Searching for Alternatives in Spatial Reasoning: Local Transformations and Beyond

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Abstract

Searching for alternative solutions of an indeterminate reasoning task is an important and necessary step in order to draw certain inferences as in the case of deduction. To elucidate the underlying mental representations and processes of the search for alternatives in spatial reasoning, an experiment was conducted that used specific material stemming from AI research of Qualitative Spatial Reasoning. The results showed that searching for alternative solutions can be best explained as a revision process starting with an initial mental model of the premises. Proceeding from one solution to an alternative is apparently achieved by local transformation. Interestingly, local transformations have a "logic of their own": They can lead to systematic errors of omission and to errors of commission.

Spatial Reasoning and Mental Models

Dealing with spatial problems is a frequent and important challenge in everyday as well as in professional life. It occurs across various fields like spatial navigation or spatial configuration and design. In this paper, we will concentrate on a special sort of spatial problem solving, namely reasoning based on spatial relational descriptions. This type of reasoning can be investigated with recourse to several background theories of thinking developed in cognitive psychology. According to previous research in spatial reasoning (Byrne & Johnson-Laird, 1989; Evans, Newstead, & Byrne, 1993) and according to our own previous findings (Knauff, Rauh, & Schlieder, 1995; Knauff, Rauh, Schlieder, & Strube, 1998; Rauh & Schlieder, 1997) the most promising and most successful framework is the theory of mental models.

Mental Model Theory as Framework

The core assumption of the mental model theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991) states that when we reason we build an integrated representation of the situation that the premises describe. This integrated representation—the mental model—is in certain aspects analogous to the state of affairs and, as a consequence, lacks the information whether relationships are explicitly mentioned in the premises and or are implicitly determined by the representational format.

A further consequence of the assumption of integrated representation becomes evident when certain kinds of inferences have to be drawn. Take deductive inference for example: To test whether a contingent relationship in the initial mental model is necessarily true, the reasoner has to test all the alternative models of the premises. If a contradictory example is found, the putative conclusion will be rejected; if not it will be accepted as a valid conclusion.

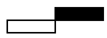
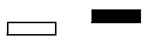
The search for alternative models takes place during what we call the phase of model variation. It seems to be a deliberate mental process so fragile that it causes many systematic reasoning errors. There are errors of omission, i.e. inferences that could have been validly drawn, and there are errors of commission, i.e. inferences that are not justified by the premises.

Therefore, model variation has attracted much attention, but little is empirically known about how the mental search for alternative models is accomplished by the human process of reasoning. For a precise investigation of the model variation phase, there is the need for relational material with a rich inherent structure and unambiguous semantics.

Spatial Reasoning with Interval Relations

Traditional investigations of spatial reasoning used relations like *left-of*, *right-of*, *in front of*, and *behind*. As argued elsewhere (Knauff et al., 1998), these spatial relations have no clear semantics. Therefore, studies of reasoning using these spatial relations are problematic because it is unclear whether the results obtained can be attributed to the inference processes, or are due to the ambiguity of these relations. To remedy this situation, we use Allen's (1983) set of 13 qualitative interval relations that enables one-dimensional spatial reasoning. These relations have clear geometric semantics based on the bounding points of the intervals, i.e. their starting points and ending points. They also have the property of being jointly exhaustive and pairwise disjoint (JEPD)—a property that also reduces the risk of misinterpretations. In Table 1, we shortly introduce these relations together with verbalizations that we use in our experiments.

Table 1: The 13 qualitative interval relations, associated natural language expressions, and a graphical example (adapted and augmented according to Allen, 1983).

Relation symbol	**Natural language description**	**Graphical example**
X < Y	X lies to the left of Y	
X m Y	X touches Y at the left	
X o Y	X overlaps Y from the left	
X s Y	X lies left-justified in Y	
X d Y	X is completely in Y	
X f Y	X lies right-justified in Y	
X = Y	X equals Y	
X fi Y	X contains Y right-justified	
X di Y	X surrounds Y	
X si Y	X contains Y left-justified	
X oi Y	X overlaps Y from the right	
X mi Y	X touches Y at the right	
X > Y	X lies to the right of Y	

With these relations, reasoning tasks known as three-term series problems can be constructed. One example is "*X overlaps Y from the left. Y surrounds Z.*" The example also shows that there are many three-term series problems generated from these relations that have more than one solution. To be precise, there are 42 three-term series problems that have three solutions, 24 that have five solutions, 3 that have nine, and another 3 that have thirteen solutions. We utilize this property in order to construct indeterminate three-term series problems to investigate precisely the phase of model variation. In the next section, we will present a more formal analysis of these tasks. From this analysis and the revealed properties of the different tasks, hypotheses can be derived that we will test in a model variation experiment.

A Formal Framework for Model Variation

In principle, there are two ways to construct alternative models of the premises. The first consists of repeating the complete construction of alternative models one after another (*model iteration*). We will examine the more plausible variation strategy that consists of generating alternative models by *locally* transforming the *initial model* (see also Schlieder, 1998), i.e. the first model constructed during model variation (*model revision*).

In this view, any sequence of models $M_0, M_1, ..., M_n$ corresponds to a *sequence of transformations* $T_1, T_2, ..., T_n$, where the output model M_i of T_i is the input model of T_{i+1}. The set $\{M_0, ..., M_n\}$ is *ordered* by the sequence $T_1, T_2, ..., T_n$.

Since models of a three-term-series problem are completely determined by only one relation, namely the one between *X* and *Z*, we can treat models and relations equivalently. Seen this way, a transformation is a transition from one relation r_1 to another relation r_2, or, in short, $r_1 \rightarrow r_2$.

Conceptual Neighborhoods

Freksa (1992) introduced the notion of conceptual neighborhood between interval relations. Formally, the three conceptual neighborhoods are defined by the graphs in Figure 1. Two relations are *neighbors* iff they are connected by an edge of the corresponding graph.

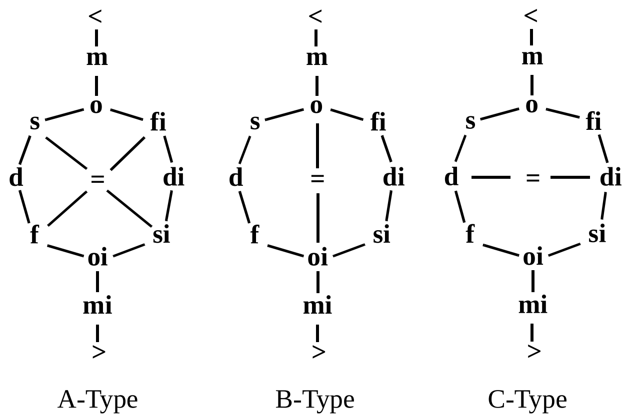


Figure 1: Freksa's (1992) conceptual neighborhoods.

The common generic principle underlying the three types of neighborhood reads as follows: Interval relations r_1 and r_2 are said to be *conceptual neighbors* if a model of intervals X and Y satisfying X r_1 Y can be continuously transformed into a model of intervals X' and Y' satisfying X' r_2 Y' such that during the transformation no model arises in which a relation different from r_1 and r_2 holds (see Schlieder & Hagen, in press). Their peculiarities arise from different transformation processes. The A-neighborhood is based on a transformation that can be described as the movement of one single bounding point of one interval whereas the B-neighborhood relies on the movement of a complete interval of fixed length. The transformation defining the C-neighborhood consists of keeping the center of the changing interval fixed and varying its length. The types of transformations defining the A(B,C)-neighborhoods will be called *A(B,C)-transformations*.

Local Transformations: Steps between A-Neighbors

An examination of sequences of A-transformations revealed a need to formally refine the conceptual framework. In order to describe the model revision process adequately the definition has to include the movement of bounding points and its direction. An A-transformation between intervals X and Y does not specify the moving bounding point since it can always be accomplished in two ways by movements of a suitable bounding point: Either by moving one bounding point of interval X in one direction or one of Y in the opposite direction (see Table 2). An A-transformation with specified moving point *p* will be called a *step (of bounding point p in direction d)*.

Table 2: The relation of A-transformations and steps.

A-transformation	**step right**	**step left**
$< \rightarrow m$	E_X	S_Y
$m \rightarrow o$	E_X	S_Y
$o \rightarrow fi$	E_X	E_Y
$fi \rightarrow di$	E_X	E_Y
$di \rightarrow si$	S_X	S_Y
$si \rightarrow oi$	S_X	S_Y
$oi \rightarrow mi$	S_X	E_Y
$mi \rightarrow >$	S_X	E_Y
$o \rightarrow s$	S_X	S_Y
$s \rightarrow d$	S_X	S_Y
$d \rightarrow f$	E_X	E_Y
$f \rightarrow oi$	E_X	E_Y
$s \rightarrow =$	E_X	E_Y
$= \rightarrow f$	S_X	S_Y
$= \rightarrow si$	E_X	E_Y
$fi \rightarrow =$	S_X	S_Y

Note that tracking sequences of interval relations does not permit the direct observation of steps. *Step-sequences,* i.e. sequences of steps that refer to the same point *p* moving in constant direction *d,* can explain errors of omission or commission that cannot be explained on the level of A-transformations. In order to show this, we need one more definition. A step-sequence S_1, ..., S_n is *extendible at the beginning (or the end)* iff there exists a step S_0 (or S_{n+1}) such that S_0, S_1, ..., S_n (or S_1, ..., S_n, S_{n+1}) is a step-sequence. If it is extendible at the beginning or at the end it is *(totally) extendible*.

Errors of Omission and Errors of Commission. Our general assumption about the implications of this formalism for the traversal of solution sets is as follows: Moving along a step-sequence, i.e. keeping the moving point and its direction constant, is easier to process than changing them or even performing a non-A-transformation.

Therefore, errors of omission should be observed more frequently if the end of a step-sequence is reached but the solution set is not completely traversed. Errors of commission, in turn, should occur more frequently with non-solutions which are a continuation of a step-sequence.

Hypotheses

In the following we present hypotheses specifying the implications of the above considerations in more detail. They can easily be verified consulting Table 2 and Figure 2, which displays solution sets of all three-term series problems with multiple models.

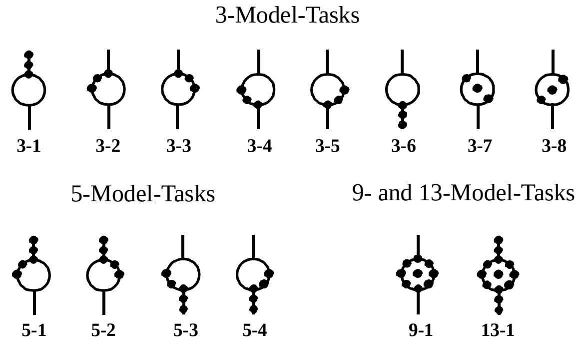


Figure 2: The solution sets of three-term-series problems with multiple models. The valid relations are represented as points at corresponding positions of Figure 1.

3-Model-Tasks. The relations determining the solution set of a 3-model-task can be ordered in two ways by sequences of A-transformations (e.g. for (3-1): $< \rightarrow m \rightarrow o$ or $o \rightarrow m \rightarrow <$). Each of these sequences can be accomplished in two ways as step-sequence (e.g. $< \rightarrow m \rightarrow o$ by steps to the right of the ending point E_X of interval X or by steps to the left of the starting point S_Y of Y). One of these sequences is extendible except for the solution sets (3-7) and (3-8) where all sequences are non-extendible. There are two interesting hypotheses concerning 3-model tasks: (1) 3-model-tasks having extendible solution sequences are prone to errors of commission, and (2) 3-model-tasks with solution sets (3-7) and (3-8) have significantly less errors of commission than the other 3-model-tasks.

5-Model-Tasks. The solution set of a 5-model-task can be ordered in two ways by sequences of A-transformations. Each of these sequences can be accomplished in two ways, as step-sequence that is non-extendible, or as a sequence S_1, S_2, S_3, S_4, where S_1, S_2 and S_3, S_4 are non-extendible step-sequences, having the same direction but referring to different bounding points of the same interval. Accordingly, we can formulate the hypothesis, that errors of omission will most frequently occur between step 2 and step 3.

9-Model-Tasks and 13-Model-Tasks. The solution set of a 9-model-task or of a 13-model-task can be ordered in multiple ways by sequences of A-transformations. Each of them fall into several step-sequences, including necessary changes of direction between them. So we expect a decreased number of correct and complete solution sequences for these tasks.

Experiment on Model Variation

Participants

24 students (12 female, 12 male) of the University of Freiburg were paid for participation.

Materials

The material consisted of the 72 indeterminate three-term series problems that can be constructed by the 12 interval relations, if the trivial "=" relation is omitted. In each three-term series problem the spatial relationship between a red and a green interval is described in the first premise, and the relationship between the green interval and a blue one is given in the second premise.

Procedure

The computer-assisted experiment was divided into three phases. During the *definition phase* participants were given the verbalizations of the interval relations together with an explanation of the semantics with respect to the ordering of starting points and ending points. Additionally, a pictorial example was displayed.

During the *learning phase*, participants read sentences describing the relation between a red and a blue interval. For each sentence they had to specify the relationship of the two intervals graphically by clicking the mouse in rectangular regions on the screen. After having confirmed the final choices, the participant got feedback about the accuracy of the configuration. If the configuration did not match the relation, additional information about the correct answer was given, i.e. a verbal description of the ordering of start points and end points. Learning trials were blocked with 13 sentences using the interval relations. If one relation was answered correctly in three consecutive blocks, the learning criterion for this relation was accomplished. As soon as the learning criterion was reached for all relations, the learning phase stopped.

During the *inference phase*, participants were given 3 practice trials, and then received the 72 indeterminate three-term series problems. After self-paced reading of the premises, the premises vanished, and the participants were asked to generate all possible relationships between the red and the blue interval. By clicking the mouse they specified the spatial relationships analogous to the interval-specifying procedure in the learning phase. After finishing the configuration, participants could either continue specifying other solutions, or stop working on the present task and go to the next three-term series problem.

We recorded premise processing times, drawing times, and, of course, the sequence of solutions by pixel coordinates and by interval relations.

Results

In the following, data analyses are applied to the constructed solution sequences. Since all participants passed the learning phase successfully, all data collected in the inference phase were included in the statistical analyses.

First, we tested the hypothesis that solution sequences followed the principles of conceptual neighborhood. All transitions in the solution sequences were analyzed for the existence of A-, B-, and C-transformations. We found that the significant majority of the transitions (3145 of 4462 [= 70.48%]) conformed to A-transformations. Transitions conformed to B- or C-transformations in 64.95% or 64.34% of all the cases, respectively. The three values are rather similar, since most transitions are consistent with all three types of conceptual neighborhood. Only transitions involving the "=" relation discriminate between different types of conceptual neighborhood (see Figure 1). Therefore, we performed an analysis for these transitions and found the frequencies listed in Table 3.

Table 3: Number of "="-transitions conforming to different types of conceptual neighborhood.

	Absolute	**Percent**
A-transformation	296	75.13%
B-transformation	49	12.44%
C-transformation	22	5.58%
Other	27	6.85%
Total	394	100%

We obtained the results in Table 4 by exclusively analyzing correct and complete solution sequences of 3-, 5-, 9-, and 13-model tasks.

The interesting fact is the nearly monotonic decrease of the number of correct and complete solution sequences in dependence of the number of models. Besides, it is noteworthy that correct and complete sequences of the 9- and 13-model problems (i) are rarely observed (as predicted by our hypothesis), and (ii) that none of these sequences conformed perfectly to any of the neighborhood transformations. We will return to the latter point below.

Table 4: Number of correct and complete solution sequences

	Percent	**A-Transf.**
3-model-tasks	52.88% (533 of 1008)	75.61% (403 of 533)
5-model-tasks	34.20% (197 of 576)	86.29% (170 of 197)
9-model-tasks	13.89% (10 of 72)	0% (0 of 10)
13-model-tasks	16.67% (12 of 72)	0% (0 of 12)
Total	43.52%	73.27%

Errors of omission. To test for the hypothesis of systematic errors of omission between step 2 and step 3, we looked at the solution generated last in the whole solution sequence for all 5-model-tasks. In Table 5 the results for the six 5-model-tasks with solution set (5-2) (see Figure 2) are listed. As stated above, we expected an increasing number of solution sequences terminating after the second step, i.e. for relation *o*.

Table 5: Frequencies of relations as last solution for 5-model tasks with solution set (5-2).

di	**fi**	**o**	**m**	<
10	7	22	8	87
7.46%	5.22%	16.42%	5.97%	64.93%

As Table 5 shows, there are indeed many solution sequences terminating with the relation *o* (22 of 134). This pattern of results was also obtained for the 5-model-tasks with the other three solution sets. The result confirms our predictions of systematic errors of omission between steps 2 and 3.

Errors of commission. According to our predictions of systematic errors of commission, the 3-model-tasks with solution sets (3-1) to (3-6) were analyzed for transitions from relation *o* (*oi*) followed by an erroneous one. The number of such transitions was 57. It turned out that 26 of them were steps with the *o* (*oi*) relation as precursor. Given that there are at least 8 other erroneous relations that are not A-transformations of *o* (*oi*), this shows that the transition from a correct solution to an erroneous one is about three times more probable if the erroneous solution is the next step in the step-sequence. The result corroborates our hypothesis of systematic errors of commission. Additionally, the 3-model-tasks with solution sets (3-7) and (3-8) had 13.5 commission errors on the average, much less than the 72.0 commission errors that could be observed on the average for the 3-model tasks with solution sets (3-1) to (3-6).

Strategies for 9- and 13-Model-Tasks. As shown in Table 4 none of the correct and complete solution sequences of the 9-model-tasks and the 13-model-tasks conformed perfectly to any of the conceptual neighborhood transformations. In an exploratory data analysis, we identified two classes of strategies for navigating through the solution set that guided the successful search for alternatives in solving 9- and 13-model-tasks.

Constant-Direction-Strategies. The first class of strategies consists of three sequences of A-transformations following one after another. The two transformations joining them are not A-transformations, but *jumps* in the graph of the A-neighborhood. (see the diagram in Figure 3)

As the pseudo code description in Figure 3 shows this strategy can be accomplished in a simple way: All steps refer to points of the same interval and proceed with the same direction. For each step the other bounding point of the interval is tested if a step leads to a valid model, and the information determining this model is stored if necessary. The jumps occur only if proceeding within a step-sequence is not possible. Then the stored information is retrieved again to construct the corresponding model to begin the next step-sequence.

The success of this kind of strategy depends highly on the choice of the initial model since the moving direction is constant and an omitted model will never be reached.

```
Choose an initial model;
Choose an interval (with bounding points p and q) that is part
of the relation between the first and the third interval;
Choose p and direction d such that step(p, d) possible;

while step(p, d) or step(q, d) possible
begin
  if step(p, d) possible then
    begin
      if M empty and step(q, d) possible then
        Store info identifying the result of step(q, d) in M;
      step(p, d);
    end
  else
    begin
      if M not empty then
         Continue with the model
            identified by M;
      else if step(q, d) possible then
         step(q, d);
    end
end
```

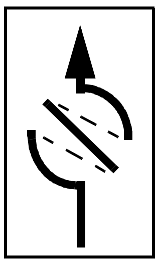


Figure 3: Constant-Direction strategies as pseudo code and a diagram of a possible path in the Freksa-graph. Details of the algorithm are specified only as far as necessary; *step(p, d)* represents a step-transformation of p in direction d, M information identifying a model.

Symmetry-Strategies. The second class of strategies is based on the use of symmetric transformations mapping relations to their inverses (*transposition*-symmetry). Their limitations and strengths concerning the traversal of the solution set arise from the fact that the solution sets of 9- and 13-model tasks fall into several disjointed subsets that are closed in relation to symmetry-transformations. An extended version involves additional *reorientation*-symmetry. This type of symmetry can be described as reflection of the graphical example in Table 1 at the vertical axis. All relations are symmetrical to themselves with respect to reorientation except the pairs *f*-*s* and *fi*-*si*. In place of the closed subsets {*f*, *fi*} and {*s*, *si*} their union now forms a closed subset.

For the traversal of the solution set of a 13-model-task following the extended type of strategy this implies that at least 5 non-symmetric transformations (out of a total of 12 necessary transformations) are needed to traverse all relations. A 9-model-task needs at least 3 non-symmetric transformations (out of a total of 8). The type of strategy that relies only on transposition requires one more non-symmetric transformation. Especially for 13-model-tasks we cannot expect complete solutions without an additional guiding principle. Furthermore, errors in finding a closed subset will lead to omitting it completely. On the other hand due to the cyclic structure of a closed subset, its traversal is insensitive to the first relation established.

General Discussion

In summary, the presented results corroborate the assumption that searching for alternatives is based on a model revision process proceeding from an initial model to alternatives by local transformations. We demonstrated and specified this for one-dimensional spatial reasoning, where local transformations appear as movements of a point along a step-sequence. Additionally, we were able to show that local transformations have a logic of their own: They can systematically suppress certain inferences on the one hand, but, on the other hand, lead to false ones. Again, we specified these conditions with the help of our relational material, and thus were able to predict errors of omission and errors of commission precisely. This point is also very important for augmenting our existing cognitive modeling of mental model construction with an empirically adequate revision process.

With respect to psychological theories of reasoning, our results are pretty much in accordance with the mental model theory. In particular, the decline of number of correct and complete solution sequences with the number of models corresponds well with mental model theory assumption that the difficulty of a reasoning task is dependent on the number of models. Likewise, the notion of local transformation only makes sense with recourse to analog representations, e.g. mental models. Therefore, our data also present a new challenge for other theories of reasoning.

Acknowledgements

This research was supported by the German National Research Foundation (Deutsche Forschungsgemeinschaft; DFG project *MeMoSpace* under contract no. *Str 301/5-1* to Gerhard Strube, Christoph Schlieder, and Reinhold Rauh). We would like to thank Goran Sunjka for his extensive help in implementing the computer-aided experiment, Katrin Balke for running the experiment, and Rebecca Ellis and Patrick Mueller for proof reading an earlier draft.

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