Subitizing, Visual Indexes, and Attention

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Abstract

Among the many fascinating findings to come out of the numerical cognition literature, subitizing – the ability to quickly, effortlessly, and accurately identify the number of items in small collections – holds a special place. Despite hundreds of studies probing this ability, the identity of the cognitive systems that explain its unique features remains unknown. One prominent account is that of Trick and Pylyshyn (1994), which is based on pre-attentive parallel individuation of visual indexes. Despite this account's promise, a few researchers have questioned its validity, due to experiments showing that attentional load influences enumeration performance, which they interpret as invalidating a pre-attentional model of subitizing. The present discussion paper offers a novel re-interpretation of some studies on the nature of the relation between subitizing and attention to help clear up in which sense subitizing depends on attentive vs. pre-attentive processes, thereby providing a novel defense of Trick and Pylyshyn's influential model.

Keywords: Subitizing; Attention; Numerical Cognition; Parallel Individuation; Multiple-Object Tracking

Introduction

While numerical cognition has only grown into a coherent and widespread area of research in the past forty years or so, at least one aspect of our numerical abilities has been studied for more than one hundred years. In the late 19th century, Jevons (1871) and Cattell (1886) observed that adult humans' ability to accurately label the number of objects perceptually presented in a visual array – its numerosity[1] – dropped when this number rose above 3. Starting at 4 stimuli, enumeration errors started creeping in, confidence in answers dropped and reaction time increased linearly in relation to the number of objects displayed thereafter. This ability to quickly and accurately apprehend the numerosity of collections of around four objects was eventually dubbed *subitizing* by Kaufman and colleagues (1949), to describe "one of several quite different ways in which people discriminate the number of objects [they are perceiving]" (519), and more specifically "the discrimination of stimulus-numbers of 6 and below." (520)

Despite general consensus that enumeration in the subitizing range recruits cognitive resources distinct from those used in enumeration of larger quantities,[2] the precise nature of the system underlying our ability to subitize – e.g. whether it is an object-tracking system, a parallel individuation system, or a dedicated system for enumerating small numerosities without counting them[3] – remains unsettled. Another issue concerns the relation between subitizing and attention, and whether or not subitizing is pre-attentive in nature. Answering this question is important as it can not only shed light on the origins of our numerical abilities, but also on how we individuate stimuli more broadly. In this paper, I offer a novel re-interpretation of some studies concerning attentional effects on subitizing in order to help identify the nature of this ability and clear up certain ambiguities regarding the cognitive systems on which it rests. More precisely, I offer a new defense of Trick and Pylyshyn's (1994) explanation of subitizing and its reliance on pre-attentional visual indexes against claims (e.g. by Vetter, P., Butterworth, B., & Bahrami, B. 2008 and Railo, H. Koivisto, M., Revonsuo, A., Hannula, M.M., 2008) that subitizing cannot be a pre-attentional process, by explaining how these criticisms conflate features of personal-level attentional processes and features of task-directed sub-personal individuation of visual indices.

I start in section 1 with a sketch of Trick and Pylyshyn's influential account of subitizing by explaining their views on visual indexes as well as Pylyshyn's work on Multiple Object Tracking (MOT hereafter), in order to then present how they exploit findings in this domain to explain features of subitizing in section 2, where I also summarize some empirical support in their favor. I then present some

[1] The term 'numerosity' has unfortunately come to mean different things to different people in recent years, some authors (e.g. Malafouris 2010; Coolidge & Overmann 2012) having taken the surprising decision to use it to describe the cognitive *system* that detects quantities of discrete objects in collections instead of these collections themselves, in opposition to what was common practice in the numerical cognition literature. Here, I use the term 'numerosity' to describe the number of distinct objects that can be perceived in a collection. This perceiver-relativity of numerosity emphasizes that numerosities are not numbers, since they are properties of physically-detectable collections of discrete objects, not abstract entities of arbitrary size.

[2] See Porter 2016, (p.3-4) for a few single-process approaches.

[3] See Samuels & Snyder 2024 (p.21) for references regarding each of these options.

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challenges to this view in section 3 before presenting my alternative re-interpretation in section 4.

Visual Indexes in Multiple Object Tracking

Taking their cue from Zenon Pylyshyn's (1989, 2001, 2007) work on visual indexing and multiple object tracking, Trick and Pylyshyn (1994) propose a model of the way in which visual objects are individuated that offers a promising explanation of the features of subitizing, including its restricted range. According to this visual indexing based account, subitizing rests on a limited-capacity cognitive system that automatically creates a small number of indexes of visual feature clusters based on their spatiotemporal continuity, allowing us to keep track of what went where in our visual field. This indexing system tags a small number of clusters of perceptual features in a way that allots them higher processing priority for attention at a later processing stage. Visual indexing "provides a means of setting attentional priorities when multiple stimuli compete for attention, as indexed objects can be accessed and attended before other objects in the visual field" (Sears & Pylyshyn 2000, 2).[4]

Pylyshyn devised many MOT paradigms to confirm the existence of such an individuating mechanism, which showed that its capacity limits correspond to those of the subitizing range.[5] In a typical MOT trial, visual stimuli (e.g. geometrical shapes) are presented on a screen and participants' attention is directed towards a particular group of objects (e.g. by having them flicker on the screen) to mark them as targets that they will have to track. Then, the target stimuli move around the screen in random motion and the subjects are tasked with correctly identifying them once they stop moving at the end of the trial. Typically, the target stimuli move among other identical non-targeted objects that act as distractors to the task.

In MOT, changing features of target stimuli (e.g. color, shape, size) or even their kind (e.g. a frog changing to a prince) does not limit our ability to track them, which suggests that individual features of objects are not required for tracking. Rather, what is important is tracking spatiotemporal continuity of feature clusters. Seen this way, visual indexes are purely referential: they point to things in the world (Pylyshyn 2007). Thus, visual indexes are limited to the demonstrative content "THIS", individuated about four times in parallel.[6] Visual indexes, then, would be part of a mechanism for demonstrative thought that participates in representation, but is not itself representational, since visual indexes track objects without representing their properties.

Evidence for this purely referential nature of visual indexes comes from the fact that if objects change features during motion (e.g. change color), when the motion stops, if an object disappears from the display, subjects can identify its last location, but not the relevant feature (Scholl, B. Pylyshyn, Z. & Franconeri, S.L. 1999). This suggests spatiotemporal continuity is processed at a higher priority level than individual features. Moreover, if objects adopt strange trajectories that take them in and out of existence in ways that do not correspond to passing behind an occlusion (e.g. by shrinking to nothing on one side of the occlusion and reappearing on the other side), tracking fails (Scholl & Pylyshyn 1999), which suggests that the system allowing us to track multiple objects distinguishes occlusion from non-existence.

The most important point to note about behavior in MOT is that it often displays high success rates for tasks involving less than 5 objects, which is the same capacity limits observed in subitizing. While the reason for this specific value of a capacity limit is not known, it is often associated with limits to working memory (see Pagano, S., Lombardi, L. & Mazza, V. 2014) or object-directed attention (e.g. Mandler & Shebo 1982; Scholl 2001). While we cannot tell why the limit is precisely where it is, there is good reason to think it is related to balancing processing load and attentional demands. Attention-constrained capacity limits of reference tokens in a pre-attentive system is to be expected, since there would be no benefit in selecting more objects pre-attentively than can be handled by attention. And yet, there is also a reason why a small number of objects must be individuated in parallel for counting tasks: this is the only way for attention to be able to orient itself spatially through the visual landscape. Otherwise, if only one object were individuated, attention could not orient itself towards the next object to be counted. On the other hand, if too many objects are individuated, the processing costs become high. Thus, having a small number of objects individuated in parallel is a good way to balance being able to compute spatial relations between these and orient attention accordingly without creating too large of a cognitive load. Importantly, an explanation of the capacity limits of working memory, multiple object tracking, or object-directed attention (whatever the relation between these turns out to be) is *not* required in order to explain the restricted range of our ability to subitize: the important part for current purposes is to explain some key features of subitizing in terms of another, more basic cognitive system,

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[4] Yantis proposed a similar system in charge of assigning tags for processing priority, based on findings indicating that the number of stimuli that can capture attention simultaneously is limited to four objects (e.g. Yantis & Johnson 1990).

[5] See for example Pylyshyn 1989, 2003. For a review of Multiple Object Tracking, see Scholl 2009.

[6] To highlight the fact that these individuation indexes ostensibly point to feature clusters like a finger points to an object, Pylyshyn dubbed them FINSTS, for FINgers of INSTantiation. Given that the relationship between finsts and object-files (Kahneman, D., Treisman, A. & Gibbs, B. J. 1992), is, to use Pylyshyn's words, "not entirely transparent" (2003, 209f), I will refrain from using this term here, to avoid confusion. Intuitively: finsts 'connect' spatiotemporal information about feature clusters to representations of objects, while object files are the equivalent of memory slots where some feature information about objects is stored. See chapter 4 of Pylyshyn 2003 for more details on this distinction, as well as Pylyshyn 2007, p.37-39.

leaving the explanation of the properties of this basic system for another time.[7]

From visual indexes to subitizing

According to Trick and Pylyshyn, subitizing occurs in two stages. In the first, 'prenumeric' stage, visual indexes are assigned in parallel to each stimulus. It is only in the second stage, dubbed 'number recognition', when subjects choose a response from their numerical lexicon, that numerical content comes in. This second part of the process does not occur in parallel, since words for numbers must be retrieved from memory, where, according to Trick and Pylyshyn, they are stored in ordinal format. Following the work of Klahr (1973), they claim that numerical recognition involves "matching each individuated item with a number name, in the order of the number name" (Trick & Pylyshyn 1994, 88). This is what explains the slight slope in response time for the subitizing range: word retrieval takes a little longer for each successive number word, since the matching from index to number word starts at 1.

Support for such a visual-index based explanation of subitizing comes from many experimental sources. One particularly compelling source of behavioral evidence involves presenting visual stimuli in the form of retinal afterimages produced by flashguns.[8] Since the perceived dots are not actual physical objects but retinal afterimages, serial visual tagging of these is prevented, as the eyes cannot move from one object to the next to individuate them in a serial counting routine.[9] When stimuli are presented in this manner, there are unusually high error rates in numerosity-identification tasks outside of the subitizing range even when afterimages persist for up to a minute, suggesting that failure to individuate objects serially prevents precise enumeration of quantities outside the subitizing range. As predicted by the visual index model, accurate enumeration is preserved for small numerosities, since no such serial processes are required in the subitizing range, given that we can individuate a limited number of visual indexes in parallel and simply read off their number without having to attend to each serially.

Further support comes from a study where participants were asked to simultaneously track multiple objects *and* subitize (Chesney & Haladjian 2011). If, as Trick and Pylyshyn proposed, subitizing recruits the same individuation mechanisms that allow MOT, then interference between these two tasks should be evident. As predicted, results showed that the number of objects in participants' subitizing range decreased in direct relation to the number of objects they were asked to track, confirming that parallel individuation of visual objects is a limited resource shared by both MOT and enumeration tasks.

Trick and Pylyshyn also confirmed that subitizing depends on parallel individuation processes via a series of behavioral experiments that tested the effects of feature arrangements on subitizing performance. The idea here is that if subitizing depends on automatic, pre-attentive individuation mechanisms, when such individuation is blocked, subitizing shouldn't be possible. One way to prevent such individuation from happening is to confound object boundaries, making it difficult to create visual indexes for these. Thus, paradigms in which the stimuli are arranged in a way that leads to grouping together features from different objects should affect our ability to subitize, since the underlying feature grouping that accounts for the rapid response in subitizing would be countered by the location and feature confounds.[10] As predicted, subitizing does not occur when displays involve concentric circles (or squares) as stimuli: concentric circles share a center area as well as a center point, which means visual indexing mechanisms struggle to individuate them as distinct objects, and serial attention is required to individuate these (Trick & Pylyshyn 1994, 97). Similarly, when attempting to quantify Os in a sea of Qs, subitizing fails because the target stimuli (the Os) and the distractors (the Qs) share too many features for the targets to be individuated in a way that they can pop-out perceptually.

To sum up, Pylyshyn's visual indexing theory offers a compelling explanation of how subitizing works that is well supported by behavioral data and is consistent with neuroimaging data supporting the two-systems approach to our numerical abilities. Despite its explanatory power, however, this account has been questioned based on findings that suggest attention is necessary for subitizing. I present some of these findings in the next section and then offer a novel re-interpretation of them that seems in line with Trick and Pylyshyn's approach in the following section.

Empirical challenges to Trick and Pylyshyn?

For some, a potential problem with seeing visual indexing as an explanation for features of subitizing is that several behavioral studies appear to have established that variations in attentional load can significantly compromise subitizing. The idea here seems to be that this would mean that the features of subitizing cannot be explained by pre-attentional processes like parallel individuation in visual indexes, since attentional processes affect it. For example, some brain studies show that subitizing recruits areas often associated with attention (Burr, D.C., Turi, M. & Anobile, G. 2010; Ansari, D. Lyons, I.M., van Eimeren, L. & Xu, F. 2007),

[7] For example, whether or not the capacity limit is best explained in terms of a limited number of slots is still up for debate (see e.g. Bays 2015; Gross & Flombaum 2017.) See Porter 2016 for more on the nature of parallel individuation and its limits.

[8] See Atkinson, J. Campbelle, F.W., & Francis, M.R.J. 1976, replicated and expanded in Simon & Vaishnavi 1996.

[9] In typical subitizing studies, subjects are prevented from counting by restricting the time during which stimuli are exposed. This method however leaves open a single-mechanism explanation of subitizing according to which subitizing is a form of very fast counting that can occur in short temporal windows. Using afterimages prevents such counting to be considered a possible alternative explanation, since it prevents serial attention.

[10] For more ways of preventing individuation, see Porter 2016.

while studies varying attentional load using dual task procedures (Vetter et al. 2008), the attentional blink (Egeth, H.E., Leonard, C.J., & Palomares, M. 2008; Olivers & Watson 2008) and inattentional blindness paradigms (Railo, et al. 2008) have all shown that subitizing is compromised when another task recruits too many attentional resources. For many of these authors, these results make it difficult to see subitizing as based in visual indexing, since this is a pre-attentional process, whereas there is evidence of interference with subitizing at the attentional level itself.

For example, consider a study that tested preattentive vs attentive models of subitizing using an inattentional blindness paradigm (Mack & Rock 1998) to determine whether variation in attentional resources available for enumeration would have an effect on subitizing (Railo et al. 2008). Inattentional blindness paradigms exploit the fact that we are often blind to stimuli that are not relevant to the task we are trying to complete. In this case, the idea is that if subitizing recruits pre-attentional resources, as Trick and Pylyshyn claim, then it should not be affected even if our attention is focused on another task, since it is *pre*-attentional.

Here, the primary task was to determine which arm of a cross was longer. As a secondary task, subjects were asked to enumerate 1-6 dots that briefly (200ms) appeared outside the area of the target stimulus in non-canonical arrangements. To vary attentional load, three conditions were used. In the inattentional condition, dots were presented without warning after a few trials, at the same time as the primary display. If participants noticed these, they were asked to say how many there were, and to rate confidence in their response on a scale of 1 to 5. In this inattention condition, attentional resources are dedicated to the primary task of comparing line lengths on the cross.[11] In a divided-attention condition, dots would appear only on certain trials, but participants knew this could happen, unlike in the inattention condition. Finally, in the full-attention condition, participants' primary task was to enumerate the dots, even though they were asked to focus gaze on the cross while enumerating.

According to these authors, "the preattentive model predicts that enumeration within the subitizing range should already be accurate in the inattention condition and performance within the subitizing range should not be affected by the manipulation of attention, whereas the attentive model predicts that the accuracies should decrease as the number of the objects increases and attention would have an effect even within the subitizing range." (Railo et al. 2008, 87) The underlying rationale here seems to be that if Trick and Pylyshyn are right and subitizing is based only on pre-attentive resources, then performance in determining the quantity of dots should not be affected if their number is in the subitizing range, while performance should be affected outside the subitizing range, given performance there depends on serial attention.

Railo and colleagues (2008) found that performance was affected in both the inattentional and divided-attention conditions, while performance matched that found in regular subitizing studies in the full attention condition. In the inattention condition, accuracy was high only for 1 and 2 dots. Given that this is well below the regular subitizing range of 3-4 objects, they conclude that "subitizing cannot be explained by purely preattentive mechanisms" (Railo et al. 2008, 100), since below-par performance was achieved by modifying *attentional* resources available for enumeration, which contradicts their interpretation of the *pre*-attentive model's predictions.

A similar take on Trick and Pylyshyn's account is illustrated by the dual-task procedure employed by Vetter et al. (2008), which adopts the load theory of attention (e.g. Lavie & Tsal 1994; Lavie, N., Hirst, A., de Fockert, J. W., & Viding, E. 2004). This approach sees attentional selection as depending on task demands. When a task does not take up too many attentional resources, it is possible to perform another, secondary task, using the leftovers. But when tasks require high attentional load, performance in secondary tasks suffer. The idea is that if there is a performance difference in the secondary task between low-load and high-load conditions, then that means that attentional resources are shared between the primary and secondary tasks, and thus that these tasks both depend on attention.

There are many ways to vary attentional load. In the present case (Vetter et al. 2008), researchers claim that if subitizing is the result of preattentive processes, then it should not be affected by the attentional load of a primary task. Thus, they varied the difficulty of a primary task – in this case, color identification – to determine whether variation in attentional load would affect enumeration performance when it is a secondary task. Here, in the low load condition, subjects merely had to determine whether or not a color was present, while in the high load condition, they had to detect two specific color–orientation conjunctions. In both load conditions, the secondary task was to identify the number of targets from a circular arrangement of high-contrast gabor patches ranging from 1 to 8 elements.

Data obtained show that both load conditions affected subitizing accuracy, with more pronounced effects in the high-load condition. Here too the authors conclude that their results challenge both "the traditionally held notion of subitizing as a pre-attentive, capacity-independent process", and "the proposal that small numerosities are enumerated by a mechanism separate from large numerosities", instead supporting "the idea of a single, attention-demanding enumeration mechanism" (Vetter et al. 2008, 1).

Subitizing and attention: where's the beef?

While the skepticism towards Trick and Pylyshyn's model in these experiments is plausible, another explanation for

[11] Of course, as Railo and colleagues admit, if participants noticed the dots, some attentional resources must still have been taken up by these.

these and similar results appears equally possible that does not invalidate the determinant role of preattentive individuation in subitizing. The novel interpretation I offer here is that attentional effects on subitizing performance are not inconsistent with Trick and Pylyshyn's account since it allows for such attentional effects via the task-relativity of pre-attentional individuation of indices. In other words, studies claiming to invalidate Trick and Pylyshyn's model by showing influence of attention on subitizing performance and interpreting this as meaning that subitizing is attentional in nature conflate sub-personal and personal processes: the influence of attention on subitizing, I claim, is due to the fact that personal-level attention-related task-setting processes modulate where sub-personal parallel indexing takes place, which in turn influences subitizing performance.

So, in line with the finding mentioned above according to which MOT and subitizing compete for indexes (Chesney & Haljian 2011), performance variations observed and reduced subitizing range in these studies could reflect the fact that some individuation indexes were taken up by the primary task, meaning that less of these remained to be exploited for enumeration tasks.[12]

This pre-attention-friendly reinterpretation of Railo and colleagues' (2008) study applies to that of Vetter and colleagues' (2008) as well: in both cases, it appears sensible to consider the possibility that the attentional effects on subitizing were due to there being less visual indexes remaining for enumeration due to the demands of the main task. This interpretation explains different effects between high- and low-load conditions: the high load condition would recruit more indexes, given that there are more perceptual variables to pay attention to, which would leave less resources for subitizing than in the low load condition. If I am right, such studies showing attentional effects on subitizing by varying attentional load are not inconsistent with Trick and Pylyshyn's claim that subitizing *depends on* preattentive individuation mechanisms. The attention required by the primary task can direct individuation in a way that affects individuation in the secondary task, even if such individuation occurs pre-attentively.

There is ample evidence that the nature of the task can determine at which stage attention is deployed, which means that some tasks can compete for the same pre-attentional resources (Wu 2014, 20). In general, the same re-interpretation of these findings based on the individuation-hungry demands of attention to a main task could also apply to other recent studies questioning the fact that subitizing relies on preattentive parallel processing, including those listed above, which space limitations prevent me from attending to in further detail.

The general problem here seems conceptual in nature: critical studies confound saying that we can explain features of subitizing by appealing to features of pre-attentive parallel individuation in visual indexes, as Trick and Pylyshyn do, and saying that subitizing *is itself* a pre-attentive phenomenon, as critics of the pre-attentive interpretation seem to be saying. The problem is, these are two radically different claims: on the one hand, we are saying that the key features of subitizing – including its speed, accuracy, confidence levels, and limited range – are *explainable* due to the pre-attentive parallel individuation processes on which it is based, while on the other, we are interpreting subitizing as being itself something that happens at the pre-attentive level.

Of course, even if my reinterpretation hits the mark, the recently available brain data indicating that attentional networks are activated in the subitizing range[13] appear to constitute rather telling proof that attention *does* affect subitizing. Happily, there is a sense in which it is not debatable that subitizing requires attention: subitizing requires a conscious, personal-level decision, that of enumerating a number of perceptually-presented objects. People do not go about subitizing in the same way they go about breathing, building red blood cells, or individuating visual indexes at the sub-personal level: one must *intend* to enumerate in order to subitize.[14] Given that enumerating is intentional and goal-driven, chances are that subitizing requires attention.[15] Indeed, this is why subjects in the experiments above could perform enumerative or other tasks at all: they had to pay attention to the right stimulus, and complete the tasks asked of them by researchers.

[12] Another possibility here which was suggested to me by an anonymous reviewer is that such studies show competition for temporal resources: the idea here would be that setting up indices takes time and so competing tasks do not allow enough time for subjects to set up enough indices for all tasks, thereby affecting performance. Space limitations prevent me from addressing this interesting and plausible possibility in full, but I will allow myself to say that such a time-based explanation does not seem to explain why the subitizing limit would share features of MOT or other working memory limits, since many experimental tasks showing these limits do not seem to impose time limits, including the after-image paradigm mentioned here.

[13] E.g. Pagano et al. 2014; Piazza, M., Fumarola, A., Chinello, A. and Melcher, S. 2011; Burr et al. 2010; Ansari et al. 2007.

[14] Sadly, on top of the confusion regarding its (pre-)attentional status, there appears to be confusion regarding a related aspect of subitizing, namely, whether the term is supposed to describe our *ability* to enumerate quickly, or the cognitive *system* on which this ability rests. While little ambiguity seems present in Kaufman and colleagues' initial phrasing (quoted above), gradual spread of the use of this term seems to have allowed it to pick up some extra semantic baggage over time, leading many (e.g. Núñez 2017) to use it to describe abilities present in animals and pre-verbal infants. While this terminological confusion does warrant further attention, it need not concern us here, as the literature targeted remains faithful to the original meaning of the word.

[15] While it could be argued that intentional, goal-driven action does not imply attention, this appears especially unlikely in the case of subitizing, given its numerical nature. And besides, as Wu points out, goals usually do involve attention: "the subject's goals pervasively influence attention, so much so that some theorists have questioned whether there is attention without the influence of goals" (Wu 2014, 38).

Thus, the fact that it has been shown that subitizing requires attention is not altogether much of a surprise, nor need it undermine the validity of Trick and Pylyshyn's account, as long as it is possible to explain attentional effects by appealing to preattentive interactions, as I have proposed. Indeed, such interactions seem unavoidable, given the intimate relation between pre-attentive and attentive processes, including sharing capacity limits, as alluded to above. So, while there is evidence that attention is necessary for subitizing, it seems easy to accept this as confirmation that attentional processing relies on pre-attentional selection, and that tasks compete both for the attentional resources and the pre-attentional selecting and indexes that must accompany any attentional processing.[16]

Indeed, the fact that neuroimaging data show some kinds of attention are involved in subitizing can actually be considered support for Trick and Pylyshyn's view. For example, evidence was found that subitizing recruits brain areas associated with stimulus-driven attention (Ansari et al. 2007). This accords well with Pylyshyn's visual indexes, which "are assigned primarily in a stimulus-driven manner, so that salient feature characteristics or changes are automatically indexed" (Sears & Pylyshyn 2000, 2). Thus, it seems false to talk of challenging "the traditionally held notion of subitizing as a pre-attentive, capacity-independent process" (Vetter et al. 2008, 1).

Conclusion

If my reinterpretation of certain studies of the relation between attention and subitizing makes sense, empirical literature showing that attentional demands influence subitizing performance is not a problem for Trick and Pylyshyn, since their claim has never been that subitizing *itself* is preattentive, but rather that its special features can be explained in terms of features of a preattentive visual indexing mechanism on which it is based.[17]

Even if my proposed reinterpretation works, however, the consensus now seems to be that individuation is one of attention's main roles (Cavanaugh 2011). Worse, the very notion of a distinction between preattentive and attentive processing has been questioned, as has the possibility of preattentive processing (e.g. Joseph, J. S., Chun, M. M., & Nakayama, K. 1997; Duncan & Humphreys 1989. See also Vetter et al. 2008, 1). This means that it may be impossible for Trick and Pylyshyn's visual indexes to be accurately described in terms of preattentive individuation processes, since there may be no such thing.[18]

Such implications show how investigation into the nature of subitizing raises complicated conceptual questions, including that of the relationship between attentive and preattentive processes, and how objects are individuated. Unfortunately, as has often been remarked (e.g. Wu 2014), attention itself is a particularly confused notion, and a number of important questions in attention research have not found solutions because "the key concepts (selection, automaticity, attention, capacity, etc.) have remained hopelessly ill-defined and/or subject to divergent interpretations" (Allport 1993, 188, cited in Wu 2014).

Despite the problematic conceptual confusion around the nature of attention - which we will not address here for obvious space-related reasons - the claim being made here is relies on a relatively benign conceptual distinction. Even if the distinction between pre-attentive and attentional processes is not clear, it is clear that some processes labelled pre-attentive are sub-personal and beyond cognitive control, while others are not. In the former category we find parallel individuation of visual indexes, while in the latter category we find subjects' ability to choose tasks that will direct where such parallel individuation takes place. The question of where to draw the conceptual line at pre-attentive vs attentive – if anywhere – is secondary to the main issue here, which is whether characteristics of subitizing can be explained in terms of personal-level attention or in terms of sub-personal individuation, regardless of their precise relation.

In this sense, the novel interpretation offered here shows Trick and Pylyshyn's account still holds, since it actually predicts impact of attention-driven task-related individuation of visual indexes at the sub-personal level. The important insight that Trick and Pylyshyn offer is that subitizing relies on an individuation mechanism that *automatically* tags a limited number of objects for further processing, depending on the *chosen* task: "A critical aspect of the explanation is the assumption that individuation is a distinct (and *automatic*) stage in early vision and that when the conditions for automatic individuation are not met, then a number of other phenomena that depend on it, such as subitizing, are not observed." (Pylyshyn 2003, 175, emphasis added) So while there remains a number of controversies surrounding our ability to enumerate, including the number of cognitive systems involved and the role of attention, the interpretation offered here shows that Trick and Pylyshyn's visual indexing account still affords a plausible and well-supported explanation for the unique features of subitizing.

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[16] See Dehaene & Cohen 1994, who noted years ago that attentional processing goes hand in hand with pre-attentional processing. See also Trick & Pylyshyn 1994 and Gliksman, Y., Weinbach, N., & Henik, A. 2016, for evidence that cueing can influence performance in the subitizing range.

[17] The same reasoning applies to the claim that object selection in MOT could be directed by attentional processing (Oksama & Hyona 2004): participants in MOT are asked to pay attention to objects and track them, so there is clearly an attentional component here too. But this attentional component's essential contribution to these processes must not be overstated: if I intend to take a deep breath and then attend to my breathing, does that make breathing an attention-dependent process? Of course not.

[18] Illustrating that progress can be made without settling the preattentive-attentive divide, some authors (Pagano et al. 2014; Mazza 2017) simply speak of an 'attention-based' individuation mechanism behind subitizing.

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