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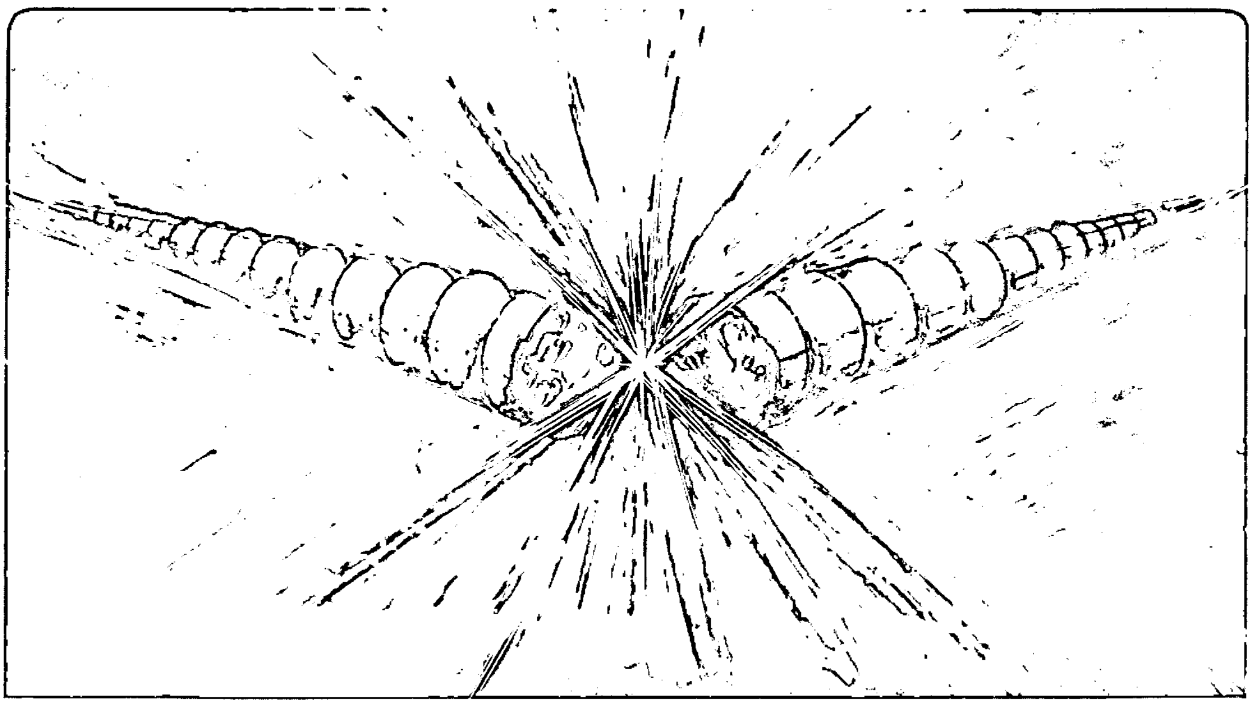
August 1983



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CHARACTERISTIC TIME STRUCTURE IN SLOW RESONANT EXTRACTED BEAM*

LBL-16003

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Summary

The phase diagram for resonant extraction is normally shown as a sharp line, passing through the unstable fixed point, dividing the stable region from the unstable region. This line is a sharp boundary line in the limiting mathematical case where the particle passes through all points on the line. In the physical case, a specific particle occupies only a limited number of phase points along the line. As a result, the line becomes a band of finite width containing a number of regions of stability and instability. The width of this band and the time structure extracted from this band is related to the tune shift and shape of the perturbation used in the extraction system.

This paper discusses the physical reasons for this effect and presents computer calculations showing the time structure for the NU equals two-thirds extraction system in the Bevalac. Photomultiplier pictures of beam structure taken before and after recent changes in the perturbation magnet show similar changes in time structure corresponding to changes in operating value of the NU shift used for extraction.

Introduction

A normal phase diagram for slow resonant extraction at the Bevalac is shown in Fig. 1. The beam inside the closed area is in the stable region. Any beam just outside the closed area is unstable and would move around the stable region to the unstable fixed point FP and then move radially inward along the separatrix (s). The task in designing a resonant extraction system is to select the field tune (NU) and the shape of the perturbation required to produce the necessary growth rate and derivative at the septum of the extraction magnet. This involves studying the motion of the particle along the separatrix. Any characteristic time structure in the beam spill associated with the extraction mechanism is determined by the beam response in the shaded region of the stable area, as shown in Fig. 1, which feeds the beam into the separatrix. This is the region of study in this report.

In the limiting mathematical model, where all phase points on the boundary of the phase diagram are swept out, the shaded area is stable. In the actual case, only a limited number of phase points along the line are touched. A particle precessing around the phase diagram and passing through the region of the perturbation is left with a net gain or loss in amplitude unless it passes symmetrically through the perturbation. This is easy to show for a simple model where a particle makes a single precession through the region of the perturbation. The question then is to determine if this effect is coherent over many turns. The results presented in this paper show that there are regions where this effect is coherent.

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As a result, there is a band as shown by the shaded area in Fig. 1 where some particles grow slowly until they move outside of the outer boundary and then move rapidly along the sepatrix to the septum of the extraction magnet. Other adjacent particles may remain stable. In the case of the Bevalac, some of these delay times can be several milliseconds. These time constants must be considered when designing feedback spill controls for beam extraction.

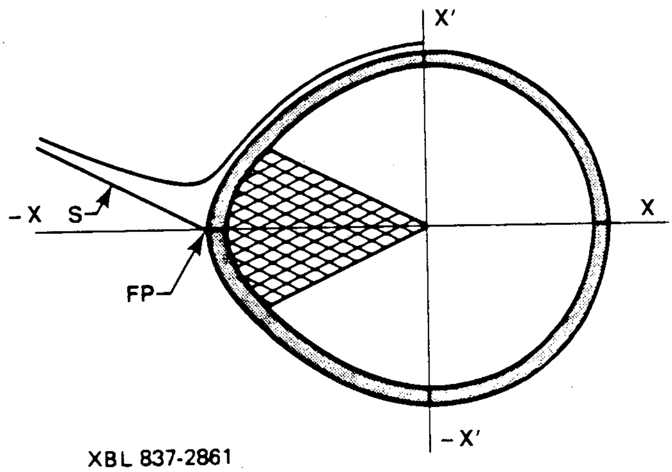

Fig. 1. Phase diagram for resonant extraction. Shaded area shows region of bands of instability.

Resonant Extraction

I will now examine this effect for the Bevalac slow extraction system. This system is designed to work at NU equals two thirds, with the perturbation providing a NU shift of 0.02 and a Bevalac tune of NU equal to 2/3 - 0.02. With these values, a particle will have one traversal of the perturbation per three turns around the machine when in the cross-hatched area of Fig. 1. The rest of the time the particle will make two traversals of the perturbation per three turns around the machine, once in the positive x' region and once in the negative x' region. The particle will precess in the counterclockwise direction for NU less than two thirds.

The Bevalac synchrotron is a weak focusing machine with four quadrants and four straight sections. The matrix transformation for one-half straight section-quadrant-one-half straight section was calculated. These results were used to evaluate the terms in the phase amplitude solution to Hill's equation. This solution was then applied to one turn around the Bevalac. In the literature on betatron oscillations, most authors derive a quantity MU which is given by cos MU = 1/2 trace of the matrix transformation for a section of the accelerator. The matrix transformation for a section is calculated by taking the product of matricies for each element in the section. The quantity MU is called the constant phase advance for that section of the accelerator. What is not clearly stated is that in all cases with drift spaces this constant phase advance MU is a mathematical parameter and not the physical phase of the particle. It is therefore necessary to be careful when physical phase information is needed. In this case, I have derived the first derivative for the phase-amplitude solution to Hill's equation. This derivative is not in general derived in the literature.

The solution to Hill's equation is then:

$$x = W(s)\,A\cos(\Psi + \delta) \qquad (1)$$

$$x' = W(s)\,A\,(\sin MU/K^{1/2}\ b)\,\sin(\Psi + \delta) \qquad (2)$$

where $x' = (1/NU)(dx/d\theta)$; $W(s)$ is the amplitude function in the solution to Hill's equation of the form $x = W(s)\,e^{i\Psi(s)}$. The matrix transformation is of the form

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

The choice of $(1/NU)(dx/d\theta)$ for the derivative is to directly display the amplitude of the betatron oscillation in the phase diagram, that is both x and x' have dimensions of distance.

Computer Calculations

Equations 1 and 2 were solved in a computer program which added in the appropriate value for the perturbation each turn around the Bevalac. If the particle moved on to the separatrix and came out to a selected radial position, the calculation was terminated and the number of turns from the start, the displacement, and the derivative were printed. In addition, one hundred bin counters were set. Each sixty turns in the calcuation incremented the counter index by one. When the calculation was terminated, whichever bin counter was indexed at that time had one count added. This gave a time profile of beam structure in terms of sixty turns per bin which is about 30 μs for normal Bevalac fields. The calculation was also terminated at 6000 turns (3 ms) and the particle considered stable.

Scans were made in a series of 100 steps over various widths at the edge of the stable regions for two values of delta NU (0.02 and 0.05).

For reference purposes in the following sections, the radial width of the circulating beam in the Bevalac is about 30 cm (full width) at intermediate field values.

Results

Spatial Structure

The band of instability for a NU shift of 0.02 is about 1.27 cm and about 0.254 cm for a NU shift of 0.005. The spatial bands of instability are shown in Fig. 2 and Fig. 3 for these two cases. In the case of delta NU of 0.02 (Fig. 2), only the first 0.254 cm is shown out of the 1.27-cm band. The abscissa is position in parts 1 to 100 of the width scanned. The ordinate is in time bins 1 to 100, each bin being 60 turns (30 μs). If the particule goes to bin 100, it is considered stable.

In the NU shift of 0.02, 47 percent of the beam was unstable in the 1.27-cm width. For the NU shift of 0.005, 56 percent of the beam was unstable in the 0.254-cm width. In the first 0.254 cm, 78 percent of the beam was unstable for the case of a NU shift of 0.02.

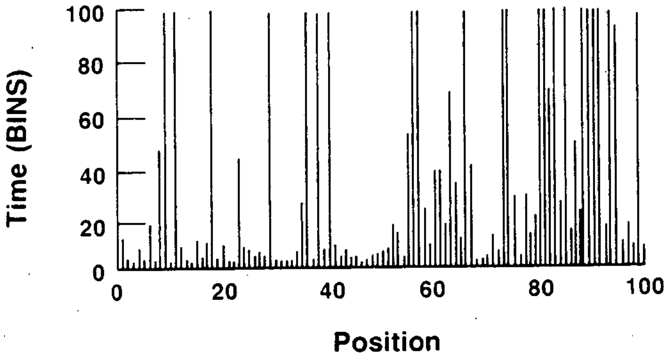

Fig. 2: NU shift 0.02

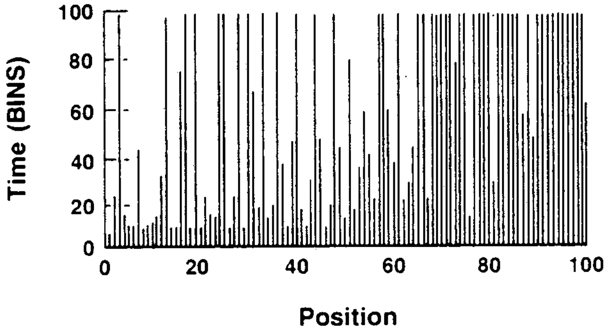

Fig. 3: NU shift 0.005. Time to extract particles vs position relative to edge of beam

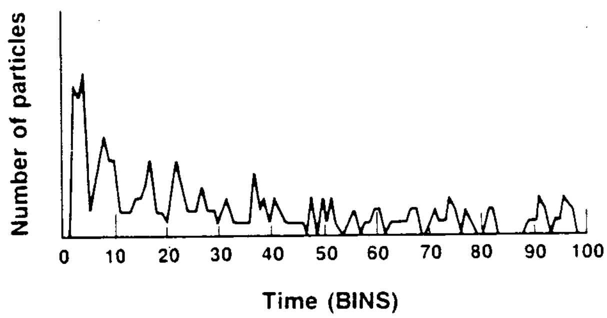

Fig. 4: NU shift 0.02

Time Structure

The time structures for the two cases are shown in Fig. 4 and Fig. 5. The time base is in terms of 100 bins. In this case, 100 bins is 6000 turns or about 3 ms total time. Each bin is then about 30-μs wide. In the case of NU shift 0.02, the beam is more

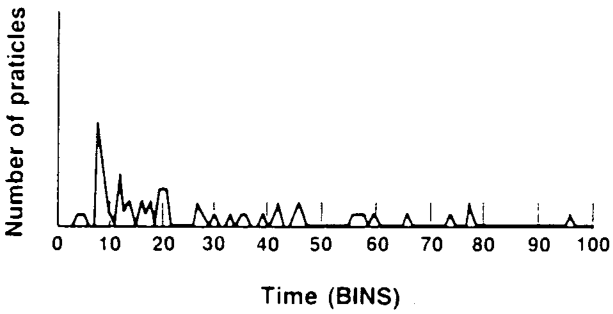

Fig. 5: NU shift 0.005 XBL 837-2852

uniformly spread over the entire period (3 ms) than in the case of a 0.005 NU shift.

Modulation

A sinusoidal modulation to the displacement can be added representing radial field motion caused by ripple in the magnetic guide field. This is shown in Fig. 6 and Fig. 7 with a maximum radial motion of plus

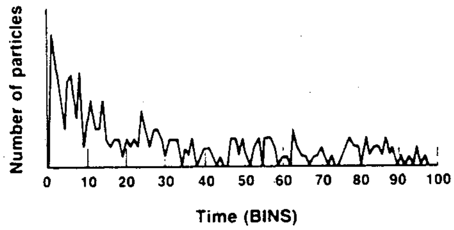

Fig. 6: NU shift 0.02. Radial modulation of position

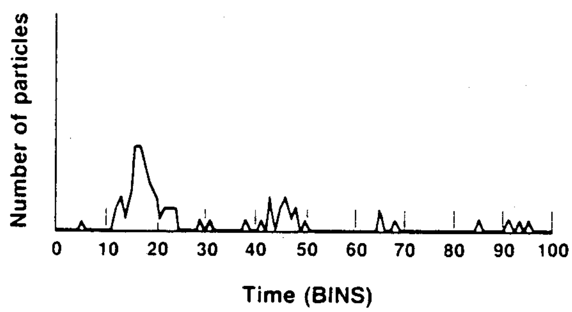

Fig. 7: NU shift 0.005. Radial modulation of position

or minus 0.0254-cm modulation with four cycles of oscillation in 6000 turns. In the case of a NU shifts

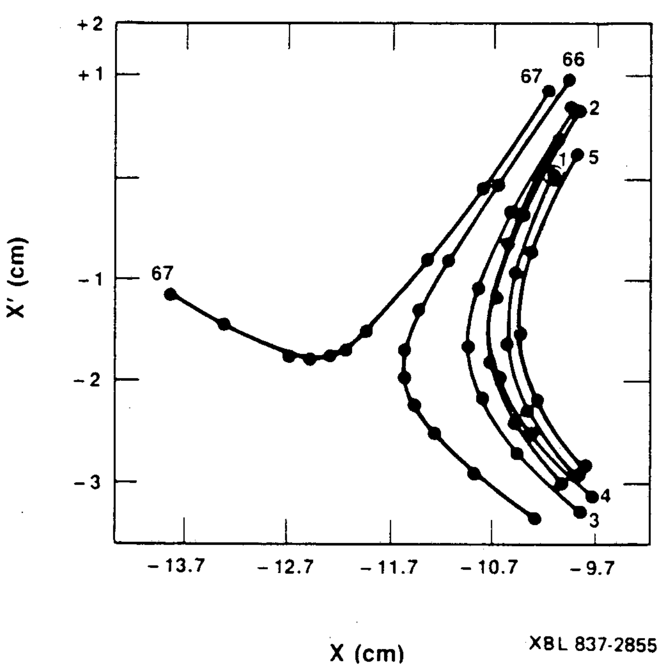

Fig. 8: Trajectories near unstable fixed point

of 0.005 (Fig. 7), the time structure shows this four-cycle modulation very clearly. With a NU shift of 0.02, the four-cycle modulation does not appear (see Fig. 6).

Extraction Path

A typical beam path in phase space on successive turns in the region of the unstable fixed point is shown in Fig. 8. Only a limited number of trajectories near the unstable fixed point are shown. Note that trajectories 2 and 3 have successively larger displacements. Trajectory 4 has reduced amplitude just inside trajectory 2, and trajectory 5 has a smaller displacement than the initial trajectory. The trajectories vary (not shown) until the particle finally is extracted on the 67th trajectory. This is about 2600 turns around the accelerator or ~1.3 ms.

The initial conditions for this particle is about 0.64 cm inside the standard mathematical boundary separating the stable and unstable regions. Particles started at 0.00254 cm on either side of this particle are stable to 6000 turns.

Conclusions

These examples are all based on the assumption that the closed orbit of the beam is at the x equals zero position and the perturbation is applied suddenly. Particles outside the normally stable region are lost rather quickly and are ignored.

In the actual physical case, we would start with the partially filled space as shown in Fig. 2 and Fig. 3, and shift the perturbation or orbit to produce beam extraction. The specific details of the time structure would be somewhat different than shown here. There is still a band containing regions of stability and instability along with the associated time structure.

To make a more detailed calculation to compare to observed structure would be very difficult. The ripple on the guide field causes orbit distortions as well as radius of curvature changes. We have no detailed knowledge of this resultant motion to use in a calculation. The important point is to be aware of the existance of this effect and the resultant time constants associated with the extraction.

The new cryogenic liner in the Bevatron required a new perturbation magnet. We are at present operating with a smaller NU shift than with the original magnet. The time structure in the present extracted beam spill is at a higher frequency and is more sensitive to magnet ripple than with the original extraction system (see Figs. 6 and 7). The frequency response of the feedback extraction system was unable to control the spill. The frequency response of the feedback system has been increased and is now better able to control the spill. However, we need to increase the NU shift we are at present operating with to decrease the sensitivity of the extraction mechanism to noise. This requires taking operating time from the experimental program to make the necessary NU measurements. Because of the limited hours of operation for the past few years from budgetary constraints, the experimental program has taken top priority.

Basically I have been discussing time structure in the resonant extraction system at the Bevalac. But I would like to point out that in general there is a certain danger in describing a basically delta-type physics event in terms of a continuous mathematical function if we only look at the physical results of the continuous function solution. I am sure there are more of these effects in the field of accelerator physics and suspect there are many more in other fields of physics.

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