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‡ CAPTURE WITH SIX NUCLEON FORM FACTORS\*

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ABSTRACT

The matrix element for  $\mu$  capture ( $\mu^- + p \rightarrow n + \nu$ ) including not only the effects of vector, axial vector, weak magnetism, and induced pseudoscalar but also two "second-class" couplings has a small dependency on these hitherto undetected couplings. Capture in  $\mu$ -mesic hydrogen from the S states with both  $F = 1$  and  $F = 0$  is computed, and angular distributions of recoil neutrons and capture rates are given as functions of the six coupling constants. It is found that the second-class terms may contribute fully as much as weak magnetism and induced pseudoscalar terms.

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## INTRODUCTION

Study of  $\mu$ -meson capture in hydrogen ( $\mu^- + p \rightarrow n + \nu$ ) might allow detection of some as yet unobserved terms in the interaction Hamiltonian. The capture rates and angular distributions of recoil neutrons are affected not only by known vector and axial vector couplings, but also by the presumed induced-pseudoscalar<sup>1</sup> and weak-magnetism<sup>2</sup> couplings and two hypothesized "second-class" couplings.<sup>3</sup> The weak-magnetism and induced pseudoscalar effects are predicted by definite theories. The second-class interactions are allowed by the invariance principles known to govern the weak interactions. They would be expected if one accepts the principle of the renormalizability of first-order weak interactions. On the other hand, complete absence of second-class interactions would indicate a relationship between the weak interactions and isotopic spin such as the conserved vector current theory, which predicts that the vector interactions are all first-class. A simple theory that has vector = axial vector coupling and no others has the peculiar feature that no capture takes place in a  $\mu$ -mesic atom in the hyperfine triplet state.<sup>4</sup> For this reason, the  $F = 1$  capture rate is a good measure of the deviation from this simple theory due to inequality of vector and axial vector coupling constants and (or) the presence of any other couplings. Capture by individual protons provides a clearer test of the theory than capture by more complex nuclei, since, in analysis of capture on the complex nuclei, one is beset by

uncertainties in computing the  $\mu$ -meson wave function and nuclear wave function for the initial state and the nuclear wave function for the final state. Moreover, if one employs a nuclear model with a core of nucleons of zero total angular momentum with one proton in orbit around it, the spin of the core protons is correlated neither to the spin of the nucleus nor to the spin of the meson. This means that, although the probability of capture by the one outermost proton may be highly sensitive to the hyperfine configuration of the meson, the probability of capture by any of the many core protons is completely insensitive to the hyperfine configuration, and captures by the core largely obscure the hyperfine effect.<sup>5</sup> On the other hand, for hydrogen there is no uncertainty of nuclear structure. Mu wave functions have been computed for muons in liquid hydrogen,<sup>6</sup> and the capture rates are known as a certain combination of the singlet and triplet rates.<sup>7</sup> However, the liquid hydrogen capture is dominantly singlet. Elementary angular momentum conservation shows that for hydrogen the recoil neutrons from the hyperfine singlet state have spin antiparallel to their direction of flight.<sup>8</sup> The triplet rates and angular distributions might be picked out by counting only those neutrons polarized parallel to their line of flight. Unfortunately, counting rates are too low to allow such an experiment now, and no one has yet thought of a way to clearly distinguish the triplet rate.

### INTERACTION HAMILTONIAN

The most general form of the effective Hamiltonian in a theory with a two-component neutrino without derivatives in leptonic fields and with V and A coupling is, as Weinberg<sup>3</sup> has pointed out,<sup>9</sup>

$$H_{\text{int}} = \bar{\Psi}_\nu \gamma_\lambda (1 + \gamma_5) \Psi_\mu \bar{\Psi}_n (f_V \gamma^\lambda + f_A \gamma^\lambda \gamma_5 + i g_V \sigma^{\lambda\eta} q_\eta + i g_A \sigma^{\lambda\eta} q_\eta \gamma_5 + h_V q^\lambda + h_A q^\lambda \gamma_5) \Psi_p + \text{h.c.}$$

Here  $q$  is the momentum of the neutron minus the momentum of the proton,  $h.c.$  is the Hermitian conjugate, and the form factors  $f_V$ ,  $f_A$ ,  $g_V$ ,  $g_A$ ,  $h_V$ , and  $h_A$  are functions of  $q^2$  and are real in case the interaction is time-reversal-invariant.<sup>10</sup> To date there is no experimental evidence that the interaction has further symmetries, and one is not justified in further restricting the Hamiltonian. The coefficients  $f_V$  and  $f_A$  are the vector and axial vector coupling constants.<sup>11</sup> Modification of the vector, axial vector theory due to effects of the terms  $g_V$ , weak magnetism, and  $h_A$ , induced pseudoscalar, have been considered previously,<sup>12, 13</sup> but it is of interest to consider also the effects of the  $g_A$  and  $h_V$  terms that are distinguished as second class,<sup>3</sup> since their symmetry under  $G$ , the product of charge conjugation and charge mirroring, is opposite to the  $f_A$  and  $f_V$  terms respectively. Effects of the terms other than vector and axial vector should be larger in  $\mu$  capture than in  $\beta$  decay because of the larger momentum transfer.

### MATRIX ELEMENT

The capture of the meson bound in an  $S$  state was computed for each of the two hyperfine states in the approximation that the matrix element for capture from a plane-wave state is a constant over the range of momenta of the  $p$  and  $\mu$  in the bound system. Full relativistic formalism was used throughout. The matrix element was computed twice, once by the method of traces and once by choosing an explicit representation, and multiplying  $\gamma$  matrices and spinor wave functions. In our notation  $\underline{P}$  is the neutron three momentum,  $P$  equals  $|\underline{P}|$ ,  $m$  is the neutron mass,  $E$  is the total neutron energy,  $\omega$  equals  $q_4$ ,  $\theta$  is the angle between the direction of quantization of angular momentum and  $\underline{P}$ , and  $a$  is the Bohr radius of  $\mu$ -mesic hydrogen.

Conservation of four momentum implies

$$P = \frac{m_p^2 - m_n^2 + m_\mu (m_\mu + 2m_p)}{2(m_p + m_\mu)}$$

We denote as  $R_{Ff}(\theta)$  the number of recoil neutrons emitted per unit solid angle per unit time for a hyperfine state of angular momentum  $F$  with  $z$  component  $f$ . The triplet angular distribution has the form

$$R_{11}(\theta) = A_{11} + B_{11} \cos \theta + C_{11} \cos^2 \theta,$$

where the coefficients  $A_{11}$ ,  $B_{11}$ , and  $C_{11}$ , are, from the matrix-element computation:

$$\begin{aligned} A_{11} = & [(2\pi)^2 a^3]^{-1} (P^2/2E) [1 + P/(P^2 + m^2)^{1/2}]^{-1} \left\{ 2E |f_V - f_A|^2 \right. \\ & + [\omega^2 (E - m) + 3P^2 E + P^2 m + 2\omega P^2 + 2P^3] |g_V|^2 \\ & + [\omega^2 (E + m) + 3P^2 E - P^2 m + 2\omega P^2 + 2P^3] |g_A|^2 \\ & - [2P^2 E + 2P^2 \omega + 2E\omega P + 2P^3] \operatorname{Re} g_V^* g_A \\ & + (\omega - P)^2 (E + m) |h_V|^2 \\ & + (\omega - P)^2 (E - m) |h_A|^2 \\ & + 2[(E + m)P - (E - m)\omega] \operatorname{Re} (f_V - f_A)^* g_V \\ & + 2[(E - m)P - (E + m)\omega] \operatorname{Re} (f_V - f_A)^* g_A \\ & + 2(\omega - P)(E + m) \operatorname{Re} (f_V - f_A)^* h_V \\ & + 2(\omega - P)(E - m) \operatorname{Re} (f_V - f_A)^* h_A \\ & \left. + 2(\omega - P)[P^2 + P(E + m)] \operatorname{Re} g_V^* h_V \right\} \end{aligned}$$



$$\begin{aligned}
 & -2(\omega - P)[P^2 + \omega(E + m)] \operatorname{Re} g_A^* h_V \\
 & -2(\omega - P)[P^2 + \omega(E - m)] \operatorname{Re} g_V^* h_A \} \\
 & +2(\omega - P)[P^2 + P(E - m)] \operatorname{Re} g_A^* h_A
 \end{aligned}$$

$$B_{11} = [(2\omega)^2 \kappa a^3]^{-1} (P^2/2E) [1 + P/(P^2 + m^2)^{1/2}]^{-1} \left\{ 2(P + E) |f_V - f_A|^2 \right.$$

$$+ (E - m)(\omega - P)^2 |g_V|^2$$

$$+ (E + m)(\omega - P)^2 |g_A|^2$$

$$- P^2 (\omega - P) \operatorname{Re} g_V^* g_A$$

$$+ (\omega - P)^2 (E + m) |h_V|^2$$

$$+ (\omega - P)^2 (E - m) |h_A|^2$$

$$+ 2P(\omega - P)^2 \operatorname{Re} h_V^* h_A$$

$$- 2(\omega - P)(E - m + P) \operatorname{Re} (f_V - f_A)^* g_V$$

$$- 2(\omega - P)(E + m + P) \operatorname{Re} (f_V - f_A)^* g_A$$

$$+ 2(\omega - P)(E + m + P) \operatorname{Re} (f_V - f_A)^* h_V$$

$$+ 2(\omega - P)(E - m + P) \operatorname{Re} (f_V - f_A)^* h_A$$

$$+ 2P(\omega - P)^2 \operatorname{Re} g_V^* h_V$$

$$- 2(\omega - P)^2 (E + m) \operatorname{Re} g_A^* h_V$$

$$- 2(\omega - P)^2 (E - m) \operatorname{Re} g_V^* h_A$$

$$+ 2P(\omega - P)^2 \operatorname{Re} g_A^* h_A \}$$

$$\begin{aligned}
C_{11} = & [(2\omega)^2 a^3]^{-1} (P^3/2E) [1 + P/(P^2 + m^2)^{1/2}]^{-1} \left\{ 2|f_V - f_A|^2 \right. \\
& + [-2P^2 - 2\omega(E - m) - 2\omega P - 2P(E + m)] |g_V|^2 \\
& + [-2P^2 - 2\omega(E + m) - 2\omega P - 2P(E - m)] |g_A|^2 \\
& + [-\omega^2 - 3P^2 - 2\omega E - 2PE] \operatorname{Re} g_V^* g_A \\
& + 2(\omega - P)^2 \operatorname{Re} h_V^* h_A \\
& - 2(\omega + 2m - P) \operatorname{Re} (f_V - f_A)^* g_V \\
& - 2(\omega - 2m - P) \operatorname{Re} (f_V - f_A)^* g_A \\
& + 2(\omega - P) \operatorname{Re} (f_V - f_A)^* h_V \\
& + 2(\omega - P) \operatorname{Re} (f_V - f_A)^* h_A \\
& - 2(\omega - P)(\omega + E + m) \operatorname{Re} g_V^* h_V \\
& + 2(\omega - P)(P + E + m) \operatorname{Re} g_A^* h_V \\
& + 2(\omega - P)(P + E - m) \operatorname{Re} g_V^* h_A \\
& \left. - 2(\omega - P)(\omega + E - m) \operatorname{Re} g_A^* h_A \right\} .
\end{aligned}$$

Evaluation of the coefficients in proton mass units gives

$$\begin{aligned}
A_{11} = & [(2\omega)^2 a^3]^{-1} (1/4\omega) \left\{ 0.128 |f_V - f_A|^2 \right. \\
& + 3.00 \times 10^{-3} |g_V|^2 \\
& + 1.58 \times 10^{-3} |g_A|^2 \\
& - 1.47 \times 10^{-3} \operatorname{Re} g_V^* g_A \\
& + 1.6 \times 10^{-3} |h_V|^2 \\
& \left. + 4.47 \times 10^{-6} |h_A|^2 \right\} .
\end{aligned}$$

$$\begin{aligned}
& + 2.69 \times 10^{-2} R_l (f_V - f_A)^* g_V \\
& + 1.83 \times 10^{-3} R_l (f_V - f_A)^* g_A \\
& - 2.88 \times 10^{-2} R_l (f_V - f_A)^* h_V \\
& - 7.96 \times 10^{-5} R_l (f_V - f_A)^* h_A \\
& - 3.20 \times 10^{-3} R_l g_V^* h_V \\
& - 3.94 \times 10^{-5} R_l g_A^* h_V \\
& + 1.60 \times 10^{-4} R_l g_V^* h_A \\
& - 1.68 \times 10^{-4} R_l g_A^* h_A \} .
\end{aligned}$$

$$\begin{aligned}
E_{11} = [(2\pi)^2 n a^3]^{-1} (1/4\pi) \{ & 0.141 |f_V - f_A|^2 \\
& + 4.47 \times 10^{-6} |g_V|^2 \\
& + 1.61 \times 10^{-3} |g_A|^2 \\
& + 7.96 \times 10^{-5} R_l g_V^* g_A \\
& + 1.61 \times 10^{-3} |h_V|^2 \\
& + 4.47 \times 10^{-6} |h_A|^2 \\
& + 1.70 \times 10^{-4} R_l h_V^* h_A \\
& + 1.59 \times 10^{-3} R_l (f_V - f_A)^* g_V \\
& + 3.02 \times 10^{-2} R_l (f_V - f_A)^* g_A \\
& - 3.02 \times 10^{-2} R_l (f_V - f_A)^* h_V
\end{aligned}$$

$$\begin{aligned}
& - 1.59 \times 10^{-3} R_i (f_V - f_A)^* h_A \\
& + 1.70 \times 10^{-4} R_i g_V^* h_V \\
& - 3.22 \times 10^{-3} R_i g_A^* h_V \\
& - 8.92 \times 10^{-6} R_i g_V^* h_A \\
& + 1.70 \times 10^{-4} R_i g_A^* h_A \} ,
\end{aligned}$$

and

$$\begin{aligned}
C_{11} = [(2\pi)^2 \pi a^3]^{-1} (1/4\pi) \{ & 1.33 \times 10^{-2} |f_V - f_A|^2 \\
& - 2.98 \times 10^{-3} |g_V|^2 \\
& + 3.86 \times 10^{-5} |g_A|^2 \\
& + 1.56 \times 10^{-3} R_i g_V^* g_A \\
& + 1.70 \times 10^{-4} R_i h_V^* h_A \\
& - 2.51 \times 10^{-2} R_i (f_V - f_A)^* g_V \\
& + 2.82 \times 10^{-2} R_i (f_V - f_A)^* g_A \\
& - 1.51 \times 10^{-3} R_i (f_V - f_A)^* h_V \\
& - 1.51 \times 10^{-3} R_i (f_V - f_A)^* h_A \\
& + 3.02 \times 10^{-3} R_i g_V^* h_V \\
& - 3.20 \times 10^{-3} R_i g_A^* h_V \\
& - 1.69 \times 10^{-4} R_i g_V^* h_A \\
& - 2.08 \times 10^{-6} R_i g_A^* h_A \} ,
\end{aligned}$$

with  $[(2\pi)^2 \pi a^3]^{-1} = 3.36 \times 10^{-12} (m_p)^3$ . It can easily be shown from symmetry considerations that the angular distribution from the other triplet state has the form<sup>14</sup>

$$R_{10}(\theta) = A_{10} + C_{10} \cos^2 \theta,$$

where  $A_{10} = A_{11} + (5/3)C_{11}$ ,  $C_{10} = -2C_{11}$ , and  $R_{00}(\theta)$  is of course isotropic.

Integration shows the capture rate for the hyperfine triplet state to be

$$\begin{aligned} \tau_1^{-1} = & [(2\pi)^2 \pi a^3]^{-1} (2\pi P^2/E) [1 + P/(P^2 + m^2)^{1/2}]^{-1} \left\{ (2/3)(3E + P) |f_V - f_A|^2 \right. \\ & + (1/3)[3\omega^2(E - m) + 7P^2E + P^2m + 4\omega P^2 + 4P^3 - 2P\omega(E - m)] |g_V|^2 \\ & + (1/3)[3\omega^2(E + m) + 7P^2E - P^2m + 4\omega P^2 + 4P^3 - 2P\omega(E + m)] |g_A|^2 \\ & - (1/3)[4P^2E + 6P^2\omega + 4\omega PE + 3P^3 - P\omega^2] \operatorname{Re} g_V^* g_A \\ & + (\omega - P)^2 [(E + m) |h_V|^2 + (E - m) |h_A|^2 + (2/3) \operatorname{Re} h_A^* h_V] \\ & - (2/3)[3E(\omega - P) - mP - 3m\omega + P(\omega - P)] \operatorname{Re} (f_V - f_A)^* g_V \\ & - (2/3)[3E(\omega - P) + mP + 3m\omega + P(\omega - P)] \operatorname{Re} (f_V - f_A)^* g_A \\ & + (2/3)(\omega - P)[3(E + m) + P] \operatorname{Re} (f_V - f_A)^* h_V \\ & + (2/3)(\omega - P)[3(E - m) + P] \operatorname{Re} (f_V - f_A)^* h_A \\ & + (2/3)(\omega - P)[3P^2 + 2P(E + m) - P\omega] \operatorname{Re} g_V^* h_V \\ & - (2/3)(\omega - P)[2P^2 + 3\omega(E + m) - P(E + m)] \operatorname{Re} g_A^* h_V \\ & - (2/3)(\omega - P)[2P^2 + 3\omega(E - m) - P(E - m)] \operatorname{Re} g_V^* h_A \\ & \left. + (2/3)(\omega - P)[3P^2 + 2P(E - m) - P\omega] \operatorname{Re} g_A^* h_A \right\}. \end{aligned}$$

In proton mass units this is

$$\frac{1}{\tau_1} = [(2\pi)^2 \mu a^3]^{-1} \left\{ \begin{aligned} &0.132 |f_V - f_A|^2 \\ &+ 1.97 \times 10^{-3} |g_V|^2 \\ &+ 1.59 \times 10^{-3} |g_A|^2 \\ &- 0.949 \times 10^{-3} \operatorname{Re} g_V^* g_A \\ &+ 1.61 \times 10^{-3} |h_V|^2 \\ &+ 4.47 \times 10^{-6} |h_A|^2 \\ &+ 5.64 \times 10^{-5} \operatorname{Re} h_V^* h_A \\ &+ 1.84 \times 10^{-2} \operatorname{Re} (f_V - f_A)^* g_V \\ &+ 1.13 \times 10^{-2} \operatorname{Re} (f_V - f_A)^* g_A \\ &- 2.91 \times 10^{-2} \operatorname{Re} (f_V - f_A)^* h_V \\ &- 5.83 \times 10^{-4} \operatorname{Re} (f_V - f_A)^* h_A \\ &- 2.18 \times 10^{-3} \operatorname{Re} g_V^* h_V \\ &- 1.09 \times 10^{-3} \operatorname{Re} g_A^* h_V \\ &+ 1.01 \times 10^{-4} \operatorname{Re} g_V^* h_A \\ &- 1.69 \times 10^{-4} \operatorname{Re} g_A^* h_A \end{aligned} \right\} .$$

A similar calculation gives the singlet capture rate. Since the decay is isotropic there are no interference effects in the angular integration of final momenta and the capture rate can be written in the comparatively simple form

$$\begin{aligned}
\tau_0^{-1} = & [(2\pi)^2 a^3]^{-1} (2\pi P^2/E) \{1 + P/(P^2 + m^2)\}^{1/2} \}^{-1} \\
& \{ [10E + 6P - 8m]^{1/2} f_V \\
& + [10E + 6P + 8m]^{1/2} f_A \\
& - [5P^2 E + 3P^2 m - 6P\omega(E - m) + 9\omega^2(E - m) - 4P^3 + 12P^2\omega]^{1/2} g_V \\
& - [5P^2 E - 3P^2 m - 6P\omega(E + m) + 9\omega^2(E + m) - 4P^3 + 12P^2\omega]^{1/2} g_A \\
& + (\omega - P)[E + m]^{1/2} h_V \\
& - (\omega - P)[E - m]^{1/2} h_A \}^2 .
\end{aligned}$$

In proton mass units this is

$$\begin{aligned}
\tau_0^{-1} = & [(2\pi)^2 a^3]^{-1} \{ 0.413 f_V + 1.08 f_A - 0.0731 g_V - 0.0413 g_A \\
& - 0.0401 h_V + 0.0021 h_A \}^2 .
\end{aligned}$$

## DISCUSSION OF RESULTS

To evaluate the significance of these results one needs at least tentative values of the form factors. As a first step in computing estimates for the form factors,  $\mu$ -e universality is commonly assumed; that is, one assumes that the coupling constants for muon interaction with a bare nucleon are equal to the constants for electron-bare nucleon interaction. There is experimental evidence from  $\mu$  decay that this is true for the axial vector constant,<sup>15</sup> but there has been no measurement for the vector constant or any of the others. If one assumes this universality, the Feynmann-Gell-Mann conserved vector current theory together with measurements of nucleon electromagnetic form factors suggests that  $g_V = 3.71 f_V (1/2 m_p)$ ,  $f_V$  is 0.972 times the vector coupling constant in  $\beta$  decay, and  $f_A$  is 0.999 times the axial coupling constant

in  $\beta$  decay.<sup>16</sup> Dispersion theoretic techniques have been used to compute what is probably the major contribution to  $h_A$ , a one-pion exchange between nucleons and leptons.<sup>17, 18</sup> The result is that  $h_A$  is  $8 m_\mu^{-1}$  times the axial vector coupling constant in  $\beta$  decay. Using these values and the measured values for the  $\beta$  decay coupling constants,<sup>19</sup> one gets

$$\begin{aligned} f_V &= +7.02 \times 10^{-6} m_p^{-2} & f_A &= +9.02 \times 10^{-6} m_p^{-2} \\ g_V &= -1.30 \times 10^{-5} m_p^{-3} & h_A &= -6.81 \times 10^{-4} m_p^{-3}. \end{aligned}$$

The three significant figures written for  $h_A$  are set down by way of example for use in subsequent calculation; it should be remembered that no more than about 10% accuracy is claimed in the pion-exchange calculation of  $h_A$ . Not even theoretical estimates have been made for the two second-class terms  $g_A$  and  $h_V$ , but let us guess with Weinberg<sup>3</sup> that they are of the order of  $f_V/m_p$ . In units in which  $m_p = 1$ , the ratios of the terms are

$$|f_V| : |f_A| : |g_V| : |g_A| : |h_V| : |h_A|$$

$$1 : 1.3 : 1.9 : 1 : 1 : 97.$$

In Table I are given the rates using these estimates for triplet capture. The singlet rate with the form factor estimates are

$$\begin{aligned} \tau_0^{-1} &= [(2\pi)^2 a^3]^{-1} |0.413 f_V + 1.08 f_A - 0.0731 g_V - 0.0413 g_A - 0.0401 h_V + 0.00211 h_A|^2 \\ &= |6.37 + 21.4 + 2.09 - (?1) - (?1) - 3.17|^2 \\ &= |26.7|^2 = 713 \text{ sec}^{-1}, \end{aligned} \tag{1}$$

where the notation of Table I has been used.

These rough estimates indicate clearly the most important general feature of the calculation of the deviation of the rates from simple  $V=A$  theory.



It is that the effects of the second-class interactions might be fully as important as the effects of weak magnetism and induced pseudoscalar interactions in  $\mu$  capture. It seems probable that this is equally true for  $\mu$  capture in complex nuclei.

The values obtained here by using only first-class interactions ( $\tau_1^{-1} = 16.9 \text{ sec}^{-1}$ ,  $\tau_0^{-1} = 713 \text{ sec}^{-1}$ ) differ from those calculated by Primakoff,<sup>13</sup> who gets  $\tau_1^{-1} = 13 \text{ sec}^{-1}$  and  $\tau_0^{-1} = 636 \text{ sec}^{-1}$ . The difference is due in part to the higher-order relativistic effects included in this calculation and in part to a difference in the experimental value of the ratio of the axial to vector coupling constants of beta decay ( $f_A^{(\beta)} / f_V^{(\beta)}$ ) used in calculating the  $\mu$  capture coupling constants. There is in fact quite a wide range in the experimental values of this ratio.<sup>20</sup> The variation of the prediction of the relativistic calculation is displayed in Table II as a function of  $f_A^{(\beta)} / f_V^{(\beta)}$  over a range that includes both the value used by Primakoff ( $f_A^{(\beta)} / f_V^{(\beta)} = 1.21$ ) and the value used above ( $f_A^{(\beta)} / f_V^{(\beta)} = 1.25$ ).

In the triplet capture rate the most important term is the induced pseudoscalar. If there are no large second-class contributions, the pure pseudoscalar term together with its interference with  $f$  and with  $g_V$  make a 90% contribution to the capture rate and so a triplet capture rate might be a good test of the  $h_A$  estimate. It is interesting to note that if the sign of  $h_A$  used is incorrect, then we have  $\tau_1^{-1} = 15.9 \text{ sec}^{-1}$  (while  $\tau_0^{-1} = 1090 \text{ sec}^{-1}$ ). However, if  $h_V$  is  $+20 f_V / m_p$ , then we have  $\tau_0^{-1} = 204 \text{ sec}^{-1}$  and  $\tau_1^{-1} = 204 \text{ sec}^{-1}$ , and this second-class interaction will dominate the capture. The axial vector form factor is most important in the singlet capture rate, but the weak magnetism and induced pseudoscalar terms make almost cancelling contributions of about 20% each. Here again, second-class interaction effects might be large. For example, for  $g_A = +20 f_V / m_p$  and  $h_V = +20 f_V / m_p$ , we would have  $\tau_0^{-1} = 2 \text{ sec}^{-1}$

(while  $\tau_1^{-1} = 252 \text{ sec}^{-1}$ ) and for  $g_A = -20 f_V/m_p$  and  $h_V = -20 f_V/m_p$ , we would have  $\tau_0^{-1} = 2690 \text{ sec}^{-1}$  (while  $\tau_1^{-1} = 185 \text{ sec}^{-1}$ ). If it ever proves possible to measure angular distributions of recoil neutrons, a measurement of  $B_{11}$  would be another check on the  $h$  terms and would be especially interesting as the first direct evidence of parity nonconservation in  $\mu$  capture.

Is there any reason why the second-class interactions might be as large as  $20 f_V/m_p$ ? At present there is no theory which predicts such effects, but it is not hard to contrive mechanisms that might give rise to these large form factors. For instance, if there were a charged meson (or  $n$ -pion resonance) scalar under parity with zero strangeness, with a weak coupling to the leptons and with mass of several  $m_\mu$ , then the second-class term  $h_V$  might well be  $20 f_V/m_p$  in the same way that  $h_A$  is approximately  $-97 f_V/m_p$ .

The theoretical arguments that are used to fix the values  $f_A, f_V, g_V, h_A$  are quite plausible and are generally accepted in predicting  $\mu$  capture rates. However, an extreme skeptic might make the following points: The value of  $f_A$  is based on a demonstration of  $\mu$ - $e$  universality in the reactions  $\pi \rightarrow \mu + \nu, \pi \rightarrow e + \nu$ . One trusts that this universality carries over to relate the two reactions  $n \rightarrow p + e + \nu, p + \mu \rightarrow n + \nu$ . The arguments used to extrapolate the form factors from their values for the momentum transfer in  $\beta$  decay to their values for the momentum transfer in  $\mu$  capture are based on approximations whose validity is not unquestioned. Universality has not been demonstrated for vector interaction of leptons with strongly interacting particles, yet this optimistic conjecture is the sole basis for our value of  $f_V$ . The value of  $g_V$  is based on the conserved weak vector current theory, which has for its only experimental support the success of its prediction that the vector coupling constants in  $n \rightarrow p + e + \nu$  and in  $\mu \rightarrow e + \nu + \nu$  are precisely equal. As a matter of fact, this support is not firm because the best evidence

indicates a small difference between the coupling constants.<sup>21</sup> The relationship of the form factor  $h_A$  to  $f_A$  is derived either by use of questionable approximations or from a theory described by the authors as "highly tentative."<sup>17</sup>

On the basis of a measurement of  $\mu$  capture rates, it will be difficult to assign values to the form factors, since a given capture rate can be fitted by many choices of form factors, which may include accidental cancellation of important effects such as occur in the singlet capture rate in Eq. (1) between  $g_V$  and  $h_A$  terms. None the less, even a rough measure of the triplet capture rate would be a valuable check of the theory, and particularly sensitive to  $h_V$ ,  $h_A$  terms, and a measure of the singlet capture rate to an accuracy of a percent would shed light on the possible existence of the  $h_A$ ,  $h_V$ ,  $g_A$ , and  $g_V$  terms.

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## FOOTNOTES

- \* Portions of this work were done under the auspices of the U. S. Atomic Energy Commission.
- † National Science Foundation Predoctoral Fellow.
1. M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).
  2. R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).
  3. S. Weinberg, Phys. Rev. 112, 1375 (1958).
  4. J. Bernstein, T. D. Lee, C. N. Yang, and H. Primakoff, Phys. Rev. 111, 313 (1958).
  5. The Pauli exclusion of neutrons tends to inhibit core capture; however, see the calculation of H. Uberall, Phys. Rev. 121, 1219 (1961).
  6. S. Cohen, D. L. Judd, and R. J. Riddell, Jr., Phys. Rev. 119, 384, 397 (1960).
  7. S. Weinberg, Phys. Rev. Letters 4, 575 (1960).
  8. S. S. Gershtein, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 463 and 993 (1958). [translation: Soviet Physics-JETP 7, 318 and 685 (1958)].
  9. Notation: Indices run from 1 to 4; metric  $g_{\lambda\eta}$  has the signature (---+);  

$$\left\{ \gamma_{\lambda}, \gamma_{\eta} \right\} = g_{\lambda\eta}, \quad \gamma_5 = i \gamma^1 \gamma^2 \gamma^3 \gamma^4, \quad \sigma_{\lambda\eta} = \frac{i}{2} [\gamma_{\lambda}, \gamma_{\eta}] \quad \hbar = c = 1.$$
  10. Dr. A. C. Zemach, Physics Department, University of California, has suggested that for free nucleons the Dirac equation may be used to re-write the  $g$  terms in the Hamiltonian:

$$\sigma_{\mu\lambda} (n^{\lambda} - p^{\lambda}) \rightarrow i \gamma_{\mu} (m_n - m_p) + i (p_{\mu} + n_{\mu}),$$

$$\sigma_{\mu\lambda} (n^{\lambda} - p^{\lambda}) \rightarrow i \gamma_{\mu} \gamma_5 (m_n + m_p) + i \gamma_5 (p_{\mu} + n_{\mu}),$$

simplifying subsequent computation.

11. Our notation differs from the standard notations:

$f_V = C_V = C_V'$ ,  $f_A = -C_A = -C_A'$ , where  $C_V$ ,  $C_A$ ,  $C_V'$ , and  $C_A'$  are defined in T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956).

12. G. Flamand and K. W. Ford, Phys. Rev. 116, 1591 (1959). (See also Ref. 5).
13. H. Primakoff, Revs. Modern Phys. 31, 802 (1959). We feel that the reduced-mass correction proposed by J. C. Sens, Phys. Rev. 113, 679 (1959) is not valid. Primakoff has apparently included the reduced-mass effect correctly in calculating his Table I.
14. E. Lubkin, Annals of Physics 11, 414 (1959), Appendix 10.
15. T. Fazzini, G. Fidecaro, A. W. Merriam, H. Paul, and A. V. Tollestrup, Phys. Rev. Letters 1, 247 (1958); G. Impeduglia, R. Plano, A. Prodell, N. Samios, M. Schwartz, and J. Steinberger, Phys. Rev. Letters 1, 249 (1958).
16. For a discussion of the derivation of these numbers see A. Fujii and H. Primakoff, Nuovo cimento, Ser. 10, 12, 327 (1958).
17. M. Gell-Mann and M. Levy, Nuovo cimento 16, 705 (1960).
18. J. Bernstein, M. Gell-Mann, and L. Michel, Nuovo cimento 16, 560 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo cimento 17, 757 (1960).
19. R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, Phys. Rev. Letters 5, 323 (1960); M. A. Clark, J. M. Robson, and R. Nathans, Phys. Rev. Letters 1, 100 (1958); M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. 110, 1214 (1958).
20. See the closing paragraph of C. S. Wu, Revs. Modern Phys. 31, 783 (1959).
21. See the summarizing discussion in Ref. 17.

Table I. Angular-distribution coefficients and triplet capture rate

Source	$A_{11}^a$	$B_{11}^a$	$C_{11}^a$	$\tau_1^{-1a}$
$ f_V - f_A ^2$	+0.197	+0.217	+0.0204	+2.54
$ g_V ^2$	+0.195	+0.000290	-0.193	+1.60
$ g_A ^2$	(+0.01)	(+0.01)	(+0.001)	(+0.5)
$R \pm g_V^* g_A$	(? 0.1)	(? 0.001)	(? 0.1)	(? 0.5)
$ h_V ^2$	(+0.1)	(+0.1)	0	(+1)
$ h_A ^2$	+0.796	+0.796	0	+9.95
$R \pm h_V^* h_A$	0	(? 0.1)	(? 0.4)	(? 0.1)
$R \pm (f_V - f_A)^* g_V$	+0.269	+0.0159	-0.251	+2.30
$R \pm (f_V - f_A)^* g_A$	(? 0.01)	(? 0.1)	(? 0.1)	(? 1)
$R \pm (f_V - f_A)^* h_V$	(? 0.1)	(? 0.1)	(? 0.01)	(? 1)
$R \pm (f_V - f_A)^* h_A$	-0.0416	-0.836	-0.791	-3.89
$R \pm g_V^* h_V$	(? 0.1)	(? 0.01)	(? 0.1)	(? 1)
$R \pm g_A^* h_V$	(? 0.001)	(? 0.1)	(? 0.1)	(? 0.1)
$R \pm g_V^* h_A$	+0.545	-0.0303	-0.573	+4.32
$R \pm g_A^* h_A$	(? 0.1)	(? 0.1)	(? 0.01)	(? 1)
Totals	+1.96	+1.63	-1.79	+16.9

<sup>a</sup>Quantities at the head of each column are equal to the algebraic sum of the terms in that column. The form factors which are the source of each term are listed at the left of each row. Parentheses indicate an order-of-magnitude guess for second-class terms, and question marks indicate unknown signs. Totals of terms without parentheses are given. Units for  $A_{11}$ ,  $B_{11}$ , and  $C_{11}$  are  $\text{sec}^{-1} \text{sr}^{-1}$  and units for  $\tau_1^{-1}$  are  $\text{sec}^{-1}$ .

Table II. Capture rates as a function of beta-decay coupling constants

$f_A(\beta) / f_V(\beta)$	$\tau_0^{-1}$ (sec <sup>-1</sup> )	$\tau_1^{-1}$ (sec <sup>-1</sup> )
1.09	576	15.0
1.13	605	15.4
1.17	640	15.9
1.21	676	16.4
1.25	713	16.9
1.29	751	17.4
1.33	790	17.9