Title
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Abstract

The purpose of this study was to find the level of abstraction that facilitates transfer in mathematical problem solving. Two experiments in this study showed that subjects who made good abstraction showed better transfer (Experiment 1), and it is possible to teach an abstracted schema quickly (Experiment 2), although a hint is necessary in testing. The abstracted schema was the idea of how to construct correct equations for target problems. This schema was at an more abstract level than the form of equations. Thus, we argue that the process named *solution compression*, in which two or more equations are considered to be constructed from one idea, is needed in order to generalize this schema and to promote transfer in mathematical problem solving.

Introduction

To become proficient in solving mathematical problems, one of the most important skills is to transfer knowledge that was acquired from examples to novel problems (target problems). Transfer is difficult to obtain (Reed, 1993). Rather small changes in a problem can greatly reduce the effectiveness of an example (Reed, Dempster, & Ettinger, 1985).

The basic mechanisms that yield transfer are analogy (Anderson, 1987; Gick & Holyoak, 1983; Pirolli & Anderson, 1985; Reed, 1993; Ross, 1984) and abstraction (Gick & Holyoak, 1983; Ross & Kennedy, 1990; Suzuki, 1995; Suzuki & Kuriyama, 1996). In this study, using algebra word problems, we try to form transfer through abstraction. We use target problems that have elements of the correct equation which cannot be generated by analogy. Each of the target problems in this study can be solved with similar equations to the one used in the example problem. However, a problem solver has to modify the solution of the example because part of the equation is changed in the target problems. We will call these target problems *similar* target problems. In contrast to this study, most of the studies on transfer have used *isomorphic* target problems that can be solved with the same equation as the one used in the example.

Holyoak, Novick, & Melz (1994) insisted that the process named *adaptation* was needed in order to solve similar target problems. Adaptation will be required whenever the underlying structures of the example and the target problems are not completely isomorphic. If the target problem has elements that do not correspond to anything in the example, additional target-generated inferences (i.e., adaptation) will be required to supplement those generated by analogy. Holyoak et al. (1994) reanalyzed the results of Reed (1987) and showed that the difficulty of transfer to similar target problems was due to the difficulty of the adaptation. Novick & Holyoak (1991) also obtained similar results. Although most of the works on transfer have used isomorphic problems, there is no guarantee that the underlying structures (i.e., the form of equations) of an example and a target problem are fully isomorphic; indeed, this will seldom be the case for mathematical problems. Moreover, similar problems are often more difficult than isomorphic problems (Reed, 1987). Thus, it is quite meaningful to explore what knowledge should be acquired and how the knowledge is acquired in order to promote transfer to the similar target problems. This is the basic aim of this study.

If problem solvers cannot solve the similar target problem by analogy, it is expected that an appropriate abstraction from example problems or their solutions facilitates transfer (Gick & Holyoak, 1983). However, there is a little evidence that the abstraction promotes transfer in mathematical problem solving (Reed, 1993). Gick & Holyoak (1983) and Catrambone & Holyoak (1989), using Dunker's radiation problem and its isomorphic problems, showed that an appropriate schema abstraction promotes transfer. Although their results are impressive, it is unclear whether they could be duplicated for complex problems such as algebra word problems (Reed, 1993). Indeed, a negative result that failed to support the schema abstraction hypothesis was provided (Reed, 1989). Moreover, it is not isomorphic problems but similar problems that we will use. Ross & Kennedy (1990), using isomorphic probability problems, showed that an abstraction plays an important role in transfer, that is, the use of earlier examples promotes generalizations about problem types and affects later performance. However, Ross & Kennedy (1990) did not examine the contents of the knowledge (schema) that the students acquired. In other words, it is unclear what and how knowledge is acquired. Bernardo (1994) showed that the problem-type schemata included both problem-specific and abstract information. However, it is unclear whether these schemata facilitate transfer because he used priming paradigm (he examined, not problem solving, but recognizing problem sentences).

The purposes of this study are (1)to show evidence that an appropriate abstraction facilitates transfer to similar target problems, (2)to explore what abstract schema should be acquired, and (3)to discuss the processes of schema abstraction and illustration. We have carried out two experiments in order to achieve these purposes.

Overview of Study

Both of the experiments in this study provide evidence an appropriate abstraction from examples facilitates transfer: in order to promote transfer, it was useful to induce or to teach the idea of how to construct correct equations This result is quite meaningful because little evidence has been presented that show this effect of some abstraction. In contrast to many previous studies which considered the forms of correct equations themselves as the abstract structures of algebra word problems (e.g., Reed, 1987, 1989; Reed et al., 1985, Ross & Kennedy, 1990), we dealt with the idea of how to construct correct equations as an abstract structure (schema).

What are the properties of the idea ? Are they different from those which are included in the schema induced from puzzle-like problems (e.g., Dunker's radiation problem) (see Gick & Holyoak, 1983) ? Why is it useful to induce or teach the idea in order to facilitate transfer to similar target problems ? What abstract schemata are acquired and how ? We will discuss these questions in general discussion, and argue that (1) to induce or teach the idea means to form more abstract schema than the schema induced from puzzle-like problems, (2) this type of abstract schema is useful for complex problems such as algebra word problems, (3) three types of the processes of acquiring the abstract schemata are considered. We will call the process to make more abstract schema, such as the idea of how to construct equations, *solution compression*.

Experiment 1

This experiment is a correlational study that shows inducing an abstract schema from examples is a predictor of the amount of transfer.

Method

Participants. 50 high-school students participated in a collective paper-and-pencil test during their normal mathematics classes.

Materials. The following three problems were used.

Pond Problem: Three person A, B, C went around a pond, starting at the same point, to the same direction. They started at the same time. Person A was walking at 70m per minute, B was running at 150m per minute, and C was going around by bicycle. It takes five minutes from starting for C to catch up with A. Four minutes after C had caught up with A, C caught up with B. Find the average speed of C. (The correct equation: $5x - 70 \times 5 = 9x - 150 \times 9$)

Clock Problem: There is a round clock that has 60 graduations keeping equal angles. Now, it is twelve o'clock, the minute hand is on top of the hour hand. When will the minute hand be on the hour hand next again ? (The correct equation: $6x - 0.5x = 360$)

Train Problem: A man walked along with rails at an average speed of 4 km/h. One group of the trains running in the same direction of the person passed him every 10 minutes. Also, another group of the trains running in the opposite direction to the person met him every 8 minutes. All the trains ran with keeping a constant distance and at the same speed. Find the average speed of the trains. (The correct equation: $10x - 4 \times 10 = 8x + 4 \times 8$)

The participants were given the pond problem first, and then the clock problem second, the train problem last. The clock problem and the train problem were the example and the target problem, respectively. The clock problem was the target problem for the pond problem, but was the example problem for the train problem.

The forms of the correct equations for these three problems are different from each other. Many previous studies have considered the forms of correct equations themselves as the abstract structures of algebra word problems (e.g., Reed, 1987, 1989; Reed et al., 1985, Ross & Kennedy, 1990). The form of correct equation for the pond problem (example) is Rate C $\times$ Time C - Rate B $\times$ Time B = rate C $\times$ Time C Rate A $\times$ Time A. This form of the equation has been considered to be the abstract structure and to have eight slots. The form of correct equation for the clock problem (target) is Rate H $\times$ Time H Rate M $\times$ Time M = the angle of a round. The right side of the equation for the target problem is changed from the one for the example. Students often show difficulty in generating altered quantities even if the forms of the equations become easy by this change (Reed, 1987). A possible remedy for generating altered quantities is to build instruction around procedures that can generate quantities that are in the target problem but not in the example (Reed, 1993). Procedures can be attached to slots in a schema for the purpose of generating values to fill the slot (Bobrow & Winograd, 1977). Reed & Bolstad (1991) tried to facilitate transfer by using this method: they taught students the example solution, the form of the equation, and the set of procedures. However, this method was not useful for promoting transfer to similar target problems. Moreover, it is difficult to use this method for making transfer to the train problem because the form of equation is extremely changed (see the right side of the equation). Indeed, previous studies on transfer did not consider problems such as the train problem to be a target problem for examples such as the pond problem and the clock problem because of the extremely changed form of equation (e.g., see Reed, 1987, 1993). Our original attempt is to try to obtain transfer to such problems as the train problem.

In contrast to these previous studies, we consider an idea of how to construct correct equations as an abstract schema induced from example. The idea is as follows: when an object catches up with another object, a difference between the two distances is equal to a particular distance. Although this idea may look like the form of equation itself, there are some important differences. First, this idea teaches us how to find the solutions to the problems. In other words, students can learn from the idea to construct the left side of the equation first, then to find the right side. But the form of equation itself does not tell us this order. Second, this idea is available for all three problems, but it is difficult to find common form of the three equations. Especially, the form of the equation for the train problem is different from the one for the pond problem or the clock problem. Indeed, the above-mentioned method by Reed et al. (1991) is not available for facilitating transfer to the train problem from the examples in our study. In short, this method cannot accept the changing of arithmetic signs (changing from − to +) in the right sides of the equations because the method does not consider the arithmetic signs as slots. Third, the idea is a more abstract schema than the form of the equations because two or more slots of the equations are changed to one component. For example, the right side of the equation for the pond problem, having four slots, is changed to one component: a particular distance.

Procedure. As mentioned above, the participants tried to construct correct equation for the pond problem first, and then the clock problem second, the train problem last. They were given five minutes for each problem. In testing, the following hint was given to the students: this problem is similar to the problem which has just been learned.

After the attempt to solve the pond problem, the solution was provided. The participants were required to understand the solution for four minutes. The same procedure was repeated after the attempt to solve the clock problem. For the clock problem, the provided solutions had the following contents: in comparison with the hour hand, the minute hand will go around the clock one time more, thus, the difference between two angles ($6x - 0.5x$) is equal to the angle around the clock, that is, 360. Similar explanation was provided for the pond problem.

The participants were asked the following question after they understood the solution to the pond problem: please describe what you learned (or found) from the problem as to (1) what is important in order to solve this problem, (2) why did you fail to solve this problem if you failed to do so, (3) what do you need to notice when you meet similar problems. These questions are used in *cognitive counseling* for learners to induce useful lessons after problem solving (Ichikawa, 1991). Four minutes were spent on this task. The participants' responses to this question were expected to reflect the contents of schemata that the students acquired from the example. We assigned the participants to two groups based on the responses to this question. The participants who referred to the above-mentioned idea of how to construct the correct equation were assigned to the *good-abstraction* students' group. The participants who did not refer to the idea were assigned to *poor-abstraction* students' group. We expected that the *good* students would show better transfer to the clock problem than the *poor* students did.

Then a test that measured the degree of the understanding of the provided solution was done for four minutes. The test asked the students what each element of the correct equation (for example, 350, 9, 9x-350, and so on) represented. An example of an item of this test was as follows: what does "$5x-350$" represent ? The participants were prohibited from seeing the solution. If a participant correctly answered one of the items, he or she received one point. This test was marked on a maximum scale of seven points. The reason that we carried out this test was to check the above-mentioned assignment of the participants. It was possible that the *good* students were able to understand the solution to the pond problem better than the *poor* students did. That meant that the assignment was not based on the participants' responses to the question about acquired schemata. To deny this possibility, we had to check the degree of the participants' understanding. It was also possible that the *good* students had better ability in mathematics than the *poor* students did. To check this possibility, we collected the participants' score of a test of mathematics that had been done two weeks before this experiment was carried out.

The procedure similar to the one for the pond problem was repeated for the clock problem before the participants tried to solve the train problem. In short, the participants were asked the question about their schemata acquired from the clock problem, and given the test of understanding, after the solution to the clock problem was learned. The test of understanding the solution to the clock problem was marked on a maximum scale of five points. The participants were reassigned to the two groups, that is, *good* students and *poor* students, based on the responses to the question about acquired schemata. We expected that the *good* students would show better transfer to the train problem than the *poor* students did.

The participants were required to solve the pond problem and the clock problem again, one week after their first attempts. Five minutes were given for each problem.

Results and Discussion

Data from seven participants were discarded because they succeeded in solving the pond problem.

Table 1 shows the proportions of participants who succeeded in constructing correct equations for each problem. The participants could attempt to solve the clock problem by using the schemata acquired from the pond problem (see the second column in Table 1) and to solve the train problem by using the schemata acquired from the pond problem and the clock problem (see the third column). When they were required to solve the pond problem and the clock problem again, they could solve these problems by using the schemata acquired from the pond problem and the clock problem (see the fourth and fifth column). The hypothesis is that the *good-abstraction* students show better transfer than the *poor-abstraction* students. We also expected that the *good* students would show better performance on the retest one week later. The overall tendency of the data supported these expectations.

Transfer to the Clock Problem. The *good* students showed better transfer to the clock problem (See table 1). In contrast to 27 % of the *poor* students ($N = 11$) succeeding in solving this problem, 56 % of the *good* students ($N = 32$) constructed the correct equations. This difference is significant ($\chi^2(1) = 2.75, p < .05$, one tailed). This result supports the above-mentioned hypothesis.

There were no difference as to the scores of understanding the solution for the pond problem. The mean scores of the *good* and *poor* students were 6.78 and 6.55, respectively ($t(41) = 0.96, p = .34$). The difference as to the scores of ability in mathematics was also not significant. The mean scores of the *good* and *poor* students were 28.2 and 28.3, respectively ($t(36) = 0.05, p = .96$). Thus, the data to doubt that the assignment was based on another criterion was not provided. [1]

Transfer to the Train Problem. The train problem was a very difficult problem because only six of 43 par-

[1] In Experiment 1, we collected the students who were on a certain level of achievement in their schoolwork, because we needed to show that the assignment was based, not on general math ability, but the contents of abstraction. In general, students with good abstraction skills may perform better on a general math ability test.

Groups	Clock	Train	Pond (retest)	Clock (retest)
Good	.56	.20	.87	.63
Poor	.27	.00	.46	.31

Table 1: Performance on each problem

ticipants (30 were *good*, 13 were *poor*) were able to solve this problem. Notice that all of the six students were in the *good* group (see Table 1). (Two of the six participants failed to solve the clock problem, and the rest succeeded in solving it. Of course, all of them failed to solve the pond problem.) The difference between the two groups was marginally significant ($p = 0.10$, Fisher's exact test, one tailed). This result supports the hypothesis that *good-abstraction* students show better transfer. There was no difference as to the scores of understanding the solution for the clock problem. The mean scores of the *good* and *poor* students were 4.53 and 4.23, respectively ($t(41) = 1.00, p = .32$). However, the difference as to the scores of ability in mathematics was marginally significant. The mean scores of the *good* and *poor* students were 30.3 and 23.3, respectively ($t(36) = 2.00, p = .052$). We met the limitation of correlational study. In the second assignment, it is possible that the participants were assigned to the two groups based, not on the contents of acquired schemata, but their ability in mathematics. We have to carry out a controlled experiment: Experiment 2.

Performances on Retest. As shown in Table 1, 87 % of the *good-abstraction* students [2] answered correctly to the retest of the pond problem in contrast to 46 % of the *poor-abstraction* students. There was a significant difference as to these percentages between the two groups ($p < 0.01$, Fisher's exact test, one tailed). All of the participants had failed to solve this problem on their first attempt. As to the retest of the clock problem, 63 % of the *good* students gave the correct equation in contrast to 31 % of the *poor* students. This difference was significant ($\chi^2(1) = 3.86, p < .05$, one tailed). Thus, we can say that the effect of good abstraction remained for one week.

Experiment 2

The results of Experiment 1 show that acquiring the idea of how to construct equations facilitates transfer. Although this experiment has the advantage of examining directly the schemata that students acquired (we can know the contents of schemata from participants' responses directly), there is a well-known limitation of correlational study: the correlational study can not prove a causal relationship. Thus, we will teach students the idea of how to construct equations and confirm the results of Experiment 1 by doing a controlled experiment: Experiment 2. As mentioned above, the idea is a more abstract schema than the forms of equations themselves. No researcher has tried to teach students to represent problems at a more abstract level than the forms of equations (Reed, 1993, p.62).

[2]The second assignment was used because the reassignment reflects the final contents of the acquired schemata

Method

Participants. Participants were 76 high-school students.

Materials. Participants were given the clock problem as an example and the pond problem as a target problem.

Procedure. Participants were randomly assigned to three conditions, that is, *control* condition, *formula* condition, and *compression* condition.

At first participants were required to solve the example. The correct solution to the example was shown to the participants after they tried to solve it for five minutes. The contents of the solution was the same as the one given in Experiment 1.

After the solution instruction, an explanation about this problem was presented. In the control condition, the subjects were taught that we call these kinds of problems "catching-up-with" problems. In the formula condition the following explanation was added to the one given in the control condition: the formula of the equation of this kind of problem is Rate A $\times$ Time A - Rate B $\times$ Time B = a particular distance. In the compression condition the following explanation was added to the one given in the control condition: the point of this kind of problem is that the difference between the two distances is equal to a particular distance.

The participants then proceeded to the target problem. They were given five minutes to solve it without any hints. After this first attempt, the participants were given four minutes to solve this problem again with the hint: this problem is similar to the clock problem which has just been learned.

Results and Discussion

Data from four participants were discarded because they succeeded in solving the example. Table 2 shows the number of participants who succeeded in constructing the correct equation to the target problem.

There were no differences among the three conditions without hint ($\chi^2(2) = 3.32, p > .10$). This result is in line with the results of previous studies that showed the difficulty of transfer without hint (e.g., Gick & Holyoak, 1983). The proportions of participants who succeeded in solving the target problem with hint were significantly slanted ($\chi^2(2) = 6.54, p < .05$). Thus we can say that participants succeeded in constructing correct solution more in the formula condition and compression condition than in control condition (see Table 2).

This means that the appropriate abstraction with hint facilitates transfer in the complex domain such as mathematical problem solving.

Although the participants in the formula condition showed good transfer, note that they were not taught the form of the equation itself. (see the right side. see also discussion in the explanation of materials in Experiment 1.) Participants in the formula condition would be able to interpret the explanation about the example just as the explanation in the compression condition. In Experiment 1, we have already shown that the *good-abstraction* students who had succeeded in making good abstract

Conditions		Before hint		After hint	
		Success	Fail	Success	Fail
Control	(N=26)	2	24	3	23
Formula	(N=23)	6	17	8	15
Compression	(N=23)	3	20	10	13

Table 2: The number of participants who can solve the target problem.

schema themselves showed good transfer. No *good* students referred to the form of equation itself. Therefore, we can conclude that students have to acquire the knowledge that is instructed in the compression condition in order to promote transfer.

General Discussion

What Knowledge Should Be Acquired ? Compressed Solution

Both experiments in this study show that acquiring the idea of how to construct correct equations facilitates transfer to similar target problems. This finding is quite meaningful because there is little evidence to show that a schema abstraction from examples facilitates transfer in mathematical problem solving. Moreover we succeeded in facilitating transfer to similar target problems which is difficult to obtain (Reed, 1987, Reed & Bolstad, 1991).

In this section, we will examine what the properties of the idea are, in other words, what knowledge should be acquired in order to promote transfer, especially to similar target problems. The previous studies have considered the forms of equations as the abstract structures of problems, but it must be difficult to memorize the form of an equation itself because it sometimes has many slots and there are various forms of equations in the domain of algebra word problems. Moreover, transfer to similar target problems is difficult to obtain even if students learn the forms of equations (Reed & Bolstad, 1991). The idea of constructing equations is a more abstract schema than the forms of equations as we have already mentioned (see Materials in Experiment 1). Students can construct various forms of equations from one idea as shown in Experiment 1. In short, two or more slots of an equation are compressed into one component. For example, the right side of the equation of the train problem, having four slots, are compressed into one concept: a particular distance. We will call this compression in abstraction "*solution compression*". The compressed solution in this study teaches students to construct the left side of the equation first, then to find the right side.

Solution compression is defined as compressing a whole procedure of a solution. Mathematical problems often have a long and complex solution procedure. It is difficult to memorize the whole procedure. Indeed, the transfer difficulty seems to stem from a tendency by many learners to memorize a solution procedure from examples rather than a more meaningful organization (Catrambone, 1996). Thus, students have to compress the procedure. This teaches us that a compressed solution is different from the abstract schema that is induced from a puzzle-like problem. For example, consider the transfer from Dunker's radiation problem to the fortress problem (Gick & Holyoak 1983). The tumor is changed to a "target" in the abstract schema, that corresponds to the form of equation in an algebra word problem. The notion of "target" is directly changed to "fortress" in the fortress problem. Although it is needed to create super-ordinate concepts (e.g., "target") to describe a more general schema (Reed, 1993), we could memorize the whole solution procedure to a puzzle-like problem. It is usually short and simple. For example it seems to be relatively easy to memorize the following abstract solution: if a target is difficult to overcome because a large force cannot be aimed at the target from one direction, divide the force into convergent small forces. Thus, the solution compression is not needed.

Another difference between compressed solution from mathematical problem and abstract schema from puzzle-like problem is found in the process of illustrating abstract schemata. Only one step is needed in the illustration of an abstract schema acquired from a puzzle-like problem. As to the above-mentioned example, the notion of "target" is directly changed to the "fortress". In contrast to this case, at least two steps are needed in order to solve a similar target problem. In our study, the idea of constructing equations would be expanded to a form of equation at first (the form of equation corresponds with abstract schemata from puzzle-like problems), and then the values to fill the slots of the equation are generated.

Student can construct the form of equation for a similar target problems before they generate values to fill the slots of the equation, by acquiring the idea of constructing equations. This is the reason why the compressed solution, the idea of constructing equations, are useful for facilitating transfer to similar target problems that cannot be solved just like the example.

We may be able to say that to acquire the compressed solution allows students to set a subgoal. For example, in this study, students can set the following subgoal: construct the left side of the equation at first and then find the right side. Prior work with the subgoal learning demonstrated that if a student learns the subgoal structure for solving problems in a domain, then he or she is more likely to adapt old procedures for novel problems (Catrambone, 1994, 1995, 1996) although the subgoal is set at a more abstract level in our study.

How Is the Knowledge Acquired ?

We have discussed what schema should be acquired. Now we will speculate how the schema or compressed solution is acquired. We do not have enough data to clarify the process of acquiring the compressed solution yet. However, we can begin to speculate on it.

We can consider three ways as processes of solution compression. The first is instruction by others (teachers, printed materials, and *etc.*). Experiment 2 is an example of this case. We taught students the compressed solution directly. This instruction was effective for facilitating transfer. The second is inducing schemata by learners themselves. As shown in Experiment 1, Ichikawa (1991) employed the questions to have learners induce useful lessons after problem solving. Ichikawa (1991) called this activity to induce lessons by learners themselves "lesson induction". The *good* students in Experiment 1 were considered to do good lesson induction, or to generate good self-explanations (Chi, Bassok, Lewis, Reimann, & Glaser, 1989). Note that many participants were able to acquire the compressed solution from only one problem: the pond problem (see the first assignment in Experiment 1). Abstract schemata have often been considered to be induced from making an analogy between two problems (Gick & Holyoak, 1983; Novick & Holyoak, 1991; Ross & Kennedy, 1990), but the results of Experiment 1 shows that doing lesson induction or self-explanations from only one example problem is also a good way to acquire an abstract schema. The third is to use analogy from different domain. Suzuki(1995) has taught his participants the solution for the "work" problem by using the following analogy: two persons start to eat a stick candy from each side. The participants who were given this analogy showed good transfer. An important point of the solution to the work problem becomes clear by this analogy. The point, compressed solution, is that the sum of the amount of work (stick candy) that each person did (eat) is equal to the whole amount of the work. In many previous studies, Analogies were always made between an example and a target problem (Gick & Holyoak, 1983; Novick & Holyoak, 1991; Ross & Kennedy, 1990). In contrast to these studies, Suzuki (1995) used an analogy between an example problem (work problem) and another example in a different and familiar domain (eating stick candy).

In this study, we have shown that an appropriate abstraction, to acquire the compressed solution, facilitates transfer to similar target problems. The compressed solution is a more abstract schema than the form of equation itself, which has been considered to be the abstract structure of solutions to algebra word problems. Our study contributes to clarifying what schema should be acquired in order to promote transfer, but do not have enough data yet to clarify the process of acquiring the compressed solution. Further research is needed in order to examine the process of acquiring the compressed solutions.

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