Title

Why Streaks Are Special: The Time of Patterns

Journal

Proceedings of the Annual Meeting of the Cognitive Science Society, 32(32)

ISSN

1069-7977

Authors

Sun, Yanlong
Wang, Hongbin

Publication Date

2010

Peer reviewed

Why Streaks Are Special: The Time of Patterns

Yanlong Sun (Yanlong.Sun@uth.tmc.edu)
Hongbin Wang (Hongbin.Wang@uth.tmc.edu)

University of Texas Health Science Center at Houston
7000 Fannin St. Suite 600
Houston, TX 77030 USA

Abstract

People seek for patterns and pay particular attention to streaks even when they are generated by a random process. The present paper examines statistics of pattern time in sequences generated by Bernoulli trials. We demonstrate that streak patterns possess some statistical properties that make them uniquely distinguishable from other patterns. Because of the uncontaminated continuity, streak patterns have the largest amount of self-overlap, resulting in the longest waiting time and the largest variance of interarrival times. We then discuss the psychological implications of pattern time such as in memory encoding and perception of randomness.

Keywords: Perception of randomness; representativeness; hot hand belief; gambler's fallacy; waiting time; patterns.

Introduction

When faced with temporal sequences of events, people often attempt to make sense out of apparent patterns even when they are completely random. Among the most perceptible patterns, "streaks" or "runs," defined as continuous series of the same outcomes, are notorious for they not only yield counterintuitive statistical properties but also inspire extensive investigations on the biases in human perception of randomness and probabilistic judgment and reasoning.

One well-known example is the hot hand belief. Many basketball fans believe that some players have the "hot hand" and tend to make successful shots in streaks. However, in a seminal study, Gilovich, Vallone, and Tversky (1985) find no significant statistical evidence to distinguish the actual shooting sequences from the sequences of Bernoulli trials. This finding has been controversial but withstood several critical attacks (for a comprehensive summary on the hot hand study, see, Bar-Eli, Avugos, & Raab, 2006). In explaining the hot hand belief, Gilovich et al. (1985) use the representativeness heuristic, which has also been used to explain the gambler's fallacy (Tversky & Kahneman, 1974). By such heuristic, people expect the essential characteristics of a chance process to be represented not only by the entire global sequence but also by local subsequences. For instance, when tossing a fair coin, a streak of four heads—which is quite likely in even relatively small samples—would appear to be non-representative.[1] Thus, in the gambler's fallacy, a tail is "due" to balance a streak of heads. In the hot hand belief, a streak of successful shots may lead people to reject the randomness of sequences and signal the existence of a hot hand. Several researchers have questioned the representativeness heuristic for its incompleteness in accounting for two opposite psychological dispositions, but their arguments are still based on the evidence that the hot hand belief is false (e.g., Ayton & Fischer, 2004; Burns, 2004). Together, the hot hand belief and the gambler's fallacy have been considered as two outright fallacies in people's perception of streak patterns, and this stance has a great impact on studies in other disciplines such as behavioral finance and economics (e.g., Camerer, Loewenstein, & Prelec, 2005; Gilovich, Griffin, & Kahneman, 2002; Rabin, 2002).

Moreover, studies on people's judgment and generation of random sequences show that people expect fewer and shorter streaks when observing sequences produced by an independent and identically distributed process (i.i.d.) and they tend to avoid long streaks when instructed to generate such sequences (e.g., Budescu, 1987; Falk & Konold, 1997; Nickerson, 2002; Olivola & Oppenheimer, 2008). Besides behavioral evidence, the salience of streak patterns is also indicated by the results from a functional magnetic resonance imaging (fMRI) study (Huettel, Mack, & McCarthy, 2002). In a "pattern violation task," participants were informed of the random order of the sequences. However, greater activation was found in prefrontal cortex (PFC) when participants observed violations of streak patterns (e.g., [AAAA] vs. [AAAB]) than violations of an alternating pattern (e.g., [ABABAB] vs. [ABABAA]) in a random binary sequence. In addition, the amplitude of fMRI hemodynamic responses (HDR) started increasing at lengths 2 for streak patterns (i.e., [AAB]) but only started increasing at lengths 6 and larger for alternating sequences (i.e., [ABABAA]). (Oskarsson, Van Boven, McClelland, & Hastie, 2009, provided a comprehensive review on judgments of random and nonrandom sequences of binary events.)

Given the unique role of streaks in people's perception and judgment of temporal sequences, an inevitable question is what is so special about streaks? To answer this question, we have to examine the statistics of patterns more carefully

[1] The probability of observing four heads in a row at least once in 20 tosses is 0.48.

since they are widely known for producing counterintuitive results (for the same reason, many people are surprised by the results of the runs test in the hot hand study).

It would seem too obvious to mention once again the unique composition of a streak: a streak is composed of an *uncontaminated* run of the same elements, which makes it exceptionally stand out from other non-streak patterns (such as alternation and symmetry) or any composition without an apparent order. While this property does not affect *how often* a streak occurs, it does affect *when* a streak *first* occurs. To exemplify, we compare two patterns HHH and THH (where H = heads and T = tails in tossing a fair coin). Governed by the independence and stationarity assumptions of Bernoulli trials, these two patterns have the same *probability of occurrence* in any three consecutive tosses (hence the fallacy in the gambler's fallacy and the hot hand belief). However when the coin is tossed repeatedly, the *probability of first occurrence*—the probability that a pattern first occurs at the n^{th} toss, given that the pattern has not occurred before—can be different for different patterns (see Figure 1). For example, both patterns THH and HHH are equally likely to occur or not occur in the first three tosses. If THH has not occurred before, it will have a probability of 0.125 to first occur at the 4^{th} toss. In contrast, if HHH has not occurred before, its probability of first occurrence at the 4^{th} toss is only 0.0625, half of that for THH (for a method of calculating the probability of first occurrence, see Sun, Tweney, & Wang, 2010a). Overall, it will on average take 14 tosses to observe the first occurrence of HHH but only take 8 tosses to observe the first occurrence of THH. Moreover, if we monitor these two patterns simultaneously, it is more likely that we first encounter THH than we first encounter HHH (the odds are 7:1). In other words, it appears that the first occurrence of the streak pattern HHH has been "delayed."

The time it takes for the first occurrence of a pattern (measured by the number of trials) is a statistical property known as *waiting time*. Compared to the long history of studies on the gambler's fallacy (see, Ayton & Fischer, 2004), the development of waiting time and its related properties is fairly new (see, Gardner, 1988; Graham, Knuth, & Patashnik, 1994). Most recently, this development has received attention in psychological literature. Hahn and Warren (2009) argue that given people's limited exposure to the environment (i.e., the number of coin tosses is finite), the longer waiting time of streak patterns would have made them less likely to be observed, thus, "there is something right about the gambler's intuition that the longer the run, the more likely, by contrast, is a sequence with a final tails" (p. 458). Sun et al. (2010a) criticize Hahn and Warren's interpretation, and argue that it is the particular composition of patterns, rather than the length of the global sequence, that plays an essential role in both the statistics of waiting time and people's perception of randomness (also see Sun, Tweney, & Wang, 2010b).

Notwithstanding the debate above, the unique composition of a streak and its "delayed" first occurrence may provide a new prospective in the investigations on human perception of randomness. Particularly, different compositions of patterns may be directly related to memory encoding due to the limited working memory capacity (e.g., Falk & Konold, 1997; Olivola & Oppenheimer, 2008). For example, a streak of HHH can be easily memorized as "3Hs." In addition, different waiting times in effect indicate different variances in the distribution of pattern occurring times (Sun & Wang, 2010), and this fact may have direct consequence in people's intertemporal choices as it has been suggested that human brains are sensitive to time discounting (e.g., Ainslie & Monterosso, 2004; McClure, Ericson, Laibson, Loewenstein, & Cohen, 2007; McClure, Laibson, Loewenstein, & Cohen, 2004). In the following, we demonstrate some interesting properties in the statistics on the time of patterns and discuss the psychological implications.

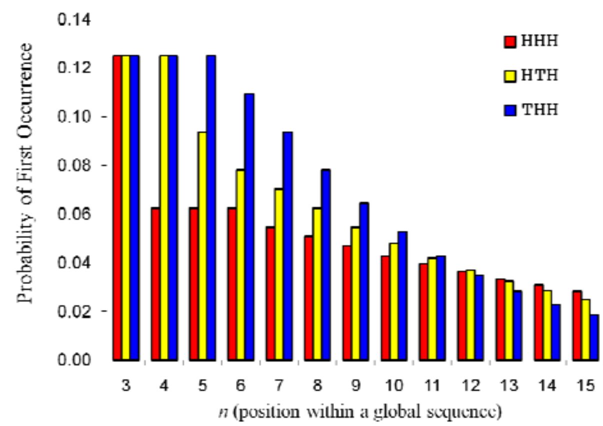

Figure 1: Probabilities of first occurrence for patterns HHH, HTH and THH when a fair coin is tossed repeatedly.

Mean Time and Waiting Time

The time of patterns has been studied by several different methods and different terminologies exist (e.g., Graham, et al., 1994; Li, 1980; Ross, 2007). To be consistent, here we clarify some basic concepts. In a process of coin tossing, the *interarrival time* (T) is the number of trials (tosses) between any two successive occurrences (arrivals) of the pattern, and the *first arrival time* (T^*) is the number of trials required to encounter the first occurrence of the pattern since the beginning of the process[2]. Then, *mean time* ($E[T]$) is the expected value of the interarrival time, and *waiting time* ($E[T^*]$) is the expected value of the first arrival time. We also distinguish the variance of interarrival time and the variance of the first arrival time by $Var(T)$ and $Var(T^*)$,

[2] T and T^* may have different distributions, so that the process of counting patterns is also called a general renewal process or a delayed renewal process (Ross, 2007).

respectively. To simplify the discussion, here we only discuss the case of a fair coin (i.e., $p_H = p_T = ½$) and focus on pattern length $r = 3$. Unless specified, the discussion in the following will extend to patterns for all $r \geq 3$. (A more general treatment can be found in Sun & Wang, 2010.)

Overlap and Waiting Time

We first note that when generated by an independent Bernoulli process, a pattern will have a mean time that is the inverse of its probability of occurrence. Thus, any pattern of the same length will have the same mean time. For example,

$$E[T_{HHH}] = E[T_{HTH}] = E[T_{THH}] = (1/2)^{-3} = 8.$$

However, waiting time can be different for different patterns. Compared to other patterns of the same length, streak patterns always have the longest waiting time. For example,

$$E[T^*_{HHH}] = 14,\ E[T^*_{HTH}] = 10,\ \text{and},\ E[T^*_{THH}] = 8.$$

Table 1 lists the mean and variance of interarrival time T and the first arrival time T^* for all possible patterns of length 3. Extra caution should be taken to properly explain these results. An example is given in Figure 2, which depicts pattern time in two different contexts where individual patterns are monitored either independently (panel A) or simultaneously (panels B and C). Note that the colored circles in Figure 2 highlight the position where individual patterns have occurred and they actually represent the values of an "indicator variable" for pattern occurrence. In addition, arrows represent the minimum interarrival time between successive occurrences of patterns—the "minimum succeeding distance" for a pattern to occur given a previous occurrence of either the same pattern or another pattern.

Table 1: Mean and variance of interarrival time T and the first arrival time T^* for patterns of length $r = 3$. Note that for non-overlapping patterns such as HHT, the two pairs of statistics are identical (shown in bold).

Patterns	$E[T]$	Var[T]	$E[T^*]$	Var[T^*]
HHT	**8**	**24**	**8**	**24**
HTT	**8**	**24**	**8**	**24**
THH	**8**	**24**	**8**	**24**
TTH	**8**	**24**	**8**	**24**
HTH	8	56	10	58
THT	8	56	10	58
HHH	8	120	14	142
TTT	8	120	14	142

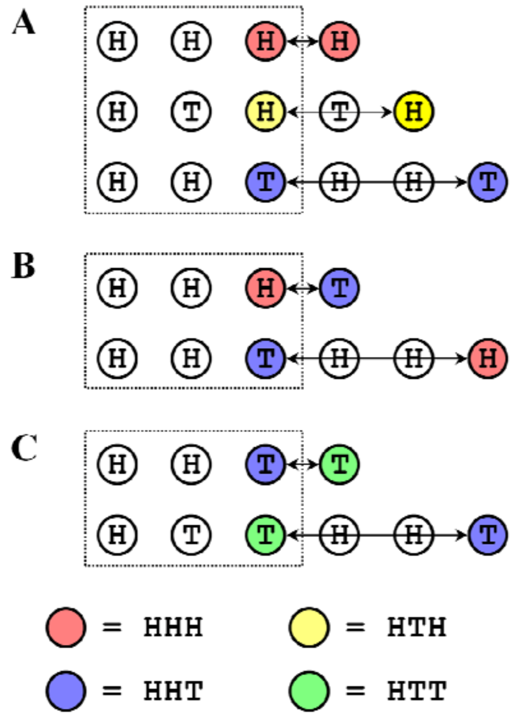

Figure 2: Visualization of pattern occurrences. Each circle represents the outcome of a single toss and the colored circle indicates one occurrence of the corresponding pattern. Arrows represent the "minimum succeeding distance" between successive occurrences of patterns, which also inversely indicate the levels of self-overlap (A) and inter-overlap (B and C).

Figure 2A illustrates the essence of waiting time as it is defined independently for each individual pattern. When the coin has been tossed exactly 3 times, the probability of occurrence for any pattern is the same, 1/8 (also see Figure 1)[3]. However, interesting phenomena will happen at the 4th toss (or, an observational window of size 3 starts moving from the beginning towards the end of the sequence one position a time). For example, if pattern HHH has occurred at $n = 3$, it can have an immediate reoccurrence at $n = 4$. In contrast, if pattern HTH has occurred at $n = 3$, its earliest next occurrence will have to be 2 tosses away at $n = 5$. More extremely, if we are monitoring pattern HHT and it has occurred at $n = 3$, then its earliest next occurrence will have to be 3 tosses away at $n = 6$. An intuitive explanation for this is that the reoccurrences of HHH can *self-overlap* with each other thus tend to be mostly clustered and the

[3] Alternatively, we can imagine that a coin is tossed repeatedly and a long sequence of heads and tails is generated. Then, an observational window of width $r = 3$ randomly lands on any position of the sequence and captures exactly 3 trials. Given the independence assumption of Bernoulli trials, the probability that the observational window will capture any pattern is the same 1/8, as if the process starts from scratch (i.e., the window lands at the beginning of the sequence).

reoccurrences of HHT cannot overlap thus tend to be mostly dispersed.

Figure 2A in effect illustrates all 3 possible levels of self-overlap for patterns of length 3, since reoccurrences of reciprocal patterns self-overlap in the same way (e.g., HHH and TTT, HTH and THT), and reoccurrences of HHT, HTT, THH, and TTH do not self-overlap. Comparing Figure 2A with Table 1, we can see that with other factors remaining constant, the self-overlapping property of a pattern completely determines the pattern's waiting time $E[T^*]$ and the variance of interarrival time $\mathrm{Var}(T)$. Since a streak pattern is an uncontaminated run of the same elements, by such unique composition, a streak pattern will have the largest amount of self-overlap and consequently, the longest waiting time.[4] As a comparison, non-streak patterns such as HTH or HHT only have partial self-overlap or no self-overlap at all so that they will have shorter waiting times. Moreover, waiting time grows approximately exponentially as the amount of self-overlap grows. As a consequence, the difference in waiting time between HHH and HTH is much greater than that between HTH and HHT.

The difference in waiting time can be viewed as one type of *precedence relationships* in which individual patterns are monitored independently and only the self-overlap within each pattern is considered. For example, suppose two players are betting on two patterns HHH and HHT, respectively, then each player tosses a coin of her own in isolation (i.e., the "solitaire game" in Sun, et al., 2010a). Because of the different waiting time, the player who bets on HHT will be more likely to get her desired pattern earlier than the player who bets on HHH.

Inter-overlap and Nontransitivity

The result above might give an impression that pattern HHT is always more likely to occur earlier than pattern HHH, thus the gambler's fallacy might actually have a valid statistical basis. However, such precedence relationship may not hold if two patterns are monitored simultaneously in the same sequence and both self-overlap and inter-overlap are involved (the "interplay game"). The fact is that although HHT is faster than HHH in the solitaire game, in the interplay game, HHT overlaps with the end of HHH (two positions) more than HHH overlaps with the end of HHT (none) (see Figure 2B). Overall, it can be calculated that in the interplay game, we are equally likely to first encounter HHH as to first observe HHT.

Figure 2C shows another comparison between HHT and HTT. Despite that these two patterns have the same waiting time of 8 tosses, because of the different amount of inter-overlap, the odds of HHT preceding HTT against HTT preceding HHT are 2:1. This indicates that the precedence relationship in the interplay game is *nontransitive*. That is, for pattern length $r \geq 3$, if one player first chooses any one of the patterns, the other player can always choose another pattern of the same length to ensure a better than even chance to win. In other words, the interplay game only favors the player who chooses later.

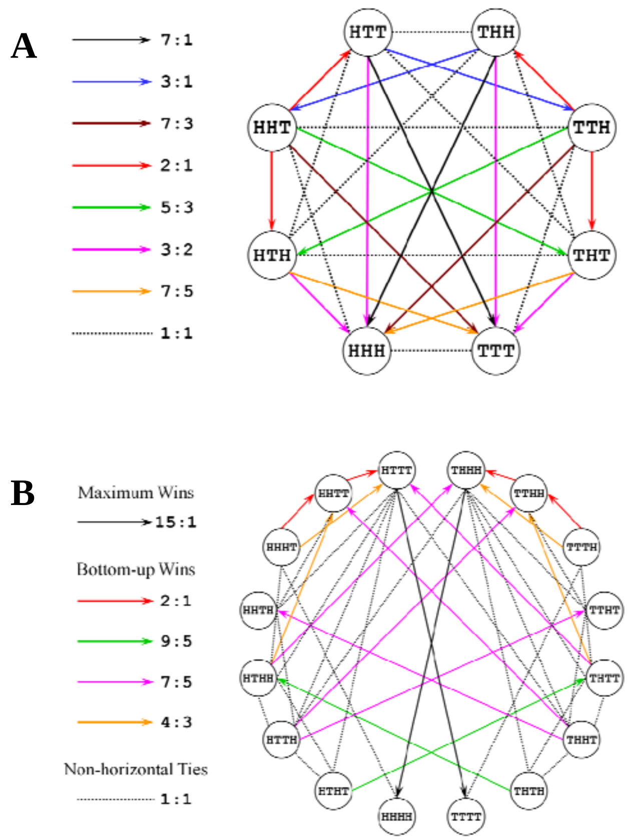

Figure 3: Pair-wise precedence in the interplay game and the corresponding odds. Arrows originate from the faster patterns and point to the slower patterns. A: pattern length $r = 3$. B: pattern length $r = 4$. Only the relationships in legends are connected between patterns and all other connections are either downward wins or horizontal ties. Note that in both A and B, no arrow originates from streak patterns.

Figures 3 shows the pair-wise precedence relationships in the interplay game for pattern length $r = 3$ and 4. Close examinations of Figure 3A confirm that the precedence relationship does not exactly follow the order of waiting time listed in Table 1. Particularly, a pattern with a shorter waiting time may not be necessarily encountered earlier than a pattern with a longer waiting time. Nevertheless, it appears that streaks are still the slowest patterns—at best, a streak pattern can tie with its "end-reversal" counterpart or its reciprocal streak (e.g., HHH vs. HHT, or, HHH vs. TTT), and it can never "beat" any other pattern. In other words, nontransitivity in the interplay game does not mean the

[4] It might seem counterintuitive that overlapped occurrences (hence faster reoccurrences) are associated with a longer waiting time. However, a reoccurrence of the pattern has to be based on a previous occurrence. Since a pattern of length $r \geq 2$ is more likely to have not occurred in the first r tosses than it has occurred, faster reoccurrences actually signify a delay in the waiting time.

equivalence (or indifference) between patterns in a circular fashion. Considering all possible pair-wise comparisons, streak patterns are still unique for their delayed first occurrences.[5] This fact holds for all pattern length $r \geq 2$. Figure 3B shows the pair-wise comparison in the interplay game when pattern length $r = 4$, in which streak patterns are still at the bottom of the game.

Discussion

We have examined several types of pattern time statistics in different contexts and demonstrated that streak patterns indeed possess some unique statistical properties. Here we discuss their psychological relevance and implications in the investigations on human perception of randomness.

First, the particular unbroken continuity of a streak leads to the maximum amount of self-overlap. As a consequence, successive occurrences of a streak tend to be clustered and such tendency would make it harder for human memory to keep an exact count of the actual number of occurrences. By contrast, successive occurrences of other patterns have to be either partially or completely separated (e.g., Figure 2A) and much more evenly distributed (indicated by the small variance of interarrival time in Table 1). For example, in Figure 2A, if the observational window is in size 4 instead of 3, two consecutive instances of 3Hs can be captured by one window. If the memory is encoded as the number of the windows containing the streak (at least once), two instances of 3Hs captured in the same window would have the same weight as one instance of 3Hs. Alternatively, two instances of 3Hs could be replaced by one instance of 4Hs. In either case, the remembered number of occurrences of 3Hs will be less than it actually is.

Moreover, compared with all other patterns, a streak is the slowest pattern to occur, determined by either self-overlap alone (solitaire) or a combination of self-overlap and inter-overlap with another pattern (interplay). In other words, as a random sequence unfolds over time, we are more likely to first encounter another pattern other than a streak. The only exception is the case in interplay where a streak can tie with its end-reversal counterpart or another streak (e.g., Figure 3). Even in this exception, a streak retains an inferior status because of the "minimum succeeding distance" (see Figure 2B). Although it is equally likely HHH preceding HHT as HHT preceding HHH, if HHH occurs first, HHT can immediately follow. If HHT occurs first, the next best shot for HHH has to be 3 tosses away. That is, the discrepancy in the minimum succeeding distance can obscure people's experience of HHH more than it does to HHT.

Together, although streak patterns have the same mean time as any other pattern, their longest waiting time and maximum clustering tendency can leave them underrepresented in people's experience thus make them appear rare or "non-representative" in recollection. Actually, a recent study by Olivola and Oppenheimer (2008) seems to confirm our speculation: when participants recalled the studied binary sequence, the lengths of streaks present in the original sequence were underestimated. Even more interestingly, Olivola and Oppenheimer found that when a streak was present *early* or late in a 25-event sequence, the overall sequence was judged as less likely to be random, compared to when the same streak occurred in the middle of the sequence. It appears that people may actually have an intuitive and accurate response to waiting time such that a streak is unlikely to occur early in sequences generated by a random process.

It should be noted that besides the delayed first occurrence, the particular composition of streaks can manifest itself in many other forms. One example is the probability of occurrence at least once and its complementary "probability of nonoccurrence," whose roles in affecting people's perception of event likelihood have been discussed (Hahn & Warren, 2009; Sun, et al., 2010a). Another example is the shear disparity in the variance of interarrival times between different patterns. When time is essential in predicting future events, different levels of variance may have direct consequences in people's risk preference (e.g., Lopes, 1996; Markowitz, 1991; Sun & Wang, 2010).

Last but not least, in the examples discussed throughout the paper, the sequences of coin tosses are generated by Bernoulli trials (hence inter-event independent and memoryless). However, the process of counting patterns, particularly streak patterns, are not exactly memoryless (this is implied by the unequal mean time and waiting time, see Table 1). As human memory plays essential roles in predicting and planning future events, studies on such process can be useful in order to untangle the interaction of human memory and perception of randomness. Among these different statistics of the similar nature, people can be more sensitive to one form of manifestation than to another or even completely indifferent. We may not be able to use these statistics to vindicate a certain type of bias or fallacy. Nevertheless, these statistics can aid us to better understand the task environment so that we may eventually be able to more precisely pinpoint the source of the error.

Acknowledgments

This work was partially supported by an AFOSR grant (FA9550-07-1-0181), an ONR grant (N00014-08-1-0042), and a Vivian Smith Foundation grant to HW. We would like to thank Ryan D. Tweney, Haiqing Wei, Xuebo Liu, and Franklin Tamborello for helpful discussions and comments.

[5] Guibas and Odlyzko (1981) and Graham et al. (1994) provide strategies to construct a "winning pattern" to beat a given pattern for pattern length $r \geq 3$, in which a streak can never be constructed as a winning pattern.

References

Ainslie, G., & Monterosso, J. (2004). A marketplace in the brain? *Science, 306*(5695), 421-423.

Ayton, P., & Fischer, I. (2004). The hot hand fallacy and the gambler's fallacy: two faces of subjective randomness? *Memory & Cognition, 32*(8), 1369-1378.

Bar-Eli, M., Avugos, S., & Raab, M. (2006). Twenty years of "hot hand" research: Review and critique. *Psychology of Sport & Exercise, 7*(6), 525-553.

Budescu, D. V. (1987). A Markov model for generation of random binary sequences. *Journal of Experimental Psychology: Human Perception and Performance, 13*(1), 25-39.

Burns, B. D. (2004). Heuristics as beliefs and as behaviors: The adaptiveness of the "hot hand". *Cognitive Psychology, 48*, 295-331.

Camerer, C., Loewenstein, G., & Prelec, D. (2005). Neuroeconomics: How neuroscience can inform economics. *Journal of Economic Literature, 43*(1), 9-64.

Falk, R., & Konold, C. (1997). Making sense of randomness: Implicit encoding as a basis for judgment. *Psychological Review, 104*(2), 301-318.

Gardner, M. (1988). *Time travel and other mathematical bewilderments*. New York: Freeman.

Gilovich, T., Griffin, D., & Kahneman, D. (Eds.). (2002). *Heuristics and biases: The psychology of inuitive judgment*. New York: Cambridge University Press.

Gilovich, T., Vallone, R., & Tversky, A. (1985). The hot hand in basketball: On the misperception of random sequences. *Cognitive Psychology, 17*, 295-314.

Graham, R. L., Knuth, D. E., & Patashnik, O. (1994). *Concrete mathematics*. Reading MA: Addison-Wesley.

Guibas, L. J., & Odlyzko, A. M. (1981). String overlaps, pattern matching, and nontransitive games. *Journal of Combinatorial Theory, series A, 30*, 183-208.

Hahn, U., & Warren, P. A. (2009). Perceptions of randomness: Why three heads are better than four. *Psychological Review, 116*(2), 454-461.

Huettel, S. A., Mack, P. B., & McCarthy, G. (2002). Perceiving patterns in random series: Dynamic processing of sequence in prefrontal cortex. *Nature Neuroscience, 5*(5), 485–490.

Li, S.-Y. R. (1980). A Martingale approach to the study of occurrence of sequence patterns in repeated experiments. *The Annals of Probability, 8*(6), 1171-1176.

Lopes, L. L. (1996). When time is of the essence: Averaging, aspiration, and the short run. *Organizational Behavior and Human Decision Processes, 65*(3), 179–189.

Markowitz, H. M. (1991). Foundations of portfolio theory. *Journal of Finance, 46*(2), 469-477.

McClure, S. M., Ericson, K. M., Laibson, D. I., Loewenstein, G., & Cohen, J. D. (2007). Time discounting for primary rewards. *Journal of Neuroscience, 27*(21), 5796-5804.

McClure, S. M., Laibson, D. I., Loewenstein, G., & Cohen, J. D. (2004). Separate neural systems value immediate and delayed monetary rewards. *Science, 306*(5695), 503-507.

Nickerson, R. S. (2002). The production and perception of randomness. *Psychological Review, 109*(2), 330-357.

Olivola, C. Y., & Oppenheimer, D. M. (2008). Randomness in retrospect: Exploring the interactions between memory and randomness cognition. *Psychonomic Bulletin & Review, 15*(5), 991-996.

Oskarsson, A. T., Van Boven, L., McClelland, G. H., & Hastie, R. (2009). What's next? Judging sequences of binary events. *Psychological Bulletin, 135*(2), 262-285.

Rabin, M. (2002). Inference by believers in the law of small numbers. *The Quarterly Journal of Economics, 117*(3), 775-816.

Ross, S. M. (2007). *Introduction of probability models* (9th ed.). San Diego, CA: Academic Press.

Sun, Y., Tweney, R. D., & Wang, H. (2010a). Occurrence and nonoccurrence of random sequences: Comment on Hahn and Warren (2009). *Psychological Review, 117*(2), 697-703.

Sun, Y., Tweney, R. D., & Wang, H. (2010b). Postscript: Untangling the gambler's fallacy. *Psychological Review, 117*(2), 704-705.

Sun, Y., & Wang, H. (2010). Gambler's fallacy, hot hand belief, and time of patterns. *Judgment and Decision Making, 5*(2), 124-132.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science, 185*, 1124–1131.