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Political Expenditures and Power Laws: A Spatial Model of the Lobbying Process

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A familiar refrain heard during most campaigns is, “There’s too much money in politics!” Whether that is indeed the case or not, money certainly plays a key role in the American political process, whether in the form of campaign contributions from special interest groups, campaign contributions from individuals or other lobbying expenditures by special interest groups. Furthermore, the amount of money explicitly tied into the political process through one of those three channels is undoubtedly increasing up to the present. In the last full presidential election cycle (2003-04) the Democratic and Republican party raised a record setting \$1.5 billion in campaign contributions alone. Ironically, this was the first election cycle in which the Bipartisan Campaign Finance Reform Act of 2002 which restricted campaign contributions was in effect. Meanwhile special interest groups spent over \$4 billion in various lobbying expenditures during the same two year period.<sup>i</sup> The message is quite clear: there is a substantial amount of money in politics, and voters and politicians on both sides of the aisle have felt compelled to reform the giving process.

Setting aside the issue of campaign finance, I concern myself with the other, larger component of money in politics, lobbying expenditures. As long as government policies and regulatory actions can be targeted to distinct groups, these interests will have incentives to lobby the government for preferential treatment. This is an inescapable feature of modern democracies, yet the public holds lobbyists in such a dim view that over nine out of ten Americans believe it should be illegal for lobbyists to give any item of value to politicians.<sup>ii</sup> Grossman and Helpman (2001) cleanly identify three basic motives for lobbying – gaining access to politicians, providing credibility for favored policy positions, and direct influence on policy. However, the effects of lobbying on policy (and ultimately social welfare) are ambiguous. Targeted transfers may or may not be inefficient, while competition among special interest groups could potentially produce more or less efficient redistributive policies. Given lobbyists’ key role in policymaking, their ever increasing expenditures, and the public’s poor opinion of them, political finance reform is an issue of central importance. Ideally, we would like to reform spending in politics in a way to maximize social welfare. If, however, our understanding of lobbying is flawed, then policy reforms may be inefficient at best, and socially detrimental at worst.

There is a very well developed theoretical literature on special interests and lobbying stretching back nearly a half century. Olson’s (1965) seminal work identified obstacles to collective action and underscored the differences between individual and group interests, even among like-minded constituents. Stigler (1971) suggested that lobbying, particularly with respect to regulation and redistribution, was motivated by rent seeking behavior, and this line of thought was more rigorously followed by Peltzman (1976) and Becker (1983). Winston and Maheshri (2008) model interest group behavior in a dynamic setting, where an agency problem between special interests and the constituents that govern them can lead to inefficient methods of redistribution.

Broadly speaking, there have been two general approaches to describing special interest group (inter)action theoretically. Becker (1983, 1985) models interest group competition between representative taxed and subsidized groups as a reduced form game. Special interests make expenditures on political pressure which in turn develop their political influence and generate rents from the government. Grossman and Helpman (1996, 2001) and Grossman, Helpman and Dixit (1998) have applied the common agency model of Bernheim and Whinston (1986) to a strategic game between interest groups and politicians involving political contributions contingent on actual policies drives lobbying behavior.

In both approaches, very little attention is given to the distribution of lobbying expenditures by interest groups. This is unfortunate, because the distribution of lobbying expenditures (rather than the magnitude of these expenditures) is of first order concern to policy makers. Broadening the base of political participation and dissuading or restricting one group from dominating all government interactions are priorities to political reformers.<sup>iii</sup> Becker simply assumes away the distribution through the use of representative agents, and the structure of the common-agency model of Grossman and Helpman has a tendency to yield knife-edge strategies in which one group does all of the giving in equilibrium. Neither of these theoretical results can be corroborated in lobbying data. In fact, they are directly refuted. The distribution of lobbying expenditures simply cannot be characterized by a single group taking full action, nor is it characterized by lumpy point masses of groups with different policy interests. Instead, I note a conspicuous empirical regularity, namely that the distribution of lobbying expenditures follows a power law. This casts serious doubt on the ability of strategic models of lobbying to generate realistic predictions.

In general, the literature on the subject has relied too much on highly stylized models of decision making to describe special interest behavior. In a recent survey on the state of political economy research, Timothy Besley (2004) notes that there is no clear correct theoretical framework for understanding special interest politics, and in fact “there is no reason to believe that any single theoretical approach will dominate.” Indeed, one of the goals of this paper is to provide a substantively new and different approach to understanding decision making by special interest groups. I analyze the distribution of lobbying expenditures by special interests in all sectors and industries. While the main contribution is a theoretical description of a general set of processes consistent with specific behavior, I stress that all of the analysis is empirically motivated. That is, only after that lobbying expenditures follow a power law do I propose a new, spatial model of lobbying behavior that may be driven by general, plausible heuristics that special interests use to guide their actions.

The striking predictions by this model of the distribution of lobbying expenditures stand in stark contrast to the predictions by widely accepted strategic models in the style of Grossman and Helpman. That fact corroborates my approach and implies an “impossibility theorem” of sorts with great policy relevance: practically no political reform of modest scale will have any effect on the shape of the distribution of lobbying expenditures. Furthermore, the analysis shares key similarities to models of widely disparate phenomena in the physical, biological and social sciences; this cross-disciplinary universality is intellectually satisfying in its own right.

The paper is organized as follows. In section II, I develop the theory of power laws and allude to their prevalence elsewhere in the scientific world. In section III, I use actual data on US special interest groups to identify a broad, empirical regularity in the distribution of their lobbying expenditures, which naturally gives rise to a spatial model of the lobbying process laid out in section IV. In section V, I discuss the policy implications of these findings and stress the superiority of this approach in describing aggregate special interest behavior relative to the stylized, strategic workhorse models in this field. Supplemental mathematical background is provided in two appendices.

## Power Laws

The term *power law* is given to a general class of distributions with a salient feature: *scale invariance*. This feature possesses great intuitive appeal. Consider some data generating process. Then it is said to be scale invariant if the probability density of the data is similar at all scales. That is, if we observed the density over some domain of the process and compared it with the density of another domain of the process which was scaled up (or down) by a constant factor, then the densities on the two domains would be proportional to one another. In simpler terms, the same fundamental forces generate the data at all scales.

Power laws are of great scientific interest in large part due to their universality. They appear so widely not only in physics, biology and earth sciences, but also in demography, economics, finance, and social networking (Newman 2006). This list is by no means exhaustive. Models of ferromagnetism, percolation and biological speciation (Yule 1925) imply power laws arising in substantively different settings by fundamentally different mechanisms. Data on the intensity of solar flares (Lu and Hamilton 1991) and armed conflict (Roberts and Turcotte 1998) follow power laws, as does city growth (Gabaix 1999), firm size (Axtell 2001), stock market volatility (Gabaix et. al. 2003) and telephone call frequency (Ebel et. al. 2002).

This variety of different environments all gives rise to power laws, which are, in some sense limiting distributions of a general class of stochastic processes (those with scale invariance). I broadly define two of power law generating processes as dynamic and spatial. As the name suggests, dynamic processes evolve over time and often contain some component of a random walk. For example, power laws can be deduced from stochastic accumulation or disintegration of different quantities (such as cities growing with random migration and biological genera fragmenting into new species through random mutation).

Spatial processes, meanwhile, need not be dynamic and are intimately related to fractals. The self similarity of fractals is broadly analogous to the scale invariance of power law distributions, and in spatial models, this invariant behavior arises at particular critical points (similar to the fractional dimension which a fractal occupies). Examples of these spatial processes include percolation (as water boils, there is a point between the liquid and gaseous phase in which the sizes of bubbles are distributed according to power laws) and the evolution of forest fires (periodic, stochastic fires arrange smaller groups of trees in a specific manner until the entire forest is vulnerable to a single fire).

More formally, a probability density  $\pi(x)$  is scale invariant if

$$\pi(bx) = g(b)\pi(x) \tag{1}$$

for all values of  $x$  and  $b$ , and some function  $g$ . We say this distribution follows a power law, because it is necessarily the case that we can write the density as

$$\pi(x) = Cx^{-\gamma} \tag{2}$$

for some exponent  $\gamma$  and constant  $C$ .<sup>iv</sup> As reflected by the functional form of equation (2), these distributions have a striking geometric property. Namely, when plotted on logarithmic axes, the graph of  $\pi(x)$  will be a downward sloping straight line with slope  $-\gamma$ . A graph of,  $\Pi(x)$  the cumulative density of  $\pi(x)$  will be a downward sloping straight line with slope  $-(\gamma+1)$ .<sup>v</sup> The constant (or invariant) slope at all scales of the variable on the  $x$  axis echoes the notion of scale invariance.

To the empiricist, there is a simple test of whether data follows a power law or not. One needs only to rank the variable of interest in the data in order of largest to smallest and then plot the logarithm of the value of the variable on the  $x$  axis and the logarithm of the rank of the variable on the  $y$  axis. In some sense, this is a plot of  $\Pi(x)$ . If this plot yields a straight line, then the data follows a power law with an exponent roughly equal to the slope of the line minus one. A precise maximum likelihood test of power law behavior can also be performed. Details are given in appendix A.

### General Empirical Findings

According to the Federal Lobbying Disclosure Act of 1995, all lobbyists with expenditures exceeding \$20,000 are required to file semi-annual reports with the Senate Office of Public Records. All filings by lobbyists can be traced to individual clients (trade groups, unions, firms, etc.) The Center for Responsive Politics has enumerated all lobbying expenditures by interest groups of over \$20,000 annually beginning in 1998. I use these data to explore the distribution of lobbying expenditures and to statistically test for scale invariance.

**Table 1. Summary Statistics**

Variable	Mean	S.D.	Min.	Max.	Data Source
Annual lobbying expenditures (in thousands of dollars)	315.8	981.4	10	30796.6	Lobbying Database, Center for Responsive Politics (CRP)
Number of actively lobbying interest groups within industry	77.38	101.4	4	767	Lobbying Database, CRP
Number of industries	76				
Number of sectors	12				
Years in sample	1998-2006				

All monetary variables are in 2006 dollars.

Summary statistics for the lobbying data are provided in table 1. The dataset is large and comprehensive; the large number of observations allow for very precise distributional estimates. All monetary amounts are reported in 2006 dollars using the average CPI from the US Bureau of Labor Statistics. Of particular note is the wide range of values that lobbying expenditures for individual interest groups assumes (from \$20 thousand to over \$30 million). The fact that these values span several orders of magnitude indicates that there is no “typical scale” of lobbying. This is an important clue towards scale invariance. Furthermore, there is great heterogeneity in the number and size of contributors in each industry. If power law coefficients are similar across industries, then this could mean that the distribution of lobbying expenditures is unrelated to the underlying structure of who gives.

As explained above, there are two traditional ways to test whether expenditures follow a power law. The first is a maximum likelihood estimate of the power law coefficient as derived in appendix A. This is, by definition, the most efficient test of power law behavior; however, it only provides an unconditional, univariate estimator. The second is an OLS regression of log-rank of expenditures on log-expenditures.<sup>vi</sup> The slope coefficient (in absolute value) in this regression represents the power law exponent. If it is estimated very precisely, then we can be confident that the data are generated by a power law process. Although this is not technically the most efficient test of power law behavior, Gabaix and Ibragimov (2007) provide a simple correction – simply subtracting  $\frac{1}{2}$  from the rank before running the regression – which materially improves the quality of the estimates.

The benefit of the latter approach is that it allows for simple multivariate analysis. More specifically, statistical tests of common power law exponents can be conducted simply even if the intercepts of the tails of the distributions are different. If a common power law exponent is suspected for the lobbying expenditures of two different industries, then we can simply test the significance of the slope coefficient in a log-log OLS regression of the corrected rank on expenditures if we include industry specific fixed effects. In addition, when dealing with panel data, it is possible to account for temporal effects as well.

**Table 2. Common OLS Power Law Exponent Estimates<sup>vii</sup>**

Left hand side variable is  $\ln(\text{industry rank} - 0.5)$ .

<b>Right Hand Side Variable(s)</b>	$\hat{\gamma}$	<b>S.E.</b>	<b>Naïve S.E.<sup>viii</sup></b>	<b>R<sup>2</sup></b>
$\ln(\text{industry share of expenditures})$	0.85	0.005	0.04	0.70
Above plus year fixed effects	0.84	0.005	0.04	0.71
Above plus sector fixed effects	0.81	0.005	0.03	0.87
Above plus industry fixed effects	0.80	0.005	0.03	0.92
52929 observations				

Estimates of the overall power law exponent for different specifications are provided in table 2. This is a common exponent for all groups. The key insight to take

away from this table is that the power law exponent is estimated extremely precisely as evidenced by the minuscule standard errors. Furthermore, additional fixed effects do not materially change the estimate of the exponent very much although they help to explain more of the variation in the data. This is extremely strong evidence of power law behavior in lobbying expenditures across industries.

Of course, the distribution of lobbying expenditures *within* industries is also of great interest. Due to the large number (76) of distinct industries in the sample I do not provide exponent estimates for each industry. However, all of these estimates are highly statistically significant with the most generous, naïve standard errors on the order of 4% of the estimates. With such precisely estimated exponents, I can test whether the distribution of lobbying expenditures is well predicted by sector and industry specific fundamentals.

### Table 3. Industry Level Power Law Exponents and Industry Structure

Dependent variable is  $\hat{\gamma}$  as computed following the method of Gabaix and Ibragimov (2007) for each industry. Yearly fixed effects are included in these regressions. All estimates of  $\hat{\gamma}$  are highly significant at with over 99% confidence.

Variable	(1)	(2)	(3)	(4)	(5)	(6)
Number of active interest groups within industry*	-0.02 (0.03)	--	--	--	-0.02 (0.02)	--
Total industry expenditures on lobbying (dollars)*	--	0.03 (0.02)	--	--	--	0.03 (0.02)
Number of active interest groups within sector (thousands)*	--	--	0.001 (0.08)	--	0.05 (0.05)	--
Total sector expenditures on lobbying (millions of dollars)*	--	--	--	(0.06) (0.04)	--	0.01 (0.03)
Sector fixed effects	Yes	Yes	No	No	Yes	Yes
Yearly fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Num. Observations	684	684	108	108	684	684

Standard Errors clustered by sector are provided in parentheses.

\* indicates variable has been transformed by natural logarithm.

In table 3, I present alternative specifications of power law exponent regressions. The idea behind these tests is to see whether sector and industry level lobbying information can shed any light on the power law distribution of lobbying expenditures. The regressions are consistent and indicative of the fact that there is little predictive power in basic industry and sector level lobbying fundamentals. The right hand side variables are transformed by natural logarithms to provide for the best fit possible. Still, all coefficients are statistically insignificant. While this is an admittedly rudimentary test – it is loosely proving a negative – it is surely indicative of the fact that the distribution of lobbying expenditures within industries is not governed by industry lobbying structure.

These results, broadly speaking, are well supported by the anecdotal evidence presented in table 4. Here, I provide three examples of groups of industries with nearly

identical power law exponents. In each group, there are a number of disparate industries from a wide variety of sectors, each of which assumes the same power law distribution. As an example, it is highly unlikely that electronics manufacturers, poultry and egg farmers and the clergy all have similar political access, costs and benefits of lobbying, and industrial structure, yet their lobbying expenditures are distributed nearly identically. This is highly suggestive that the aggregate lobbying behavior is governed by other forces common to all industries.

**Table 4. Selected Industries Grouped by Industry Level Power Law Exponent<sup>ix</sup>**

This is not an exhaustive list of industries in the sample.

$\hat{\gamma}$	Industries
0.68-0.71	TV/Movies, Pharmaceuticals, Food Products Manufacturing, Lodging Tourism, Home Builders, Printing and Publishing, Food Stores, Beer, Wine and Liquor, Oil and Gas, Forestry and Forest Products, Securities and Investments, Telecommunications, Savings and Loans
0.78-0.82	Electronics Manufacturing, Trucking, Poultry and Eggs, Computers and Internet, Real Estate, Live Entertainment, Mining, Health Professionals, Clergy, Lawyers and Law Firms, Business Services, General Contractors, Dairy, Livestock
0.98-1.07	Sea Transport, Fruits and Vegetables, Environmental Services, Fisheries and Wildlife, Lobbyists, Special Trade Contractors, Education, Non-profit Institutions

All exponent estimates are highly statistically significant at over 99% confidence.

### A Spatial Model of the Lobbying Process

I have identified two general types of processes which give rise to power law behavior: those that are accumulative and those that are spatial. In deciding how to best model lobbying decisions, I first rule out the former. Accumulative processes are those in which growth rates of various quantities (in this case, levels of lobbying expenditures) are proportional to initial levels. As time elapses, the distribution of these quantities then assumes a power law over some relevant domain, usually the rightmost tail of the distribution.

One characteristic of accumulative processes that we would expect to observe is that the rank ordering of the largest variables tends to remain fairly stable over time. In particular, those special interests who lobbied the most in a particular year would be almost certainly expected to be the ones who lobby the most in subsequent years. A brief examination of the US data, however, soundly refutes this prediction. The most active special interest group within an industry in a particular year is also the most active special interest group in the following year only 60% of the time. If an interest group is one of the five most active groups within an industry, they are in the top five in the following year only 65% of the time. If we expand the subsample, the probability that a top ten



most active special interest group maintains their top ten ranking in consecutive years rises only to 67%. This casts serious doubt on an accumulative process driving lobbying behavior.<sup>x</sup>

Spatial processes are less intuitively appealing in how they give rise to power law behavior, and mathematically, they can take on a number of forms. I propose an extremely general class of spatial processes which do yield power law distributions. Underlying this explanation is the idea that special interest groups, while not necessarily acting completely independently of other competing groups, make their lobbying decisions based on simple heuristics, or “rules of thumb.” Generally speaking, they can be described as follows:

1. Interest groups give their money to their favorite politician and those politicians most similar to them. They are less likely to give to politicians with vastly divergent policy positions.
2. Interest groups give their money in support of their favorite policy prescription, and those policies most similar to their favorite. They are less likely to support vastly different policy alternatives.

These simple assumptions of what drives lobbying decisions can be nicely embedded in a policy space which captures the three motives – access, credibility and influence – for lobbying. Consider a two dimensional square lattice where politicians are ordered along one dimension by their prior policy positions on a particular issue, and policy alternatives are ordered along the second dimension. Lobbying can then be thought of as a process in which a special interest group puts money on particular squares of the lattice, with each chosen square representing a contribution to a particular politician for a particular policy alternative. This compactly captures all three motives for lobbying. Interest groups lobbying for access would place money on several squares corresponding to a particular politician. Credibility for an interest group’s policy preferences could be signaled by money placed on several squares corresponding to a particular policy alternative. Direct influence would be captured by money placed on squares corresponding to favored politicians and favored policies.

The two “rules of thumb” above imply that the squares which a particular interest group places money on (that is, the politicians to whom they lobby for particular policy alternatives) will always form a contiguous subspace of the lattice. The number of squares, or politician-policy combinations, which are selected by the special interest is then proportional to the total amount spent on lobbying by the interest group.

For a simple analogy, think of the politician-policy lattice as a large roulette table. Assuming that they have nonzero lobbying expenditures, special interests place some chips (spend money on lobbying) on that square which represents their bliss-points (their favorite politician in support of their favorite policy). In addition, they place other chips on squares adjacent to their bliss-point. The area of the roulette table covered in their chips is then proportional to their total lobbying expenditure.

I prove below that following the rules above, the distribution of the total number of squares upon which money is placed (total lobbying expenditures) follow a power law distribution for large enough areas (i.e., the right tail of the distribution). In the proof below, the politician-policy space is continuous. In the illustrative roulette table analogy

and forthcoming simulations, the politician-policy space is discrete, but if this discrete space is large enough then the intuition will follow to a good approximation. Using the parlance of spatial models developed by physicists, I refer to lobbying a particular politician-policy combination as “shading” it (so named for coloring in squares on a lattice).

Consider a process in which shaded regions, or clusters, of politician-policy space successively grow following a general probabilistic rule: at iteration  $n$ , each cluster grows stochastically. The probability of growth decreases in some arbitrarily general manner so that as  $n$  gets large, the cluster stops growing with probability equal to one. This shading process takes place  $Z$  times in parallel yielding  $Z$  individual clusters (representing the  $Z$  different interest groups making their lobbying decisions simultaneously). I ask the following question: “What is the distribution of limiting cluster sizes that this process produces?” Since clusters represent a sum of combinations for which the  $z^{\text{th}}$  interest group has spent money lobbying for, I am really asking “What is the distribution of lobbying expenditures that this process produces?”

To answer, I proceed in two steps. First, I examine one particular cluster (interest group) and derive the limiting distribution of its size (lobbying expenditure). Then I aggregate this distribution over all  $Z$  clusters in order to derive the approximate limiting distribution of the entire process. Supplemental proofs are included in appendix B. A word of caution is in order – the derivation below holds only for large clusters (large lobbying expenditures).

*Step 1. Derivation of the limiting distribution of a single cluster*

The details of the shading process are as follows. At the  $n^{\text{th}}$  iteration, the cluster stochastically increases by a factor of  $k$ , which is drawn from the density  $K_n(k, x)$ . Because clusters cannot decrease in size,  $k \geq 1$ . Also, because clusters do not grow without bound,  $k \leq \bar{k}$ . This density can vary across iterations, but I make two assumptions on it:

$$K_n(k, x) = \kappa_n(k) \chi_n(x), \text{ and}$$

$$\chi_n(bx) = h(b) \chi_n(x) \text{ for all } b.^{\text{xi}}$$

The first assumption is that  $K_n$  is multiplicatively separable. The second assumption is a technical condition which generalizes the scale invariance of the process. It is satisfied by all functions  $\chi_n(x) = ax^b$  for instance. The assumption that clusters stop growing with probability equal to one can now more formally be written as

$$\lim_{n \rightarrow \infty} K_n(1, x) = 1.$$

More simply, the probability that a cluster stays fixed (grows by a factor of 1) approaches 1 as the process unfolds.

Define an integer valued function  $\pi_n$  such that  $\pi_n(x)$  is the probability that the cluster is of size  $x$  at iteration  $n$ . Let  $\pi(x) = \lim_{n \rightarrow \infty} \pi_n(x)$ .<sup>xii</sup> I now show that  $\pi$  obeys a power law, or  $\pi(x) = Cx^{-\gamma}$  for some values of the exponent  $\gamma$  and constant  $C$ . By lemma 1, this is equivalent to showing that  $\pi(bx) = g(b)\pi(x)$ . It suffices to show that

$\pi_{n+1}(bx) = g(b) \cdot \pi_{n+1}(x)$  for large values of  $n$  and  $b \leq 1$ . I prove this by induction on  $n$  with two assumptions: the standard inductive assumption and that  $x$  is sufficiently large. In proving the base case (i.e.,  $\pi_j(bx) = g(b) \cdot \pi_j(x)$  for some  $j$ ), I rely on a polynomial approximation. Both the inductive step and base case holds best at large  $x$ .

For the base case, I show that  $\pi_j(bx) = g_0(b)\pi_j(x)$  for some  $j$ . This obviously does not hold for all functions  $\pi_j$ , but it does hold for a large, general class of functions. In particular, if for some  $j$ ,

$$\pi_j(x) = \begin{cases} P_\pi(x) & 0 \leq \bar{k}^j \\ 0 & \text{otherwise} \end{cases}$$

where  $P_\pi$  is some polynomial function then  $\pi_j(bx) = P_\pi(bx) = b^k P_\pi(x) + o(x^{k-1})$ .<sup>xiii</sup> For large enough  $x$ , the remainder term is dominated and the equality stands. The base case is then satisfied using the function

$$g_0(b) = \begin{cases} b^k & b \leq 1 \\ 0 & b > 1 \end{cases}.$$

For the inductive step, I show that the proposition holds at iteration  $j+1$  assuming that it holds at iteration  $j$ , which was just proven in the base case. First, note that

$$\pi_{j+1}(x) = \int_1^\infty \pi_j\left(\frac{x}{k}\right) \mathbf{K}_{j+1}(k, x) dk. \quad (3)$$

Evaluating  $\pi_{j+1}(bx)$  and substituting in for  $\mathbf{K}_{j+1}(k, bx)$  I can rewrite equation (3) as

$$\pi_{j+1}(bx) = \int_1^\infty \pi_j\left(\frac{bx}{k}\right) h(b) \mathbf{K}_{j+1}(k, x) dk. \quad (4)$$

Now, substituting the base case, I can simplify equation (4) as  $\pi_{j+1}(bx) = g(b)\pi_{j+1}(x)$ , where  $g(b) = g_0(b)h(b)$ . This completes the proof.

Note that I go through induction to refrain from making the much stronger assumption that the limiting distribution  $\pi$  is well approximated by a polynomial. I simply assume that at some iteration  $j$ , the density of cluster sizes are well approximated by a polynomial. In other words, at some point, the system becomes “scale free,” in the sense that the inductive hypothesis begins to hold. This is similar to a “continuous phase transition” in physics, in which observable quantities obey power law distributions as conditions reach a certain point. Since this holds for large values of  $x$ , the right tail of the distribution of cluster sizes approximates a power law.

### *Step 2. Derivation of the limiting distribution of Z clusters*

Now, assume there are  $Z$  processes going on in parallel (that is,  $Z$  interest groups are making their lobbying decisions). Then the  $z^{\text{th}}$  process generates limiting clusters according to the distribution  $\pi_z(x) = C_z x^{-\gamma_z}$ . The question posed is equivalent to computing  $\Pi(x)$ , the probability that a cluster from any of the  $Z$  processes is of size  $x$ . In the interest of full generality, suppose that the coefficients  $C_z$  are distributed according to some density  $f$  (with cumulative density  $F$ ) and that the function  $\mathbf{K}$  is defined in such a

way for each of the interest groups that the exponents  $\gamma_z$  are distributed according to some density  $g$ . We can say a few things about this density  $G$ . First of all, it takes nonzero values only for  $\gamma > 0$ .<sup>xiv</sup> Let us assume that it also takes nonzero values only for  $\gamma \leq \bar{\gamma}$ .<sup>xv</sup> Then  $G$  is roughly equal to

$$G(\gamma) = \begin{cases} P_G(\gamma) & 0 \leq \gamma \leq \bar{\gamma} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Where  $P_G$  is a polynomial approximation in the specified interval.

The aggregate density of clusters is given by

$$\Pi(x) = \int_0^{\infty} \int_0^{\bar{\gamma}} Cx^{-\gamma} g(\gamma) d\gamma dF. \quad (6)$$

Using the approximation from equation (5), equation (6) neatly simplifies to

$\Pi(x) = C'x^{-\gamma'}$ , where the primes indicate new constants.<sup>xvi</sup> This is simply a statement of the fact that the aggregate density of clusters follows a power law.

Admittedly, this result is proven only for the right tail of the distribution, and is an aggregate approximation. Still, it is illustrative of the general class of processes which could give rise to observed power laws.

## Discussion

This formulation of lobbying behavior has significant and unique implications, both for academics and policy makers. For the former group, this modeling technique is a significant departure from the standard game theoretic approach to modeling lobbying and other complex decision making. For the latter group, this model implies that standard political reforms which target lobbying expenditures are not likely to have any effect on the distribution of lobbying within and across industries. In some sense, they are doomed to fail since they do not address the appropriate determinants of lobbying decisions.

I repeatedly stress that this approach is empirically driven. It is only in response to the inability of existing models of special interest groups to describe aggregate lobbying trends that I posit this spatial approach. More precisely, the evidence on aggregate lobbying expenditures is simply inconsistent with any reasonably simple Nash equilibrium in lobbying decisions. As such, I depart from that solution concept and instead provide an alternative mechanism for lobbying expenditures based on sensible, general heuristics rather than strategic optimization of well defined objectives. The spatial model described above predicts the power law in lobbying expenditures that is borne out of the actual data. In contrast, existing common agency based models tend to either predict atomistic distributions of lobbying expenditures where one dominant group spends all of the money in an industry, or all groups spend equally. Neither of these two equilibria are well supported by the data.

Furthermore, the existence of this observed power law should be of great interest to policy makers. Recall from tables 3 and 4 that the power law exponent in each industry does not seem to be correlated with industry fundamentals. In some sense, this is a statement that the relative distribution of lobbying expenditures is not a function of

the costs and benefits of lobbying for groups in a particular industry. (As stated before, it would be a stretch at best to claim that the costs and benefits of lobbying for the clergy are in the same proportion as they are for electronics manufacturers and poultry farmers.) That is not to say that the costs and benefits of lobbying do not affect decisions by special interest groups; on the contrary, lower costs and higher benefits are likely to be associated with greater lobbying activity by special interest groups. However, the relative distribution of expenditures is likely to remain unchanged. As this is a quantity of first order importance to policy makers, the implication is clear: policies which seek to affect the shape of the distribution of lobbying expenditures by altering the costs and benefits of lobbying are likely to fail.

A natural question is then, “What sorts of policies could have an impact on the shape of the distribution of lobbying expenditures?” First of all, remember that the spatial mechanism only generates a power law in lobbying expenditures in the right tail of the distribution (for large amounts of expenditures). Hence, if the costs of lobbying became so onerous relative to the benefits that special interest groups dramatically reduced their activity, then the mathematical approximations in the model would grow tenuous. So very extreme policies aimed at curbing influence could in fact have the effect of reshaping the distribution of lobbying expenditures. Secondly disaggregating the legislative process could also have a similar effect. By this, I mean that having more individual votes on appropriations and regulations could conceivably change the politician-policy space from a two dimensional manifold to a multi-dimensional manifold. One of the implicit assumptions in this model of lobbying is that politicians and policies can be ordered in a manner consistent with single peaked preferences (this ensures that clusters are connected subspaces). With a complicated multidimensional policy space, it becomes increasingly likely that interest groups’ preferences are no longer single peaked over all policies.

## **Conclusion**

In recent years, research in political economy has become almost singularly focused on strategic models of behavior which generate well defined Nash equilibria. While this is certainly of value in providing a systematic approach to understanding interactions between political actors, it often fails to describe adequately aggregate behavior in a manner consistent with empirical observations. I provide an alternative approach to modeling lobbying decisions based on simple, sensible heuristics instead of strategic optimization. While I say very little about how individual interest groups will act, I can make predictions of aggregate behavior which are very well supported by actual lobbying data.

My empirical contributions are clear: I identify a power law in lobbying expenditures, and I provide evidence suggesting that the shape of the distribution of these expenditures is uncorrelated with industry fundamentals and industry specific costs and benefits of lobbying. The policy implications of these facts are that most modest lobbying reforms will have little effect on the relative amounts spent by interest groups on lobbying; hence, they will fail at one of their primary objectives.

Broadly defined extensions to this research are twofold. First, I believe that this paper underscores the importance of basing theoretical models in the social sciences on actual empirical observations. If predictive power is a standard by which economic models are to be judged, then this is incumbent. Second, while equilibrium concepts based on optimization of well defined objectives often perform admirably to describe many economic phenomena, they do not have a monopoly on explanatory ability. Echoing Timothy Besley, they are just one of many tools which we can use to understand the world.

## Appendix A

This derivation of the maximum likelihood estimate of the power law exponent is in the spirit of (Newman 2006).

Consider the arbitrary power law density  $\pi(x) = Cx^{-\gamma}$ . In order to estimate the power law exponent, we need to identify the minimum scale at which the power law arises. Often times, this is simply the smallest observation in the sample, denoted  $x_{\min}$ . Because any probability density must integrate to 1, we can determine the coefficient  $C$  as follows:

$$1 = \int_{x_{\min}}^{\infty} Cx^{-\gamma} dx = \frac{C}{1-\gamma} (x^{1-\gamma}) \Big|_{x_{\min}}^{\infty}, \text{ therefore}$$

$$C = (\gamma - 1) x_{\min}^{\gamma-1}, \text{ and}$$

$$\pi(x) = \frac{\gamma - 1}{x_{\min}} \left( \frac{x}{x_{\min}} \right)^{-\gamma}. \quad (A1)$$

We are trying to compute a maximum likelihood estimate of the parameter  $\gamma$ , or

$$\hat{\gamma}_{ML} = \arg \min_{\gamma} \prod_{i=1}^n \pi(x_i; \gamma).$$

As is often the case, it is easier to take logarithms and minimize the log-likelihood function. Plugging in equation (A1) and taking logs, we have

$$\hat{\gamma}_{ML} = \arg \min_{\gamma} \left[ n \ln(\gamma - 1) - n \ln(x_{\min}) - \gamma \sum_{i=1}^n \ln \left( \frac{x_i}{x_{\min}} \right) \right]. \quad (A2)$$

Setting the derivative of the argument with respect to  $\gamma$  equal to zero, we get

$$\frac{n}{\hat{\gamma}_{ML} - 1} - \sum_{i=1}^n \ln \left( \frac{x_i}{x_{\min}} \right) = 0, \text{ so}$$

$$\hat{\gamma}_{ML} = 1 + n \left( \sum_{i=1}^n \ln \left( \frac{x_i}{x_{\min}} \right) \right)^{-1}, \quad (A3)$$

which is in fact identical to equation (3) in the text.

Estimating the standard error on  $\hat{\gamma}_{ML}$  is done by computing the width of the likelihood function as a function of the parameter  $\gamma$ . First, we exponentiate the argument in equation (A2) to obtain the (non log-)likelihood function. For clarity, let  $a = x_{\min}^n$ , and let  $b = \sum_{i=1}^n \ln \left( \frac{x_i}{x_{\min}} \right)$ , neither of which are functions of the . Then we can rewrite the likelihood function as  $\ell = ae^{-b\gamma} (\gamma - 1)^n$ . To obtain the variance of  $\gamma_{ML}$ ,  $\sigma_{ML}^2$ , we first need to compute the mean and the mean square of  $\gamma$ , which are respectively given by

$$\frac{\int_1^{\infty} e^{-b\gamma} (\gamma-1)^n \gamma d\gamma}{\int_1^{\infty} e^{-b\gamma} (\gamma-1)^n d\gamma} = \frac{n+1+b}{b}, \text{ and} \quad (A4)$$

$$\frac{\int_1^{\infty} e^{-b\gamma} (\gamma-1)^n \gamma^2 d\gamma}{\int_1^{\infty} e^{-b\gamma} (\gamma-1)^n d\gamma} = \frac{n^2 + 3n + b^2 + 2b + 2nb + 2}{b^2}. \quad \text{xvii} \quad (A5)$$

$\sigma_{ML}^2$  is then simply equal to the difference of (A4) and (A5), or  $\frac{n+1}{b^2}$ . Substituting back for  $b$ , this gives us

$$\sigma_{ML}^2 = (n+1) \left( \sum_{i=1}^n \ln \left( \frac{x_i}{x_{\min}} \right) \right)^{-2}. \quad (A6)$$

For large values of  $n$ ,  $n+1 \cong n$ , so we can rewrite equation (A6) neatly in terms of the parameter estimate  $\hat{\gamma}_{ML}$  given in (A3) as

$$\hat{\sigma}_{ML}^2 = \frac{(\hat{\gamma}_{ML} - 1)^2}{n}, \quad (A7)$$

which is again identical to equation (4) in the text.



## Appendix B

*Lemma 1.* For any differentiable  $\pi(x)$ ,  $\pi(x) = Cx^{-\gamma}$  if and only if  $\pi(bx) = g(b)\pi(x)$  for all  $b$  and functions  $g$ .

*Proof.* The “only if” proposition is trivially true with  $g(b) = b^{-\gamma}$ . We now prove the reverse direction.

Set  $x = 1$ . Then  $g(b) = \frac{\pi(b)}{\pi(1)}$ , so  $\pi(bx) = \frac{\pi(b)\pi(x)}{\pi(1)}$ . As this holds for all values of  $b$ ,

we can differentiate both sides with respect to  $b$  to get  $x\pi'(bx) = \frac{\pi'(b)\pi(x)}{\pi(1)}$ .

Setting  $b = 1$ , we have  $\pi'(x) = \frac{\pi'(1)\pi(x)}{\pi(1)x}$ . But this is a simple, separable first order

differential equation with solution  $\ln \pi(x) = \frac{\pi(1)}{\pi'(1)} \ln x + \ln C$ . Exponentiating both sides,

we get  $\pi(x) = Cx^{-\gamma}$ , where  $\gamma = -\frac{\pi(1)}{\pi'(1)}$ , and the proof is complete.

It is actually sufficient for the “if” proposition to hold only for  $b < 1$ . Suppose  $\pi(bx) = g(b)\pi(x)$  for  $b < 1$ . Define  $c = \frac{1}{b} > 1$ . Then the following is true:

$$g(b)\pi(x) = \pi(bx) = \pi\left(b^2 \frac{x}{b}\right) = g(b^2)\pi\left(\frac{x}{b}\right) = g(b^2)\pi(cx).$$

Hence,  $\pi(cx) = \frac{g(b)}{g(b^2)}\pi(x)$ . More succinctly,  $\pi(cx) = h(c)\pi(x)$  where

$$h(c) = \frac{g(c^{-1})}{g(c^{-2})}.$$

As a postscript, we can solve for the coefficient  $C$  by setting  $x = 1$ , finding  $C = \pi(1)$ .

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<sup>i</sup> Campaign figures are from the Federal Election Commission. Lobbying expenditures come from the Lobbying Database maintained by the Center for Responsive Politics. The lobbying expenditures only count contributions over \$20,000, hence this is a lower bound. All monetary values hereafter are in 1996 dollars.

<sup>ii</sup> In addition, two thirds of Americans believe that lobbyists should not be allowed to contribute to political campaigns. According to an ABC News/Washington Post poll conducted on January 5-8, 2006.

<sup>iii</sup> Senator John McCain, cosponsor of the McCain-Feingold Bipartisan Campaign Finance Reform Act himself stresses, “Our law was not designed to lower spending in elections... It was, however, designed to ensure that the money political groups spend in federal elections is limited to reasonable, small contributions from individuals.” (*USA Today*, November 4, 2004).

<sup>iv</sup> For a proof of this assertion, see Lemma 1 in the appendix.

<sup>v</sup> Plotting the cumulative density is superior to plotting a histogram of the density itself since cumulative distributions do not discard any data that would have been lost in the binning process.

<sup>vi</sup> Instead of using log-expenditures on the right hand side, power law exponents are sometimes better estimated using log-share of expenditures as the independent variable. For a brief discussion, see Gabaix (1999).

<sup>vii</sup> Parameter estimates and standard errors are calculated following the method of Gabaix and Ibragimov (2007) for the OLS estimates.

<sup>viii</sup> Naïve standard errors are simply the traditional OLS standard errors clustered by industry.

<sup>ix</sup> Parameter estimates are calculated following the method of Gabaix and Ibragimov (2007) for the OLS estimates.

<sup>x</sup> This also provides evidence against the most disarmingly simple explanation of the observation of power law behavior in lobbying expenditures. Axtell (2001), among others in a tradition dating back to the 1950s, notes that firms sizes within US industries follow power laws. One might think that if firms spend an amount on lobbying directly proportional to their size, then the power law observed in this paper would be a simple artifact of market structure. This is easily refuted by the fact that the rankings of large firms tend to remain quite stable from year to year, whereas the rankings of active special interest groups vary considerably, as explained above. Recently Rossi-Hansberg and Wright (2007) have shown that the distribution of establishment (firm) sizes in the American economy does not – and should not be expected to – follow a power law.

<sup>xi</sup> Note that together these two assumptions imply that  $K_n(k, bx) = h(b) K_n(k, x)$ .

<sup>xii</sup> By assumption,  $\pi$  is a finite function since clusters stop growing with probability one.

<sup>xiii</sup> We define  $\pi_j$  in pieces because any polynomial approximation of a positive function of  $x$  must approach infinity for large  $x$ , but the probability that the process generates an infinitely large cluster being must be zero. This piecewise definition is ok because we need only prove the assertion for  $b > 1$ .

<sup>xiv</sup> This is because  $\pi_z(x)$  is a probability density, hence it must integrate to 1.

<sup>xv</sup> This is empirically sensible because power law densities are rarely observed in nature to have exponents larger than 3.

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<sup>xvi</sup> With a polynomial representation of the density  $g$ , the power law functional form is preserved over successive integrations by parts.

<sup>xvii</sup> In calculating the mean and mean square of  $\gamma$ , we assume that  $\gamma > 1$ . Looking at equation (A3), it is clear that this must be the case for any maximum likelihood estimate of  $\gamma$ .