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The Counting Numbers Are a Cultural Idea System

A Comment on Overmann 2015

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Cognitively recognizing and representing the numerosity of a collection of objects is not just a human capacity, but is within the range of primate cognitive abilities, as shown by a female chimpanzee taught to associate Arabic number symbols with quantities of up to seven objects. Cognitively, she appeared to use subitizing for small quantities (one to three objects) and estimation for larger quantities (four to seven objects; Murofushi 1997). Human numerosity, though, is more complex, because it involves not only subitizing and estimation but counting as well (Dahaene 1992), thus suggesting that human numerosity has both biological and cultural components, with counting belonging to the cultural component. The fact that subitizing is not an estimation procedure (Revkin et al. 2008) suggests, at first glance, that it is a precursor to counting, hence counting systems with only three or four named counting numbers may, as Overmann discusses, be representing numerosity distinctions within the subitizing range. If so, counting might be an extension of and elaboration upon subitizing. Alternatively, there could be a sharp break between subitizing, viewed as a form of pattern matching, and counting, expressed through underlying logic that makes counting an internally consistent and coherent conceptual system. Thus, the transition from subitizing to counting systems provides a test case for whether the role of culture in the transition from preexisting biologically grounded capacities to the cultural expression of those capacities is, on the one hand, essentially one of providing a community-shared overlay for an already existing biological capacity or, on the other hand, one that, though grounded in a biological capacity, involves introducing a conceptual system that is qualitatively distinct from the underlying biological capacity (cf. Carey 2009).

The answer to this question relates to fundamental issues regarding what constitutes culture that go back to Claude Lévi-Strauss's (1969 [1949]) widely referenced idea that culture begins when behavior is guided by rules rather than by biologically grounded propensities, with the incest taboos being a canonical example of this transition. Incest taboos are rules, he argued, that, while having a primate biological precursor in inbreeding avoidance (Pusey 2004), are not simply a cultural codification of an inbreeding avoidance propensity but represent a transformation of that propensity into something new and different; namely, rules about marriage are not only shared by community members in a normative sense but foundational for the structural organization of human societies. In his view, humans, and only humans, have societies structured through rules, rather than just through biological propensities and properties emergent from those propensities (contra Smaldino 2014).

The transition from numerosity based on subitizing and/or estimation to numerosity determined through counting thus provides an excellent arena for examining whether the system of counting is essentially just a cultural overlay over an already existing biological capacity of subitizing, or whether it involves a qualitative shift to a cultural idea system (see Leaf and Read 2012). It is certainly possible that, as spoken language developed, utterances relating to the quantity differences distinguished through subitizing became part of the vocabulary. How, though, would there be extension of the quantities recognized through subitizing to exact, named quantities beyond the subitizing range? The chimpanzee experiment suggests that the extension is from subitizing to estimation, not from subitizing to an extended subitizing range. In addition, the subject chimpanzee, unlike young children, required many trials to learn to match a quantity with an Arabic number symbol and did so without learning from previous trials; that is, she did not seem to be learning the concept of a counting number, but just learning a matching process between a visually recognized quantity and a symbol that served as a marker for that quantity. If subitizing is more than a pattern-matching process, then we would expect to find some evidence of the chimpanzee learning to treat each new quantity as an expansion of what has already been learned, as occurs with children when learning the counting numbers. However, this does not happen.

Furthermore, subitizing involves nonconscious recognition of exact quantities in the one to three (or four) range that occurs more rapidly than when counting takes place, suggesting that counting involves more than the pattern matching of subitizing. In addition, 3-year-old humans implicitly recognize that counting involves a one-to-one matching process, because they accurately recognize when someone who is counting makes an error in one-to-one matching (Gelman and Meck 1983), yet chimpanzees shift to an estimation procedure for quantities beyond the subitizing range.

In addition, the counting systems that are said to be just linguistic labels for quantities in the subitizing range are of the form "one, two, three, many," where "many" means something like "a quantity whose count would be greater than the largest named counting number," which is what the word infinity means in the poetic expression "the infinite ways that I love you" in *Alaska Love Poem* (Green 2010). Here "infinite" does not have the mathematical meaning of a cardinality greater than for any finite set, but of a finite

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quantity that is uncountable because there is no counting number name for a quantity that large.

Altogether, observations like the above indicate that something is missing in a subitizing account of counting numbers. What is missing is the logic that determines what constitutes a counting number. Let us now turn to the logic of the counting numbers, but without employing the counting numbers or making reference to the mathematical formalism of the Peano Axioms (Peano 1889 [1967]) through which the underlying logic of the natural numbers has been formally expressed.

We begin by defining a same size relation for comparing "this collection" of material objects with "that collection" of material objects in the following manner. Remove an object from this collection and an object from that collection. Continue removing objects in this manner until either this collection and/or that collection is empty. If both collections are emptied in the same round of this removal process, then we will say that this collection and that collection have the same size.

A key property of the same size relation is that it is an equivalence relation defined over an ensemble consisting of various collections of material objects; that is, the same size relation is reflexive (a collection of objects has the same size as itself), symmetric (if this collection has the same size as that collection, then that collection has the same size as this collection), and transitive (if this collection has the same size as that collection and that collection has the same size as another collection, then this collection and another collection have the same size).

Next we make use of a mathematical theorem stating that an equivalence relation determines a partition of an ensemble of collections of material objects into subensembles with the property that each collection of material objects in the ensemble is in precisely one subensemble and all collections in the same subensemble, and only these collections in the ensemble, have the same size. Furthermore, it says any collection in a subensemble of collections can represent the size common to all collections in that subensemble. Then, to construct a counting number, we need only distinguish a single collection of material objects and then define a counting number to be the size of that distinguished collection.

This is precisely the way the Iqwaye of Papua New Guinea defined (and represented) the counting number that determines whether there are enough males to raid a neighboring group (Mimica 1988). The Iqwaye kept a string of shells, and the size of the collection of males (in the sense of cardinality) making up the raiding party had to be of the same size as or greater than the collection of shells in the shell string. They used a matching procedure to see whether this was the case. We define a counting number, such as a baker's dozen, in a similar manner, although in this case the size of a reference collection is expressed using the symbolic system of natural numbers (see Peano 1889 [1967]).

The logic of the same size relation can be extended to form the concept of smaller and larger counting numbers than a given counting number, and this may be used to arrange ordinally whatever counting numbers have been defined. In addition, the same logic may be used to determine whether there is a counting number between two already defined counting numbers, which leads to the conceptual formation of an ordinally arranged counting number list without any "gaps" between a counting number and its ordinal successor in the list. Inclusion of "many" (or its equivalent) gives conceptual closure to a list of named counting numbers. Counting numbers defined in this way incorporate the principles of one-to-one correspondence, stable order, cardinality, abstraction, and order irrelevance used by Gelman and Gallistel (1978) in their definition of "counting."

In sum, the underlying logic for the counting numbers stems from the same size relation, and this refers equally to short named counting number sequences, such as "one, two, three, many," and to longer sequences of named counting numbers. Counting numbers are a cultural construct in their own right and not simply a cultural overlay for numerosity expressible at a biological level.

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