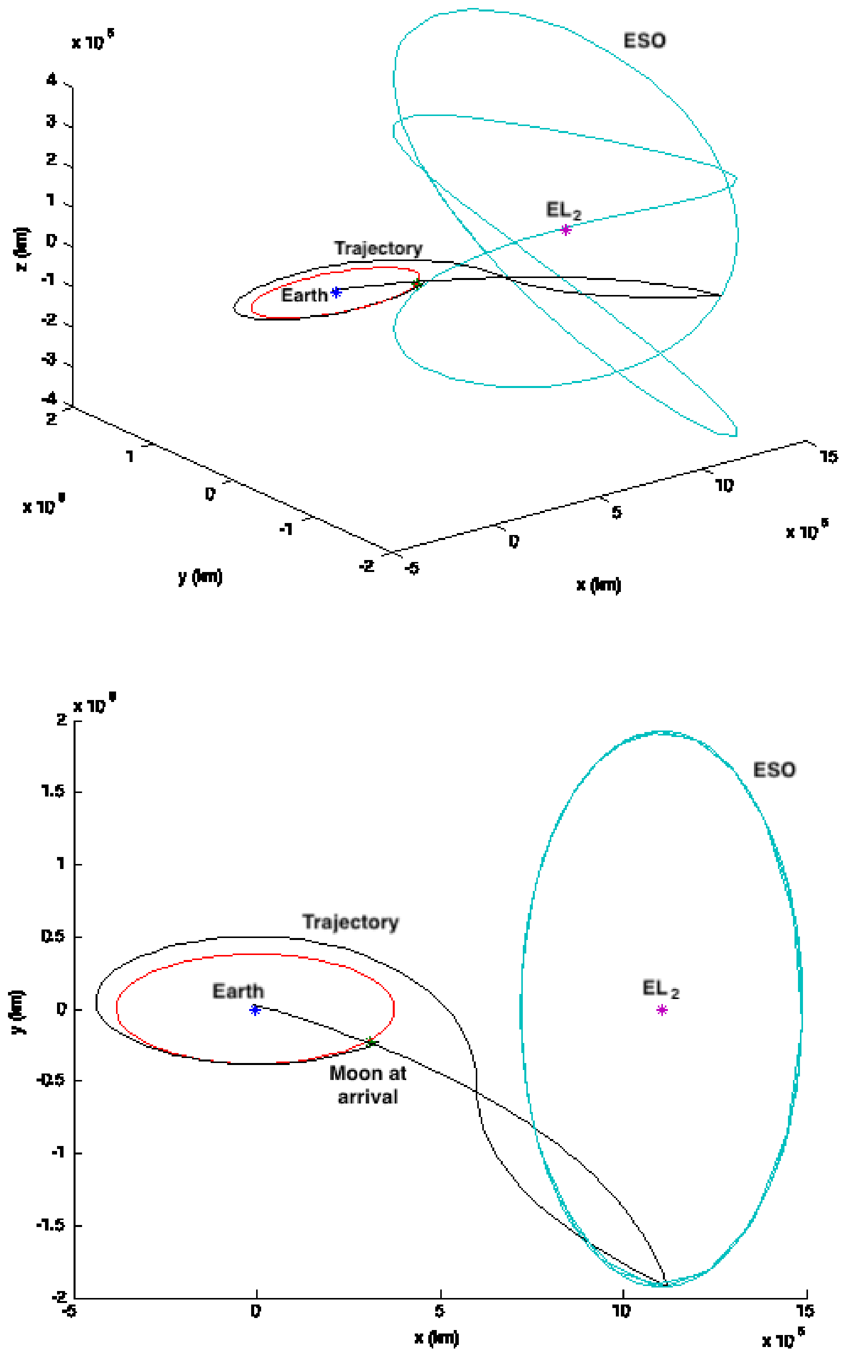

Figure 4.5: View of the completed trajectory. 3D view (top) and planar view (bottom).

Table 4.1: Computed values for the low energy transfer.

Δv_{MI1} (m/s)	Δv_{MI2} (m/s)	Δv_{TOTAL} (m/s)	Transfer time (days)
2765	711	3476	139.6

4.1 Comparison of Low Energy Transfer with Other Transfer Options

In this section, the low-energy transfer is compared with other transfer options for the example transfer considered in the previous section.

Table 4.2: Comparison of low-energy transfer with other transfer options.[a]

Transfer	Δv_{TOTAL} (m/s)	Transfer time (days)
Low-Energy	3476	139.6
Direct Transfer	4068	8.8
Fast Direct Transfer	4015	6.2
Direct Transfer with bridge	4014	48.8
Complex Transfer	3599	25.7
Efficient Transfer	3590	38.3
Long Duration Transfer	3948	33.2
Long Duration Transfer with Lunar Flyby	3776	50.9

[a]The values of Δ v compared do not account for the impulse required to place the vehicle into the LEO. The values of the transfers aside from the low-energy transfer are obtained from [2].

As one can see, the low-energy transfer has the lowest Δv budget at the cost of transfer time. It might not be the most efficient transfer in terms of a ratio between transfer time and propellant mass required. For example, one of the trajectories in the group of efficient transfers saves 100 days in the mission at the cost of only 100 m/s more (see Table 4.2). However, the low-energy transfer does not focus on the best Δv - transfer time ratio but on minimizing Δv.

Direct transfers are the ones that have the highest Δv budget but the lowest transfer time. This is ideal for manned missions. But CubeSat missions look for the lowest Δv budget possible so direct transfers are not suitable. One can also observe how direct transfers with bridge are far from ideal. The Δv budget is high and the transfer time is relatively high as well. This is mainly due to one of the problems of direct transfers – the reduced launch windows. This problem sometimes makes direct transfers increase the transfer time if the relative position of the Earth and the Moon (or other potential final targets) is not ideal.

Transfers in the group of complex transfers show similar values as transfers in the efficient transfers group. However, the values presented in Table 4.2 do not account for the Δv budget required to park at the initial LEO. These transfers require a high initial altitude which translates into a higher Δv budget.

Lastly, long duration transfers reduce the Δv budget in comparison to direct transfers but the difference in transfer time is too large for such a small reduction of Δv budget. They can include lunar flybys, since they travel away from the Earth-Moon system. A flyby can reduce even more the Δv budget, but it is not comparable to a low-energy transfer.

Chapter 5

Conclusions

This work has used a method to compute a low-energy transfer from an Earth orbit to a lunar orbit. This is particularly important for CubeSats missions and other missions with low-mass spacecrafts that lead to low propulsion capability.

During the computation, the values that most affect in the Δv budget are the following. First of all, the altitude of the initial parking orbit has been determined to be a variable to consider when reducing the Δv budget. A high altitude parking orbit at the Earth would imply a higher Δv budget during launch. Second, the point chosen at the Earth Staging Orbit is critical. With an accurate location of the point, one can achieve the minimum impulse required to target the ESO from the LEO. The neighborhoods of this point were studied showing that this point was a local minimum. Lastly, reaching the final destination asymptotically reduces the amount of energy required to zero. The final target orbit naturally captures the CubeSat, ideally avoiding the need for a propulsive maneuver at the destination.

In the case study, this work has shown how a low-energy transfer can save over 500 m/s in impulse. In comparison to the transfers to the Moon used in the Apollo missions or other transfer options, the low-energy transfer achieves the minimum Δv budget. The transfer time can be up to 6 months, while a direct transfer can do the job in 6 days. But the savings in propellant mass required are an enabling capability for the use of CubeSats swarms as space telescopes.

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Chapter 6

Appendix

6.1 Finding the Lagrange Points

Numerically solving the equilibrium points of the CRTBP equations (see Equation 2.1) 5 solutions are obtained. Those are the 5 Lagrange points of the system.

The following Matlab code[8] solves the equilibrium points of any system of 2 massive bodies using the CRTBP theory. The required input is the three-body constant μ and gives the result normalized in the CRTBP – the more massive body is placed at $x = -\mu, y = 0, z = 0$ and the less massive body is placed at $x = 1 - \mu, y = 0, z = 0$.

$$\mu = \frac{M_{lower}}{M_{lower} + M_{higher}} \tag{6.1}$$

```

clear all;

global ilp mu

clc; home;

fprintf('\n               program crtbp1\n');

fprintf('\n< equilibrium coordinates and energy >\n\n');

while(1)
    fprintf('\nplease input the value for the mass ratio\n');

    mu = input('? ');

    if (mu > 0)
        break;
    end
end

xm1 =   mu;

xm2 = 1   mu;

% L1 libration point
```

```
ilp = 1;

xr1 =  2;

xr2 = +2;

rtol = 1.0e 8;

[xl1, froot] = brent ('clpfunc', xr1, xr2, rtol);

yl1 = 0;

r1sqr = (xl1  xm1)^2 + yl1^2;

r2sqr = (xl1  xm2)^2 + yl1^2;

e1 =  0.5 * (xl1^2 + yl1^2)  (1  mu) / sqrt(r1sqr)  mu / sqrt(
    r2sqr);

% L2 libration point

ilp = 2;

xr1 =  2;

xr2 = +2;
```

```
rtol = 1.0e 8;

[xl2, froot] = brent ('clpfunc', xr1, xr2, rtol);

yl2 = 0;

r1sqr = (xl2   xm1)^2 + yl2^2;

r2sqr = (xl2   xm2)^2 + yl2^2;

e2 =  0.5 * (xl2^2 + yl2^2)   (1   mu) / sqrt(r1sqr)   mu / sqrt(
    r2sqr);

% L3 libration point

ilp = 3;

xr1 =  2;

xr2 = +2;

rtol = 1.0e 8;

[xl3, froot] = brent ('clpfunc', xr1, xr2, rtol);

yl3 = 0;
```

```
r1sqr = (xl3   xm1)^2 + yl3^2;

r2sqr = (xl3   xm2)^2 + yl3^2;

e3 =  0.5 * (xl3^2 + yl3^2)   (1   mu) / sqrt(r1sqr)   mu / sqrt(
    r2sqr);

% L4

xl4 = 0.5   mu;

yl4 = 0.5 * sqrt(3);

r1sqr = (xl4   xm1)^2 + yl4^2;

r2sqr = (xl4   xm2)^2 + yl4^2;

e4 =  0.5 * (xl4^2 + yl4^2)   (1   mu) / sqrt(r1sqr)   mu / sqrt(
    r2sqr);

% L5

xl5 = 0.5   mu;

yl5 =   0.5 * sqrt(3);
```

```
r1sqr = (xl5   xm1)^2 + yl5^2;

r2sqr = (xl5   xm2)^2 + yl5^2;

e5 =  0.5 * (xl5^2 + yl5^2)   (1   mu) / sqrt(r1sqr)   mu / sqrt(r2sqr);

% print results

fprintf('\n                              program crtbp1\n');

fprintf('\n                < equilibrium coordinates and energy >\n\n');

fprintf('\nmass ratio = %12.10e\n', mu);

fprintf('\nlocation       x coordinate        y coordinate                energy\n');

fprintf('\n    L1          %10.6f          %10.6f          %12.10e\n', xl1, yl1, e1);

fprintf('\n    L2          %10.6f          %10.6f          %12.10e\n', xl2, yl2, e2);

fprintf('\n    L3          %10.6f          %10.6f          %12.10e\n', xl3, yl3, e3);
```

```
fprintf('\n    L4           %10.6f           %10.6f           %12.10e\n',
   xl4, yl4, e4);

fprintf('\n    L5           %10.6f           %10.6f           %12.10e\n\n'
   , xl5, yl5, e5);
```

6.2 Computation of Periodic Orbits

The present section shows the computation of trajectories in the CRTBP system. With the precise initial conditions it plots periodic orbits between the two massive bodies. The following table shows some examples of initial conditions to plot periodic orbits.

Table 6.1: Some examples of initial conditions to plot periodic orbits in the CRTBP.

Orbit	x	y	z	u	v	w
1	0.300	0	0	0	-2.536	0
2	2.841	0	0	0	-2.748	0
3	0	0	0	0	2.066	0
4	-2.500	0	0	0	2.100	0
5	0.952	0	0	0	-0.958	0
6	3.148	0	0	0	-3.077	0

The following Matlab code[8] plots some examples of periodic orbits as well as draws any orbit in the CR3BP for given initial conditions.

```
clear all;

global mu

clc; home;

fprintf('\n                program g3body\n');

fprintf('\n< graphics display of three body motion >\n\n');

while(1)

    fprintf('\n <1> periodic orbit about L1\n\n');

    fprintf(' <2> periodic orbit about L2\n\n');

    fprintf(' <3> periodic orbit about L3\n\n');

    fprintf(' <4> user input of initial conditions\n\n');

    fprintf(' selection (1, 2, 3 or 4)\n\n');

    icflg = input('? ');

    if (icflg >= 1 && icflg <= 4)
```

```
        break;

    end

end

switch icflg

    case 1

        % periodic l1 orbit (tr 32 1168, pp 25,29; 74)

        y(1) = 0.300000161;
        y(3) = 0;
        y(2) = 0;
        y(4) =  2.536145497;

        ti = 0;
        tf = 5.349501906;

        mu = 0.012155092;

        % set plot boundaries

        xmin =  1.5;
        xmax = +1.5;
```

```
        ymin =  1.5;
        ymax = +1.5;

    case 2

        % periodic l2 orbit (tr 32 1168, pp 31,34; 126)

        y(1) = 2.840829343;
        y(3) = 0;
        y(2) = 0;
        y(4) =  2.747640074;

        ti = 0;
        tf = 2 * 5.966659294;

        mu = 0.012155085;

        % set plot boundaries

        xmin =  3;
        xmax = +3;
        ymin =  3;
        ymax = +3;

    case 3

        % periodic l3 orbit (tr 32 1168, pp 37,39; 63)
```

```
        y(1) =  1.600000312;
        y(3) = 0;
        y(2) = 0;
        y(4) = 2.066174572;

        ti = 0;
        tf = 2 * 3.151928156;

        mu = 0.012155092;

        % set plot boundaries

        xmin =  2;
        xmax = +2;
        ymin =  2;
        ymax = +2;

    case 4

        % user input of initial conditions

        fprintf('\nplease input the x component of the radius
           vector\n');

        y(1) = input('? ');

```

```
    fprintf('\nplease input the y component of the radius vector\n');

    y(3) = input('? ');

    fprintf('\nplease input the x component of the velocity vector\n');

    y(2) = input('? ');

    fprintf('\nplease input the y component of the velocity vector\n');

    y(4) = input('? ');

    while(1)

        fprintf('\nplease input the value for the earth moon mass ratio\n');

        mu = input('? ');

        if (mu > 0)
            break;
        end
    end

```

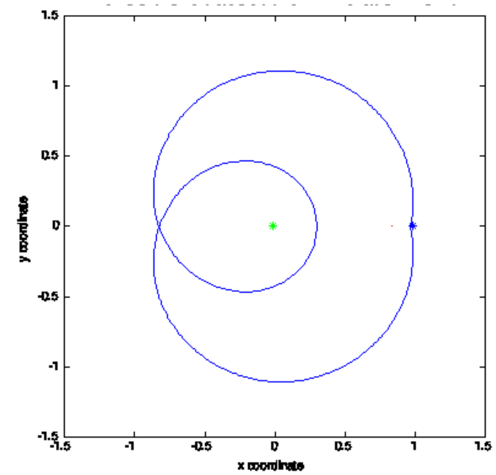
```
        while(1)

            fprintf('\nplease input the final time\n');

            tf = input('? ');

            if (abs(tf) > 0)
                break;
            end
        end

        % default values for plot boundaries

        xmin =  2;
        xmax = +2;
        ymin =  2;
        ymax = +2;

end

% request the integration step size

while(1)

    fprintf('\n\nplease input the integration step size\n');

    fprintf('(a value of 0.01 is recommended)\n');
```

```
    dt = input('? ');

    if (abs(dt) > 0)

        break;

    end

end

% set ode45 options

options = odeset('RelTol', 1.0e 10, 'AbsTol', 1.0e 10);

% initialize

t2 =  dt;

npt = 0;

fprintf('\n\n  working ...\n');

while(1)

    t1 = t2;

```

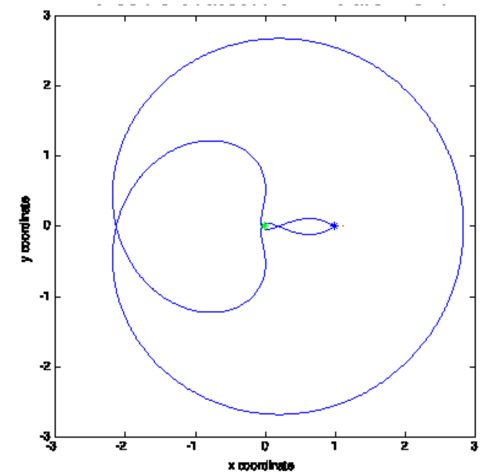
```
    t2 = t1 + dt;

    [twrk, ysol] = ode45(@crtbp_eqm, [t1, t2], y, options);

    npt = npt + 1;

    xplot(npt) = ysol(length(twrk), 1);

    yplot(npt) = ysol(length(twrk), 3);

    y = ysol(length(twrk), 1:4);

    % check for end of simulation

    if (t2 >= tf)

        break;

    end

end

% plot trajectory

plot(xplot, yplot);

axis ([xmin xmax ymin ymax]);
```

```
axis square;

ylabel('y coordinate');

xlabel('x coordinate');

% label locations of Earth and Moon

hold on;

plot( mu, 0, '*g');

plot(1   mu, 0, '*b');

% label libration points

switch icflg

    case 1

        plot(0.836892919, 0, '.r');

        title('Periodic Orbit about the L1 Libration Point', '
           FontSize', 16);

    case 2
```

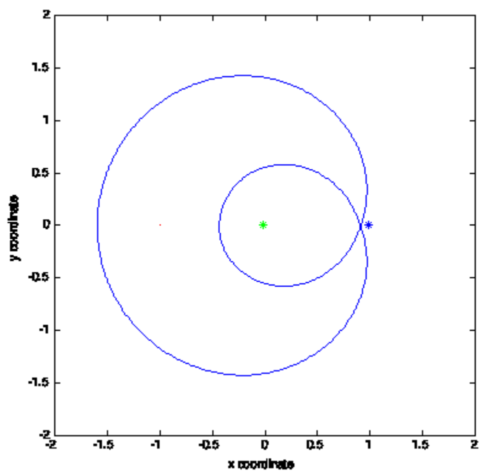
```
        plot(1.115699521, 0, '.r');

        title('Periodic Orbit about the L2 Libration Point', '
            FontSize', 16);

    case 3

        plot( 1.005064527, 0, '.r');

        title('Periodic Orbit about the L3 Libration Point', '
            FontSize', 16);

    case 4

        title('User Defined Initial Conditions', 'FontSize', 16);
end

% create eps graphics file with tiff preview

print  depsc  tiff  r300 g3body.eps
```



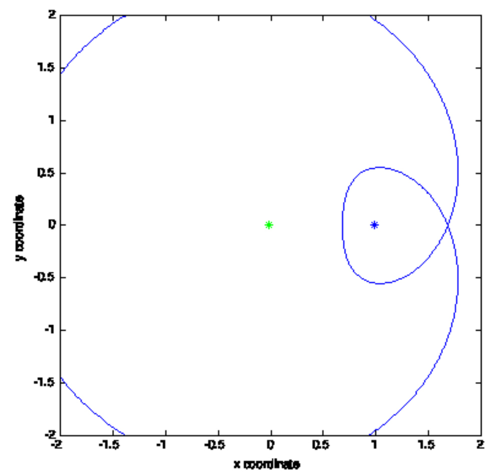
(a) Orbit 1



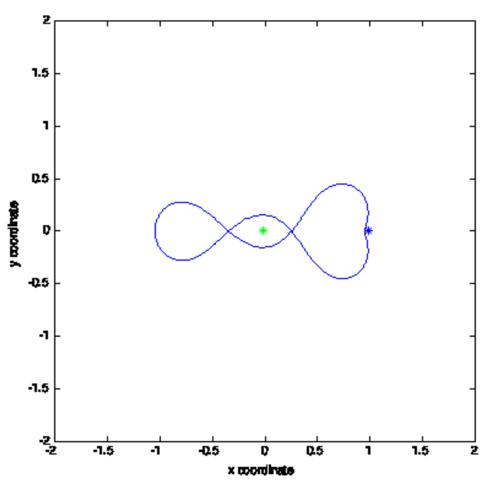
(b) Orbit 2



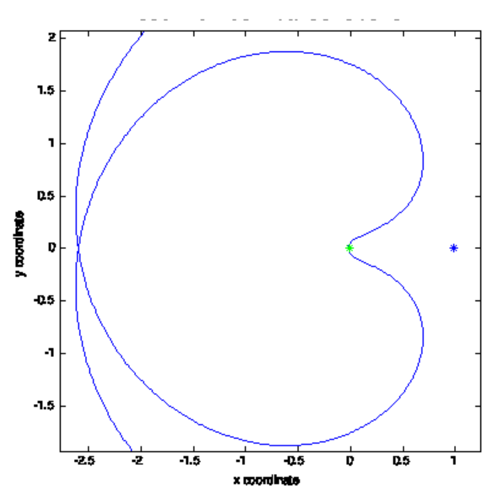
(c) Orbit 3



(d) Orbit 4



(e) Orbit 5



(f) Orbit 6

Figure 6.1: Plots of the orbits in Table 6.1