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Three-Dimensional Magnetic Field Produced by an Axisymmetric Iron Yoke*

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Abstract—A computational procedure, in which separate analyses are performed for conductor and high permeability iron yoke, has been developed for calculating the three-dimensional (3-D) magnetic field components. Whereas the field components of the isolated 3-D current array can be evaluated at all desired points through the use of a 3-D Biot-Savart law program, we have developed a method for calculating the supplemental field that will arise as a result of the presence (coaxially) of a surrounding high-permeability magnetic yoke with an axially-symmetric bore. We may speak of this supplemental field as an "image field" although we shall realize that it may be possible in general to attribute it only to a distinctly diffuse distribution of "image current" or magnetic moments. The boundary associated with the "image field" is such that at each point along the boundary of the high permeability iron the total scalar potential shall be constant, e.g. $V^i = -V^d$ (where i=image and d=direct). When we describe both potentials as a series of "harmonic components" then the nature of the boundary condition is such that a de-coupling of one harmonic from another is preserved, and therefore it is also true that $V^i(n) = -V^d(n)$ at the iron interface, where n is a harmonic number. When we solve the appropriate differential equations for these scalar potential functions throughout the iron-free region with the proper applied boundary condition for the scalar potential of each harmonic number, we achieve upon summation the appropriate field contribution of the surrounding high-permeability iron.

I. INTRODUCTION

The magnetic elements (dipoles, quadrupoles, etc.) of a particle accelerator or transport line frequently are formed from suitably located current windings surrounded by a ferromagnetic yoke. This yoke can act to provide shielding, to influence the I. strength of the fields within the device, and to some extent control the quality of the field produced by the magnet. The strength of the total field in the region occupied by conductors of course can be relevant to structural issues in the design and to thresholds for the quenching of superconducting devices, while the quality of the field within the bore becomes highly significant with respect to beam transport.

When designing magnetic elements of substantial length, useful field information is conveniently obtainable from a two-dimensional (2-D) analysis of a proposed design, aided in the analysis perhaps by a simple image-current representation of the contributions from an iron yoke or application of a simple 2-D relaxation computation in x, y space through use of a program such as POISSON. Under such circumstances it is useful and conventional to represent the field in terms of "harmonic components" (as sextupole, decapole, etc. for an element with dipole symmetry), in which the spatial components of field for each harmonic component (of harmonic order n) will be separately proportional to the sine or cosine of n times the azimuthal position angle.

The design of end windings and the character of the consequential "end field", however, can in practice become of very significant importance with regard both to issues of a technological nature and to those concerned with beam dynamics. We indicate here a convenient means for obtaining the relevant characteristics of such 3-D end fields under conditions that (i) the surface of the yoke shall be axially symmetric (interface radius r_b, that may be a function $r_b(z)$ of the longitudinal coordinate z in order to permit a flared bore or other termination to the yoke) and (ii) the ferromagnetic material may be regarded as characterized by a high permeability (as would be permissible in such designs as occur in superconducting magnets in which iron saturation is greatly reduced by the presence of "collars" of sufficient size to separate substantially the yoke material from the conductors).

II. ANALYSIS OF FIELD CONTRIBUTIONS FROM A 3-D YOKE

The magnetic field that arises "directly" from the conductors in a 3-D design of course can be calculated by a straight-forward integration of the law of Biot and Savart. The results of such computations of the direct field, $B^{(d)}$, at the the iron interface can be resolved into harmonic components $\left(e.g., B_{\theta,n}^{(d)}\right)$ and these results employed as input into a sequence of computations to evaluate, for one harmonic number at a time, the harmonic contributions of the yoke to the total field.

Specifically, these latter computations visualize the field of the yoke to be represented as the negative gradient of a scalar potential function $V^{(i)}(r,\theta,z)$ that is described as a sum of individual harmonic components--thus:

$$V^{(i)}(r,\theta,z) = \sum_n V_n^{(i)}(r,z) \sin n\theta$$

(employing exclusively the sine functions for the contributions of "non-skew" magnetic elements). Determination of the functions $V_n^{(i)}(r,z)$ then becomes reduced to the sequential solution (in r,z space) of partial differential equations for the individual harmonic components of potential for a curl-free divergence-free field,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial V_n^{(i)}}{\partial r}\right) + \frac{\partial^2 V_n^{(i)}}{\partial z^2} = \left(\frac{n^2}{r^2}\right)V_n^{(i)}, \qquad (1)$$

subject to the appropriate boundary conditions

$$V_n^{(i)} = 0 \quad \text{at } r=0$$

and

$$V_n^{(i)}(r_b,z) = \frac{r_b}{n} B_{\theta,n}^{(d)}(r_b,z)$$

at the iron interface radius ($r=r_b$) in order that the tangential component of total field at the interface vanish identically for each n component:

$$B_\theta^{(d)}(r_b,\theta,z) = \sum_n B_{\theta,n}^{(d)}(r_b,z)\cos n\theta = -B_\theta^{(i)}(r_b,\theta,z)$$

We have undertaken to modify for this application a production version of the 2-D relaxation program POISSON for a scalar potential [1], primarily by replacing the source term in the polar-coordinate differential equation for a scalar potential by a term proportional to $(n/r)^2 V_n^{(i)}$. It should be noted in this connection that, for consistent use of the editor in processing the relaxation solutions to provide interpolated or interpolated/differentiated results, one preferably then should employ as interpolation polynominals expressions that themselves represent solutions of the modified differential equation.[2]

III. ILLUSTRATIVE APPLICATIONS

We now illustrate the technique summarized above by its application to the SSC main quadrupole. We calculated the iron field contribution at the end region of the 2 layer superconducting quadrupole. The "end" windings, schematically shown in Fig. 1 as line currents, were used to compute $B_\theta^{d}(r_b,\theta,z)$ around the iron boundary and subsequently to Fourier analyze it and convert it to a set of potentials for each harmonic n.
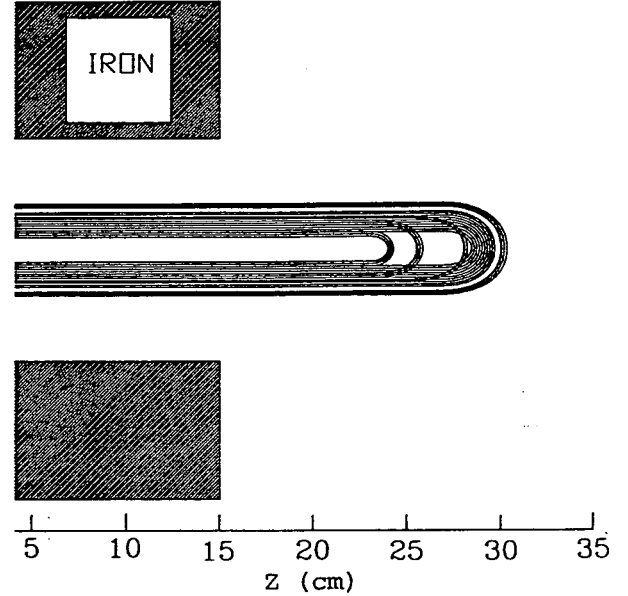

Fig. 1. Schematic of the two layer SSC quad end region.

Six sets of potentials (n = 2, 6, 10, etc.) have been included in 6 different input files submitted to the modified POISSON solver. Plots of equal potential lines from the solution of 3 such sets are shown in Fig. 2.

The scalar potential solutions were subsequently processed using the formulation below to reconstruct the iron contributions to the field components.

$$B_r^{(i)}(r,\theta,z) = \sum_n -\frac{\partial V_n^{(i)}(r,z)}{\partial r}\,\sin(n\theta)$$

$$B_\theta^{(i)}(r,\theta,z) = \sum_n -\frac{n}{r}V_n^{(i)}(r,z)\,\cos(n\theta)$$

$$B_z^{(i)}(r,\theta,z) = \sum_n -\frac{\partial V_n^{(i)}(r,z)}{\partial z}\,\sin(n\theta)$$

2

n=2

n=6

n=10

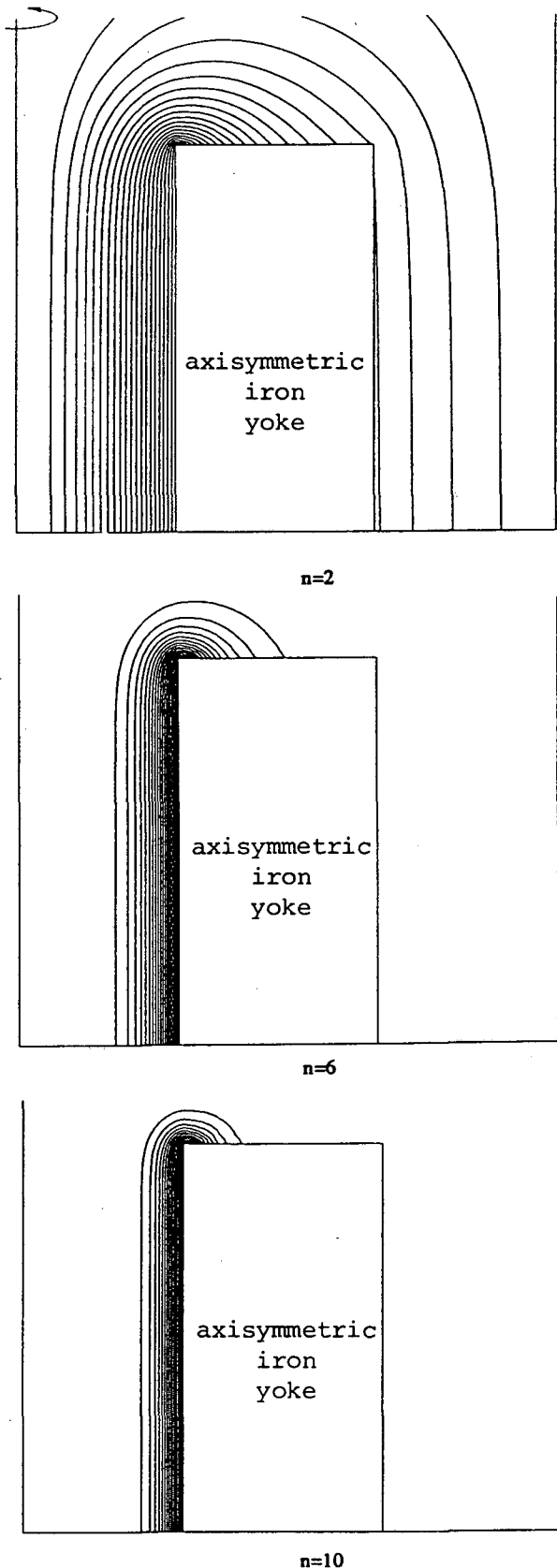Fig. 2. Solutions of equal potential lines for the first three harmonics, n=2,6, and 10.

Finally, we plot in Fig. 3 the iron contribution to B_r(n=2, θ=45, r=o) and B_z/r^2 (n=2, θ=45) in the limiting case r→o.

a

b

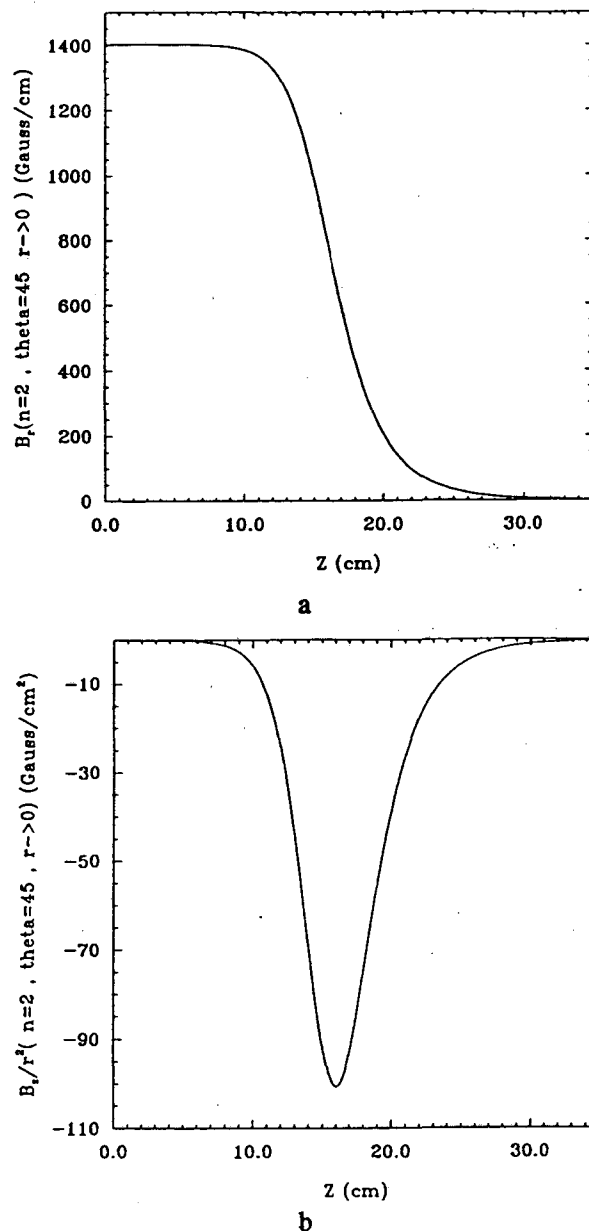Fig. 3. Field components produced by the iron (n=2); the radial direction (a) and the axial direction (b)

IV. REFERENCES

[1] "Computer Codes used in Particle Accelerator Design", AT-6, Los Alamos National Lab.
[2] See Victor Brady, "3-Dimensional Field Components" LBL Internal HIFAR Note-261, esp. Table on p. 4.

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