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Strategies for Mitigating Impacts of Near-Side Bus Stops on Cars

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Abstract

We consider a bus stop that is located a short distance upstream of a signalized intersection. A bus that dwells at this so-called “near-side stop” can impede queued cars upstream as they discharge during their green time at the intersection. Added car delays and residual queues can result. All else equal, the closer the stop’s location to the intersection, the greater the potential damage to car traffic.

We have formulated models for locating these near-side stops to achieve target levels of residual queueing among cars. Kinematic wave theory was used to this end. This same approach was also used to develop a strategy for further mitigating residual car queues by temporarily detaining some buses from reaching the stop. This bus-holding strategy can be applied selectively, so that the times that “held” buses depart from the stop are not affected. The strategy therefore will not delay buses over the longer run. Our assessments indicate that this holding strategy can significantly reduce instances of car delays and residual queueing, especially for stops that are located very close to their intersections.

Keywords: near-side bus stops, bus holding, kinematic wave theory, car queues

1 Introduction

Bus stops on city streets are often located short distances from signalized intersections. This is done to improve accessibility; e.g., by enabling bus-users to readily transfer between different bus lines (TRB, 1996). Often, the choice is made to place the bus stops upstream of their nearest intersections (Kim and Rilett, 2005), and buses that use these so-called “near-side stops” often dwell in a travel lane when loading and unloading their passengers. A dwelling bus will therefore constrain any queued cars upstream when they discharge into the intersection during green times. This can create added car delays and residual car queues at the intersection downstream.

Despite these concerns, much of the literature in this realm has focused only on how the stops affect the operation of buses (e.g., Gibson, 1996; Furth and SanClemente, 2006; Kim and Rilett, 2005; Zhou and Gan, 2005). There are instances in which computer simulation was used to study the impacts to car traffic for select sets of inputs (Joyce and Yagar, 1990; Zhao et al., 2007). However, we are unaware of any previous work to formulate analytical models for predicting these impacts for more general cases.

In light of this, we explore strategies for deploying near-side bus stops that limit the damage done to cars. The strategies are developed using kinematic wave theory (Lighthill and Whitham, 1955; Richards, 1956; Newell, 1993). It is assumed that the car demand for the intersection approach is fixed and sufficiently low that, in the absence of a dwelling bus, car queues are fully served in each signal cycle. When the stop is occupied, it is further assumed that cars can maneuver around the dwelling bus without delay whenever the stop is not engulfed by a car queue from the downstream intersection. A problem will arise when the queue has expanded beyond the stop: as noted above, a dwelling bus will thereafter constrain the discharging car flow from the queue upstream.

The following section unveils the conditions under which a dwelling bus will not prevent a car queue from fully dissipating within a single green period, such that residual queues do not form. We call these the “first-best conditions.” The less stringent conditions under which residual car queues form, but dissipate within the next signal cycle, the so-called "second-best conditions," are unveiled as well. We then explore how these first- and second-best conditions can be satisfied by judiciously selecting the location of a near-side stop relative to the intersection downstream.

In certain instances, these locations may be prohibitively far from the intersection. Section 3 therefore examines a strategy for managing buses so that first- and second-best conditions can sometimes be achieved even when a stop is located in close proximity to its downstream intersection. The strategy entails detaining select buses in reaching the stop, without postponing the buses' departures from that stop. This bus holding strategy can greatly diminish car delays, as is demonstrated in Section 4 via some illustrative examples.

The strategies for locating near-side stops and for dispatching buses to them are of practical value. These matters are discussed in Section 5, as are our ideas for enhancing the strategies in future work.

2 Strategies for Locating Stops

The framework of our theoretical analysis is presented in Section 2.1. First- and second-best conditions are developed in Sections 2.2 and 2.3, respectively. The bus-stop locations required to induce these conditions are explored parametrically in Section 2.4.

2.1 Modeling Framework

Consider a near-side bus stop that is located a distance, d, from the downstream intersection; see Fig. 1. We denote: q as the fixed car inflow in the subject direction[1]; L_C as the fixed signal cycle length; G as the fixed green time; $g = G/L_C$ as the green ratio; Q as the capacity of the approach, and Q_B the capacity of the bottleneck created by a dwelling bus. We assume that the buses' dwell times are of random duration that never exceed L_C; and that their headways are random and large enough that each bus arrival can be treated independently.

Recall too our assumption that the approach is under-saturated, i.e.,

$$q \leq gQ; \tag{1}$$

and that a dwelling bus will not restrict the inflow of un-queued cars to the intersection,[2] i.e.,

$$q \leq Q_B. \tag{2}$$

[1] Left-turn traffic can probably be excluded from q when these left-turners enjoy a protected signal phase and their own turn lane(s).

[2] If $q > Q_B$, then a bus dwelling in a travel lane will create a bottleneck for cars as they approach the intersection, regardless of when and where the bus dwells. In this case, the remedy could entail having buses dwell "offline" in bus bays.

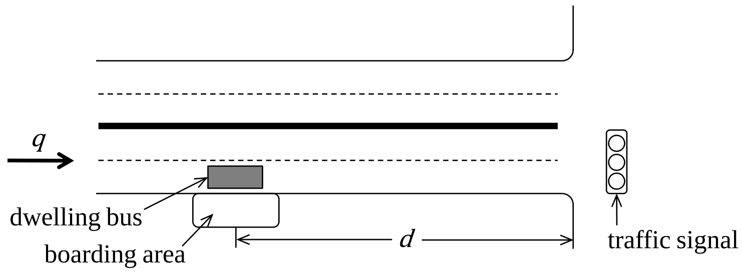


Fig. 1 Near-side bus stop

Traffic states on the subject approach are modeled using a triangular fundamental diagram in a moving-time coordinate system, whereby time travels forward over space at a free-flow vehicle pace (see Newell, 1993). For illustration, suppose that point *A* in Fig. 2 denotes the state of freely flowing cars, *C* the state when cars discharge at the approach capacity, *B* and *D* the traffic states downstream and upstream of the bottleneck created by a dwelling bus, respectively, and *J* the jam state of cars. We denote w as the backward wave speed in a car queue; w_{AD}, w_{AJ}, and w_{BJ} as the speeds of the waves that separate the other car states shown in Fig. 2. We obtain:

$$w_{AD} = \frac{w(Q_B - q)}{Q - Q_B}\,; \tag{3}$$

$$w_{AJ} = \frac{wq}{Q}\,; \tag{4}$$

$$w_{BJ} = \frac{wQ_B}{Q}\,. \tag{5}$$

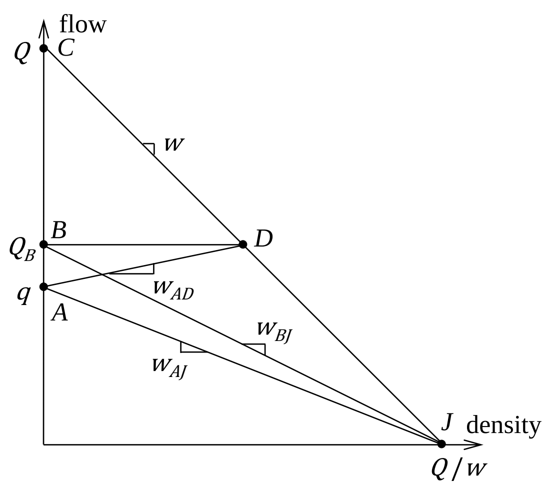


Fig. 2 Fundamental diagram for the approach in moving-time coordinates

2.2 First-Best Conditions

Following the above framework, the states of car traffic upstream of an intersection can be shown in a time-space diagram. For example, Fig. 3 shows the states in the absence of a dwelling bus, where the circled letters, *A*, *J*, and *C*, denote the car states that correspond to those labeled on the fundamental diagram in Fig. 2.[3] The horizontal line at distance d from the intersection marks the bus-stop location. Note that a bus can only arrive at the stop in one of the periodic time windows that are encircled by dotted ellipses in Fig. 3. At other times, the bus would be blocked from reaching the stop by a queue of (stopped) cars in state *J*.

We denote as t_a the time interval between the bus' arrival at the stop and the last signal transition from red to green; and S as the time required of the bus to serve boarding and alighting passengers at the stop. We assume the value of S is bounded in an interval, $[S_{min}, S_{max}]$. As an example, the solid, bold horizontal line segment in Fig. 3 represents the time that a dwelling bus spends serving its passengers. We have the following proposition:

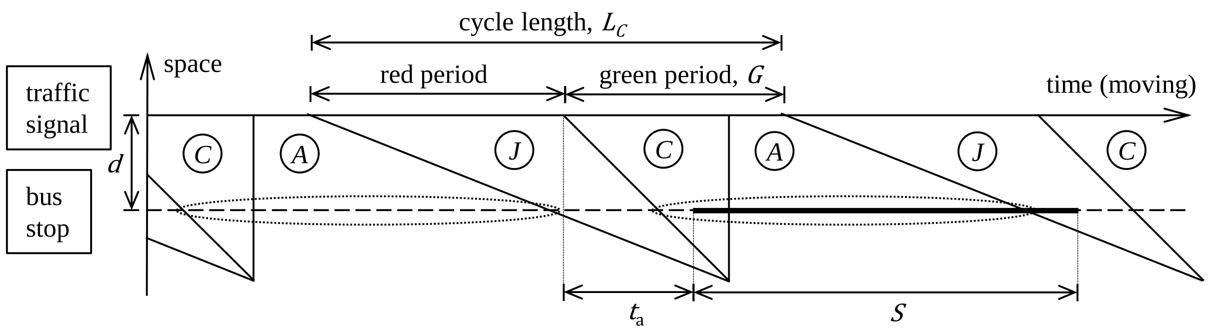


Fig. 3 Time-space diagram in the absence of a dwelling bus

Proposition 1. No residual car queue will occur at the end of any green signal if and only if one of the following three conditions holds:

(a) The bus stop is located no less than d_1 (see Fig. 4) from the intersection, i.e., $d \geq d_1$, where
$$d_1 = \frac{q - gQ_B}{Q - Q_B} w L_C \; ; \tag{6}$$
(b) $d < d_1$, $t_a \leq gL_C - \tau_1$ (i.e., the bus arrives to the left of line U_1V_1 in Fig. 4), and $S \leq \tau_1$, where
$$\tau_1 = \frac{gQ - q}{Q - Q_B} L_C \; ; \tag{7}$$
or

(c) $d < d_1$, $t_a > gL_C - \tau_1$ (i.e., the bus arrives to the right of U_1V_1 in Fig. 4), and
$$S \leq L_C + \tau_1 - t_a + \frac{d}{w}. \tag{8}$$

The proof of this proposition is furnished in Appendix A.

Note that if $q \leq gQ_B$, then we have $d_1 \leq 0$ from (6), and $\tau_1 \geq gL_C$ from (7). This means that a dwelling bus may impose limited delays to cars as they discharge into the intersection, but no residual car queue will exist at the end of a green time. This would be true regardless of where the stop is located and the periods of time when buses serve passengers there.

[3] The waves that separate states *C* and *A* are depicted in the diagram as if they travelled at infinite speed; i.e., these waves are shown as vertical lines. This is due to our use of a moving coordinate system for time.

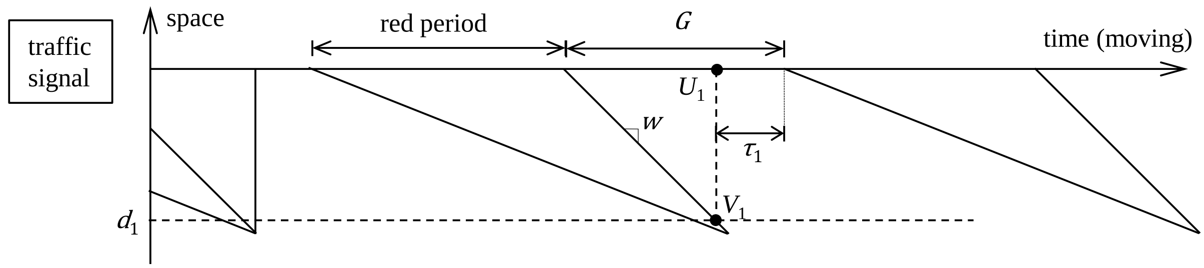


Fig. 4 Illustration of Proposition 1

2.3 Second-Best Conditions

In many cases, however, the first-best conditions are not feasible, e.g., because of limited street block length upstream of the intersection, as we shall see in Section 2.4. Thus, in this section we examine the second-best conditions, under which some queued cars might have to stop twice at the intersection due to the constraints on discharge flow imposed by a dwelling bus. However, under these conditions, the residual queue will clear within two signal cycles.

Proposition 2. No residual car queue will persist for more than one signal cycle if and only if one of the following four conditions is satisfied (see Fig. 5):

(a) The bus stop is located no less than d_2 from the intersection, where

$$d_2 = \frac{2q - g(Q+Q_B)}{Q-Q_B} w L_C \; ; \tag{9}$$

(b) $d < d_2$, $t_a \leq gL_C - 2\tau_1$ (i.e., the bus arrives to the left of U_2V_2), and $S \leq 2\tau_1$;

(c) $d < d_2$, $gL_C - 2\tau_1 \leq t_a \leq gL_C - \tau_1$ (i.e., the bus arrives between U_2V_2 and U_1V_1), and

$$S \leq (1-g)L_C + 2\tau_1 + \frac{d}{w} \; ; \tag{10}$$

or

(d) $d < d_2$, $t_a > gL_C - \tau_1$ (i.e., the bus arrives to the right of U_1V_1), and

$$S \leq L_C + 2\tau_1 - t_a + \frac{d}{w}. \tag{11}$$

The proof of this proposition is furnished in Appendix B.

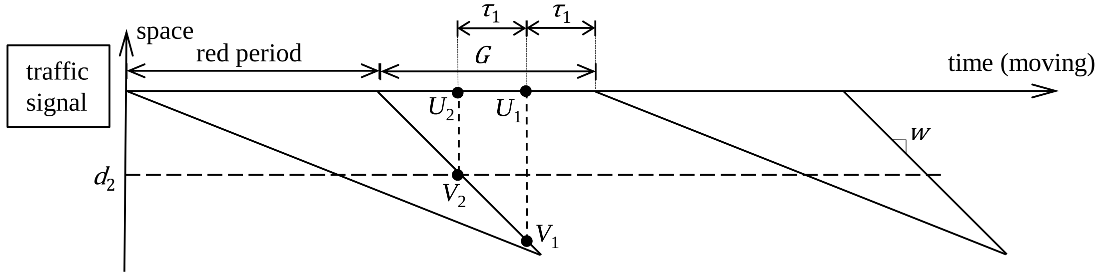


Fig. 5 Illustration of Proposition 2

Note that if $2q \leq g(Q + Q_B)$, we have $d_2 \leq 0$ and $2\tau_1 \geq gL_C$; i.e., the second-best conditions will always be achieved, regardless of when and where a bus dwells to serve its passengers.

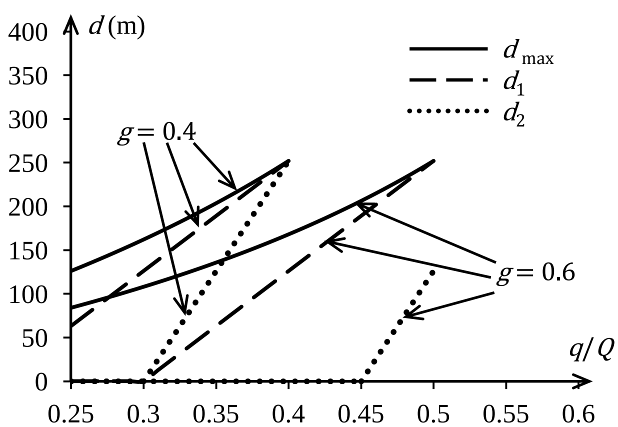
These second-best conditions give transit planners more flexibility to choose the location of the bus stop, as shown next.

2.4 Parametric Analysis

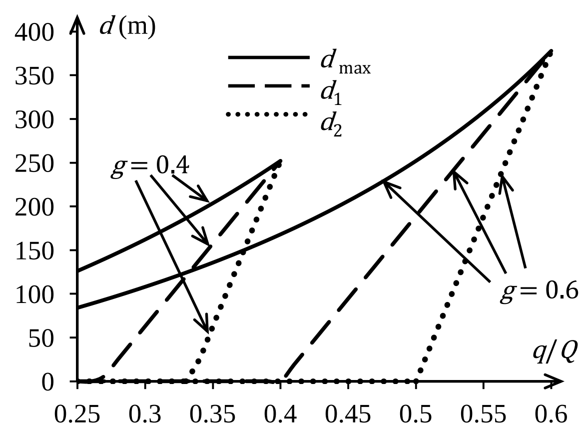
We now use (6) and (9) to examine the distance requirements for some typical cases. Figs. 6a and b present values of d_1 (dashed curves) and d_2 (dotted curves) for a range of inflow-to-capacity ratios for cars, q/Q. These figures were constructed for $w = 7$ m/sec and $L_C = 90$ sec. Note that cases are explored for capacity ratios $Q_B/Q = 0.5$ and 0.667 in Fig. 6a and b, respectively; and for $g = 0.4$ and 0.6 in both figures. Further note the solid curves in both figures. These curves display values of d_{max}, defined as the maximum distance that a car queue will reach when not impacted by a dwelling bus. Thus, a near-side stop located d_{max} from the intersection will not impose any delays on cars. This distance is given by:

$$d_{max} = \frac{(1-g)q}{Q-q} w L_C \ . \tag{12}$$

The figures show that dashed lines often lie well below their solid counterparts. Thus, we see the greater flexibility one has in locating a stop by forgoing efforts to eliminate car delays entirely and choosing first-best conditions instead. Interestingly, even greater vertical displacements can occur between a dotted line and its dashed counterpart. Hence we see that still greater flexibility that can come by accepting second-best conditions.[4] However, the figures also show how both d_2 and d_1 can approach d_{max} as q/Q increases. Note too that required distances in these instances can be quite long relative to the length of a typical city block.



(a) $Q_B/Q = 0.5$



(b) $Q_B/Q = 0.667$

Fig. 6 Comparison of d_1, d_2, and d_{max}

It thus seems that realizing even second-best conditions is not always feasible by the judicious choice of stop location alone. Fortunately, first- and second-best conditions can still be realized in many instances by using the strategy described next.

[4] We shall also note that d_1, d_2, and d_{max} all increase monotonically with car demand q/Q. While selecting the location of a near-side bus stop under variable traffic demand (e.g., in a typical day), it might be suitable to consider the highest demand.

3 A Strategy for Holding Buses

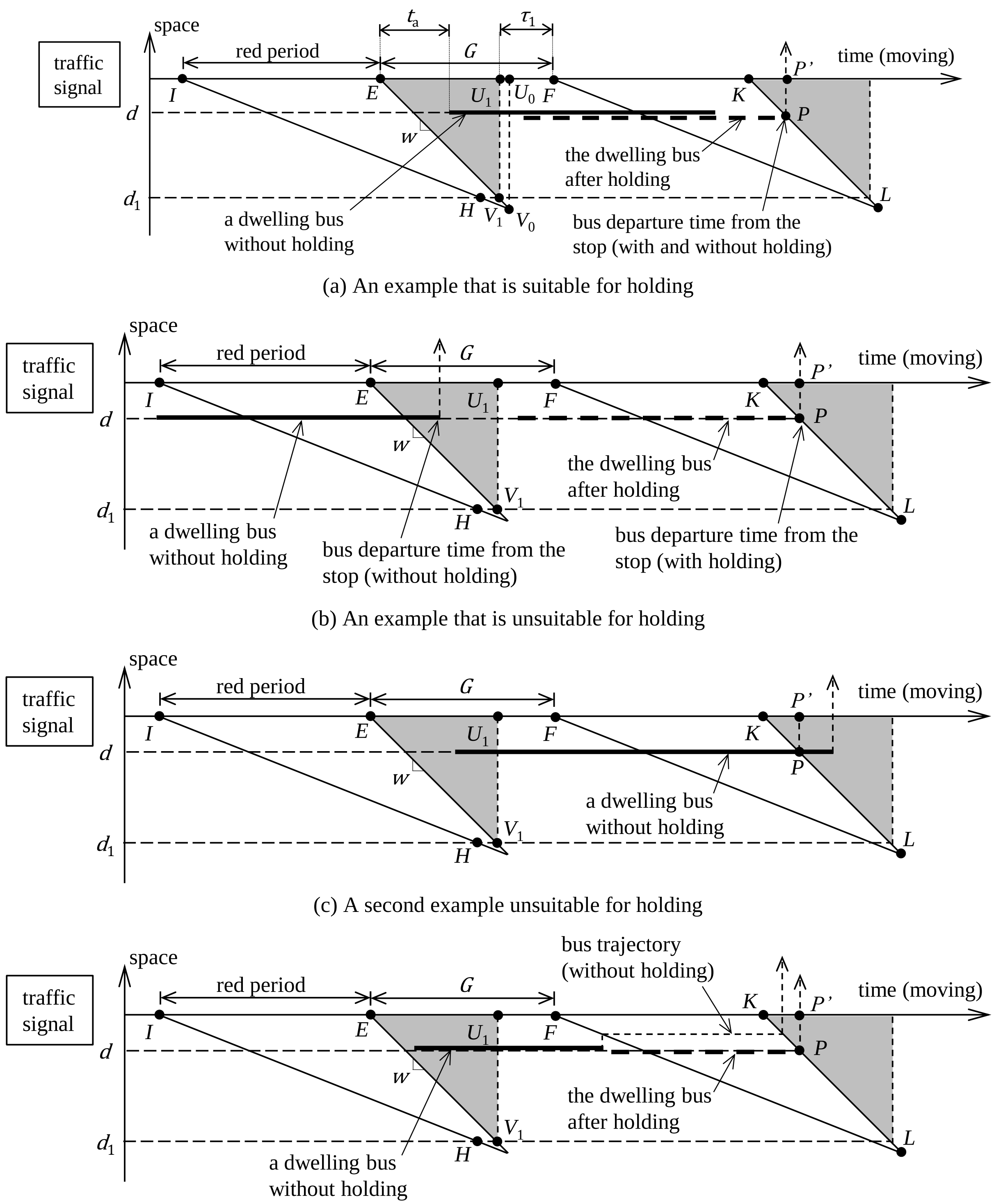
The propositions of the previous section show that residual car queues can be induced not only by the choice of d, but by other factors as well, including a bus' arrival time at the stop, denoted t_a. Fortunately, it is sometimes possible to alter a bus' arrival to mitigate the damage that would otherwise be done to car traffic, and to do so without delaying the bus in the longer run.

The action would be simple: when a bus' predicted arrival time at a stop would be damaging to cars, this arrival would, in some instances, be postponed to a more opportune time. Drivers of these "held" buses would be instructed to temporarily halt (or slow-down) in their lane in advance of the stop. The strategy is technically feasible: bus arrival times can be reliably predicted over the short run (e.g., when buses are equipped with GPS devices); and the holding instruction could be delivered in automated fashion via wireless communication technology. Recall that a bus that is halted or slowed in its lane at a distance of at least d_{max} from the intersection will not, in itself, impede cars given our assumption of a sufficiently low car inflow, q. And all passengers would soon thereafter be served at the stop. Very importantly, the holding strategy could be imposed selectively, so that buses are never delayed in departing from the stop. Thus, the only stake-holders who would be made worse-off by the strategy are those passengers on a "held" bus who alight at the coming stop. As we shall see, their delays would be modest; i.e., less than a signal cycle length. Details are offered below.

Consider the time-space diagram in Fig. 7a. By virtue of Proposition 1, we find that residual car queues will occur at the intersection if any portion of a bus' dwell time at the stop falls within the shaded areas of the figure. The solid horizontal line represents a bus that arrives at the stop located distance d from the downstream intersection. Left to its own devices, suppose that the bus would arrive at a time t_a after the start of the previous green phase, and the period that it would spend serving boarding and alighting passengers there is denoted by the solid horizontal line. Note that the bus' arrival lies within a shaded area, and would thus be damaging to car traffic. Further note that the bus finishes serving passengers at a time that lies to the left of point P in the figure. Yet, the bus cannot depart the stop immediately thereafter. It must wait instead for part of the car queue to dissipate during the next green time. The bus therefore departs the stop at the time denoted by point P, and departs the downstream intersection at the time denoted by P' (shown in moving time).

Note now the bold, dashed horizontal line in Fig. 7a. It is merely the solid line shifted forward in time. This shifted dashed line does not lie within a shaded area of the figure. Moreover, the period spent serving passengers at the stop ends by point P. The bus still departs the stop at the time coinciding with P, and then departs the intersection at the time coinciding with P'. Thus, by deferring the bus' arrival at the stop to a time that lies to the right of the vertical line U_1V_1 in the figure, interruptions to discharging cars would be diminished or even eliminated.[5] Yet, the bus can still serve all of its passengers and then depart the stop without encountering delay in the longer run.

[5] Consideration of Fig. 7a shows that if the bus arrival can be postponed to a time that coincides with the line U_1V_1, then the disruptions to cars will be diminished and residual queues will not form at the intersection. And if the bus arrival can be further postponed to a time that lies at or to the right of the vertical line U_0V_0 as in the figure, then car interruptions would be eliminated entirely.



(a) An example that is suitable for holding

(b) An example that is unsuitable for holding

(c) A second example unsuitable for holding

(d) A third example unsuitable for holding

Fig. 7 Illustration of damaging bus arrivals and their remedies

Regrettably, not all damaging bus arrivals can be so easily remedied. Imagine a bus that arrives at the stop at an earlier time that lies to the left of line *IH*, and with a dwell time that extends into the shaded area bounded by line EV_1 (see the solid, horizontal line in Fig. 7b). In this case, postponing the bus might be objectionable: Fig. 7b shows that a postponement would delay the early bus' departure from the stop until the time corresponding to point *P*. Advancing the bus' arrival at the stop might preclude the dwell time from penetrating a shaded region in the figure. Advancing a bus arrival is probably infeasible, however.

Fortunately, bus dwell times in cases like this latter one might often persist for only short initial periods of car discharge; i.e., note how the solid horizontal line in Fig. 7b penetrates a shaded region for a limited time. This occurs, in part, because bus passengers can be served during the traffic signal's red phases (Newell, 1989). All else equal, the shorter the periods that buses constrain car queues, the lower the damages. Thus, we focus now on the cases like the one in Fig. 7a, and explore the range of bus arrival times for which our holding strategy can be used without imposing long-run delays on buses.

Since candidate arrivals for this strategy fall in shaded areas like EU_1V_1, we have the constraints:

$$t_a \geq \frac{d}{w}, \tag{13}$$

and

$$t_a \leq gL_C - \tau_1 . \tag{14}$$

Further, a bus would not qualify for holding if its dwell time (in the absence of holding) could possibly extend into the shaded area to the right of line *KL*. In this instance, the bus' departure time from the stop would fall beyond point *P*; see the example shown again with a solid, horizontal line in Fig. 7c. If this bus were to have been held, its departure from the stop would only be further delayed in time. Thus, we have the constraint:

$$t_a + S_{max} \leq L_C + \frac{d}{w} . \tag{15}$$

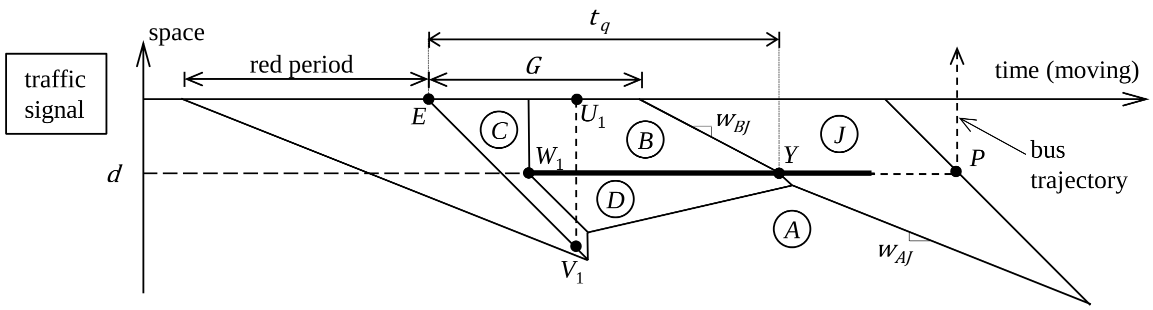
Similarly, a bus would not qualify for holding if its (un-held) dwell time could possibly end before the queue of jammed cars expanded beyond the bus stop; see the solid, horizontal line in Fig. 7d. In this case, the bus would depart the stop to join the tail of the car queue somewhere downstream, and eventually depart the stop at a time in advance of point *P*. An example trajectory for this bus is shown in Fig. 7d with dashed lines. Note that holding this bus would delay what would otherwise have been an early departure from the stop. Thus we have:

$$t_a + S_{min} \geq t_q , \tag{16}$$

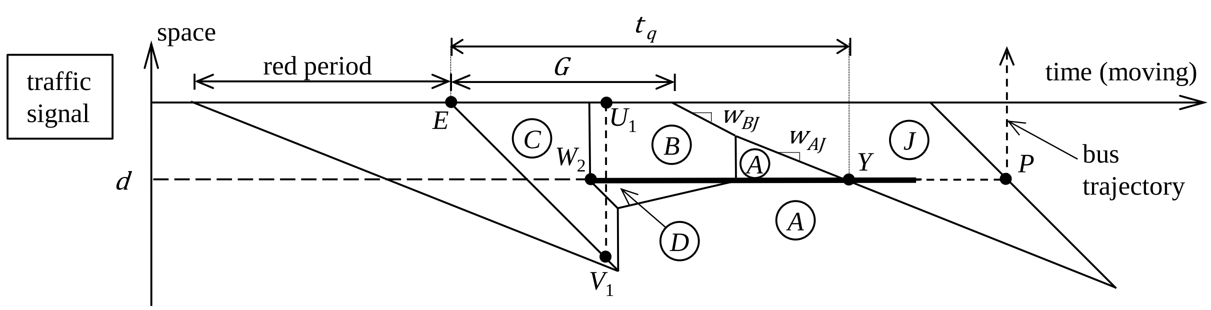
where t_q is the time interval from the start of the previous green phase (labeled point *E* in Figs. 7a~d) to the time when the tail of the car queue expands to the bus stop.

Two cases are to be considered when determining t_q. These are shown in Figs. 8a and b. In the first case (Fig. 8a), the bus arrives at the stop at the point labeled W_1, which is relatively soon after the start of the green. The resulting queue of discharging cars that forms upstream of the dwelling bus (a queue of state *D*) persists when the car queue from the following red period (of state J) eventually reaches the bus-stop. This meeting of the two queued car states is labeled *Y* in the figure. The bus would be a candidate for holding only if the time required to serve its passengers extends to *Y* or beyond, but not beyond *P*. A bus trajectory for this case is exemplified by short, dashed lines in Fig. 8a. Note how the departure time from the stop in this case (point P) could be unaltered by temporarily holding the bus' arrival.

The second case (Fig. 8b) is like the first, except that the bus arrival at the stop (point W_2) occurs later in time. As a result, the car queue of state D dissipates prior to the next arrival of the jammed queue at the bus stop, again labeled Y. As in the previous case, the bus is a candidate for holding if its time spent serving passengers extends at least to Y, but not beyond P.



(a) The first case



(b) The second case

Fig. 8 Determining t_q

From the geometries of Figs. 8a and b, we find the values of t_q that render buses candidates for holding:

$$t_q = \begin{cases} \frac{dQ}{wQ_B} + gL_C, & \text{if } \frac{d}{w} \le t_a \le \frac{q-gQ_B}{Q-Q_B} L_C - \frac{dQ(Q_B-q)}{wQ_B(Q-Q_B)} \\ \frac{dQ}{wq} + \left(g\frac{Q_B+q}{q} - 1\right) L_C + \frac{Q-Q_B}{q} t_a, & \text{if } \frac{q-gQ_B}{Q-Q_B} L_C - \frac{dQ(Q_B-q)}{wQ_B(Q-Q_B)} \le t_a \le \frac{q-gQ_B}{Q-Q_B} L_C \end{cases}. \quad (17)$$

The top and bottom equations of (17) correspond to the cases in Figs. 8a and b, respectively.

Constraints (13) ~ (17) collectively bound a region, like the shaded one in Fig. 9. The boundary line *RT* is from (14); *OR* is from (15); *MN* and *TM* are determined from (16), where the values of t_q come from (17); and (13) is a slack constraint in this example. The slope of *TM* is non-negative if $q + Q_B - Q \le 0$, and negative otherwise; see (16) and the bottom equation of (17). An example of this latter case is shown with dotted line *MT'* in the figure.

Buses with arrivals that fall within this shaded region are suitable for holding. An example arrival of this kind is shown in Fig. 9 with an "X". The arrow originating from the "X" depicts the maximum duration over which this bus arrival can be held.

The shaded region unveils useful insights. For example, moving the bus arrival to the right of line U_2V_2 (such that the "X" lies between that line and boundary OR) would, by virtue of Proposition 2, produce second-best conditions. Recall that moving the arrival all the way to the boundary OR would mean lower delays for cars (though greater delay for bus passengers who alight at the stop). Thus, we see from the figure that the range of arrivals suitable for holding expands as d diminishes. This means that the holding strategy can be especially beneficial to cars when bus stops lie close to their downstream intersections. As we saw in Section 2, obtaining second-best conditions can, in the absence of bus-holding, be impossible for a stop placed close to the intersection.

Holding buses can also benefit cars when the stop is located well upstream of the intersection. For example, if a bus stop location were to fall within the spatial range that corresponds to boundary RT in Fig. 9, then first-best conditions can be obtained by pushing bus arrivals to that boundary. This is true by virtue of Proposition 1.

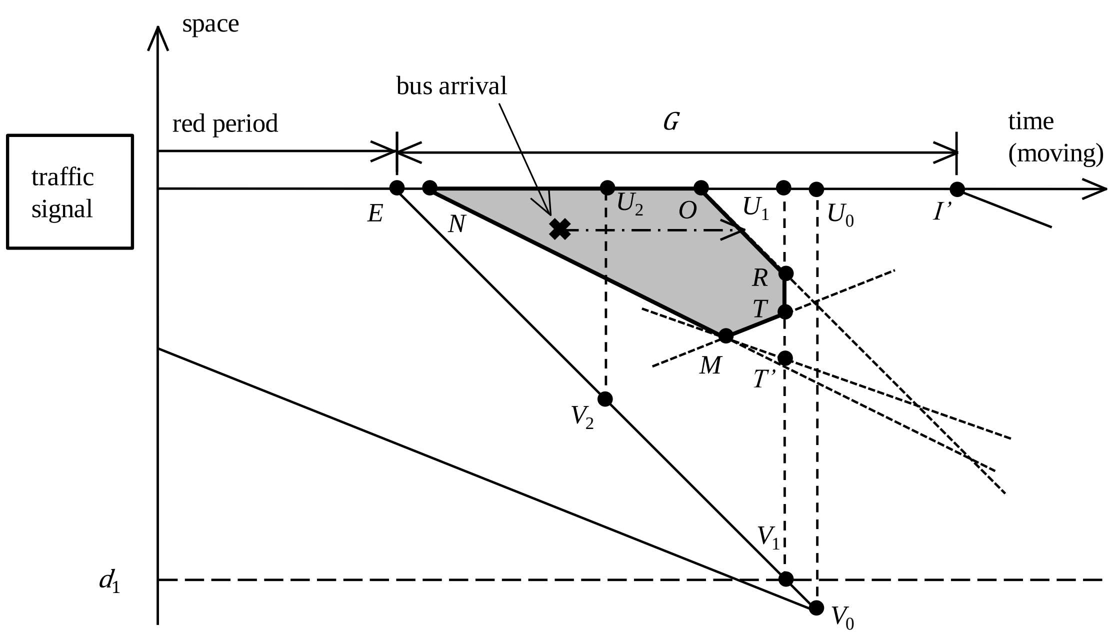


Fig. 9 The ranges of bus arrivals suitable for holding

Finally, if the shaded region in Fig. 9 can be enlarged, and if bus arrivals that fall within the region can be pushed to the line U_0V_0, we find that a dwelling bus would not induce any delays to cars at all. Interestingly, assessments of (13) ~ (17) show that the size of the shaded region increases when $S_{max} - S_{min}$ decreases. Thus, cars can benefit more from the bus-holding strategy if the variation in the time required to serve bus passengers is somehow kept small. This and other properties of the holding strategy are illustrated next by means of examples.

4 Effects of Bus Holding on Car Delays

We evaluate impacts of bus holding via the simulation of select scenarios. The well-known Cell Transmission Model (Daganzo, 1994) is used for this purpose. Each simulation depicts traffic operation over a number of consecutive signal cycles that is sufficient to capture the full effects of a dwelling bus.

Each scenario reflects distinct q, d, or distribution of S, but all scenarios feature a bus that seeks to arrive and dwell at a near-side stop at a time that is uniformly distributed within the first cycle. For each scenario, 10,000 simulations were performed. From these 10,000 realizations, we obtained the average *additional* car delay created by the dwelling bus, both with and without bus holding. We take as fixed inputs for all scenarios: $Q = 1$ vehicle/sec, $Q_B = 0.5$ vehicle/sec, $L_C = 90$ sec, $g = 0.5$, and $w = 7$ m/sec.

For our first battery of simulations, S was uniformly distributed with $S_{min} = 35$ sec and $S_{max} = 55$ sec. Fig. 10 presents the expected additional car delays for the range of d shown and for $q/Q = 0.4$ and 0.47. The latter ratio is just below 0.5, which is the upper bound due to our choices of Q, Q_B and g; see (1) and (2). The dashed curves in the figure display the additional car delays when the bus holding strategy is used. Their solid counterparts display these added delays in the absence of bus holding.

Note from Fig. 10 that the vertical deviation between a dashed curve and its solid counterpart unveils the car delay that is saved by the holding strategy. Further note how these deviations: are maximum when $d = 0$; diminish rapidly as d increases; and disappear for sufficiently large d. All this is consistent with the findings reported in Section 3.

Fig.10 also shows how bus holding can provide greater benefit to cars when car demand, q, is relatively high. For example, when $q/Q = 0.47$ and $d = 0$, the car delay saved via bus holding is 1022 car-sec. This is a large amount considering that in the absence of a dwelling bus, the car delay is only 898 car-sec per cycle.

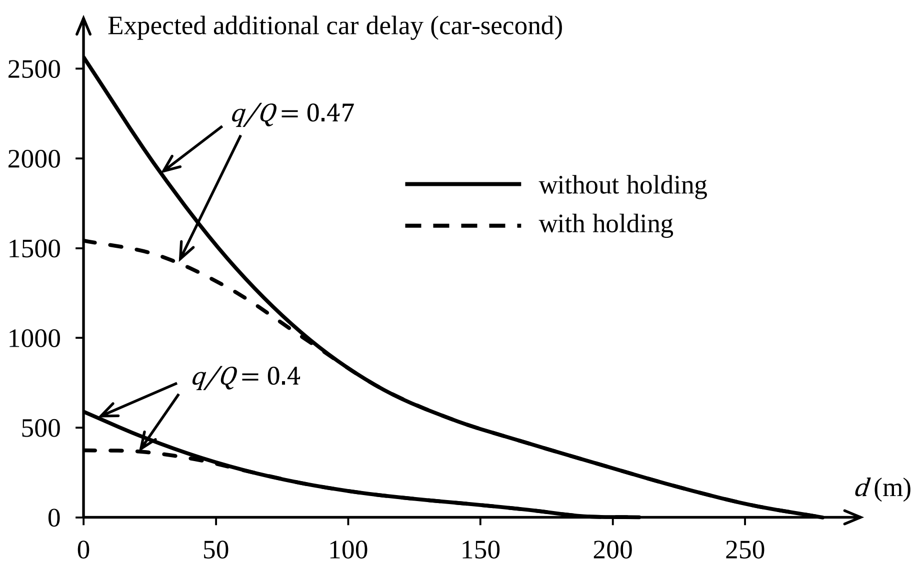


Fig. 10 Expected additional car delay versus d

Moreover, the simulation also shows that the expected bus holding times for the two cases of Fig. 10 are 7.4 sec and 5.6 sec respectively. Note how marginal they are, compared to the corresponding savings in car delays (1022 car-sec and 215 car-sec respectively; see Fig. 10). Thus, the holding strategy can significantly reduce the total delay of both car and bus passengers even if the number of bus passengers alighting at the coming stop is large.

We next explore how the impacts of the bus holding strategy depend upon the variations in bus dwell time. To this end, we fix $d = 0$ and the average bus dwell time as 45 sec, but allow the variation in bus dwell time, $S_{max} - S_{min}$, to range from 0 to 60 sec. Outcomes are shown in Fig. 11.

Note from this figure that the car delay saved by bus holding is greatest when the bus dwell time is deterministic; i.e., when $S_{max} - S_{min} = 0$. Note too how these savings: diminish as $S_{max} - S_{min}$ increase; and eventually vanish when the difference is sufficiently large. Hence we see the value of limiting the variation in bus dwell time.

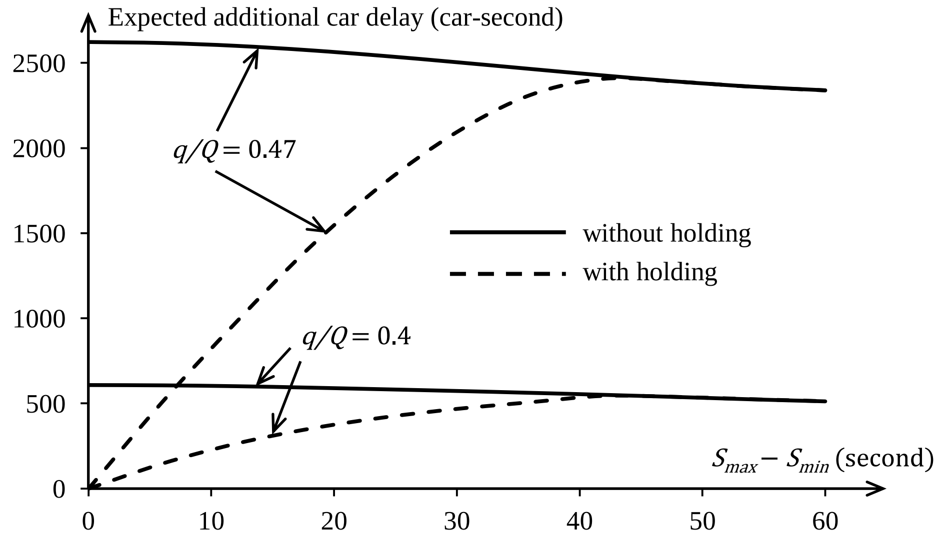


Fig. 11 Expected additional car delay versus $S_{max} - S_{min}$

Additional considerations for selecting bus-stop locations and for implementing the bus holding strategy are discussed next.

5 Conclusions

When a car queue spills-over beyond a near-side bus stop, a bus dwelling there in its lane will constrain the queued cars upstream during the green time. Kinematic wave theory was used to explore ways of mitigating this damage. Our models can be used to determine a stop's location to meet a specified objective. For example, one might choose to eliminate car delays completely by placing the stop upstream of the fully-expanded car queue. Or, one can limit these delays by locating the stop so that a residual car queue created by a dwelling bus will not arise (first-best conditions), or will arise, but dissipate within two cycles (second-best conditions).

Of course, the choice of a stop's location entails a trade-off between limiting the damage to car traffic and maintaining accessibility to bus users, e.g., to those who transfer between bus lines. This latter consideration favors placing a stop a short distance from an intersection, making the distances required even to achieve second-best conditions undesirable.

One might therefore choose to diminish the required distances between a stop and its intersection by either increasing the traffic signal's green ratio or decreasing its cycle length (see again (6) and (9)). Or, one can jointly choose a stop location and employ a strategy for deferring some bus arrivals at that stop. A bus that halts (or slows-down) in its lane in advance of the stop would be harmless under the low to moderate car demands for which our strategies are appropriate; and the holding strategy would be imposed selectively, so that buses would not fall behind their long-run schedules. Simulations indicate that the favorable impacts of bus holding can be considerable, particularly when car demand is relatively high or when the variation in bus dwell time can be kept small.

As a practical matter, bus passengers might by-and-large object to temporarily halting upstream of a stop. This could be the case even though the only passengers made worse-off would be those who alight at that coming stop; and even though the holding strategy can make it possible to locate stops closer to intersections to enhance bus-user access. These objections might be lessened by having select buses slow, rather than halt, in advance of the stop.

In some cases (e.g., when buses carry relatively few onboard occupants), a transport authority might elect to impose small delays on buses in the interest of mitigating car congestion. Candidate buses for this policy would include those that might finish serving their passengers before the car queue expands beyond the stop. Left to their own devices, some of these buses would depart the stop and join the tail of the expanding car queue somewhere downstream. Each such bus would eventually discharge through the intersection during the following green time. Should these buses instead be held upstream, this could be done in such way that each still discharges from the intersection during the following green. But some of these buses would do so from a position that is further back in the car queue; i.e., these buses would suffer delay, but the delay would be modest.

We note that our strategies are suitable for only a select range of car demand. Moreover, the choice of stop location will be based on a single choice for this demand. Unfortunately, our assessments indicate that stop location is sensitive to the magnitude of the car demand used as input. Thus, the strategies, which were developed by deterministic means, might not be robust to increases in this demand over time or shorter-run random fluctuations in demand. This matter will be explored in future work. We also intend to explore cases when car arrivals to an intersection are batched, e.g., due to the effects of another signalized intersection upstream.

Acknowledgement

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Appendix A

Proof of Proposition 1 in Section 2.2

The dwell time of a bus can be divided into a few segments (see Fig. A1 for an example), some of which are inside the dotted ellipses (we term these segments "blocking segments"), while others are outside. Since we assume $S \leq L_C$, a dwell time can include at most two blocking segments as in Fig. A1. To prove Proposition 1, we first prove the following lemma in regard to blocking segments.

Lemma 1. No residual queue will occur at the end of any green signal if and only if each blocking segment of the bus dwell time either:

(a) starts to the left of U_1V_1 (see Fig. A2), and is no longer than τ_1; or
(b) starts to the right of U_1V_1.

These conditions are illustrated in Fig. A2, where each bold line represents a typical blocking segment that satisfies the above conditions.

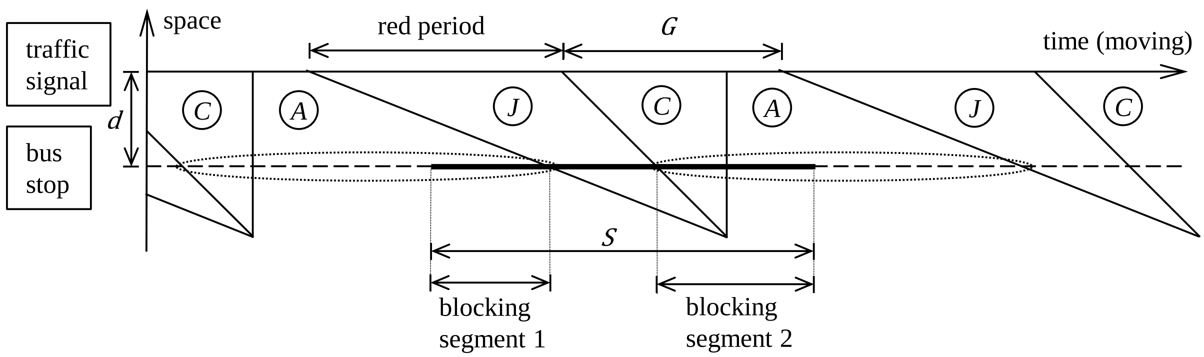

Fig. A1 Blocking segments

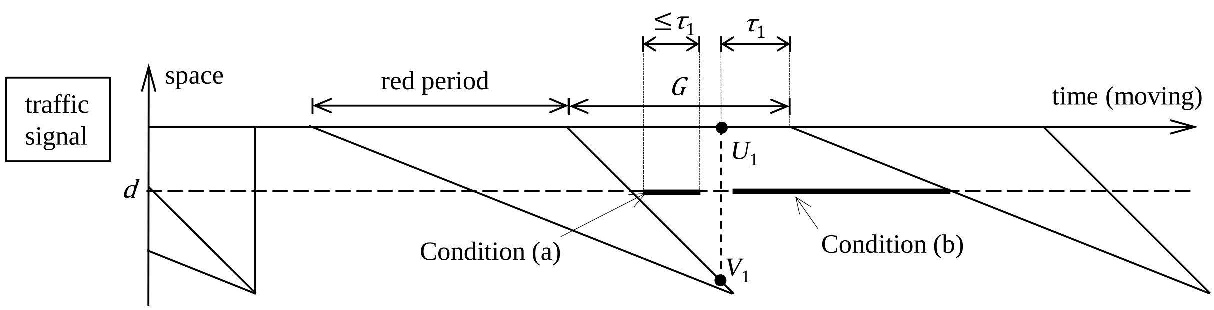

Fig. A2 Lemma 1

Proof of Lemma 1:

First consider the case when the blocking segment starts to the left of U_1V_1, as shown in Fig. A3.

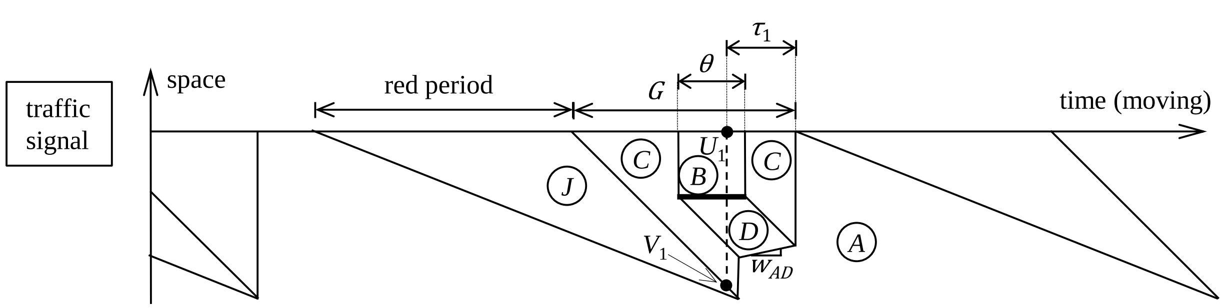

Fig. A3 Condition (a) of Lemma 1

From Fig. A3, it can be seen that a segment of the green period is blocked by the dwelling bus, during which time cars can discharge only at rate Q_B. We denote the length of that segment as θ. To guarantee that all queued cars can discharge by the end of the green period, θ must satisfy:

$(gL_C - \theta)Q + \theta Q_B \geq qL_C$, i.e.,

$$\theta \leq \frac{gQ-q}{Q-Q_B} L_C = \tau_1. \tag{A.1}$$

Equation (A.1) implies that τ_1 is the maximum portion of the green period that can be blocked by a dwelling bus if no residual queue is induced.

Since the blocking segment starts to the left of U_1V_1, θ will exceed τ_1 if the blocking segment is greater than τ_1 (note that the duration between U_1V_1 and the end of the green period is τ_1). Hence in this case, no residual queue is left at the end of green if and only if condition (a) of Lemma 1 holds.

In the other case, i.e., when the blocking segment starts to the right of U_1V_1 (Fig. A4), state B will persist for at most τ_1, thus (A.1) is satisfied with certainty, and no residual queue will persist at the end of the green period. Note that after the queue dissipates, the dwelling bus will no longer be an active bottleneck (recall Equation (2) in Section 2.1). Thus the length of the blocking segment is irrelevant. This proves condition (b). ■

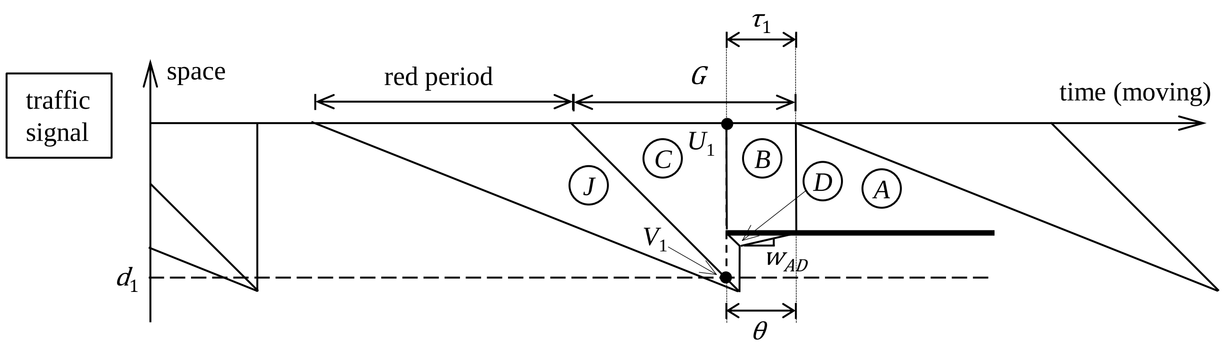

Fig. A4 Condition (b) of Lemma 1

Now we can prove Proposition 1.

To prove condition (a) of Proposition 1, by condition (b) of Lemma 1, it suffices to show that the length of U_1V_1 is d_1. From Fig. 4 in Section 2.2, we have:

$$\overline{U_1V_1} = (gL_C - \tau_1) \cdot w = \frac{q-gQ_B}{Q-Q_B} wL_C = d_1 .$$

When $d < d_1$, and if the bus arrives to the left of U_1V_1, then condition (b) of Proposition 1 immediately follows from condition (a) of Lemma 1.

If the bus arrives to the right of U_1V_1, its dwell time can contain at most two blocking segments; as explained below. The first blocking segment, $\overline{LM}$ in Fig. A5, satisfies condition (b) of Lemma 1, and thus will not induce any residual queue. In the mean time, the second blocking segment, $\overline{PR}$, has to satisfy condition (a) of Lemma 1. So,

$$S = \overline{KN} - \overline{KL} + \overline{NP} + \overline{PR} \leq L_C + \tau_1 - t_a + \frac{d}{w} .$$

This proves condition (c) of Proposition 1. ■

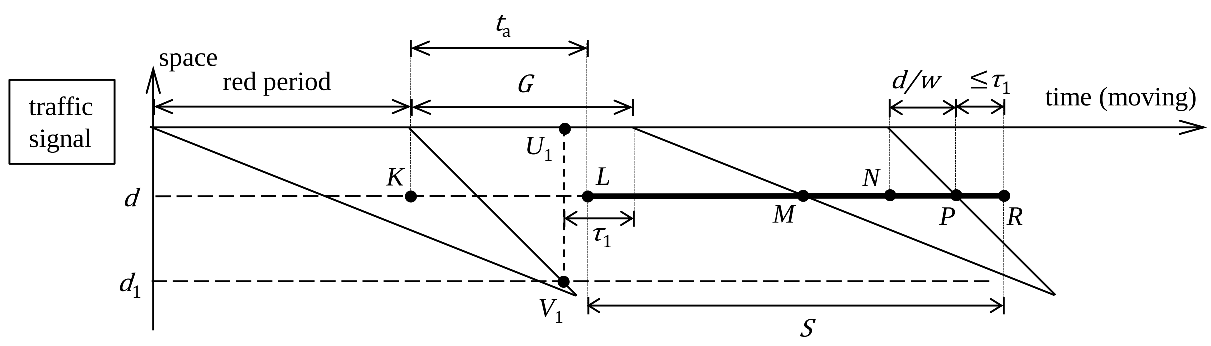

Fig. A5 Condition (c) of Proposition 1

Appendix B

Proof of Proposition 2 in Section 2.3

We first prove the following lemma:

Lemma 2. If the bus arrives between U_2V_2 and U_1V_1 (see Fig. 5 in Section 2.3), then the second-best condition is achieved if and only if
$S \leq (1-g)L_C + 2\tau_1 + \frac{d}{w}$.

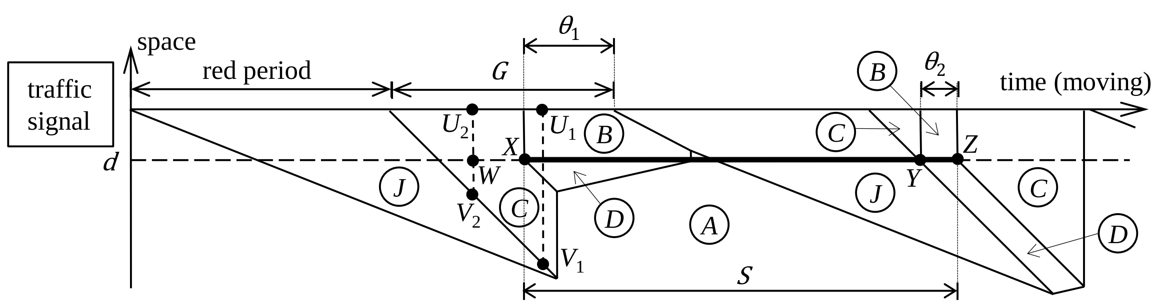

Fig. B1 Lemma 2

Proof of Lemma 2:

A time-space diagram of this case is shown in Fig. B1. From this figure, it can be seen that the portion of green periods during which cars can only discharge at Q_B, denoted as $\theta_1 + \theta_2$, should be no greater than $2\tau_1$. From this we have: $\overline{YZ} = \theta_2 \leq 2\tau_1 - \theta_1 = \overline{WX}$. Thus,

$S = \overline{XY} + \overline{YZ} \leq \overline{XY} + \overline{WX} = \overline{WY}$

$= L_C + \frac{d}{w} - (gL_C - 2\tau_1)$

$= (1-g)L_C + 2\tau_1 + \frac{d}{w}$. ■

Now we prove the conditions of Proposition 2 one by one.

For condition (a), we first verify that $d_2 \equiv \overline{U_2V_2}$:

$$\overline{U_2V_2} = (gL_C - 2\tau_1) \cdot w = \frac{2q - g(Q + Q_B)}{Q - Q_B} wL_C = d_2 \ .$$

We examine three cases:

Case I: the bus arrives between U_2V_2 and U_1V_1, and $d \geq d_2$. Since we assume $S \leq L_C$, we have

$$S \leq L_C = (1 - g)L_C + 2\tau_1 + \frac{d_2}{w} \leq (1 - g)L_C + 2\tau_1 + \frac{d}{w} \, .$$

So from Lemma 2, we know that the second-best condition can be achieved.

Case II: the bus arrives to the right of U_1V_1, and $d \geq d_1$. This case satisfies the first-best conditions (see condition (a) of Proposition 1) and is therefore trivial.

Case III: the bus arrives to the right of U_1V_1, and $d_2 \leq d \leq d_1$. The bus dwell time in this case can contain at most two blocking segments (see the definition of "blocking segments" in Appendix A). Due to condition (b) of Lemma 1 (also given in Appendix A), the first blocking segment will not induce any residual queue at the end of the green period. The second blocking segment itself can be seen as a (shortened) dwell time which starts between the counterparts of U_2V_2 and U_1V_1 in the next cycle (see Fig. B2). Hence, it follows from Case I above that the residual queue induced by the second blocking segment will never persist for more than two signal cycles. This completes the proof of condition (a).

Condition (b) of Proposition 2 becomes obvious by noting that $2\tau_1$ is the maximum portion of two consecutive green periods that can be blocked by a dwelling bus, should no residual queue be present at the end of the second green period.

Condition (c) immediately follows from Lemma 2.

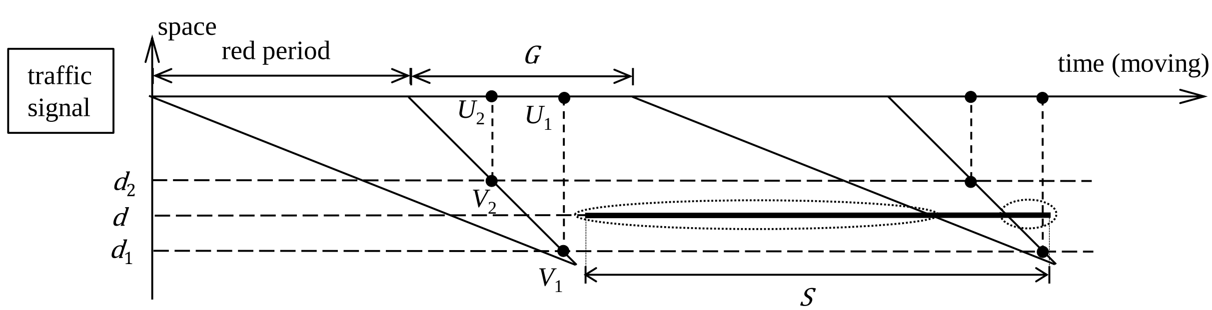

Fig. B2 Case III of condition (a) of Proposition 2

For condition (d), since the bus arrives to the right of U_1V_1, the first blocking segment will not cause any residual queue. Thus by condition (b) of Proposition 2, the second-best conditions can be achieved in this case if the second blocking segment is no greater than $2\tau_1$ (see Fig. B3). Hence,

$$S = \overline{KN} - \overline{KL} + \overline{NP} + \overline{PR} \leq L_C + 2\tau_1 - t_a + \frac{d}{w} \, . \qquad \blacksquare$$

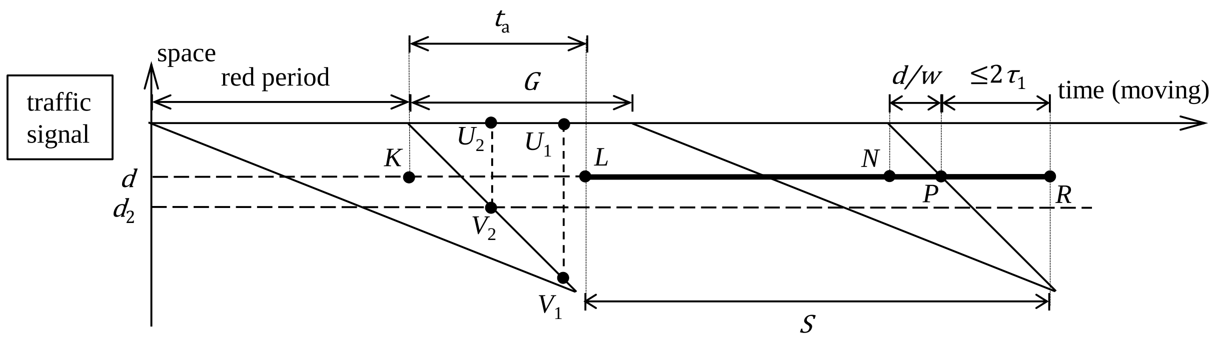

Fig. B3 Condition (d) of Proposition 2

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