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THE DECAY $p \rightarrow e^+ \gamma$ IN GRAND UNIFIED GAUGE THEORIES[☆]

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The rate for proton decay to the $e^+ \gamma$ mode is calculated. Although its branching ratio is of order α , its ratio to observable $e^+ \pi^0$ correlated decays is 1/8 to 1/40. Observable numbers of $p \rightarrow e^+ + \gamma$ decays can occur and would provide constraints on the proton wave function at the origin and grand unified parameters independent of hadron final state uncertainties.

The fact that gauge theoretic unification of electro-weak and strong interactions almost invariably leads to nonconservation of baryon and lepton numbers is currently receiving widespread attention [1]. Several experiments are now in design to search for nucleon decay^{†1}. In the next few years these experiments will yield invaluable information on the possible interactions of gauge fields with masses $> 10^{14}$ GeV which are currently considered to be responsible for proton decay. Theoretical calculations on the expected nucleon lifetime, branching ratios into various decay modes, etc. have already been reported by various authors [2,3].

Theoretical estimates of the expected lifetime in the SU(5) theory are in the range $10^{30} - 10^{31}$ yr^{‡2}. The UC Irvine-Michigan-Brookhaven (IMB) proton decay experiment with 10000 tons of water active detection mass (6×10^{33} nucleons) would then have 6000-600

decays per year in the above lifetime range. A detailed study of the underlying gauge interactions through an analysis of various dominant and "Cabibbo" suppressed as well as electromagnetic decay modes is then feasible.

We are thus motivated to consider the reaction:

$$p \rightarrow e^+ + \gamma. \quad (1)$$

Although one immediately recognizes that the overall rate for this reaction will be suppressed relative to modes involving mesons it is also relatively free of nuclear absorption and free from complications from strong interactions. These properties could make (1) a cleaner channel for an experimental analysis of proton decay. In addition, since the existing incidental limits on baryon nonconservation imply that the limiting background for detection of baryon decay are those induced by cosmic ray neutrinos an important figure of merit that need be kept in mind in comparing different nucleon decay modes is the ratio: (signal from nucleon decays/background from cosmic ray neutrinos) for that particular decay mode. We find a significant gain of this ratio for $p \rightarrow e^+ + \gamma$ versus the widely discussed $p \rightarrow e^+ + \pi^0$ mode.

We shall take the effective interaction lagrangian responsible for $p \rightarrow e^+ + \gamma$ via exchanges of vector gauge fields to have the general form [4]:

$$\mathcal{L} = 2^{-1/2} G_x \epsilon_{ijk} \bar{u}_k^C \gamma_\mu (c + d\gamma_5) u_j \bar{e}^C \gamma^\mu (a + b\gamma_5) d_i, \quad (2)$$

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^{†1} The following experiments are in various stages of construction: Irvine-Michigan-Brookhaven; Harvard-Purdue-Wisconsin; Minnesota; Frascati-Milano.

^{‡2} Reines has recently suggested that the deep mine experiment analyzed in ref. [5] may indeed have seen nucleon decay implying a lifetime close to the existing lower limit of $\approx 10^{30}$ yr.

where G_x is the effective four-fermion point coupling:

$$G_x/\sqrt{2} = g_x^2/8M_x^2, \quad (3)$$

g_x being the dimensionless coupling characterizing the interactions of the leptons and quarks with the lepto-quark gauge fields and C is the charge conjugation operator such that $\bar{u}^C = u^T C$. In the SU(5) theory of Georgi and Glashow [1], with the usual assumption that charge $\pm 4/3$ (X) fields have the same mass as charge $\pm 1/3$ (Y) fields, we have $a = 1$, $b = -3$, $c = 1$, and $d = -1$.

We shall adopt a simple nonrelativistic bound state model for the nucleon. Let p_1, p_2, p_3 be the three momenta of the three quarks with

$$P = p_1 + p_2 + p_3, \quad p = p_1 - p_2,$$

$$\mathcal{O} = \frac{1}{3}(2p_3 - p_1 - p_2).$$

Then after integrating out the center of mass motion of the three quarks the amplitude for the process can be expressed in the form:

$$\langle f|i \rangle = \int d^3p d^3\rho \sum \langle f|p_1 p_2 p_3 \rangle \phi(p, \rho), \quad (4)$$

where Σ stands for appropriate sums over spin, isospin and color indices and $\phi(p, \rho)$ is defined by:

$$\langle p_1 p_2 p_3 |i \rangle = \delta^3(P - p_1 - p_2 - p_3) \phi(p, \rho). \quad (5)$$

Now, $\phi(p, \rho)$ is expected to be varying much more rapidly with momenta compared to the amplitude $\langle f|p_1 p_2 p_3 \rangle$. Thus we may set:

$$\begin{aligned} \langle f|i \rangle &\simeq \overline{\langle f|p_1 p_2 p_3 \rangle} \int d^3p d^3\rho \phi(p, \rho) \\ &= (2\pi)^3 \overline{\langle f|p_1 p_2 p_3 \rangle} \psi(0, 0), \end{aligned} \quad (6)$$

where $\psi(0, 0)$ is the Fourier transform of $\phi(p, \rho)$ evaluated at the origin of the relative coordinates

$$r = r_1 - r_2, \quad s = r_3 - \frac{1}{2}(r_1 + r_2).$$

Fig. 1 shows the four Feynman diagrams responsible for reaction (1). The gauge invariant amplitude for the process, for the general case of an off shell photon, may be written as:

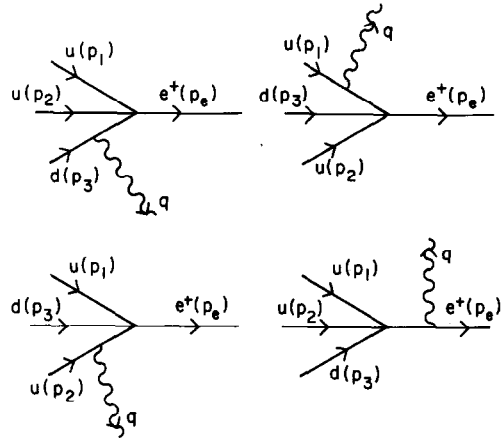


Fig. 1. Diagrams for the process $u + u + d \rightarrow e^+ + \gamma$.

$$\begin{aligned} A &= 12A_c [G_x/(\sqrt{2}m_p)] \psi(0, 0) [m_p m_e / (2q^0 E_p E_e \\ &\times (2\pi)^6]^{1/2} (2\pi)^4 \delta^4(P - p_e - q) \epsilon^\mu(q) v_e^T(p_e) C \\ &\times [A_1(q^2 \gamma_\mu - \not{q} q_\mu) + A_2(q^2 \gamma_\mu - \not{q} q_\mu) \gamma_5 \\ &+ A_3 \sigma_{\mu\nu} q^\nu / m_p + A_4 \sigma_{\mu\nu} q^\nu \gamma_5 / m_p] u_p(P). \end{aligned} \quad (7)$$

Using the interaction given in (2) we find, in the limit, $m_e \rightarrow 0$:

$$A_1 = -3(e_u + e_d)bc/(m_p^2 - 2q^2), \quad A_2 = -aA_1/b, \quad (8)$$

$$A_3 = im_p(e_u - e_d)bc/(m_p^2 - 2q^2), \quad A_4 = -aA_3/b, \quad (9)$$

where $e_u = 2e/3$, $e_d = -e/3$ are the charges of the u and d quarks in units of the charge e of e^+ . In (7) A_c is the enhancement due to exchanges of (e.g. SU(3) \times SU(2) \times U(1)) gauge fields at the proton mass. Its numerical value has been estimated to be ≈ 3.5 [4].

The width for the decay $p \rightarrow e^+ + \gamma$ is found to be:

$$\Gamma_\gamma = 18\alpha G_x^2 |\psi(0, 0)|^2 c^2 (a^2 + b^2) A_c^2 / m_p. \quad (10)$$

For purposes of numerical evaluation we shall take a simple harmonic oscillator potential responsible for the binding of the three quarks inside the proton. Then $\psi(r, s) = \psi_0(r) \phi_0(s)$ and $\phi_0(s=0) = (16/9)^{3/8} \psi_0(r=0)$. We find

$$\begin{aligned} \Gamma_\gamma &= (44\alpha) \hbar (10^{30} \text{ yr})^{-1} g_x^4 A_c^2 c^2 (a^2 + b^2) u^2 / m_{14}^4 \\ &= (440\alpha) \hbar (10^{30} \text{ yr})^{-1} g_x^4 A_c^2 u^2 / m_{14}^4 \quad [\text{for SU(5)}], \end{aligned} \quad (11)$$

where ^{#3}

$$u = |\psi_0(r=0)|^2 / 10^{-2} \text{ GeV}^3 \quad (12)$$

and

$$m_{14} = m_x / 10^{14} \text{ GeV} . \quad (13)$$

To estimate the branching ratio for $p \rightarrow e^+ + \gamma$ we need to calculate the total decay width of the proton. We assume that that can be approximated by diquark annihilation reactions of the type ^{#4}

$$u + u \rightarrow e^+ + \bar{d} , \quad u + d \rightarrow e^+ + \bar{u} , \quad u + d \rightarrow \bar{\nu}_e + \bar{d} . \quad (14)$$

As a specific example we confine ourselves to SU(5). The expressions that we derive for these reactions can of course be used for modes with muons as well. However, they will be "Cabibbo suppressed" in SU(5) and we shall ignore them in our estimate of the total decay width. The widths for reactions (14) are then found to be [m is the mass of the final antiquark in (14)]:

$$\begin{aligned} \Gamma_{uu,e}^p &= \Gamma_0 (4 + 9m^2/2m_p^2) , \\ \Gamma_{ud,e}^p &= \Gamma_0 (19 + 45m^2/4m_p^2 - 27m/m_p) , \\ \Gamma_{ud,\nu_e}^p &= \frac{19}{5} \Gamma_0 (1 + 45m^2/76m_p^2 - 27m/19m_p) , \end{aligned} \quad (15)$$

where

$$\begin{aligned} \Gamma_0 &= (20/27\pi) G_x^2 A_c^2 |\psi(0)|^2 m_p^2 (1 - 9m^2/4m_p^2)^2 \quad (16) \\ &\simeq 17\hbar (10^{30} \text{ yr})^{-1} g_x^4 A_c^2 u / m_{14}^4 . \end{aligned} \quad (17)$$

The sum of the expressions above yields the "total" decay width of the proton:

$$\Gamma_{\text{tot}}^p \simeq \frac{134}{5} \Gamma_0 (1 - 81m/67m_p + 45m^2/67m_p^2) . \quad (18)$$

For $m = m_p/3$, $\Gamma_{\text{tot}}^p \simeq 18\Gamma_0$, that is,

^{#3} Values of $|\psi(0)|^2$ in the non-relativistic quark model which are potential independent are obtained through rates and models for non-leptonic hyperon decays and electromagnetic hyperfine splittings. These range from 0.0115 GeV³ [7] to 0.004 GeV³ [8] to 0.001 GeV³ [9] although the latter calculation is strongly dependent on weak form factors. For our estimates we use 0.004 GeV³ which is also consistent with a harmonic oscillator Regge trajectory slopes and potential model relations to $\langle p^2 \rangle$.

^{#4} This assumption has also been made by Buras et al. and by Jarlskog and Yndurain [2]. However, unlike these authors we are not letting the final quark mass in reaction (14) be zero.

$$\Gamma_{\text{tot}}^p \simeq 310\hbar (10^{30} \text{ yr})^{-1} g_x^4 A_c^2 u / m_{14}^4 . \quad (19)$$

Using $g_x^2/4\pi \simeq 0.02$, $A_c \simeq 3.7$ and ^{#3} $u \simeq 0.4$ this gives for SU(5)

$$\Gamma_{\text{tot}}^p = 320\hbar (10^{30} \text{ yr})^{-1} m_{14}^{-4} . \quad (20)$$

From (20) the current experimental limit on the lifetime of the proton [5] ($\Gamma_p \gtrsim 10^{30} \text{ yr}$) implies

$$m_x \gtrsim 3.4 \times 10^{14} \text{ GeV} . \quad (21)$$

Combining (11) and (19) the branching ratio for $p \rightarrow e^+ + \gamma$ [in SU(5)] is found to be:

$$B_\gamma \simeq |\psi_0(r=0)|^2 \text{ GeV}^{-3} = 0.01u \simeq 0.004 . \quad (22)$$

For a lifetime range of 10^{30} yr to 10^{31} yr this gives 15 to 2 events/yr in the IMB experiment. Although, as expected, this branching ratio is quite small, for experimental purposes it needs to be compared with the branching ratio B_π for the reaction $p \rightarrow e^+ + \pi^0$. Estimates of the latter vary from about 8% to 40%. In addition only 1/3 of the π^0 's produced in a nucleus survive without nuclear interaction to be correlated with the e^+ [6]. For the rates in water of correlated final states,

$$R_\gamma/R_\pi = R(p \rightarrow e^+ + \gamma)/R(p \rightarrow e^+ + \pi) = \frac{1}{8} \text{ to } \frac{1}{40} . \quad (23)$$

The background to the $e^+ \pi^0$ mode arises from cosmic ray $\bar{\nu}_e$'s producing $\bar{\nu}_e + N \rightarrow e + (\Delta \rightarrow p\pi^0)$. With a proton lifetime of 10^{30} yr this provides less than one background event/year for the IMB experiment within the energy and angle constraints for an $e^+ \pi^0$ from proton decay. The back-to-back $e^+ \gamma$ background is analogously dominated by $\bar{\nu}_e + N \rightarrow e + (\Delta \rightarrow p\gamma)$. The ratio of these backgrounds is that of the branching ratios $\Gamma(\Delta \rightarrow p + \gamma)/\Gamma(\Delta \rightarrow p + \pi^0) \simeq 1\%$ and therefore there is no background to $e^+ \gamma$ in the present experiments.

Since there are many uncertain parameters in calculations of the overall proton decay rate it is important to note that an experimental determination of the $p \rightarrow e^+ \gamma$ branching ratio via eq. (22) determines $(\psi(0))^2$ by itself and can be used to directly eliminate its uncertainty in the calculation. Even without the total decay rate the $e^+ \gamma$ rate [eq. (10) or (11)] constrains the proton wave function or the grand unified couplings independent of the hadronic parameters that enter in the formation of exclusive hadronic final state modes.

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