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Essays in Macroeconomics and Trade

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Essays in Macroeconomics and Trade

by

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Abstract

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In the current thesis I investigate the impact of sectoral structure of the economy on some aspects of its short-run fluctuations and long-run trends.

In the first chapter – "Cost Structure and Price Rigidity across Sectors" – I model a mechanism through which the structure of costs of producers can affect producers' decisions on the frequency of adjusting prices. First, I establish an empirical observation that sectors which are characterized by either a higher share of labor or more diversified structure of bundles of intermediate goods are characterized by more rigid prices. Then I build and solve a partial equilibrium model that describes optimal price-setting strategies of firms in different sectors. The model provides an explanation for heterogeneity in price rigidity across different sectors. The calibrated model can be used for predicting how changes in the production processes and in the structure of costs can affect the heterogeneity of price rigidity across sectors and, hence, the aggregate price rigidity in the economy.

In the second chapter – "Technological Spillovers and Dynamics of Comparative Advantage" – I investigate the question of the evolution of sector productivity and comparative advantage under the presence of cross-sector technological spillovers. For that I develop a dynamic model of international trade with cross-sector spillovers. In addition to the standard effect of comparative advantage on labor allocation, the model accounts for the effects of labor allocation on the sector productivity and comparative advantage. The core mechanism is a combination of an idea-generating process within each sector and technological spillovers across sectors. I establish necessary and sufficient conditions for the existence and uniqueness of a balanced growth path and describe the conditions under which a welfare-improving industrial policy is possible. I calibrate the model using the US patent data to parametrize the strength of technological spillovers and use the model to describe the optimal industrial policy.

To my parents, Vladimir and Zhanna.

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Chapter 1

Cost Structure and Price Rigidity Across Sectors

1.1 Introduction

Price rigidity plays a crucial role in transforming nominal shocks into changes of output and employment. Modeling this transformation mechanism is important for understanding business cycles because, according to [29], roughly 28% of output volatility is attributed to nominal shocks.

Heterogeneity of sectors in terms of price rigidity – a well-documented fact, see e.g. [7] – is an important part of this transformation mechanism. The intuition works as follows: the longer it takes for prices to adjust to their long-run equilibrium level, the more prolonged are the deviations of output and employment from their natural levels. If all producers in the economy have the same price rigidity then strategic complementarity plays a minor role in the adjustment process. Namely, firms that adjust their prices will do so by a larger amount knowing that others will adjust their prices shortly as well. Losses in profits due to large changes in relative prices will be short-lived and, thus, will not prevent adjusting firms from large adjustments. The opposite happens when firms are heterogeneous in terms of price rigidity. Now the adjusting firms know that there is a group of firms who will keep their prices constant for a while. This means that, say, a large upward adjustment in prices will entail long-lasting losses in market shares for the adjusting firms. As a result, even the firms with low rigidity will decide to adjust their prices gradually and, thus, deviations of output and employment from their natural levels can be propagated. According to [24] and [2] the presence of heterogeneity across firms in terms of price rigidity can increase the share output volatility attributed to monetary shocks from 1% to 20-23%, which is close to the above mentioned 28%.

Several empirical facts on the heterogeneity of price rigidity across sectors have been established, outlining the possible mechanism to explain different price-setting behavior across sectors. Competitiveness and variability of markups, share of energy, labor and imported inputs in costs, inattention are among the most potential channels for explaining heterogeneity of price rigidity (see, e.g. [8]). While the market share retention ([16]) and inattention mechanism ([21]) were studied in details, the heterogeneous structure of costs as a source of heterogeneous price rigidity remains underinvestigated. This gap is important, because according to surveys (e.g.[30]) the change in costs is named a number-one reason for adjusting prices, while the nominal fixed-term contracts (a form of menu costs) – the most significant reason for non-adjustment. To sum up, to generate a realistic responsiveness of output to monetary shocks one needs to model heterogeneity in price rigidity. The latter requires taking the structure and dynamics of costs into account.

In the current chapter I investigate the relationship between the structure of costs across sectors and the price rigidity of these sectors. The chapter is structured as follows: Section 2 introduces and solves a partial equilibrium model that allows to derive optimal price-setting strategies for sectors with different compositions of inputs. Section 3 establishes the evidence on composition of costs across sectors, on differences in the patterns of price adjustment and tests the predictions of the model. Section 4 outlines some extensions and possible implications for the model and directions for further research. Section 5 concludes.

1.2 Model

In the current section I construct and solve the partial equilibrium model that can result in different optimal price-setting strategies of firms in different sectors depending on their cost structures. Partial equilibrium characteristics of the model come from a simplified approach to modeling labor and capital markets, as well as consumption. The model provides a tractable explanation for heterogeneity in price rigidity across different sectors. The calibrated model can be used for predicting how changes in production processes and reallocations in cost structures can affect the aggregate price rigidity in the economy.

The model setting is similar to the one established in [1] and [28]. The economy consists of J sectors. There is a continuum of producers in each sector, each of them producing a unique variety of products (the mass of producers in each sector is normalized to 1). For the time being let's denote the producer of a particular variety with z. Every producer z in sector j at time period t can produce output $y_{t,j}(z)$ according to

$$y_{t,j}(z) = A_{t,j}\left(L_{t,j}(z)\right)^{s_j} M_{t,j}(z)^{1-s_j}, \tag{1.1}$$

where $A_{t,j}$ is a sector-specific total factor productivity (TFP), $L_{t,j}(z)$ – amount of employed labor, s_j – share of labor costs in total costs. Log of sectoral TFP evolves as a random walk:

$$\ln A_{t,j} = \ln A_{t-1,j} + u_{t,j}, \tag{1.2}$$

where $u_{t,j} \sim iid$, $E(u_{t,j}) = 0$, $var(u_{t,j}) = \sigma_j \; \forall t, j$. Assuming $A_{0,j} = 1$, we can rewrite $A_{t,j} = W_{t,j}$, $W_{t,j} = \sum_{\tau=1}^{t} u_{\tau,j}$, where $W_{t,j}$ can be thought of a Wiener process.

I assume the perfectly elastic supply of labor and fixed wage across all sectors, $w_{t,j} = w$ (wage serves as a numeraire). As follows from the production function specification, there is no capital in the model (or one can think about intermediate goods as capital that depreciates within one period). The interest rate is constant and is defined by a discount factor β from the conventional $(1+r)\beta = 1$.

$M_{t,j}(z)$ is a sector-specific bundle of intermediate goods that is used as an input:

$$M_{t,j}(z) = \left[\prod_{i=1}^{J} \left(\xi_i^j\right)^{-\xi_i^j} \left(m_{t,i}^j(z)\right)^{\xi_i^j}\right]. \tag{1.3}$$

$m_{t,i}^j(z)$ is the physical amount of input bought by producer z in sector j from sector i at period t. ξ_i^j is a weight of input i in sector j's bundle of intermediate inputs:

$$\sum_i^J \xi_i^j = 1 \forall i, j.$$

It is exactly the matrix of parameters $\xi_{(.)}^{(.)}$ that characterize the input-output structure of the economy in the model. If any element ξ_i^j is close to 1 it means that sector j depends heavily on inputs from sector i and, hence, shocks to sector i will significantly affect sector j.

Output of each producer z within sector j is aggregated into a sectoral aggregate product which, in turn, can be used either for consumption or as an input for production in all sectors, including j itself. The demand elasticity of substitution across varieties within each sector is equal to θ. Here I make an assumption that elasticity across varieties is the same for each sector (θ enters without sector subscripts). One more assumption is that elasticity of substitution is the same for final and intermediate demand. The price of sectoral aggregate product, in other words, price at which input $m_j^k(.)$ is sold from industry j to any industry k, is equal to:

$$P_{t,j} = \left(\int_0^1 P_{t,j}(z')^{(1-\theta)} dz'\right)^{\frac{1}{1-\theta}} \tag{1.4}$$

The sectoral price indexes are then combined into the prices of intermediate input bundles for industries as follows:

$$X_{t,j} = \prod_{i=1}^{J} \left(P_{t,i}\right)^{\xi_i^j} \tag{1.5}$$

Solving the cost-minimization problem of a producer in sector j at period t we can derive the cost function

$$TC(Y_{t,j}, \{P_{t,i}\}_{i=1}^{J}, A_{t,j}, w) = \frac{Y_{t,j}}{s_j}\left(\frac{s_j}{1-s_j}\right)^{1-s_j}(w)^{s_j}(A_{t,j})^{-1}(X_{t,j})^{1-s_j}$$

Differentiating this expression w.r.t. Y_t^j we obtain the marginal cost function for *every* producer z in sector j:

$$\psi_{t,j} = \underbrace{\frac{1}{s_j}\left(\frac{s_j}{1-s_j}\right)^{1-s_j}(w)^{s_j}}_{\equiv\Omega^j}(A_{t,j})^{-1}(X_{t,j})^{1-s_j} = \Omega^j\left(A_t^j\right)^{-1}(X_{t,j})^{1-s_j} \tag{1.6}$$

Now let's turn to the demand side of the model. Demand of producer z in sector j for input aggregate from sector i is equal to

$$m_{t,i}^j(z) = \xi_i^j\left(\frac{P_{t,i}}{X_{t,j}}\right)^{-1}M_{t,j}(z) \tag{1.7}$$

From equation (1.7) one can immediately see the interpretation of ξ_i^j as a share of intermediate costs of industry j spent on inputs from industry i:

$$\xi_i^j = \frac{m_{t,i}^j(z)P_{t,i}}{M_{t,j}(z)X_{t,j}}.$$

Total demand for output of firm z in sector j is compounded from the final and intermediate demand:

$$y_{t,j}(z) = \left(\frac{P_{t,j}(z)}{P_{t,j}}\right)^{-\theta}\underbrace{\left(C_{t,j} + \sum_{i=1}^{J}\int_0^1 m_{t,j}^i(z')dz'\right)}_{\equiv Y_{t,j}} = \left(\frac{P_{t,j}(z)}{P_{t,j}}\right)^{-\theta}Y_{t,j}, \tag{1.8}$$

where $C_{t,j} = \left(\int_0^1 C_{t,j}(z')^{\frac{\theta-1}{\theta}}dz'\right)^{\frac{\theta}{\theta-1}}$ is the total final demand for aggregate product of sector j. Here I make use of the above mentioned assumption on the same elasticity of substitution between sectoral aggregates in final and intermediate demand, θ. If we consider the expression for $Y_{t,j}$ in Equation (1.8) we can observe that the price $P_{t,j}(z)$ set by producer z does not directly affect $Y_{t,j}$; there is an indirect effect since $P_{t,j}(z)$ enters the sector's j aggregate price index, but since we have a continuum of producers in each sector this effect is negligibly small. In other words, when choosing the optimal price the producer z considers aggregate demand for sectoral output $Y_{t,j}$ to be constant.

The next part of the model introduces the sources of price rigidity. Following the setting in [27] I assume that there is a Calvo price adjustment mechanism ([3]) – with probability $(1-\delta_j)$ any producer in sector j is able to adjust his prices. Every time the producer adjusts his prices he should pay a fixed cost F_j ("menu cost"). Unlike in the original Calvo price adjustment mechanism, the probability of price adjustment $(1-\delta_j)$ is chosen by firms endogenously. In other words, producers can choose the frequency of price adjustment, but not the periods in which they adjust their prices. Notice also that all producers in sector j that have an opportunity to adjust their prices are essentially identical – they observe the same costs of inputs $X_{t,j}$ and experience the same productivity shocks $A_{t,j}$ – thus, in every period t share $(1-\delta_j)$ of producers in sector j adjust their price to same level $\check{P}_{t,j}$. Sector j's price index evolves as

$$(P_{t,j})^{1-\theta} = (1-\delta_j)(\check{P}_{t,j})^{1-\theta} + \delta_j(P_{t-1,j})^{1-\theta}. \tag{1.9}$$

Sectoral price indexes aggregate into the economy-wide index, analogy of PPI in the real world:

$$P_t = \prod_{i=1}^{J} (P_{t,i})^{\varepsilon_i}, \tag{1.10}$$

where ε_i is the weight of sector i's output in total output. The real profit at period t for any producer z in sector j can be written down as:

$$\pi_{t,j}(z) = \left[(P_{t,j}(z))^{1-\theta} - \psi_t^j\,(P_{t,j}(z))^{-\theta}\right](P_{t,j})^{\theta}\,P_t^{-1}Y_{t,j}, \tag{1.11}$$

where $P_{t,j}(z)$ is the actual price for producer z's output in period t. If prices where fully flexible it would be optimal for producers to adjust every period and set prices equal to the optimal flexible price ${P^*}_{t,j}$ which is the markup over the marginal costs:

$${P^*}_{t,j} = \frac{\theta}{\theta-1}\psi_{t,j} \tag{1.12}$$

Since prices are sticky, the *actually* set prices by producers in sector j in period t, $\check{P}_{t,j}$, will be different from ${P^*}_{t,j}$. To derive the expression for $\check{P}_{t,j}$ let's solve the optimization problem that is encountered by a firm whenever it has a chance to reset its price. Using the second-order Taylor approximation for firm's profit around the steady state, we obtain an expression for firm's real profit losses at any period t due to setting a sub-optimal price $\check{P}_{t,j}$ which is different from the flex-price optimum ${P^*}_{t,j}$ (lower-case letters denote the log-deviation from the steady state, $x_t = \ln X_t - \ln \bar{X}$):

$$\hat{\pi}\left(\check{p}_{t,j}, \{p_{t,i}\}_{i=1}^{J}, y_{t,j}, a_{t,j}\right) - \hat{\pi}\left({p^*}_{t,j}, \{p_{t,i}\}_{i=1}^{J}, y_{t,j}, a_{t,j}\right) \approx \frac{|\pi_{11}|}{2}\left(\check{p}_{t,j} - {p^*}_{t,j}\right)^2, \tag{1.13}$$

where $\pi_{11} = \frac{\partial^2\pi}{\partial {p_t^j}^2}$ is estimated at the steady state. The full derivation of Equation (1.13) is be provided in Appendix A. A producer that adjusts his price at period t sets the price

$\check{p}_{t,j}$ at the level that minimizes his expected losses from keeping the price constant in all consequent periods:

$$\check{p}_{t,j} = \underset{\check{p}_{t,j}}{argmin}\ E_t \sum_{\tau=0}^{\infty} (\beta\delta_j)^{\tau} \frac{|\pi_{11}|}{2} \left(\check{p}_{t,j} - {p^*}_{t+\tau,j}\right)^2 \tag{1.14}$$

or, by considering the FOC for Equation (1.14) and setting it equal to zero

$$\check{p}_{t,j} = (1-\beta\delta_j) E_t \sum_{\tau=0}^{\infty} (\beta\delta_j)^{\tau} p*_{t+\tau,j} \tag{1.15}$$

Finally, taking into account that every price adjustment for a firm in sector j involves a real fixed cost F_j, the expected profit losses of a price-adjusting firm can be shown to be a function of a price adjustment frequency adopted in industry j and in all other industries, except j ($\Delta_j = \{\delta_i\}_{i \neq j}$):

$$L_{t,j}(\delta_j, \Delta_j) = \frac{1-\beta\delta_j}{1-\beta} \left[F_j + E_t \sum_{\tau=0}^{\infty} (\beta\delta_j)^{\tau} \frac{|\pi_{11}|}{2} \left(\check{p}_{t,j} - {p^*}_{t+\tau,j}\right)^2 \right] \tag{1.16}$$

A detailed derivation of Equation (1.16) can be found in Romer (1990).

Plugging Equation (1.15) into Equation (1.16) one can rewrite the total expected losses of sector j as

$$\begin{aligned} L_t^j(\delta_j, \Delta_j) = \frac{1-\beta\delta_j}{1-\beta} F^j + \frac{|\pi_{11,j}|}{2(1-\beta)} \left[(1-\delta_j\beta) \sum_{\tau=0}^{\infty} (\delta_j\beta)^{\tau} E_t \left({p^*}^j_{t+\tau}\right)^2 \right] - \\ - \frac{|\pi_{11,j}|}{2(1-\beta)} \left[(1-\delta_j\beta) \sum_{\tau=0}^{\infty} (\delta_j\beta)^{\tau} E_t \left({p^*}^j_{t+\tau}\right) \right]^2 . \end{aligned} \tag{1.17}$$

So far the model can be summarized in the following way: producers in every sector j minimizes the total expected losses (Equation (1.16)) subject to the following four log-linearized constraints:

$$\begin{aligned} {p^*}_{t,j} &= -W_{t,j} + \sum_{i=1}^{J} \xi_{i,j}(1-s_j) p_{t,i} &(1.18)\\ p_{t,j} &= (1-\delta_j)\check{p}_{t,j} + \delta_j p_{t-1,j} &(1.19)\\ W_{t,j} &= W_{t-1,j} + u_{t,j} &(1.20)\\ \check{p}_{t,j} &= (1-\delta_j\beta) \sum_{\tau=0}^{\infty} (\delta_j\beta)^{\tau} E_t {p^*}_{t+\tau,j} &(1.21) \end{aligned}$$

Equation (18) is a log-linearized version of Equation (1.6), Equation (19) comes from Equation (1.10), while Equation (21) is just a rewritten Equation (1.15). All variables are in log-deviations from their steady states. ${p^*}_{t,j}$ – optimal flexible price, $\check{p}_{t,j}$ – actually set price, $p_{t,j}$ – sector j price index, $W_{t,j}$ – sector j's TFP level.

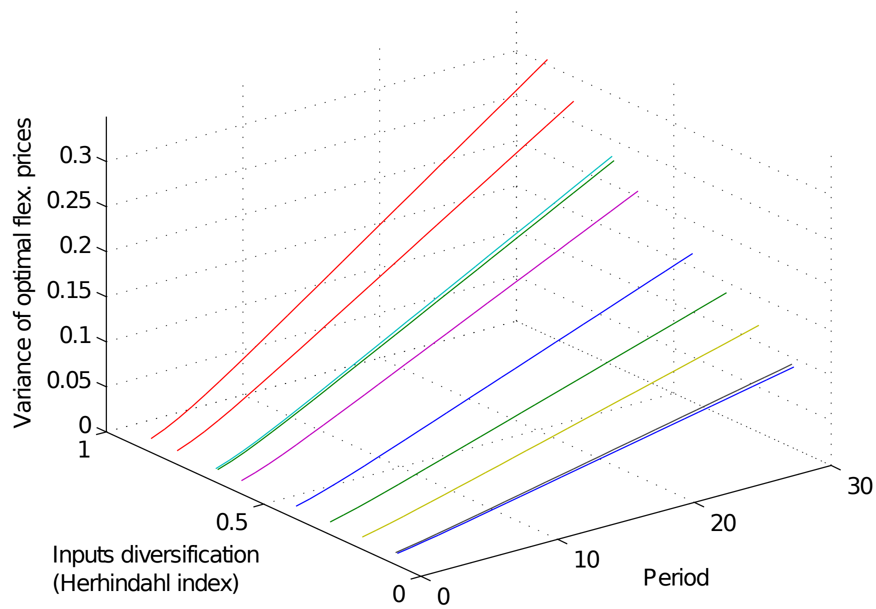
There are J such systems of equations (one for every sector). The system is forward-looking. To solve it I rewrite it in the matrix form and use the Blanchard-Kahn method. All the derivations are omitted here for the sake of brevity and are provided in Appendix B.

The intuition behind the anticipated result can be explained with Equation (1.16). Firms in every sector choose the optimal frequency of price adjustment $(1 - \delta_j)$ to minimize the total expected losses taking the vector of price adjustment frequencies in other sectors (Δ_j) as given. The equilibrium of the model is a Nash equilibrium, when every sector chooses its optimal probability of adjustment and, given the levels of adjustment probabilities in other sectors, does not have an incentive to deviate. With sectoral productivity modeled as a random walk the variance of optimal prices ${p^*}_{t+\tau,j}$ increases over time, so that there is an optimal δ_j that minimizes the expected total losses. It works as follows. Variance of optimal price ${p^*}_{t+\tau,j}$ increases faster if the costs of a representative producer in sector j are driven by one or two shocks coming from intermediate goods than if the bundle of intermediate goods is well diversified. So, for a faster growing $var({p^*}_{t+\tau,j})$ it will be optimal to set lower weights on more distant periods (high τ), thus, choosing more frequent price adjustment (low δ_j). If the intermediate goods bundles are well diversified, then variance of optimal price increases slower over time, thus, the producers will choose lower frequency of price adjustment (higher δ_j) to economize on fixed costs. By the same intuition if the costs of sector j have higher share of labor then the variance of the optimal price of sector j, $p^*_{t+\tau,j}$ rises at lower rate since the costs of labor are modeled to be constant. These arguments are illustrated by Figure (1.1) for an economy with 10 sectors: one can solve Equations (18)-(21) for variance of optimal prices for each of 10 sectors and observe that those sectors that are characterized by a less diversified structure of intermediate goods bundle and/or smaller share of labor in their costs have more volatile optimal prices and volatility of their optimal prices grows at higher rates than for sectors with more diversified input structure (low Herfindahl index) and/or larger share of labor costs.

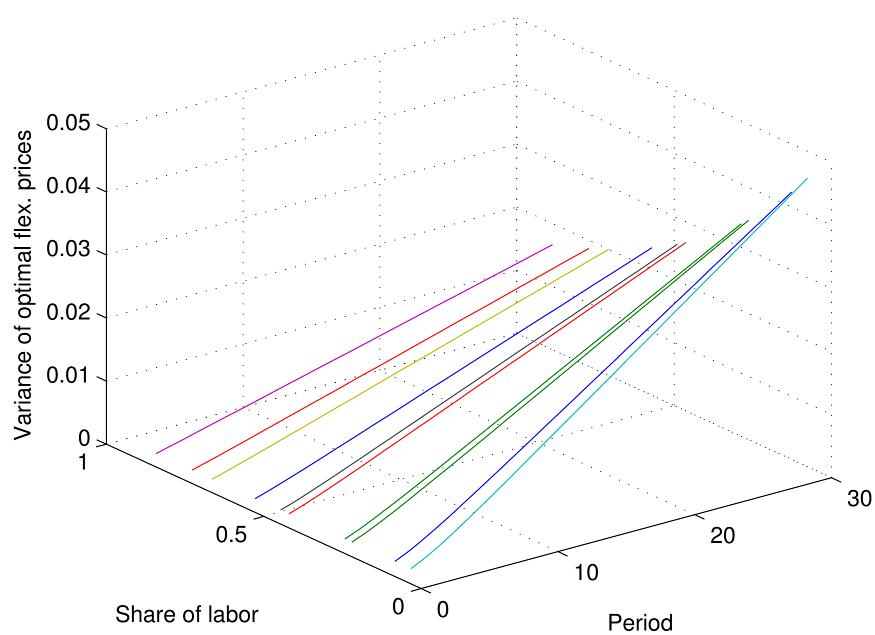
I solved the above outlined model for optimal frequencies of non-adjustment $(1 - \delta_j)$ and simulated the data from it for an economy with 10 sectors. I have done three exercises: (1) find optimal frequency of non-adjustment for sectors with different structure of intermediate goods bundle (keeping all other characteristics of sectors the same), (2) find optimal frequency of non-adjustment for sectors with different shares of labor in their costs (again, values of all other parameters kept the same) and (3) find the variance of average price adjustments depending on the level of inputs bundles diversification and share of labor. For these exercises I generated 100 observations (100 random matrices of ξ's for (1), 100 random vectors of s for (2) and 100 random vectors of ξ's and s for (3)) using the values of parameters reported in Table 1.1.

To find the equilibrium vector $\{\delta_j\}_{j=1}^{J}$ I follow the next steps:

1. Pick an arbitrary initial vector $\{\delta_j\}_{j=1}^{J}$;



(a) Impact of inputs bundle diversification on the variance of optimal flexible prices



(b) Impact of labor share on the variance of optimal flexible prices

Figure 1.1: Cost structure and variance of optimal flexible prices

2. For the current vector $\{\delta_j\}_{j=1}^J$ compute the values for $dL_{t,j}/d\delta_j$ for every sector j using formulas for $var(p^*_{t+s})$ provided in Appendix B (Appendix B also shows why $L_{t,j}$ can be expressed in terms of $var(p^*_{t+s})$, $s > 0$ if the system is at the steady stated at time t).

3. If $|dL_{t,j}/d\delta_j| > \epsilon$ for some j, where ϵ is a criteria of convergence, then update the vector $\{\delta_j\}_{j=1}^J$ in the following way: increase each element δ_j for which $dL_{t,j}/d\delta_j < 0$ and decrease those δ_j for which $dL_{t,j}/d\delta_j > 0$. Return to step 2.

To demonstrate that the loss function attains its minimum at the computed equilibrium vectors of δ's (for which the first order condition holds) let's consider the second order condition for the economy at the steady state (here I make use of the fact that $E_t(p^*_{t+s,j}) = 0 \; \forall s > 0$ if at period t economy is at the steady state; for details, again, see Appendix B):

$$\frac{\partial^2 L_{t,j}}{\partial \delta_j^2} = \frac{\beta^2 |\pi_{11,j}|}{2(1-\beta)} \left[\sum_{\tau=0}^{\infty} \left(\tau(\tau-1)(\delta_j \beta)^{\tau-2} - \tau(\tau-1)(\delta_j\beta)^{\tau-1} \right) var\left(p^*_{t+\tau,j}\right) \right]$$

It is easy to see that coefficients in front of $var\left(p^*_{t+\tau,j}\right)$ sum up to zero, so the sign of $\frac{\partial^2 L_{t,j}}{\partial \delta_j^2}$ depends on the pattern of $var\left(p^*_{t+\tau,j}\right)$ over time: it will be negative if variance monotonically decreases, zero – if it is constant and positive – if variance increases over time. From Figure (1.1) and Appendix B one can observe that $var\left(p^*_{t+\tau,j}\right)$ increases over time, thus, $\frac{\partial^2 L_{t,j}}{\partial \delta_j^2} > 0$ and the loss function indeed attains its minimum at the computed δ's.

Table 1.1: Calibration of parameters for simulations

Parameter	Value	Note
J	10	Number of sectors in the economy
s_j	0.4	Share of labor in total cost
θ	4	Price elasticity of demand
F_j	0.007	Real fixed cost of price adjustment
w_t	1	Nominal wage
$\bar{Y}_j$	1	Demand for real output of each sector
$\bar{A}_j$	1	Steady state value of TFP
σ_{A_j}	0.026	Standard deviation of productivity shocks
ξ_i^j	0.1	Share of inputs from i in input costs of j

Figure (1.2) depicts the relationship between diversification of inputs, share of labor in total costs and optimal probability of non-adjustment. As one can see from Figure (1.2) the model, indeed, generates the patterns of price adjustment across sectors that is in line with the above mentioned intuition. The more diversified structure of inputs bundle reduces the volatility of sector's marginal costs as time since last price adjustment passes and, hence, reduces the expected losses of producers from sub-optimally set prices. The latter allows industries to reset their prices with lower frequency and economize on the fixed costs of adjustment. The intuition for the labor share in costs works in the same way: if the share of input price for which is stable (labor) is higher, it means that variances of marginal costs and optimal flexible prices increase slowly over time, again, causing firms to reset prices less frequently.

Another prediction that comes from the model concerns the relationship between cost structure and the average percentage price changes across industries. Similarly to the previous prediction, the intuition is the same: the structure of costs that results in more volatile marginal costs and more volatile optimal flexible prices should lead to larger absolute values of price adjustments in the corresponding sectors. The variance of average price changes versus share of labor in total costs and extent of inputs diversification for equilibrium vectors of $\{\delta_j\}_{j=1}^{J}$ is depicted on Figure (1.3) (derivation of formulas for the variance of average price adjustments is provided in Appendix C).

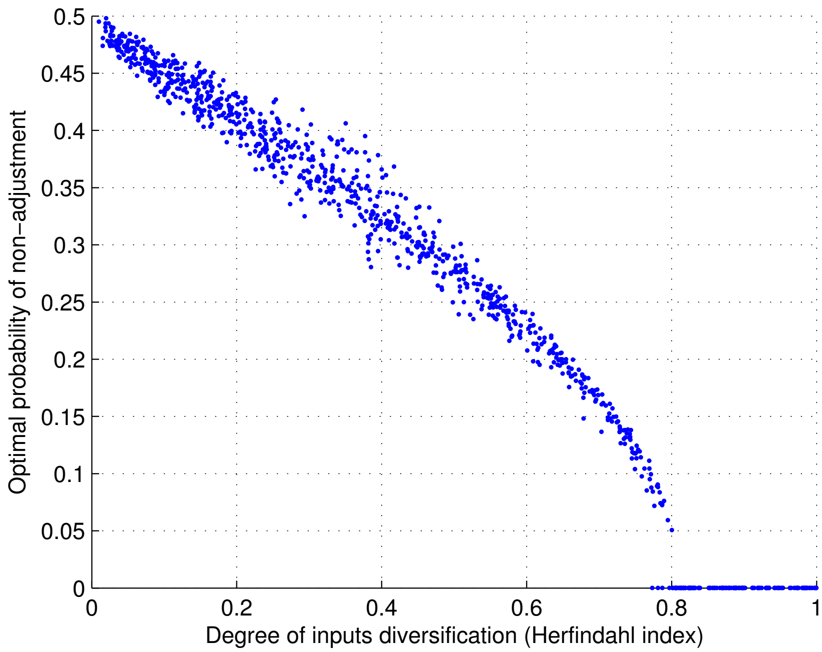
1.3 Empirical analysis

In this section I consider the data on price changes and cost structure and investigate the relationship between them. The words "sectors" and "commodities" are used below as synonyms, but all the analysis was implemented using the data on industries. The above introduced model predicts that one can expect to observe the following patterns in the data:

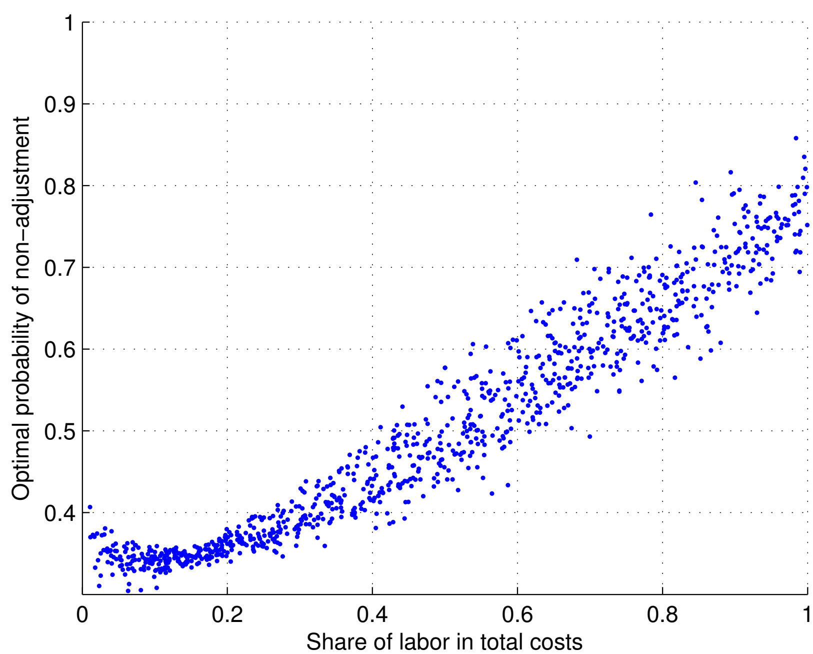
- The higher is the share of labor (value added) in total costs of a given sector, the higher should be the price rigidity in this sector;
- The more diversified is the structure of intermediate goods bundle that is used as input in a given sector, the higher should be the price rigidity in this sector.

Again, restating what has already been mentioned in the model section, the intuition behind these predictions comes as follows: if producers of every commodity are monopolistic competitors then their optimal prices in every period are equal to a markup over their marginal costs. Assuming that the markups are stable, the actually set prices should follow the pattern of marginal costs. If we accept that labor costs are less volatile than prices of commodities, then the higher share of value added in marginal costs will make optimal prices and actually preset prices more stable and, hence, the observed price adjustments will be less frequent.

The intuition for the second prediction is very similar to the intuition behind diversification: suppose every sector in every period is subject to idiosyncratic productivity shocks and

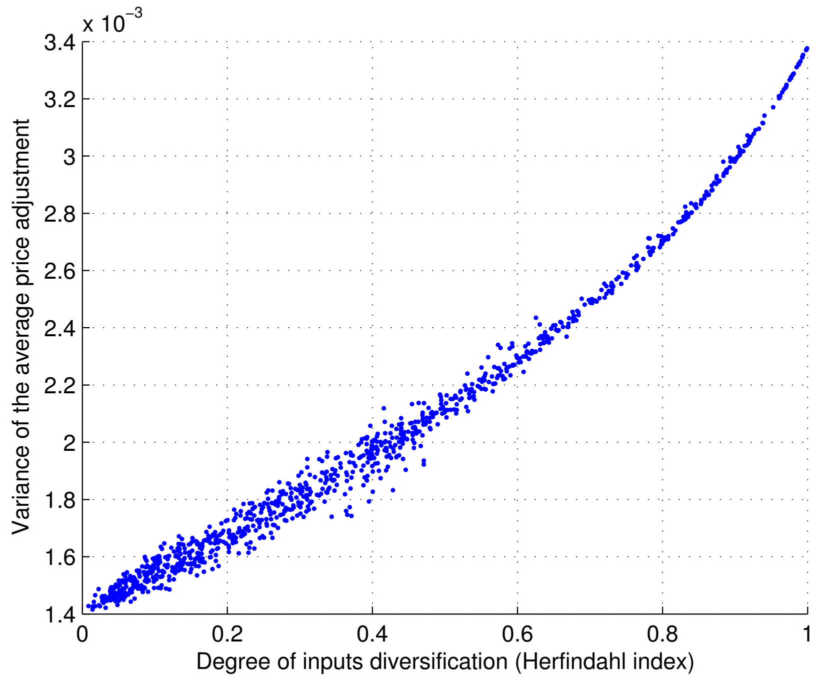


(a) Diversification of inputs and optimal probability of non-adjustment

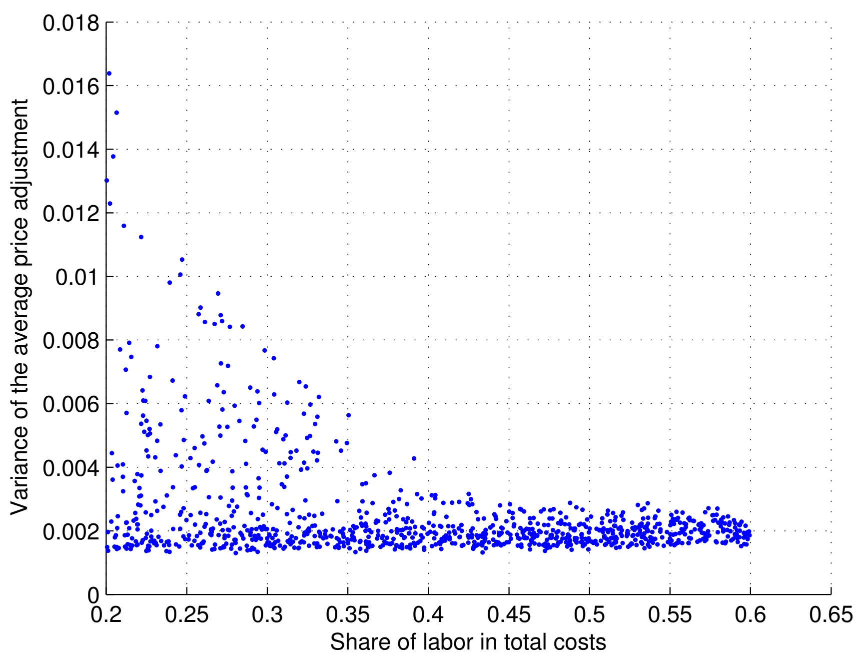


(b) Share of labor in total costs and optimal probability of non-adjustment

Figure 1.2: Cost structure and optimal probability of price non-adjustment



(a) Diversification of inputs and variance of average price adjustmen



(b) Share of labor in total costs and variance of average price adjustment

Figure 1.3: Cost structure and variance of average price adjustment

these shocks translate into the prices of produced commodities and, hence, costs of other sectors. Those producers that need to buy wast range of inputs have their intermediate goods bundles better diversified than those producers that need to buy only few inputs. As a result marginal costs and prices of the former producers will be less volatile than those of the latter.

Data on costs

To obtain the information on cost structures of different sectors I considered an *industry-by-industry* matrix of input-output flows. For further notations in this section I denote matrix of industry-by-industry input flows as Z, vector of value added as V, vector of final demand as F, and vector of total output as X. The data is taken from the benchmark input-output (IO) tables for the US for year 2002 provided by Bureau of Economic Analysis. Number of industries is equal to 426. The following variables were constructed:

- **Herfindahl index for input costs (backward linkages).** Making use of matrices Z and X we can construct a commodity-by-commodity matrix of direct inputs coefficients:

  $$A = Z \cdot \hat{X}^{-1},$$

  where $\hat{X}$ is a diagonal matrix obtained from vector X: $\hat{x}_{ii} = x_i$. Element $a_{i,j}$ of matrix A shows what value of commodity i should be supplied as an input to produce the amount of commodity j that worth 1 unit of currency. Sum $\sum_{i=1}^{N} a_{ij}$ shows the total value of intermediate goods that should be purchased by producers of commodity j to produce 1 unit of value. If we normalize each element a_{ij} by the sum of elements in the corresponding column we obtain the share of inputs supplied to producers of commodity j from producers of commodity i in the overall amount of intermediate inputs used by producers of commodity j:

  $$\bar{a}_{ij} = \frac{a_{ij}}{\sum_{k=1}^{N} a_{kj}}.$$

  The normalized direct input coefficients are used for computing the normalized "backward" Herfindahl index for every industry j:

  $$h_j^b = \frac{\sum_{k=1}^{N} \bar{a}_{kj}^2 - \frac{1}{N}}{1 - \frac{1}{N}}. \tag{1.22}$$

  Values of the index range from 0 to 1, where 0 corresponds to the case when the intermediate inputs used by producers of commodity j are distributed evenly across all N sectors (highest possible extent of diversification of inputs), while 1 means that only 1 type of intermediate goods is used as an input. Again, the intuition is that the lower this index is, the more stable is the value of sector's intermediate input bundle, the less volatile are the prices of this sector.

- **Sum of squared coefficients of direct inputs.** This measure is similar to Herfindahl index for direct inputs h^b and is computed as:

$$s_j = \sum_{i=1}^{N} a_{ij}^2$$

- **Share of wages in total output.** Expenditures on labor comprise another important part of costs – value added. The share of wages (labor compensation in IO accounts) in total output for producer j is computed as

$$w_j = \frac{W_j}{X_j}.$$

  This index will also serve as a measure of marginal costs' volatility across different sectors. The higher is the share of wages in sector j, the less volatile should be its costs.

- **Share of output used for final demand.**

$$f_j = \frac{F_j}{X_j}.$$

  This variable will be used as a proxy for "menu costs". The assumption is that it is harder to adjust prices more frequently in the final product market than in the intermediate goods markets.

There are several issues that should be kept in mind when using the IO data for constructing proxies for measures of costs diversification and costs of price adjustments. First, IO tables reflect only technological characteristics of production processes, but they do not allow us to reveal the property structure across entities. In other words if two entities in different sectors belong to one owner, the flows of products will be reflected in the IO tables, yet, prices at which these products are supplied might be more stable than market prices. If anything, this issue will tend the regression results to be downward biased in absolute values[1].

[1] Let's assume that we estimate a regression of price rigidity, R, on a measure of inputs diversification, say, Herfindahl index H. The observed diversification measure H is different from the actual one H^*: $H = H^* + \omega$. The parameter of interest $\beta(< 0)$ comes from the regression $R = \alpha + \beta H^* + u$ which now, due to measurement error, is estimated as $R = \alpha + \beta H + \underbrace{(u - \beta\omega)}_{z}$. The sign of the bias is the same as the sign of $cov(z, H) = -\beta var(\omega) > 0$ (assuming that H^*, u and ω are mutually uncorrelated) and from $\hat{\beta} = \beta + \frac{cov(z,H)}{var(H)}$ we have $|\hat{\beta}| < |\beta|$.

The second issue is related to aggregation of data that comes in the IO matrix. For instance, even if an entity uses inputs coming from only one sector it can buy this input from numerous suppliers, thus, again, decreasing the volatility of its marginal costs. Using more disaggregated data, at the level of ZIP codes, that is collected by the US Census Bureau could help to address the second issue and measure the diversification of costs more precisely. In the same manner as the previous concern, this issue will also bias the estimates downwards.

Third, share of final demand in total output might be a poor proxy for costs of price adjustments. To address this point in the regression analysis below I use another proxy for "menu costs" – share of output sold via exchange, first constructed in [26]. This measure indicates how specific (tailored for each buyer) the products are.

Finally, the whole approach that is used in this chapter is focusing attention on the supply side of price-setting mechanism. Clearly, if the demand for products of different sectors is subject to idiosyncratic shocks then the price volatility is also a function of demand shocks. Variation of variances of markups' and profit margins' across sectors is another issue that the empirical part of the chapter should address in the future work.

Data on prices

The first source of data on frequency and values of price adjustments across different sectors is the producer price indexes (PPI) provided by Bureau of Labor Statistics at monthly frequency. Number of commodity groups covered by PPI indexes is 5382. Following [4] I consider the following measures of price rigidity for this data set:

- Percent of month that are in spells (for this measure spell is considered to be a sequence of at least two months with the same price index);
- Average absolute percentage price changes: $|\Delta P_{j,t}| = |\ln P_{j,t} - \ln P_{j,t-1}|$.

The first measure is expected to be negatively related with price flexibility (i.e. higher share of periods in spells is associated with higher price rigidity), while the average absolute percentage price change is positively associated with price flexibility (and, correspondingly, negatively with price rigidity).

The second source of data on price adjustments patterns come from [25]. These data contains prices for goods and services that enter CPI at daily frequency. For this data set I take a frequency of price adjustment within a month and an average length of price spells as measures of price rigidity.

Regression analysis

To estimate the relation between different characteristics of cost structure and measures of price rigidity I estimate four regressions for each data set. Besides the above mentioned variables for cost diversification (s, h^b and w), forward linkages (f) and a proxy for specificity of products, I also included the measure of sector's monopolization as an additional control variable. It is measured as a market share of eight largest producers in a given sector. The estimates for PPI-data are reported in Table 1.2; CPI-data estimates are reported in Table 1.3.

Table 1.2: Cross-sectoral price rigidity and structure of costs, PPI-based estimates.

	(1) PCTS	(2) PCTS	(3) PA	(4) PA
Sum of squared dir. input coef-s, s	-0.37		0.04***	
	(0.36)		(0.01)	
Herfindahl index by col-s, h^b		-0.19		0.03***
		(0.20)		(0.00)
Share of wages in total costs, w	1.05***	1.06***	-0.01***	-0.02***
	(0.13)	(0.13)	(0.00)	(0.00)
Share of final demand, f	0.19***	0.19***	-0.002***	-0.001***
	(0.03)	(0.03)	(0.00)	(0.00)
Sector's monopolization proxy	0.004***	0.004***	-0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Product specificity proxy	0.36***	0.38***	-0.01***	-0.02***
	(0.08)	(0.07)	(0.00)	(0.00)
Constant	-0.39***	-0.41***	0.02***	0.02***
	(0.09)	(0.08)	(0.00)	(0.00)
N	931	931	931	931
R^2	0.304	0.304	0.484	0.489

PCTS – percentage of prices in spells,

PA – absolute average percentage price changes.

Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Coefficients at variables s and h^b show that, indeed, the relationship between rigidity of prices across sectors and the extent of inputs' diversification is positive, though, the coefficients in regressions for PCTS are statistically insignificant. This observation is in line with the above mentioned prediction: higher diversification may lead to lower values of Herfindahl indexes h^b and s, lower volatility of marginal costs, and, hence, higher price rigidity. The share of labor compensation in total costs of a sector has a positive impact

Table 1.3: Cross-sectoral price rigidity and structure of costs, CPI-based estimates.

	(1) Frequency	(2) Frequency	(3) Length	(4) Length
Sum of squared dir. input coef-s, s	94.1568***		-6.7679*	
	(12.98)		(3.05)	
Herfindahl index by col-s, h^b		85.3262***		-4.7057*
		(9.42)		(2.34)
Share of wages in total costs, w	-20.7556	-10.3301	2.9740	3.1408
	(12.91)	(12.32)	(3.03)	(3.07)
Share of final demand, f	0.19***	0.19***	-0.002***	-0.001***
	(3.13)	(2.96)	(0.73)	(0.74)
Sector's monopolization proxy	-0.0346	-0.0376	0.0035	0.0047
	(0.06)	(0.05)	(0.01)	(0.01)
Product specificity proxy	-7.4616	-7.0310	3.8736*	4.0340**
	(6.53)	(6.13)	(1.53)	(1.53)
Constant	33.7128***	26.1266***	0.5686	0.5326
	(6.56)	(6.37)	(1.54)	(1.59)
N	197	197	197	197
R^2	0.422	0.485	0.174	0.171

Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

on the price rigidity of this sector, supporting the intuition that labor costs are much less volatile than costs of intermediate inputs, thus, a higher share of labor in costs may lead to less volatile prices. The estimates at the measure of sectors' monopolization show that sectors with higher concentration of producers have more rigid prices – fewer larger player in sectors may result in more strategic price setting and, hence, higher extent of price rigidity. Share of produced goods that are supplied for final demand, f, and the share of production supplied by contracts (product specificity proxy) are positively related to price rigidity which is also in line with the above mentioned intuition.

The analysis of the above mentioned data allows me to conclude that the following predictions of the model are supported by the data:

- Higher share of labor costs in total costs is associated with lower price volatility of corresponding sectors.
- More diversified structure of used intermediate goods is associated with lower price volatility.

1.4 Applications

The introduced theoretical framework can be used for modeling the impact of changes in the cost structure on price rigidity. Analysis of the input-output data described in [20] for years 1995-2011 allows to establish the following important empirical facts which potentially may have an impact on optimal price frequency adjustments, heterogeneity of sectors in terms of price rigidity and the total responsiveness of the economy to nominal shocks

First, the share of value added in total output declines over time. Figure (1.4) depicts the ratio $\frac{VA}{X} = \frac{\sum_i \sum_s VA_{i,s}}{\sum_i \sum_s X_{i,s}}$ in which $VA_{i,s}$ denotes value added created in sector s of country i, while $X_{i,s}$ is the total output of sector s of country i. This pattern can potentially be explained by decreasing costs of market transactions and, as a result, out-sourcing of inputs and larger share of input in total costs and smaller share of labor. According to the predictions of my model, such decline in the share of value-added will result in more frequent price adjustments.

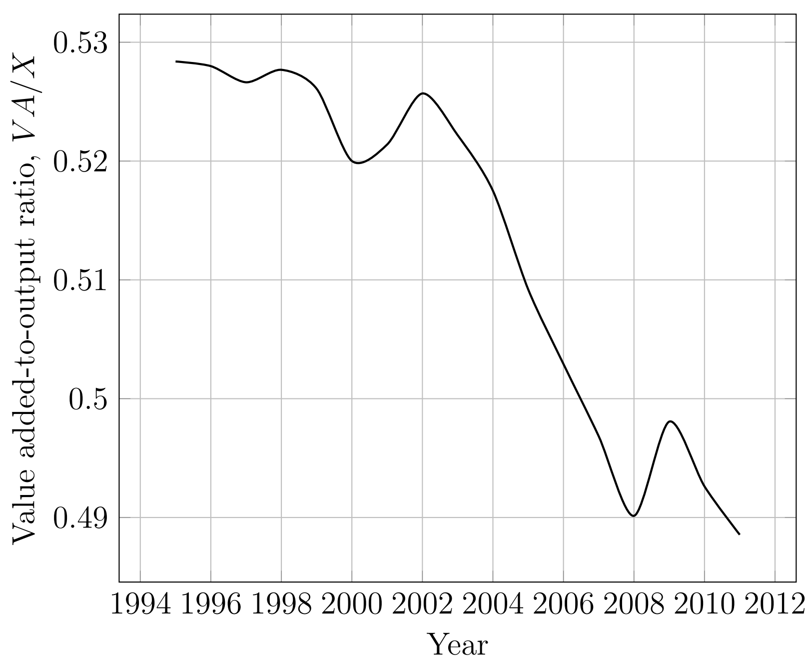


Figure 1.4: Value added-to-output ratio, World, 1995-2011.

The second feature of the data is the raise in Herfindahl index for input bundles across countries and sectors. Figure (1.5) depicts the distribution of percentage changes in Herfindahl index between 1995 and 2011. A sector in a country is a unit of observation. The index is calculated according to Equation (1.22). Increasing Herfindahl index means that the input bundles are becoming on average less diversified over time. Intuitively, this process could be

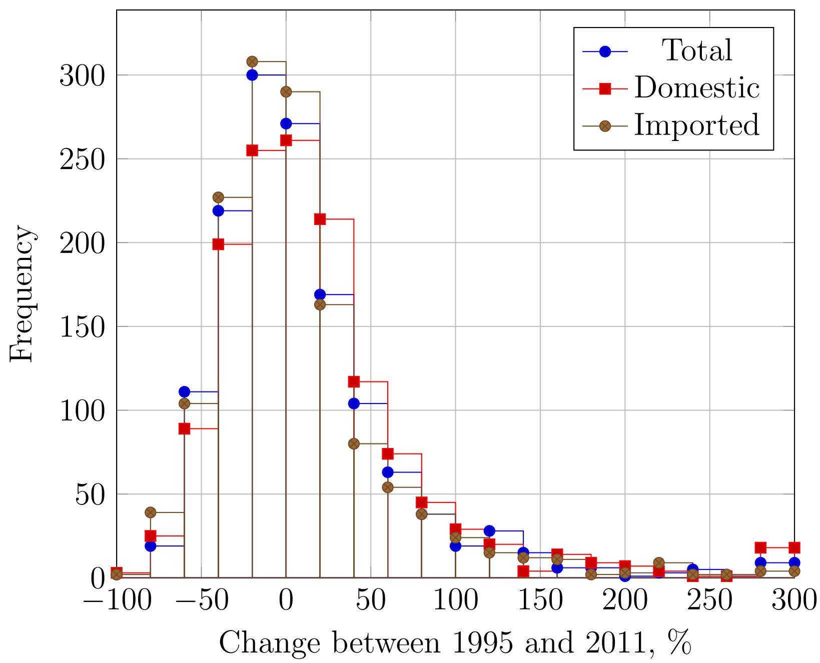


Figure 1.5: Distribution of percentage changes in Herfindahl index for input bundles, World, 1995-2011.

observed if production chains turn from "spiders" (assembling several inputs simultaneously into a final good) into "snakes" (assembling inputs into a final good sequentially). Another interesting observation follows from splitting input bundles into domestic and imported. So, concentration increases by larger amount for domestic inputs – median increase by 6.5% from 1995 to 2011 – and doesn't change for imported bundles – median change of 0.0%. The resulting increase in the concentration of overall input bundle is 1.6%. The increase in inputs concentration means that the optimally chosen frequency of price adjustments will also increase.

1.5 Conclusion

In the current chapter I demonstrate the relation between sector price rigidity and two characteristics of firms' costs – the share of labor (value added) in total costs and diversification of intermediate inputs bundle. Both, larger share of labor and higher diversification of inputs result in less frequent price adjustments and, hence, higher overall price rigidity in the economy. I suggest a model in which the extent of price rigidity is optimally chosen by producers and I show that the model explains the above mentioned empirical relations. Finally I demonstrate the changes in the structure of costs observable in the data. The latter may have a sizable impact on total price rigidity and obtaining a precise estimate of this impact is an important application of the model for future research.

Chapter 2

Technological Spillovers and Dynamics of Comparative Advantage

2.1 Introduction

Between 1980 and 1990 around $160 billion were spent by the US, Japanese, South Korean and Taiwanese governments to establish semiconductor industry in their countries. In a new wave of subsidies, the US and Chinese governments pledged to spend $150 billion and $400 billion respectively on the "green energy" sector[1]. The usual rationale for such policy emphasizes economies of scale and positive externalities generated by targeted sectors. To analyze the long-run welfare implications of the above policies, we need to model these externalities and the resulting dynamics of sectoral productivity. The key theoretical challenge here is the possibility of multiple equilibria. If productivity of a sector depends on its size – for example, through accumulation of best practices that become public knowledge within a country – then any initial comparative advantage of a sector becomes self-reinforcing. Furthermore, multiplicity of equilibria is more likely in open economies, since specialization is not thwarted by downward sloping demand. Several theoretical works as, for instance, [31] investigated balanced growth paths of open economies, yet, the derived long-run predictions in those papers depend on the starting points – multiplicity is their typical feature. A large body of empirical research documents the presence of strong technological spillovers between sectors. Accounting for them makes it even harder to model the evolution of sector-level productivities. With cross-sector spillovers each sector can reinforce not only its own productivity but also productivity of proximate sectors.

In this chapter I develop a dynamic model of international trade with cross-sector spillovers which under general conditions demonstrates a unique balanced growth path. As I show, the sufficient condition for the uniqueness is the connectedness of sectoral clusters. In addition to the standard effect of comparative advantage on labor allocation, the model accounts for

[1]Source: MacKinsey Global Institute, Breakthrough Institute

the effects of labor allocation on sector productivity and comparative advantage. The core mechanism is a combination of an idea-generating process within each sector and technological spillovers across sectors. I establish necessary and sufficient conditions for the existence and uniqueness of a balanced growth path (BGP) and describe the conditions under which a welfare-improving industrial policy is possible. I calibrate the model using the US patent data to parametrize the strength of technological spillovers and to describe the optimal policy.

The model builds on a multi-sector [9] model, as in [6], to link sector productivities to labor allocation across sectors. The reverse link from labor allocation to sector productivity is captured by assuming that employment in a particular sector exogenously generates a mass of new (publicly available) technologies useful in the sector or in others. Specifically, a new technology that emerges in an origin sector can be used in producing any variety in a destination sector with some origin-destination probability. The pattern of how technologies flow across sectors is summarized by a matrix of cross-sector spillover probabilities. The cross-sector spillover matrix can be seen as a way to formalize the idea of proximity between different sectors as in [15]. For instance, high values of spillover probabilities between origin and destination sectors mean that both these sectors are using similar technologies and are more likely to produce together in a given country. This approach allows me to describe the dynamics of the economy by a simple system of differential equations.

For the case of frictionless trade, I show that depending on the degree of connectedness between sectors the economy may have a unique or multiple BGPs. An important result is that if there are no isolated clusters, (that is, there are no groups of sectors that generate and adopt technologies only for and from members of the group), then the BGP of the model is unique. This result comes as an outcome of interaction of two forces.

The first force, centripetal, tends to equalize productivities across sectors. To see the workings of this force, assume that a country could buy at the same price a random sample of technologies for any sector. It would buy this sample for the least productive sector because in this sector a larger share of the technologies will have productivity that exceeds the sectoral productivity frontier and, thus, will be used. Buying technologies in this example is equivalent to directing labor to the least productive sectors. The centripetal force may explain why we observe weakening of comparative advantage in the data, as documented in [18].

The second force we observe is centrifugal. It comes from the fact that it is actually more expensive to buy technologies for the least productive sectors. Namely, to allow the least productive sectors to catch up, labor should be diverted to them from more productive sectors and, as a result, welfare decreases. If we have isolated clusters, then these two forces can balance each other on multiple BGPs. Under no isolated clusters the cross-sector spillovers provide technologies to the least productive sectors “for free”. As a result, centripetal force becomes stronger and the economy ends up in a unique BGP where all countries have the same relative productivities across sectors and, hence, no comparative advantage and no cross-sector trade. The model also provides a description of the transition path and for a 2-sector 2-country case has a simple phase-diagram illustration.

The model can be used to think about the welfare effect of policies that induce a reallocation of labor across sectors. If a vector of sectoral productivities is a result of sectoral labor allocation, then a country can choose its BGP as well as a transition path to it. For example, this choice is implemented by re-allocating labor across sectors and, thus, affecting the process of accumulation of new technologies across sectors. Uniqueness of BGP matters for the outcome of such policy and its information intensity. Namely, if uniqueness holds then the BGP to which a country converges doesn't depend on the initial distribution of productivities and all the policy-maker needs in order to predict the long-run implications of policy is the matrix of spillovers. In contrast, if uniqueness doesn't hold, then the policy-maker should know not only the matrix of spillovers but also the initial distribution of productivities across all sectors and countries. I demonstrate that a policy intervention can improve welfare if the spillover matrix has positive inter-sector spillovers, i.e. that some sectors generate technologies both for varieties inside and outside these sectors. Characteristics of such sectors are consistent with the notion of core sectors as in [12]. That is, core sectors generate widely applicable technologies and increase productivity of the whole economy, providing a rationale for governments to promote and even subsidize them. I also derive the criteria for defining optimal labor reallocation across sectors. For a symmetric 2-country 2-sector case this criteria has an intuitive interpretation. Namely, labor should be reallocated towards a sector that generates a larger share of technologies for destinations sectors weighted by expenditures on varieties of the destination sectors. Thus, I provide a formal framework for quantifying these technological spillovers and deriving the associated optimal policy.

Finally, I calibrate the model using the US patent data and compute the labor allocation that maximizes welfare in the BGP. The calibration part of the chapter contributes to the literature on estimating the strength of technological spillovers[2] and extends it by introducing the corrections not only for the size of the destination sector, but also for the size of the sector of the origin. The computed optimal policy improves the country's productivity in the BGP by 3.5% comparing to the no-policy BGP.

The chapter is structured as follows. Section 2 outlines the model, investigates its dynamic properties and provides the intuition for the main mechanisms. Section 3 establishes the possibility for welfare-improving economic policy and the necessary conditions for such policy. Section 4 describes the calibration of the model and estimation procedure for the spillover

[2] E.g. [11]

parameters. Section 5 presents the optimal policy based on the calibrated model. Section 6 concludes.

2.2 Model

The mechanism that maps sector productivity to labor allocation across sectors works through trade and comparative advantage. It is based on Ricardian models of trade as in [6] and [10]. The mechanism that generates the feedback from labor allocation to sector productivity is based on the exogenous process of generating new technologies and endogenous spillovers of these technologies across sectors. The flow of technologies is described by a matrix of spillover probabilities. At the aggregate level this mechanism generates equations that describe dynamics of sectors' productivity which are similar to the equations suggested in [14]. More details are provided below.

Demand

There is a discrete number of sectors and a continuum of varieties of mass normalized to 1 within each sector. The set of sectors is denoted as $\mathcal{S} \equiv \{1, \ldots, S\}$. There is also a representative household with a two-tier utility function. Varieties within each sector are aggregated according to a constant elasticity of substitution (CES) aggregator with elasticity parameter σ. Sectoral aggregates, in turn, enter the utility function as Cobb-Douglas with sector specific parameters α^s. Thus, the utility of a representative household in country i at time period t is

$$U_i(t) = \prod_{s=1}^{S} \left(C_i^s(t)\right)^{\alpha^s}, \; C_i^s(t) = \left(\int_0^1 c_i^s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}}, \tag{2.1}$$

where $c_i^s(t,\omega)$ denotes consumption of variety ω from sector s by a representative household in country i at time t.

The representative household inelastically supplies an exogenous amount of labor $L_i(t)$ that is allocated among S sectors:

$$L_i(t) = \sum_{s=1}^{S} L_i^s(t) \tag{2.2}$$

In every period the household spends its whole income $I_i(t) = w_i(t)L_i(t)$, where $w_i(t)$ is the wage rate in country i at time t. The share α^s of the income is spent on varieties from sector s. Expenditure for each variety ω from sector s is equal to

$$x_i^s(t,\omega) = \left(\frac{p_i^s(t,\omega)}{p_i^s(t)}\right)^{1-\sigma} \alpha^s I_i(t), \tag{2.3}$$

where $p_i^s(t) = \left(\int_0^1 p_i^s(t,\omega)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}}$. Each variety ω is bought from only one source – a country that supplies it at the lowest price; the set of countries is discrete and is denoted as $\mathcal{N} \equiv \{1,\ldots,N\}$. Perfect competition among producers results in pricing at marginal costs. Let $c_{ij}^s(t,\omega)$ denote the marginal cost of supplying one unit of variety ω in sector s from country i to country j.[3] The set of goods of sector s supplied from i to j at time t is $\Omega_{ij}^s(t) = \left\{\omega : c_{ij}^s(t,\omega) = \min_{k\in\mathcal{N}}\{c_{kj}^s(t,\omega)\}\right\}$. Expenditure of country j for goods of sector s that are supplied by country i at time t is

$$x_{ij}^s(t) = \sum_{\omega\in\Omega_{ij}^s(t)} x_i^s(t,\omega) \tag{2.4}$$

Utility per capita is equal to

$$\frac{U_i(t)}{L_i(t)} = \prod_{s=1}^{S}\left(\frac{\alpha^s I_i(t)}{p_i^s(t)L_i(t)}\right)^{\alpha_s} = w_i(t)\prod_{s=1}^{S}\left(\frac{\alpha^s}{p_i^s(t)}\right)^{\alpha_s} \tag{2.5}$$

Supply

As it was mentioned above, free entry and perfect competition among producers is the market setting. Production uses only one input – labor – which is transformed into output according to the function

$$Y_i^s(t,\omega) = Z_i^s(t,\omega)L_i^s(t,\omega), \tag{2.6}$$

where $Z_i^s(t,\omega)$ is a productivity of technology for producing variety ω in sector s of country i at period t. All potential entrants have access to the same technology.

Labor is homogeneous and firms take wages as given. Goods from sector s are traded between countries i and j at cost d_{ij}^s which is modelled as "iceberg" trade cost. As a result, variety ω in sector s can be supplied from country i to country j at cost

$$c_{ij}^s(t,\omega) = \frac{w_i(t)d_{ij}^s}{Z_i^s(t,\omega).} \tag{2.7}$$

Now let's turn to the mechanism that defines productivity of each variety. I assume that labor of mass 1 exogenously generates technologies[4] at rate ϕ. Each technology is characterized by a sector of its origin and productivity Q. All new technologies immediately become publicly available. Productivity of each of them is drawn from a Pareto distribution:

$$Pr(Q \leq q) = 1 - q^{-\theta} \tag{2.8}$$

[3] In this chapter I use the notations commonly used in input-output literature when the first subscript denotes the country of the origin and the second one – destination.

[4] In what follows the words "idea" and "technology" are used interchangeably and mean an invented process which can be used for producing varieties of goods.

Once any idea is generated in sector s it can be applied to any variety in this sector with probability p^{ss} and to any variety in sector r with probability p^{sr}. In this setting by time t the number of ideas that has ever been generated for any variety in sector s of country i is a random variable distributed as Poisson with parameter $T_i^s(t)$:

$$T_i^s(t) = \sum_{r=1}^{S} p^{rs} \int_0^t \phi L_i^r(\nu) d\nu + T_i^s(0) \tag{2.9}$$

The matrix of probabilities $\{p^{rs}\}_{\substack{r\in\mathcal{S}\\ s\in\mathcal{S}}}$ summarizes the information on cross-sector spillovers. The only restriction on its elements is $0 \leq p^{rs} \leq 1 \; \forall r, s \in \mathcal{S}$. Sector s with higher $\{p^{sr}\}_{r\in\mathcal{S}}$'s generates more widely-applicable technologies. Sectors r and s that are characterized by high values of p^{rs} and p^{sr} can be viewed as technologically proximate ones: high productivity in one of them helps increase productivity in the other. Intuitively, a country may want to establish or retain a sector that is characterized by high proximity or, the way it is modeled here, the one that generates more general technologies. The latter would allow the economy in the balanced growth path (BGP) to have a higher number of ideas per capita and, thus, higher welfare.

As is established in [10], for any variety ω in sector s of country i productivity $Z_i^s(t,\omega)$ is a random variable distributed as Fréchet

$$Pr(Z_i^s(t,\omega) \leq z) = e^{-T_i^s(t) z^{-\theta}}. \tag{2.10}$$

The share of country i in country j's expenditure for goods of sector s is

$$\pi_{ij}^s(t) = \frac{x_{ij}^s(t)}{\sum_k x_{kj}^s(t)} = \frac{T_i^s(t)\left(w_i(t) d_{ij}^s\right)^{-\theta}}{\sum_l T_l^s(t)\left(w_l(t) d_{lj}^s\right)^{-\theta}}. \tag{2.11}$$

Equilibrium

Assuming that trade is balanced in each period, we obtain

$$w_i(t) L_i(t) = \sum_{j=1}^{N} \sum_{s=1}^{S} \pi_{ij}^s(t) \alpha^s w_j(t) L_j(t), \tag{2.12}$$

where the left-hand side is the total income of households in country i and the right-hand side are the total expenditures of all countries for goods produced in country i. Given the total labor supply in each country at time t, $\{L_i(t)\}_{i\in\mathcal{N}}$, and the level of technology for each sector-country which in the current setting is summarized by $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$, one can solve Equation (2.12) for equilibrium wages, $\{w_i(t)\}_{i\in\mathcal{N}}$.

Since in perfect competition firms earn zero profits, total revenue of each sector in every country is equal to total costs, i.e. to labor income earned in this sector:

$$w_i(t)L_i^s(t) = \sum_{j=1}^{N} \pi_{ij}^s(t)\alpha^s w_j(t)L_j(t), \tag{2.13}$$

Having solved Equation (2.12) for wages $\{w_i(t)\}_{i\in\mathcal{N}}$, one can solve Equation (2.13) for sector labor demand $\{L_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$. Notice from Equations (2.11) and (2.12) that, conditional on the relative size of countries in terms of total labor supply, what matters for wages and labor allocation is the relative productivity across countries within sectors, $\left\{\frac{T_i^s(t)}{T_j^s(t)}\right\}_{\substack{i,j\in\mathcal{N}\\ s\in\mathcal{S}}}$. Other things being equal, countries that are relatively more productive observe larger shares of expenditures for their goods and higher welfare.

Thus, the static part of the model tells us how the state of technology $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ and total labor supply $\{L_i(t)\}_{i\in\mathcal{N}}$ affect the equilibrium allocation of labor across sectors $\{L_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$. What the dynamic part of the model adds is the evolution of technologies: given the equilibrium allocation of labor across sectors within each country, productivity of sectors evolves according to Equation (2.9) or its differential counterpart

$$\dot{T}_i^s(t) = \frac{dT_i^s(t)}{dt} = \phi\left(\sum_{r=1}^{S} p^{rs}L_i^r(t)\right). \tag{2.14}$$

Thus, the model can be summarized by Equations (2.11), (2.12), (2.13) and (2.14), where Equations (2.11), (2.12), (2.13) describe the static equilibria of the model, while (2.14) – describes its dynamics. At any point in time the model is in static equilibrium.

Definition. **Static equilibrium** of the model at time t is a set of non-negative vectors of total labor supply $\{L_i(t)\}_{i\in\mathcal{N}}$, sector productivity $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ and sector labor allocation $\{L_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ that satisfy Equations (2.11), (2.12) and (2.13).

Balanced growth path

The model is characterized by semi-endogenous growth. Thus, it can have an equilibrium in which all variables are growing at a constant rate – balanced growth path – only if the total labor supply in each country is growing at some constant rate $g > 0$.

$$L_i(t) = e^{gt}L_i(0) \tag{2.15}$$

The state of the system is characterized by two vectors – productivities $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ and total labor supply $\{L_i(t)\}_{i\in\mathcal{N}}$. The former changes endogenously, while the latter – exogenously.

Definition. **Balanced growth path** (BGP) of the model is a sequence of static equilibria that satisfies Equation (2.14) and along which each element of the vector of sector productivities $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ grows at a constant rate.

Definition. Balanced growth path is **locally stable** if the economy converges to it once it starts at any point in some ε-neighborhood of it.

Since one can show that both T's and L's along the BGP are growing at the same rate, their ratios $\left\{\frac{T_i^s}{L_i}\right\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ remain constant.

One can also show that in the BGP

$$\frac{T_i^s}{L_i} \equiv t_i^s = \frac{\phi \sum_r p^{rs} l_i^r}{g} = \frac{\phi_i^s}{g}, \tag{2.16}$$

where $l_i^r \equiv \frac{L_i^r}{L_i} \in [0;1]$, $\sum_r l_i^r = 1$ and $\phi_i^s \equiv \phi \sum_r p^{rs} l_i^r$. Dynamics of $t_\cdot^\cdot$'s outside the BGP is described by the differential equation[5]:

$$\dot{t}_i^s(t) = \phi \sum_r p^{rs} l_i^r(t) - g t_i^s(t), \ \forall i = 1, ..., N, \ \forall s = 1, ..., S. \tag{2.17}$$

Clearly, if $p^{sr} = p \ \forall s, r$ then the first term on the right-hand side of Equation (2.17) turns into ϕp and it is easy to see that the BGP is unique and stable: $t_i^s = \phi p / g$. Another observation is that the first term on the right-hand side of Equation (2.17) is bounded both from above and from below – with boundaries $\phi \max_r(p^{rs}) \geq 0$ and $\phi \min_r(p^{rs}) \geq 0$ correspondingly, while $g t_i^s(t) \in [0; \infty)$, thus, the steady state level $t_i^s \in [\phi \min_r(p^{rs})/g; \phi \max_r(p^{rs})/g]$. The last inequality guarantees that even if there exists an unstable steady state it has a stable steady state in its neighborhood to which the system will converge. In other words it can not be that $t_i^s \to \infty$. If all spillovers are non-zero – $p^{rs} > 0 \ \forall r, s$ – then we can also exclude the cases where $t_i^s \to 0$. Graphical illustration of the latter argument is provided in Figure (2.1). Two thick lines – one solid and one dashed – show two possible patterns of relation between $\phi_i^s(t)$ and $t_i^s(t)$. Although gross substitutability between sectors guarantees that more productive sectors attract more labor, the relation between $t_\cdot^\cdot$'s and $l_\cdot^\cdot$ is non-linear, so, potentially there might be multiple BGPs. One observation from Figure (2.1) is that ϕ_i^s is in limit approaching ϕp^{ss} – as sector s in country i becomes more productive, more labor is allocated to it with limit $\phi_i^s = \phi p^{ss}$ attained when sector s employs all the labor in the country, $L_i^s = L_i$. As I mentioned above, potentially, there is possibility for both stable (points A, B and D) and unstable (point C) BGPs.

[5] To clarify notations – t without sub-/superscripts denotes time, while $t_i^s(t) \equiv \frac{T_i^s(t)}{L_i(t)}$ denotes the number of ideas per capita in country i sector s at time t.

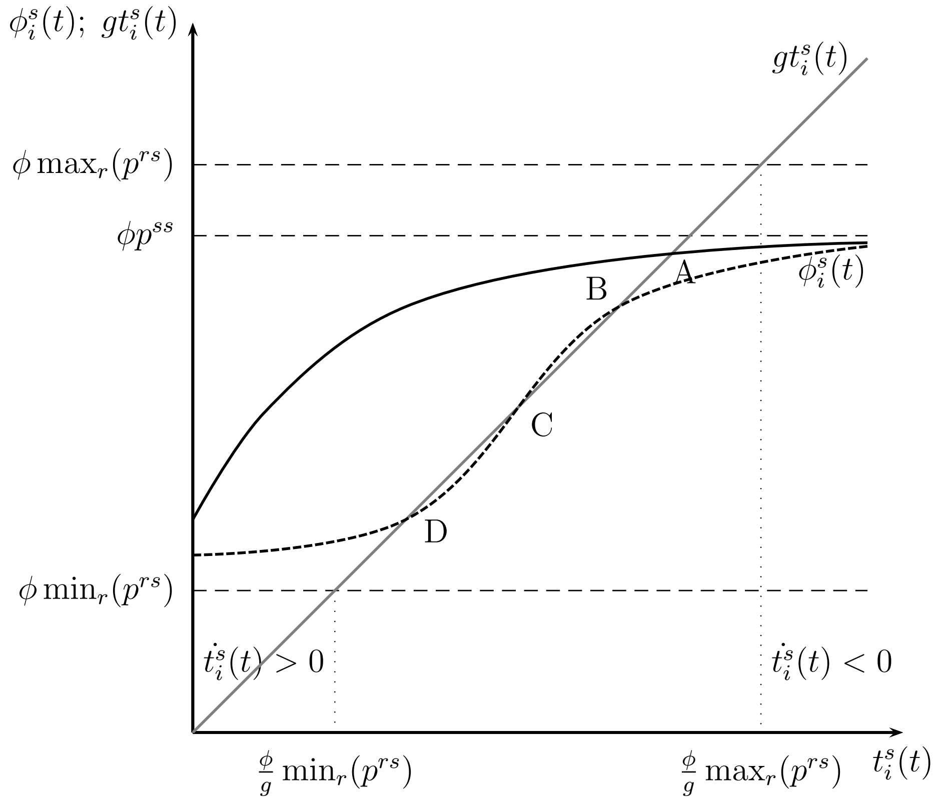


Figure 2.1: Dynamics of technological endowment per capita, $t_i^s(t)$

In general it is not trivial to show that Equations (2.17) have a unique and stable solution, thus, in what follows I will consider the above described model for some particular cases.

Autarky. Under autarky $\pi_{ij}^s = 0\ \forall i \neq j \in \mathcal{N}$ and $\pi_{ii}^s = 1\ \forall s \in \mathcal{S}$. It follows from Equation (2.13) that labor is allocated across sectors proportional to the shares of expenditure:

$$L_i^s(t) = \alpha^s L_i(t) \tag{2.18}$$

As I stated before, along the BGP all elements of $\{T_i^s(t)\}_{\substack{i=1,\dots,N\\ s=1,\dots,S}}$ are growing at the same rate g, thus, the ratios of T's across sectors and countries along the BGP remain the same:

$$\frac{T_i^s}{T_i^r} = \frac{\sum_{q=1}^S l_i^q p^{qs}}{\sum_{q=1}^S l_i^q p^{qr}} = \frac{\sum_{q=1}^S \alpha^q p^{qs}}{\sum_{q=1}^S \alpha^q p^{qr}} \tag{2.19}$$

Utility per capita (under wages normalized to 1) can be expressed as

$$\frac{U_i(t)}{L_i(t)} = \frac{1}{L_i(t)}\prod_{s=1}^S \left(C_i^s(t)\right)^{\alpha^s} = \frac{1}{L_i(t)}\prod_{s=1}^S \left(\frac{\alpha^s L_i(t)}{p_i^s(t)}\right)^{\alpha^s} = \frac{\prod_s \alpha^{s\alpha^s}}{\gamma}\prod_s \left(\frac{T_i^s(t)}{L_i(t)}\right)^{\frac{\alpha^s}{\theta}} \cdot L_i(t)^{\frac{1}{\theta}}, \tag{2.20}$$

where $\gamma \equiv \left(\Gamma\left(\frac{1-\sigma}{\theta}+1\right)\right)^{\frac{1}{1-\sigma}}$. As follows from Equations (2.16) and (2.18), $\frac{T_i^s(t)}{L_i(t)} = \frac{\phi \sum_r p^{rs}\alpha^r}{g} = const$. So, one can conclude that: 1) any economy on the BGP in autarky grows at rate $\frac{g}{\theta}$; 2) utility per capita for symmetric countries (i.e. countries with equal total labor supply) is the same; 3) relative productivity of sectors and labor allocation doesn't depend on the initial conditions. As a result, autarky is characterized by a unique and stable BGP. Another observation is that economies with more interrelated sectors (higher p^{rs}'s) have higher T/L and, thus, higher utility per capita under the same level of total labor supply.

Costless trade. Now let's consider another extreme – the case of costless trade: $d_{ij}^s = 1\ \forall i, j \in \mathcal{N},\ \forall s \in \mathcal{S}$. The immediate implication of this case is that the share of country i in expenditures for goods of sector s is the same across all destinations: $\pi_{ij}^s = \pi_{ii}^s \equiv \pi_{i\cdot}^s$. Hence, from Equation (2.13) one can obtain ratios of labor allocated across sectors r and s:

$$\frac{L_i^s(t)}{L_i^r(t)} = \frac{\alpha^s \pi_{i\cdot}^s(t)}{\alpha^r \pi_{i\cdot}^r(t)} = \frac{\alpha^s T_i^s(t) \sum_l T_l^r(t)(w_l(t))^{-\theta}}{\alpha^r T_i^r(t) \sum_l T_l^s(t)(w_l(t))^{-\theta}} \tag{2.21}$$

For any pair of countries i and j and any pair of sectors s and r

$$\frac{L_i^s(t)}{L_i^r(t)} \frac{T_i^r(t)}{T_i^s(t)} = \frac{L_j^s(t)}{L_j^r(t)} \frac{T_j^r(t)}{T_j^s(t)} \tag{2.22}$$

The second system of equations relating ratios of productivity parameters T's and labor allocation is the system for the steady state (thus, time index t is omitted):

$$\frac{T_i^r}{T_i^s} = \frac{\sum_q p^{qr} l_i^q}{\sum_q p^{qs} l_i^q}, \tag{2.23}$$

where $l_i^s \equiv \frac{L_i^s}{L_i},\ s \in \mathcal{S}$.

Before I formulate a Proposition on uniqueness of BGP, let me introduce and briefly explain the definitions used in the Proposition.

Definition. Sector $s \in \mathcal{S}$ is **stagnant** if $\sum_r p^{rs} = 0$.

A sector is defined as stagnant if it doesn't have possibilities for growth. Namely, neither technologies generated in other sectors, nor the ones generated in the sector itself can be used in a stagnant sector. Note, that this definition doesn't preclude a stagnant sector from generating technologies for other sectors.

Definition. BGP is **interior** if all elements of a sequence of vectors $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ are positive. BGP is **corner** if at least one element of this sequence of vectors is equal to zero.

Interior equilibrium means an equilibrium in which every country has non-zero productivity in every sector. In contrast, a corner equilibrium is the one in which some country has zero productivity in some sectors.

Definition. Matrix of spillovers $\{p^{rs}\}_{r,s\in\mathcal{S}}$ has **no isolated clusters** if its digraph is connected.

Absence of isolated clusters means that there are no groups of sectors that generate and receive technologies only for and from the members of the group. A simplest example of a spillover matrix with isolated clusters is a diagonal matrix – the case in which each sector generates technologies only for itself. Another way to define a matrix without isolated clusters is to say that such matrix can not be represented as a block-diagonal one by permuting rows and columns in the same order.

Finally, Proposition 1:[6]

Proposition 1. *Under zero trade costs, no isolated clusters and no stagnant sectors, the model has a* ***unique and stable*** *interior balanced growth path in which labor allocation vectors are the same across countries:* $L_i^s = \alpha^s L_i \ \forall i \in \mathcal{N},\ \forall s \in \mathcal{S}$.

To provide some intuition behind Proposition 1 let's build a phase-diagram for a simplified version of the model with 2 countries (i and j) and 2 sectors (s and r). For convenience I'll re-write the main equations of the model here. Time variable t can be omitted for brevity because we are going to consider a BGP. First, having relative productivities $t^s \equiv \frac{T_j^s}{T_i^s}$, $t^r \equiv \frac{T_j^r}{T_i^r}$ and relative size of the countries $\frac{L_j}{L_i}$ one can find relative wages $\frac{w_j}{w_i}$ from the trade balance equation:

$$1 = \left(\frac{\alpha^s}{1 + \frac{T_j^s}{T_i^s}\left(\frac{w_j}{w_i}\right)^{-\theta}} + \frac{\alpha^r}{1 + \frac{T_j^r}{T_i^r}\left(\frac{w_j}{w_i}\right)^{-\theta}} \right) \left(1 + \frac{w_j}{w_i}\frac{L_j}{L_i}\right)$$

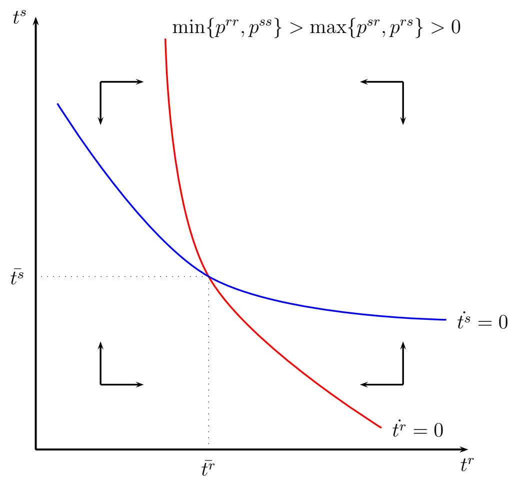
Next, labor allocation across sectors – $l_i^r \equiv \frac{L_i^r}{L_i}$ – can be obtained from

$$\frac{l_i^r}{1 - l_i^r} = \frac{L_i^r}{L_i^s} = \frac{\alpha^r}{\alpha^s} \frac{1 + \frac{T_j^s}{T_i^s}\left(\frac{w_j}{w_i}\right)^{-\theta}}{1 + \frac{T_j^r}{T_i^r}\left(\frac{w_j}{w_i}\right)^{-\theta}} \text{ and } \frac{l_j^r}{1 - l_j^r} = \frac{L_j^r}{L_j^s} = \frac{\alpha^r}{\alpha^s} \frac{1 + \frac{T_i^s}{T_j^s}\left(\frac{w_j}{w_i}\right)^{\theta}}{1 + \frac{T_i^r}{T_j^r}\left(\frac{w_j}{w_i}\right)^{\theta}}.$$

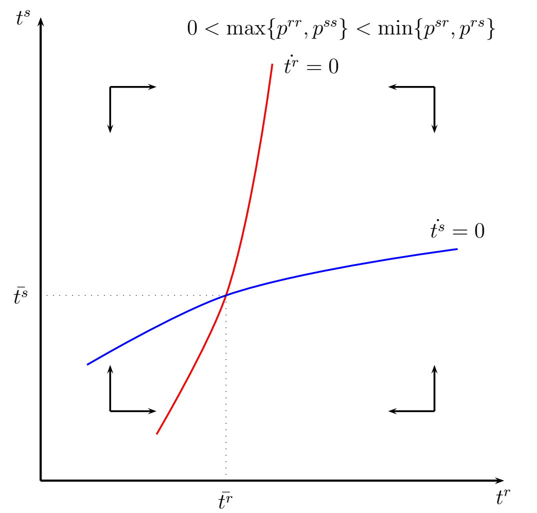
Finally, the dynamics and BGP level of relative productivities can be described by

$$\dot{t^s} = 0 : t^s \equiv \frac{T_j^s}{T_i^s} = \frac{\phi L_j (p^{rs} l_j^r + p^{ss} l_j^s)}{\phi L_i (p^{rs} l_i^r + p^{ss} l_i^s)} \text{ and } \dot{t^r} = 0 : t^r \equiv \frac{T_j^r}{T_i^r} = \frac{\phi L_j (p^{rr} l_j^r + p^{sr} l_j^s)}{\phi L_i (p^{rr} l_i^r + p^{sr} l_i^s)}.$$

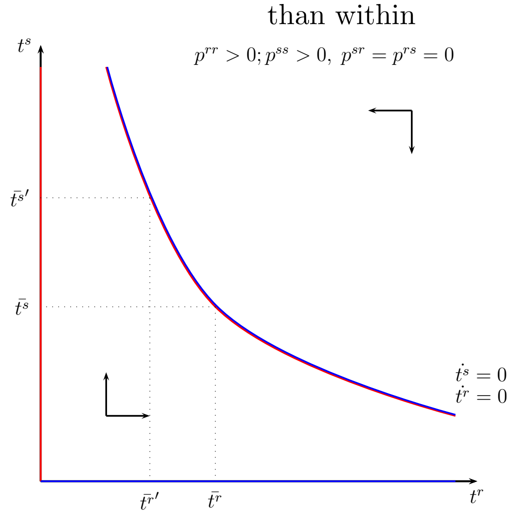
[6] The proof is provided in Appendix D. A particular extension of the proposition with positive trade costs is proved in Appendix E.



(a) Within sectors spillovers are stronger than between



(b) Between sectors spillovers are stronger than within



(c) Positive spillovers within sectors and zero – between sectors.

Figure 2.2: Dynamics of relative productivity for a 2×2 model.

If the left-hand side of the latter equations is below their right-hand side, then the current labor allocation contributes more to relative productivity of country j than country i in a given sector, hence, $\frac{T_j^s}{T_i^s}$ increases if $\frac{T_j^s}{T_i^s} < \frac{\phi L_j(p^{rs}l_j^r+p^{ss}l_j^s)}{\phi L_i(p^{rs}l_i^r+p^{ss}l_i^s)}$. The same holds for sector r. Now, one can see that the BGP is characterized by two endogenous state variables – relative productivities of countries within each sector, $t^s \equiv \frac{T_j^s}{T_i^s}$ and $t^r \equiv \frac{T_j^r}{T_i^r}$, which can be viewed as measures of comparative advantage of country j in sectors s and r correspondingly.

Figure (2.2a) illustrates Proposition 1 for this simple 2-sector 2-country case. Sub-figures (a) and (b) depict the phase diagrams for the considered economy when all spillovers are positive, $p^{rs} > 0 \; \forall r, s \in \mathcal{S}$, i.e. there are no isolated clusters. Sub-figure (a) corresponds to a more realistic case when intra-sector spillovers are stronger than cross-sector. Sub-figure (b) illustrates the opposite case. As Proposition 1 states, when there are no isolated clusters the curves $\dot{t^s} = 0$ and $\dot{t^r} = 0$ intersect only once at point $(\bar{t^r}, \bar{t^s})$ in such a way that $(\bar{t^r}, \bar{t^s})$ is a stable BGP. If the two sectors are isolated – $p^{sr} = p^{rs} = 0$ – then curves $\dot{t^s} = 0$ and $\dot{t^r} = 0$ merge into one curve, any point on which is a BGP, e.g. both points $(\bar{t^r}, \bar{t^s})$ and $(\bar{t^r}', \bar{t^s}')$ on Figure (2.2c) are on the BGP. Thus, there exist infinitely many equilibria and the initial conditions define to which BGP the economy converges.

Notice that under zero cross-sectoral spillovers there also exist unstable balanced growth paths outside the downward sloping curve $\dot{t^s} = 0, \; \dot{t^r} = 0$. If, say, country j doesn't have sector s, i.e. $T_j^s = 0$, then referring to Figure (2.2c) the state of the system can be described by a point on the horizontal axis ($t^s = 0$) that shifts over time to the right. As a result countries' production and trade patterns approach complete specialization: country j always produces only goods r, while country i produces both s and r, yet, its share in sector r is ever-shrinking. The path is unstable since any transfer of technology that turns T_j^s into a positive number will bring the system to an equilibrium on the downward-sloping curve $\dot{t^s} = 0, \; \dot{t^r} = 0$.

Now let's consider the forces that govern the dynamics of relative productivities and, hence, comparative advantage. The first force – let's name it the **country size force** – prevents the relative productivities of country j in both sectors from going to either (∞, ∞) or $(0, 0)$. This force dominates in the North-East and South-West quadrants of Figures (2.2a)–(2.2c). The explanation for it comes from the fact that each unit of labor in both countries i and j can generate technologies at the same rate ϕ, hence, the ratio of productivities on the BGP will be finite and proportional to the ratio of sizes of the two countries.

The second force – **comparative advantage centripetal force** – prevents the self-reinforcing specialization and causes a decline in comparative advantage of each country. This force dominates in the North-West and South-East quadrants of Figures (2.2a) and (2.2b). To explain the mechanism that creates it let's consider a country that can buy at the same price some mass dT of technologies with productivities drawn from the same Pareto distribution. For which sector would it buy these technologies? For the least productive one! To see why, let's assume that the country in this example is a small open economy with 2 sectors with equal shares in consumption expenditures, $\alpha^A = \alpha^B = 0.5$. If we normalize the income of the rest of the World to 1 then the income of this country can be approximated by $w_i L_i \approx 0.5\pi_i^A + 0.5\pi_i^B$. The country will invest the mass of technologies dT in a sector for which $\frac{d\pi_i}{dT}$ is the largest. One can show that the expenditure share π_i^X is an increasing and concave function in the corresponding T_i^X. Concavity comes from the fact that the more productive is the sector and the larger is its share in expenditures the harder it is to come up with a technology that would excel the existing high productivity level in this sector. Thus, the investment in the same mass dT of technologies would have the highest return in the least productive sector[7]. The comparative advantage centripetal force can explain why we observe productivity of sectors with initial comparative disadvantage growing faster and, as a result, decreasing comparative advantage across countries. The latter salient feature of the data is documented in [18].

The third force that plays an important role in the described dynamics of comparative advantage is the **comparative advantage centrifugal force**. This force emerges from the fact that, using the wording of the simple example above, it actually costs more to invest dT in the least productive sector than in the most productive. This happens because to invest dT in the least productive sector the country should divert some mass of labor to it from more productive sectors. Under no cross-sector spillovers there are multiple BGPs in which the comparative advantage centrifugal and centripetal forces equalize each other – costs or reallocating some marginal amount of labor across sectors are equal to benefits from such reallocations. Cross-sector spillovers allow to make the “investments” in the least productive sectors less costly, essentially providing technologies for them from the most productive sectors “for free” – now there is no need to divert labor to least productive sectors in order to allow them to catch up. As a result the centripetal force becomes stronger and all economies converge to the BGP without any comparative advantage. In this chapter I do not model international technological spillovers, but their presence would work in the same way as domestic spillovers – they will weaken the centrifugal force and allow the least productive sectors to catch up without diverting labor from the more productive ones; the centripetal force will remain the same.

An alternative way to convey the intuition behind Proposition 1 is to consider a simple example of 2×2 economy with and without cross-sector spillovers. Assume that we have two countries (i and j) and two sectors (r and s) for which $L_i = L_j$ and $\alpha^s = \alpha^r$ and $\frac{T_i^s}{T_i^r} = \frac{T_j^r}{T_j^s} = 10$. If $p^{ss} = p^{rr} = 1$ and $p^{sr} = p^{rs} = 0$ then the equilibrium in which $\frac{L_i^s}{L_i^r} = \frac{L_j^r}{L_j^s} = 10$ will be a BGP. Indeed, in this case sector s in i is 10 times more productive than r, it employs 10 times more labor and generates 10 times more ideas per unit of time than r. Hence, s grows at the same rate as r (productivity of sector s in i is proportional to $(T_i^s)^{1/\theta}$). So, we have illustrated that such an equilibrium is a BGP and because number “10” can be replaced by any other positive number, there is a continuum of such BGPs. Now let’s modify the assumption of zero inter-sector spillovers and set $p^{sr} = p^{rs} = 0.1$. What will happen now is that labor allocation $\frac{L_i^s}{L_i^r} = \frac{L_j^r}{L_j^s} = 10$ will generate a mass of technologies equal to 2 for sector

[7] If sectors have different shares in final consumption expenditures then the changes in expenditure shares $d\pi/dT$ should be weighted using the corresponding expenditure shares when defining the return on investment in dT.

r in country i and only 10.1 – for sector s. This will allow sector r to catch up with sector s. The opposite will happen in country j. The international spillovers will act in the same manner as inter-sector spillovers: even under $p^{ss} = p^{rr} = 1$ and $p^{sr} = p^{rs} = 0$ sector r in i disproportionally more technologies than from j than s, thus, r will grow faster up until the point when relative sizes of both sectors in both countries become the same.

2.3 Economic policy

Technology in the current model is a public good. Sectors differ in terms of how strong are the externalities that each of them generates. So, some sectors can generate technologies that are more widely used and, thus, such sectors can be considered core sectors as in [12]. As a result, there may exist a room for a welfare-improving economic policy when the government promotes the core sectors to increase productivity in the whole economy. The current section describes the necessary conditions for such policy and also gives an example of it in a form of sector-specific taxes. In what follows I assume zero discount rates, so that it is only welfare on the BGP that is taken into account as a criteria for policy optimality. This assumption allows me to provide some closed form results to describe the optimal policy.

Let's modify the above mentioned model in the following way – the government in country i taxes producers in sector s at rate τ_i^s. Thus, unit costs of producers of variety ω in sector s of country i at time t is $\frac{\tau_i^s w_i(t)}{Z_i^s(\omega,t)}$. Collected tax revenue is distributed among households as a lump-sum transfer. With this modification Equation (2.11) turns into

$$\pi_{ij}^s(t) = \frac{T_i^s(t)\left(w_i(t)\tau_i^s d_{ij}^s\right)^{-\theta}}{\sum_l T_l^s(t)\left(w_l(t)\tau_l^s d_{lj}^s\right)^{-\theta}} \tag{2.24}$$

Revenue of all producers in sector s of country i now becomes $L_i^s(t)\tau_i^s w_i(t)$. What matters for the allocation of labor in the current setting and, hence, for utility per capita, are the ratios of taxes across sectors within each country, $\left\{\frac{\tau_i^r}{\tau_i^s}\right\}_{s,r\in\mathcal{S}}$, but not absolute values of taxes $\{\tau_i^r\}_{r\in\mathcal{S}}$.

There are several explanations that justify the use of namely this policy tool. First, it is an indirect and viable tool of economic policy unlike some direct tools such as direct labor allocation across sectors. Second, in the context of an open economy this tool seems preferable to any trade policy instruments because it can be easily implemented and it doesn't discriminate between . Finally, in the context of the model, introducing sector-specific taxes is isomorphic to introducing an exogenous component of productivity for a particular sector in a particular country[8]. So, the insights obtained from modeling the impact of taxes on the

[8]Indeed under a country-sector specific productivity shifter A_i^s – exogenous component of productivity – the unit costs becomes $\frac{w_i d_{ij}^s}{A_i^s Z_i^s(\omega)}$ which is equal to the unit costs under taxation $\frac{\tau_i^s w_i d_{ij}^s}{Z_i^s(\omega)}$ if $\tau_i^s = 1/A_i^s$.

equilibrium outcomes are identical to those that would be obtained under the presence of exogenous components in sectoral productivity and comparative advantage.

Autarky

To proceed, let's again consider two extreme regimes of trade and start with **autarky**. Countries are isolated, so country indices can be dropped. Normalizing wages in a country to $w = 1$, income per capita can be written down as $\frac{I(t)}{L(t)} = \sum_s \tau^s l^s(t)$. Because of Cobb-Douglas utility at the level of sector aggregates we have $\alpha^s I(t) = \tau^s L^s(t)\ \forall s \in \mathcal{S}$, so the equilibrium labor allocation depends only on the expenditure shares $\{\alpha^s\}_{s\in\mathcal{S}}$ and taxes $\{\tau^s\}_{s\in\mathcal{S}}$. Labor demand is homogeneous in taxes of degree zero, so one can normalize taxes in one sector to 1, say $\tau^S \equiv 1$. The resulting labor allocation can be obtained as a solution of the system of equations

$$\tau^r = \frac{L^S}{L^r}\frac{\alpha^r}{\alpha^S},\ \forall r \in 1, \dots, S-1, \tag{2.25}$$

subject to the total labor supply constraint $\sum_{s\in\mathcal{S}} L^s = \bar{L}$.

Price level in sector r is equal to $p^r(t) = \tau^r \gamma \left(T^r(t)\right)^{-\frac{1}{\theta}}$, while the number of ideas per capita along the BGP is $\frac{T^r}{L} = \frac{\phi}{g}\sum_q p^{qr} l^q$. Using these two expressions the BGP level of utility per capita can be written down as

$$\frac{U}{L} = \prod_r \left(\frac{\alpha^r}{p^r}\frac{I}{L}\right)^{\alpha^r} = \frac{I}{\gamma L}\left(\frac{\phi L}{g}\right)^{\frac{1}{\theta}} \prod_r \left(\frac{\alpha^r}{\tau^r}\left(\sum_q p^{qr} l^q\right)^{\frac{1}{\theta}}\right)^{\alpha^r} \tag{2.26}$$

To find the optimal taxes I maximize U/L w.r.t τ's for any given level of L. For a 2-sector economy one can show that under equal shares in expenditures – $\alpha^s = \alpha^r = 0.5$ – sector r should be taxed at a higher rate in order to re-allocate labor to sector s if $p^{ss}p^{sr} > p^{rr}p^{rs}$, i.e. if sector s generates more widely applicable technologies than sector r. Besides, the optimal tax[9] $\frac{\tau^r}{\tau^s}$ can not be infinitely large – which would result in a collapse of the more heavily taxed sector r – first, because of love for variety and, second, because having "donor" sectors that generate more general technologies makes sense only if there are "recipient" sectors that can adopt those technologies.

An alternative way to think about industrial policy is to consider direct labor re-allocation across sectors. In this case the social planner solves the following constrained optimization problem

$$\max_{\{L^s\}_{s\in\mathcal{S}}} \gamma\left(\frac{\phi}{g}\right)^{\frac{1}{\theta}} \prod_{s\in\mathcal{S}} \left(L^s\left(\sum_{r\in\mathcal{S}} p^{rs} L^r\right)^{\frac{1}{\theta}}\right)^{\alpha^s}, \text{ s.t. } \sum_{r\in\mathcal{S}} L^r = \bar{L}, \tag{2.27}$$

[9] The optimal tax in a 2-sector economy $\tau \equiv \frac{\tau^s}{\tau^r}$ solves the FOC of maximization of the BGP level of $\frac{U}{L}$ for any level of L: $\frac{\alpha^r}{\theta}\frac{p^{rr}\alpha^r}{p^{rr}\alpha^r\tau + p^{sr}\alpha^s} + \frac{\alpha^s}{\theta}\frac{p^{rs}\alpha^r}{p^{rs}\alpha^r\tau + p^{ss}\alpha^s} + \frac{1-\alpha^s}{\tau} - \left(1+\frac{1}{\theta}\right)\frac{\alpha^r}{\alpha^r\tau+\alpha^s} = 0.$

the first order conditions for which are

$$\frac{\alpha^r}{L^r} + \sum_{s\in\mathcal{S}} \frac{\alpha^s}{\theta} \frac{p^{rs}}{\sum_{q\in\mathcal{S}} p^{qs} L^q} - \lambda = 0 \ \forall r \in \mathcal{S} \text{ and } \sum_{r\in\mathcal{S}} L^r = \bar{L} \tag{2.28}$$

For any pair of sectors r and v the optimality requires

$$\frac{1}{L^r}\left(\alpha^r + \frac{1}{\theta}\sum_{s\in\mathcal{S}} \frac{\alpha^s L^r p^{rs}}{\sum_{q\in\mathcal{S}} L^q p^{qs}}\right) = \frac{1}{L^v}\left(\alpha^v + \frac{1}{\theta}\sum_{s\in\mathcal{S}} \frac{\alpha^s L^v p^{vs}}{\sum_{q\in\mathcal{S}} L^q p^{qs}}\right) \tag{2.29}$$

From Equation (2.25) one can find the schedule of taxes that results in any particular labor allocation $\{L^s\}_{s\in\mathcal{S}}$, including the one described by Equation (2.29).

Equation (2.29) provides a criteria for optimal re-allocation of labor within any pair of sectors. If under the free market labor allocation ($L^s/\bar{L} = \alpha^s \ \forall s \in \mathcal{S}$) the right-hand side of Equation (2.29) is larger than the left-hand side then it means that sector r under free market generates a larger share of technologies for more important sectors than sector v and, thus, labor should be re-allocated from v to r. Clearly, if both sectors generate technologies of the same applicability ($p^{vs} = p^{rs} \ \forall s \in \mathcal{S}$) then labor should not be re-allocated across these two sectors comparing to the free market outcome when $\frac{L^r}{L^v} = \frac{\alpha^r}{\alpha^v}$. One can also notice that for a diagonal matrix of spillovers ($p^{qs} = p^q$ if $q = s$ and $p^{qs} = 0$ if $q \neq s$) the first order conditions turn into $L^r = \alpha^r \bar{L} \ \forall r \in \mathcal{S}$ which describes exactly the labor allocation that would took place without any policy interventions. Thus, as follows from this simple 2×2 example, the labor re-allocating policy in autarky can be welfare-improving only if there exist positive inter-sector spillovers – intra-sector spillovers alone are not enough to create a room for such policy. In fact, under diagonal matrix of spillovers the described autarkic economy is isomorphic to the one with Marshallian externalities within each sector where output in each sector s is proportional to $(L^s)^{1+\frac{1}{\theta}}$. Finally, notice that asymmetry either in expenditure shares across sectors or in spillovers is required. If $\alpha^s = 1/S \ \forall s \in \mathcal{S}$ and $p^{rs} = p^{sr} \ \forall r, s \in \mathcal{S}$ then, again, there is no room for welfare-improving policy.

Open economy

Under **costless trade** the asymmetric taxation re-allocates labor in the same manner as in autarky, yet, now this mechanism involves some additional factors. First, love for variety no longer impedes re-allocation, so the responsiveness of labor demand to taxes should be higher. Second, economic policy of trade partners comes into play. E.g. if country i through taxation re-allocates labor from a sector with narrowly-applicable technologies to a sector with widely-applicable technologies then, absent any economic policy in country j, labor in j will be re-allocated in the opposite direction. As a result, in the BGP economy i will have higher number of ideas per capita (T/L) in each sector than country j. So, i will be characterized by higher welfare, though, both economies will be growing at the same rate g/θ.

To provide a more detailed description on the role of spillovers for the implications of the outlined policy let's first consider the 2×2 model with costless trade and **zero inter-sector spillovers**: $p^{sr} = 0,\ p^{ss} > 0$. Zero inter-sector spillovers imply that on the BGP $T_i^s = \frac{\phi}{g} p^{ss} L_i^s$ which together with sector labor demand $w_i \tau_i^s L_i^s = \alpha^s \pi_i^s I$ results in either $\frac{\tau_i^s}{\tau_j^s} = \frac{\tau_i^r}{\tau_j^r}$ or some $L_{\cdot}^{\cdot} = 0$ – corner solution. In order to describe the behavior of the model under different taxes we find all $\{t^r, t^s\}$ that characterize $\dot{t}^s = 0$ and $\dot{t}^r = 0$. For $\dot{t}^s = 0$ these loci are described by $t^s = 0$ and

$$t^s = \frac{L_j^s}{L_i^s} = \frac{L_j(A + 1 + Ft^r) - L_i BFt^r}{L_i(BFt^r + 1 + Ft^r) - L_j A}, \tag{2.30}$$

while for $\dot{t}^r = 0$ – by $t^r = 0$ and

$$t^r = \frac{L_j^r}{L_i^r} = \frac{L_j(C + 1 + Ht^s) - L_i DHt^s}{L_i(DHt^s + 1 + Ht^s) - L_j C}, \tag{2.31}$$

where $A \equiv \frac{\alpha^r}{\alpha^s}\frac{\tau_i^s}{\tau_i^r} \equiv C^{-1}$, $B \equiv \frac{\alpha^r}{\alpha^s}\frac{\tau_j^s}{\tau_j^r} \equiv D^{-1}$, $F \equiv \left(\frac{\tau_j^r}{\tau_j^s}\frac{\tau_i^s}{\tau_i^r}\right)^{-\theta} \equiv H^{-1}$.

It can be shown that symmetric taxes do not eliminate multiplicity of BGPs and result in a "corner" BGP only is the economy starts with absent sectors in some countries. To be more clear let's consider symmetric countries $L_i = L_j$ with symmetric consumption shares $\alpha^s = \alpha^r$ and symmetric taxes $\frac{\tau_i^s}{\tau_i^r} = \frac{\tau_j^s}{\tau_j^r} = \frac{\tau^s}{\tau^r} > 1$ (w.l.o.g). Figure (2.3) illustrates a new equilibrium – blue lines for $\dot{t}^s = 0$ and red lines for $\dot{t}^r = 0$ – and the old equilibrium – a downward sloping gray dashed line. Both loci go through point (1,1) because symmetric taxes do not introduce any asymmetry if initially productivities were the same. Downward sloping curve is described by equation $t^s = \frac{(1+A)+t^r(1-A)}{t^r(1+A)+(1-A)}$, $A \equiv \frac{\alpha^r \tau^s}{\alpha^s \tau^r} > 1$. As one can see, there exist infinitely many interior solutions when both countries produce in both sectors. Besides, unlike in the case of no taxes, now there exist four corner solutions – two with complete specialization as before and two new, with incomplete specialization. The corner solution with complete specialization emerges when country j starts without sector r; with incomplete specialization – if j starts without s. The other two corner solutions are the symmetric cases of the above mentioned ones but when i starts either without s or r.

When both countries start with non-zero productivity in each sector the symmetric tax on sector s (equivalently, subsidy of r) has asymmetric impact on specialization and welfare. First, each point on the curve $\dot{t}^s = 0,\ \dot{t}^r = 0$ is characterized by the same total utility $U_i + U_j$ which decreases whenever $\frac{\tau^s}{\tau^r}$ deviates from 1. Second, the allocation of welfare along the curve is not the same – as the BGP point $(t^r,\ t^s)$ moves upwards U_j increases while U_i declines. Thus, if the economy started at point A where j specialized more in s while i – in r and both countries had equal welfare per capita $\left(\frac{U_i}{L_i} = \frac{U_j}{L_j}\right)$, the tax on s (subsidy for r) will improve j's relative productivity in both sectors and make it better off comparing to i, yet, it will also shrink the total size of the "pie" so that the aggregate welfare will

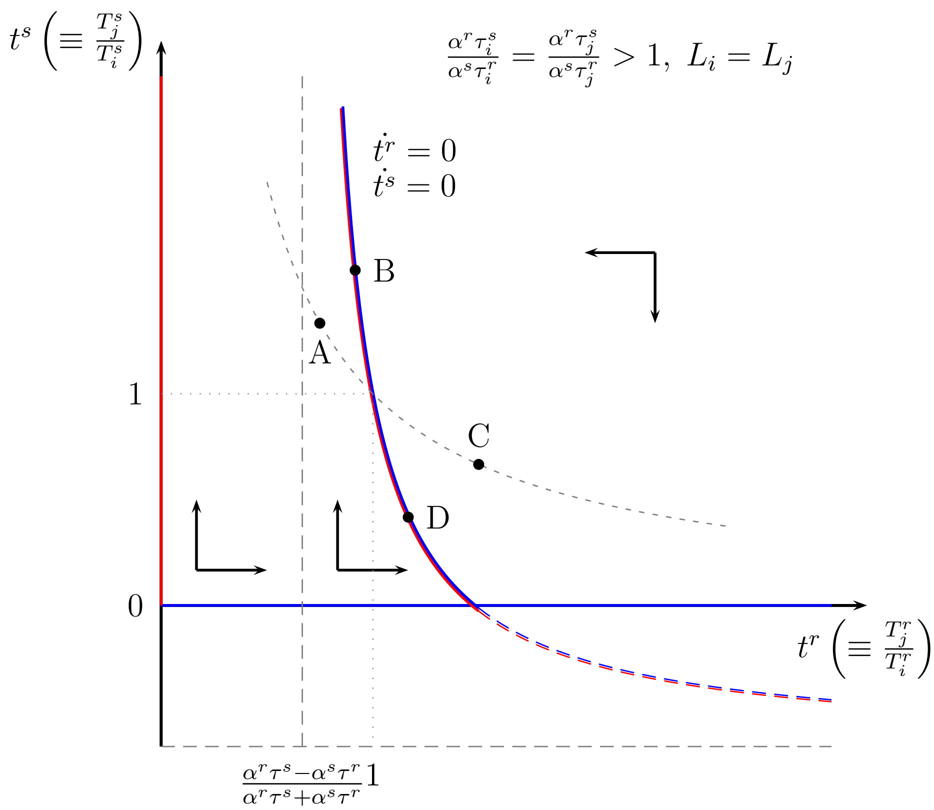


Figure 2.3: Symmetric taxes under zero inter-sector spillovers.

decline. The intuition for changes in relative productivity across countries can again be described in terms of "returns" to an additional mass of technologies in a more versus less productive sectors. The logic is similar to what we saw before: the same mass of technologies will be applied to a greater number of varieties in a less productive sector than in a more productive one. Symmetric taxes on a more productive sector s in country j re-allocate labor towards less productive sector r, increasing productivity of r by a larger factor than are the losses in productivity in s. On the contrary, country i re-allocates labor from a less productive s to a more productive r, losing a significant portion of productivity in s and gaining disproportionately less in productivity in r. As a result, relative productivity of country j increases in both sectors. It can be shown that under zero taxes $\frac{d\log U_j}{d\log \tau^s/\tau^r} = \frac{l^r_i - l^r_j}{2}$, thus, if country j specializes more in sector s it will gain (and country i will lose) from some positive tax on s/subsidy for r conditional on this tax/subsidy being symmetric across countries. Since the relation between U_j and τ^s/τ^r is not monotonic there exists an optimal level of τ^s/τ^r after which U_j will decrease.

Symmetric taxes across countries is a very particular case of policy[10], so now we consider a more general case of asymmetric taxes. For simplicity and w.l.o.g. assume $\tau \equiv \tau_j^s > 1 = \tau_i^s = \tau_i^r = \tau_j^r$, yet, avoid the assumption on symmetry in sectors' shares and country size because depending on these characteristics the outcomes of taxation will be different. Equations (2.30) and (2.31) turn into $t^s = \frac{L(\alpha+1)+t^r(L\tau^\theta - \alpha\tau^{\theta+1})}{t^r(\tau^\theta+\alpha\tau^{\theta+1})+1-L\alpha}$ and $t^r = \frac{L(\alpha^{-1}+1)+t^s(L\tau^{-\theta}-\alpha^{-1}\tau^{-\theta-1})}{t^s(\tau^{-\theta}+\alpha^{-1}\tau^{-\theta-1})+1-L\alpha^{-1}}$, where $L \equiv \frac{L_j}{L_i}$, $\alpha \equiv \frac{\alpha^r}{\alpha^s}$. Figures (2.4a)-(2.4c) depict the corresponding loci for $\dot{t}^s = 0$ and $\dot{t}^r = 0$ and the resulting patterns of specialization. Asymmetric taxes under zero cross-sector spillovers result in corner BGPs: one sector in one country collapses. Namely, if relative size of country j is smaller than the relative share of sector r – $L < \alpha$ – then taxation of s in j will result in j's complete specialization in r while i will produce both r and s. This BGP is illustrated by Figure (2.4a) where the equilibrium relative productivity stabilizes at $t^s = 0$ and $t^r = \frac{L(\alpha+1)}{\alpha-L}$. On the contrary, if j is large enough – $L > \alpha\tau$ – then the reallocation of labor from s to r inside country j will be associated with more labor allocated to s in i, so that ultimately i will produce only s while j will produce both. As Figure (2.4c) shows, in this case the relative productivities t^s and t^r approach $\frac{L-\alpha\tau}{1+\alpha\tau}$ and ∞ correspondingly. In the intermediate case – $\alpha < L < \alpha\tau$ – complete specialization will be observed: j will produce only r and i – only s: in Figure (2.4b) $t^s \to 0$ and $t^r \to \infty$.

Now let's turn to the welfare implications of each of these resulting specialization patterns. As one can show asymmetric taxes in the current setting can not make country j better off: under $\frac{L_j}{L_i} < \frac{\alpha^r}{\alpha^s}$ its utility per capita remains the same as under zero taxes, while under $\frac{L_j}{L_i} > \frac{\alpha^r}{\alpha^s}$ it decreases with τ. So we can conclude that under zero cross-sector spillovers no unilateral industrial policy can make country j better off. Coordinated symmetric policy can improve country j welfare only at a cost of country i. To summarize this section on economic policy in the open economy under zero inter-sector spillovers: we saw that no labor re-allocating policy can improve the total welfare of the World and no countries have incentives to implement such policy unilaterally. Symmetric economic policy under the diagonal matrix of spillovers can redistribute welfare across trading partners, but will unambiguously decrease the total welfare. Thus, positive inter-sector spillovers are necessary for the possibility of welfare-improving policy. It follows from the latter that under zero inter-sector spillovers it really doesn't matter in what products the country specializes.[11]

[10] Although a particular case of policy, the symmetric taxes exemplify well the case of industrial policy motivated as a "response to foreign targeting" as it is described in [17]. This example shows that country i is strictly worse off under such response to the policy of j and, as we will see later, under zero inter-sector spillovers country j would not unilaterally initiate any policy if it didn't expect a symmetric response from country i.

[11] This result, as I mentioned above, if isomorphic to the result with equal Marshallian externalities across sectors. Yet, it crucially depends on the equality of θ across sectors. If the latter are allowed to vary across sectors then then even under diagonal matrix of spillovers it would make sense to allocate labor to sectors with lower θ, i.e. to those with thicker tail of distribution of productivity.

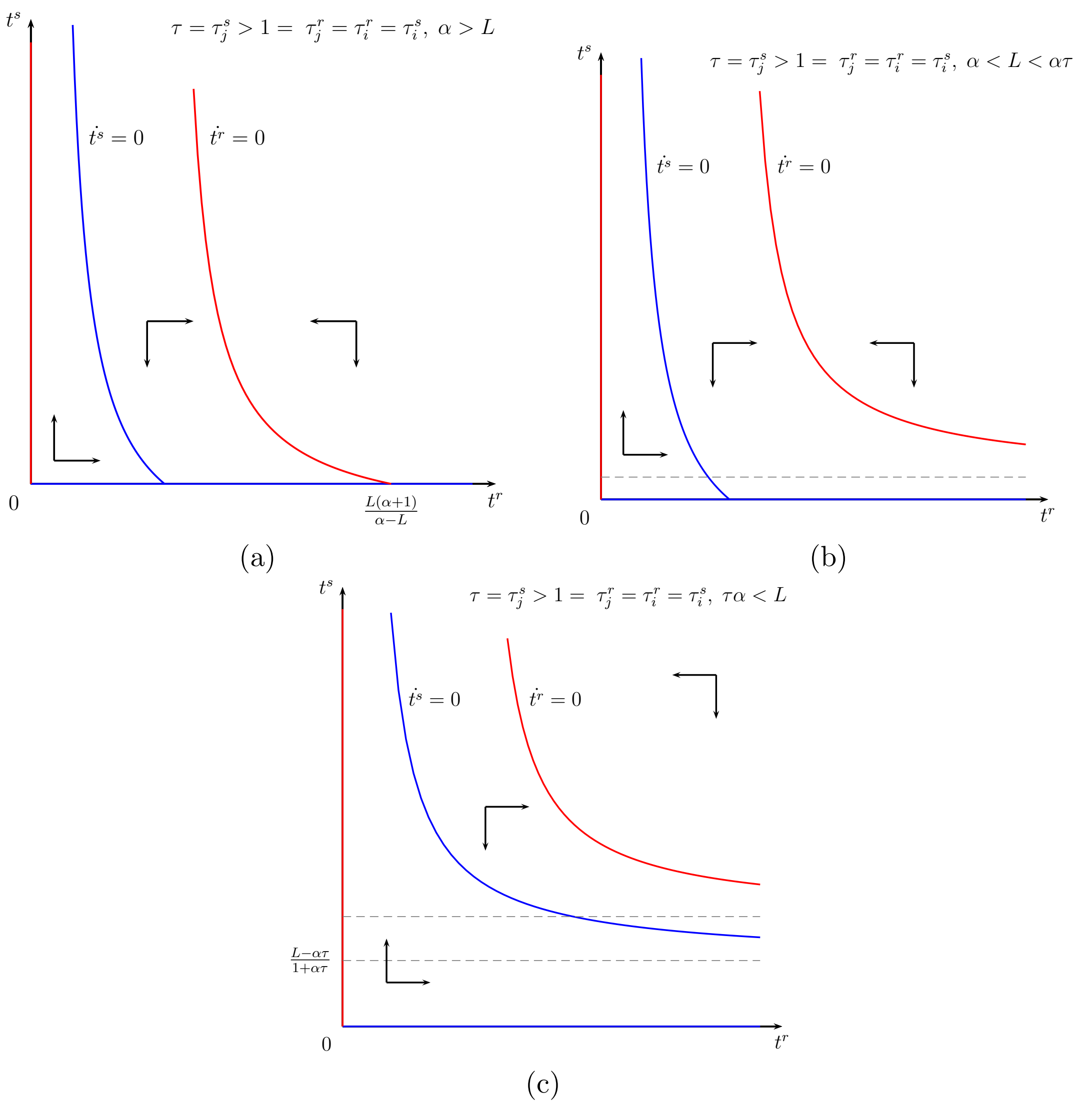


Figure 2.4: Asymmetric taxes under zero inter-sector spillovers.

The welfare implications of trade openness may be different in the case of **positive inter-sector spillovers**. This comes from the fact that under positive spillovers labor re-allocation can improve welfare. Trade openness also leads to labor re-allocation which is not necessarily aligned with the optimal one. As before, let's start with the case of symmetric taxes on sector s, $\tau_i^s = \tau_j^s > 1$. As Figure (2.5) shows, under symmetric tax both loci $\dot{t^r} = 0$ and $\dot{t^s} = 0$ will

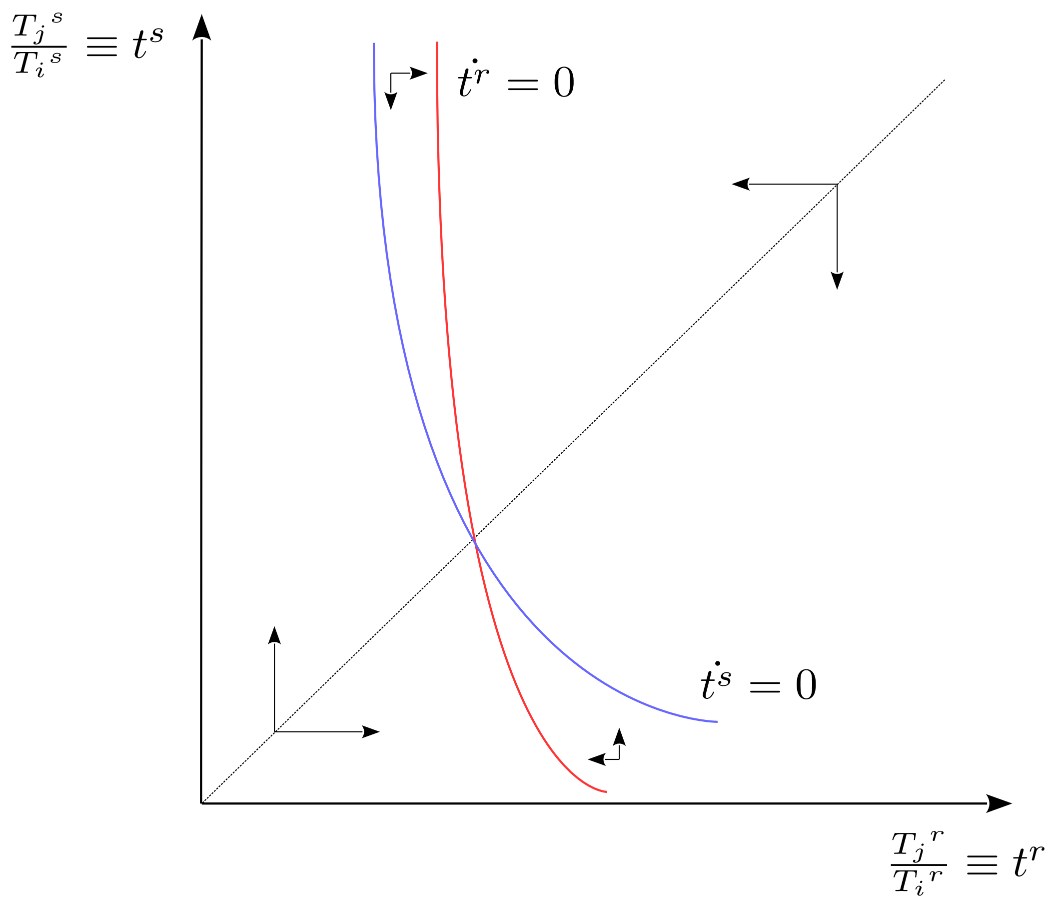


Figure 2.5: Symmetric taxes under positive inter-sector spillovers.

rotate clockwise, yet, symmetric taxes will not introduce any asymmetry to the BGP – it will remain on the 45-degree line meaning no comparative advantage in either of the countries in the long-run. What will change is the allocation of labor within each country, relative productivity between sectors within countries and, hence, intensity of spillover flows between sectors. In the same manner as re-allocation of labor towards core sectors in the autarky helped to increase welfare, symmetric taxes that favor core sectors in the open economy increase the welfare in each country and the World as a whole (the whole World can be treated as an autarky).

For asymmetric taxes let's again consider the case of country j taxing sector s: $\tau_j^s > 1 = \tau_i^s = \tau_j^r = \tau_i^r$. Now sector s has a comparative disadvantage in country j, while r – comparative advantage. This tax-wedge will shift the BGP downwards, so that in the long-run country j has a comparative advantage in sector r, while country i – in sector s. This outcome is illustrated by Figure (2.6). The straightforward result of taxation is that the

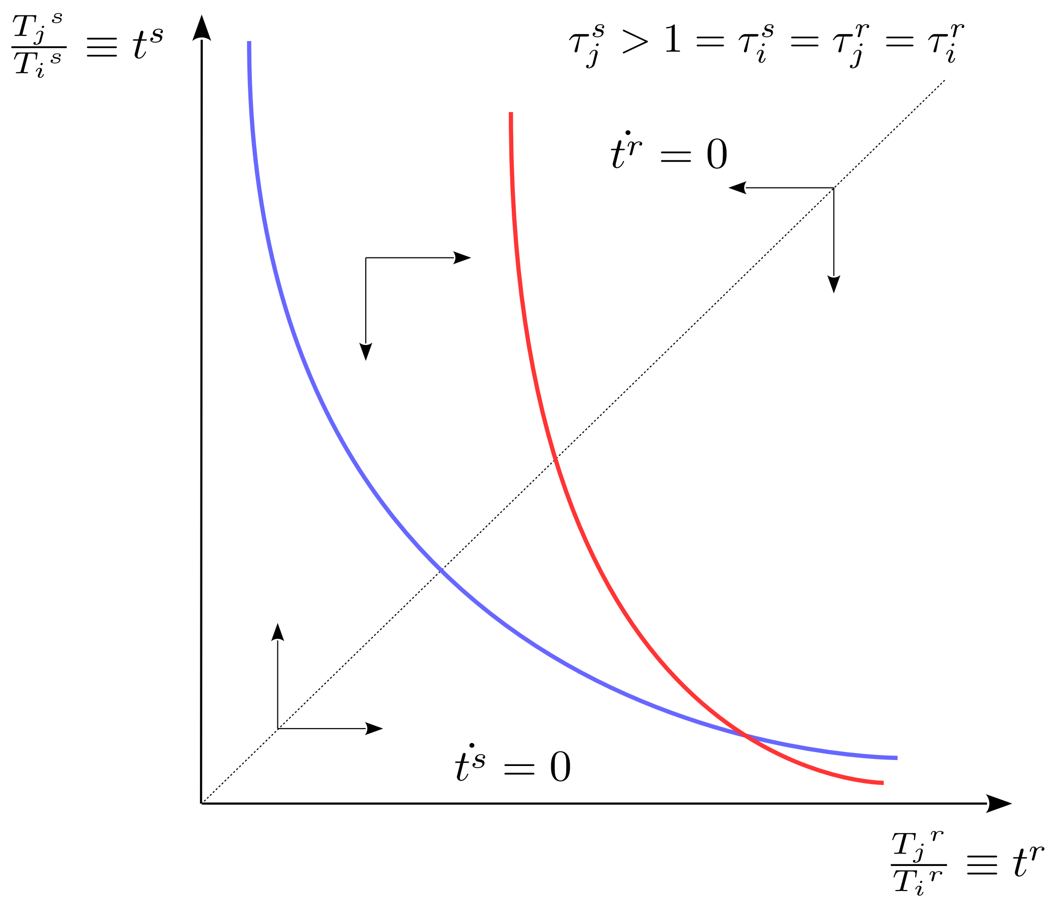


Figure 2.6: Asymmetric taxes under positive inter-sector spillovers.

employment in the taxed sectors decreases comparing to the no-tax scenario and this labor allocation is preserved on the BGP. This is especially important if we think about exogenous factors of comparative advantage (e.g. deposits of natural resources) as taxes or subsidies on particular sectors. Trade openness without any policy interventions will result in lower employment in sectors that are more heavily "taxed", i.e. those that originally are at a disadvantaged position due to exogenous factors. If the disadvantaged sectors are the core sectors, then productivity in the whole economy will decline under trade openness comparing to what it would be under the autarkic labor allocation. This mechanism is similar to the one described in [12], yet, the consequences of core sectors' shrinkage here is not a zero growth rate of the economy (in the long-run all economies are growing at rate $g/\theta > 0$ regardless of their sectoral composition), but a lower productivity and, potentially, lower welfare on the BGP.

To illustrate the last point let's add some more details to out 2-country 2-sector example. For simplicity let's consider i and j of equal size ($L_i = L_j$) with sector r and s with equal expenditure shares ($\alpha^r = \alpha^s$). Assume that sector r is a core sector, i.e. it generates more widely applicable technologies than sector s: $p^{rr} = 0.9 = p^{ss}$, $p^{rs} = 0.7$, $p^{sr} = 0.1$. Let's also assume that country i has some exogenous comparative advantage in a non-core sector s which is expressed by an equivalent exogenous subsidy $\tau_i^s < 1$. Figure (2.7) illustrates the utility per capita of country i on the BGP under autarky and frictionless trade depending on the magnitude of exogenous comparative advantage in s. The horizontal axis shows the extent of exogenous comparative advantage of s: small $\tau_i^s 1$ means that sector s receives a large exogenous "subsidy" comparing to sector r. In other words, exogenous "subsidy" means that for the same number of technologies accumulated in s and r in country i, sector s will be more productive than r by factor $1/\tau_i^s$. When country i opens to trade it observes two forces affecting its welfare. First, it observes lower prices for varieties in which country j is more productive than i. This is a standard force of comparative advantage and specialization in varieties with higher productivity which unambiguously leads to an increase in welfare of i with opening to trade. Second, the labor is reallocated to the non-core sector s with an exogenous comparative advantage: now varieties of sector r can be imported and demand for them no longer leads to the previous relatively high employment in r. This force results in a decreased productivity of economy i and a decrease in its welfare. For this example the second force dominates in the interval of $\tau_i^s \in (0.53;\ 0.76)$ which means that country i with τ_i^s in this interval may be better off in autarky than under frictionless trade. An interesting observation is the non-monotonicity of gains from trade in the strength of exogenous comparative advantage. Countries with either very low (τ_i^s close to 1) or very high (τ_i^s close to 0) exogenous comparative advantage in sector s would gain from trade. The former ones observe weak forces that pull their labor from r to s with trade openness because the exogenous advantage of s is weak. The latter ones have a significant share of labor allocated to s even in autarky, thus, trade openness doesn't have that much labor to re-allocate from r to s. It is the countries with intermediate levels of exogenous advantage in a non-core sector that may lose from openness to trade.

Unlike for the autarky, criteria for the optimal policy in the open economy does not have a closed form expression. To show this let's write down the welfare maximization problem of a social planner. Let's define the optimal tax schedule of country i as a set of taxes $\{\tau_i^s\} s \in \mathcal{S}$ that maximizes utility per capita in country i on the BGP, $u_i \equiv \frac{U_i}{L_i}$.

$$u_i \equiv \frac{U_i}{L_i} = \prod_{r \in \mathcal{S}} \left(\frac{\alpha^r}{p^r} \frac{I_i}{L_i} \right)^{\alpha^r}, \; p^r = \gamma \left(\sum_{j \in \mathcal{N}} T_j^r (w_j \tau_j^r)^{-\theta} \right)^{-\frac{1}{\theta}}, \; \frac{I_i}{L_i} = \sum_{r \in \mathcal{S}} \tau_i^r l_i^r w_i, \tag{2.32}$$

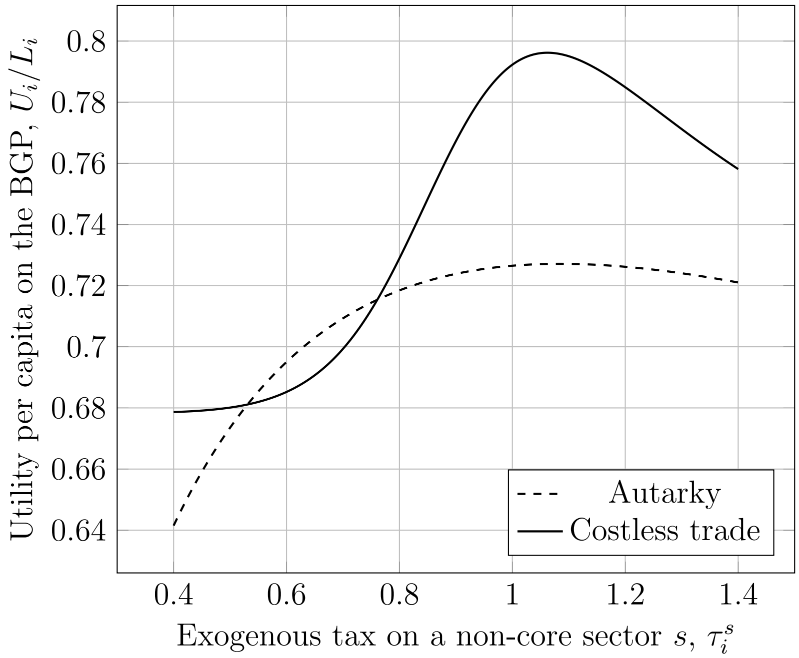


Figure 2.7: Exogenous sector productivity and gains from trade.

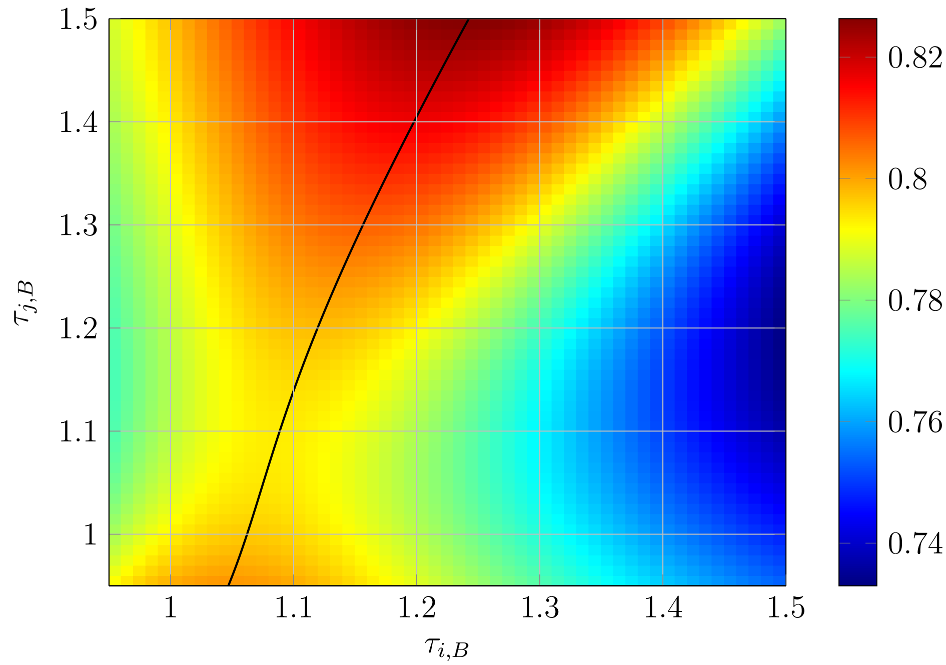
where $l_i^s \equiv L_i^s/L_i$. Substituting the last two expressions into the first one and taking logs of both sides of the resulting expression we obtain

$$\log u_i = \sum_{r\in\mathcal{S}} \alpha^r \log \alpha^r - \gamma + \log \sum_{r\in\mathcal{S}} \tau_i^r l_i^r w_i + \frac{1}{\theta} \sum_{r\in\mathcal{S}} \alpha^r \log \left(\sum_{j\in\mathcal{N}} L_j (w_j \tau_j^r)^{-\theta} \frac{\phi}{g} \sum_{s\in\mathcal{S}} p^{sr} l_j^s \right), \quad (2.33)$$

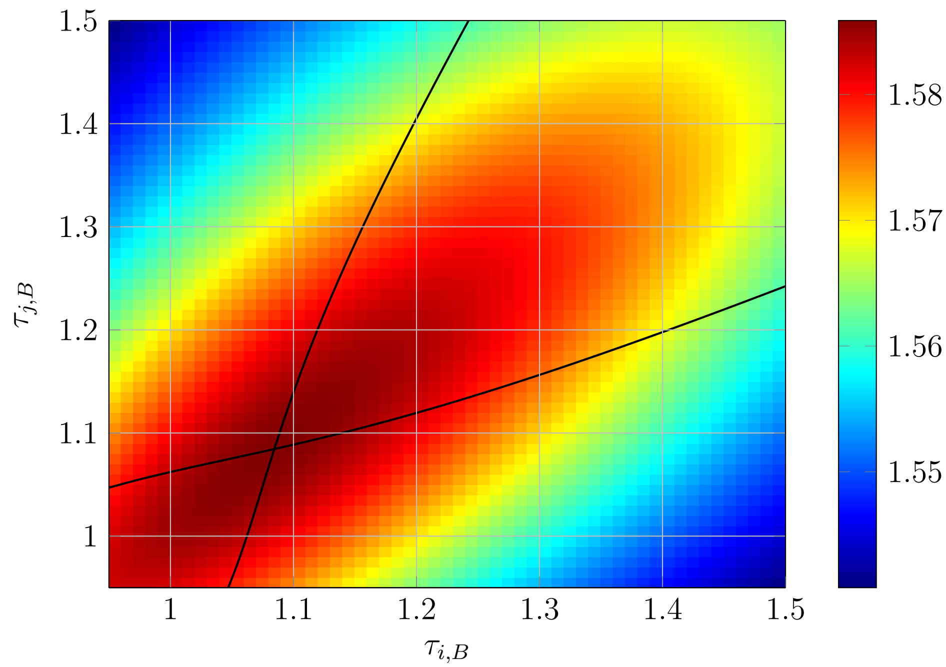
where I also made use of $\frac{T_j^r}{L_j} = \frac{\phi}{g} \sum_q p^{qr} l_j^q$. To find the optimal taxes we need to know the responses of wages and labor allocations across all sectors and countries to changes in taxes in country i – $\left\{\frac{\partial l_j^r}{\partial \tau_i^s}\right\}$, $\left\{\frac{\partial w_j}{\partial \tau_i^s}\right\}$, $i, j \in \mathcal{N}$, $r, s \in \mathcal{S}$. To find these derivatives at BGP one can use the implicit function theorem for the system of equations in l's and w's: $\tau_i^s l_i^s w_i = \pi_i^s \alpha^s I$, where $I = \sum_j \sum_s \tau_j^s l_j^s w_j$, and $\pi_j^s = \frac{T_j^s (w_j \tau_j^s)^{-\theta}}{\sum_k T_k^s (w_k \tau_k^s)^{-\theta}}$. This exercise of computing the optimal policy for an open economy will be completed in the next section using the calibrated model for the US.

To close the current section I would like to provide a numerical example that illustrates the welfare implications of industrial policy under positive inter-sector spillovers in an open economy. Consider the same two symmetric economies as in the example above. But now let's assume that both countries start with zero exogenous factors of comparative advantage and can choose different tax rates for sector s: τ_i^s and τ_j^s.[12] Figures (2.8a)–(2.8b) illustrate

[12]Taxes on sector r in both countries can be normalized to 1.



(a) Utility per capita in country i, U_i/L_i



(b) Total utility, $U_i + U_j$

Figure 2.8: Utility per capita under positive inter-sector spillovers and sector-specific taxes.

the BGP levels of utility per capita that can be attained by countries i and j depending on taxes τ_i^s (horizontal axis) and τ_j^s (vertical axis) that each of them imposes. Solid black curves in Figure (2.8a) illustrates the optimal responses of country i to taxes imposed by country j (blue lines denote the optimal responses of their trade partners). As follows from Figure (2.8a), a given country, conditional on no taxes introduced by its trade partner, has an incentive to subsidize the core sector r: given $\tau_j^s = 1$ the optimal $\tau_i^s > 1$. Same is true for country j. As one can see from Figure (2.8b) the total welfare is maximized at some $\tau_j^s > 0$, $\tau_i^s > 0$ which is similar to the prediction obtained for autarky if we consider the whole world to be an autarky – re-allocation of labor by all countries towards the core sector r increases the welfare globally. For this particular example one can compute that under autarky the optimal labor allocation is $l^r = 52\%$, $l^s = 48\%$ for both countries which is attained by tax $\tau^s = 1.0844$. If both economies can trade at zero cost and only i implements the industrial policy, then its optimal labor allocation in i turns into $l_i^r = 63\%$ and $l_i^s = 38\%$, yet, the required tax is now smaller – $\tau_i^s = 1.0622$. In other words, under costless trade country i doesn't need to produce all varieties of the non-core sector r itself and can allocate even more labor to the core sector s than in autarky. The labor demand is now more responsive to industrial policy, so the larger re-allocation is achieved with smaller taxes. One interesting question for further research is what would happen if country j can respond to taxes introduced in i and whether this game of subsidizing the core sectors has a Nash equilibrium. The second question is – provided that the Nash equilibrium exists, does it result in an optimal level of global welfare or is some coordination between the countries required for attaining the maximum of global welfare.

2.4 Calibration

In this section I quantify the matrix of spillovers and discuss calibration of other parameters. As Equations (2.29) and (2.32) show, one need parameters θ, $\{\alpha^s\}_{s\in\mathcal{S}}$, $\{p^{qs}\}_{q,s\in\mathcal{S}}$ and $\{L_i\}_{i\in\mathcal{N}}$ to characterize the optimal industrial policy (the last set is needed for the open economy case, but not for the autarky).

The parameter that describes spillovers from sector q to sector s has a straightforward interpretation – p^{qs} is a probability of an event that a random technology created in sector q is used in producing any randomly picked variety in sector s. There exists a vast literature in urban economics and economic geography that estimates the strength of technological spillovers. The most recent example is the paper by [11]. The authors of that paper measured the strength of spillovers between sectors s and q as a share of citations generated by patents in sector s that are attributed to sector q:

$$p_{EGK}^{qs} = \frac{C^{qs}}{\sum_{k\in\mathcal{S}} C_{ks}}, \tag{2.34}$$

where C^{qs} is the number of citations sent from s to q and, hence, flows of ideas from q to s.[13] Although this metric quantifies the importance of sector q as a source of ideas for sector s, it also reflect the size of sector q, not only the extent of applicability of ideas from q. As an illustration, let's assume that sector q has 99% of all available ideas (patents) while sector s – only 1%. Let's also assume that ideas from q have the same probability of being used and cited by any patent in s as ideas from s ($p^{qs} = p^{sq}$). If we measure the extent of applicability of ideas from q and from s in sector s with $p^{\cdot}_{EGK}$ we will obtain $p^{qs}_{EGK} = 0.99$ and $p^{sq}_{EKG} = 0.01$.

To obtain the estimates of $\{p^{qs}\}$ that reflect only probabilities of cross-sector spillovers but not size of sectors I derive two estimators for $\{p^{qs}\}$ which I use with the US and Japan patents data. Referring to my model, I treat each patent as a technology – when it is cited, and as a variety which receives a new technology – when it cites other patents. For estimation of $\{p^{qs}\}$ I follow two approaches that use somewhat different dimensions of the patent data and, thus, allow me to check the consistency of the estimates. The first approach – name it a **"cohort approach"** – splits all patents into cohorts based on the year of issuance and the assigned sector: patents issued in year t in sector q enter cohort (q, t). Each cohort (q, t) is characterized by the total number of patents in it $Q(q, t)$, by the number of citations sent to any previous cohort (s, t'), $C(sq, t', t)$, and received from any subsequent cohort (r, t''), $C(qr, t, t'')$, where $t' < t < t''$. Let's consider two cohorts (q, t) and (r, t'') where $t < t''$. Each idea from (q, t) can be applied to any variety in (r, t'') with probability p^{qr}. If the total number of ideas and varieties in q and r are $Q(q, t)$ and $Q(r, t'')$ correspondingly, then the number of citations from (r, t'') to (q, t) will be distributed as $C(qr, t, t'') \sim Poisson\,(p^{qr}Q(q, t)Q(r, t''))$. Figure (2.9) illustrates this example. Considering only the origin and the destination cohorts

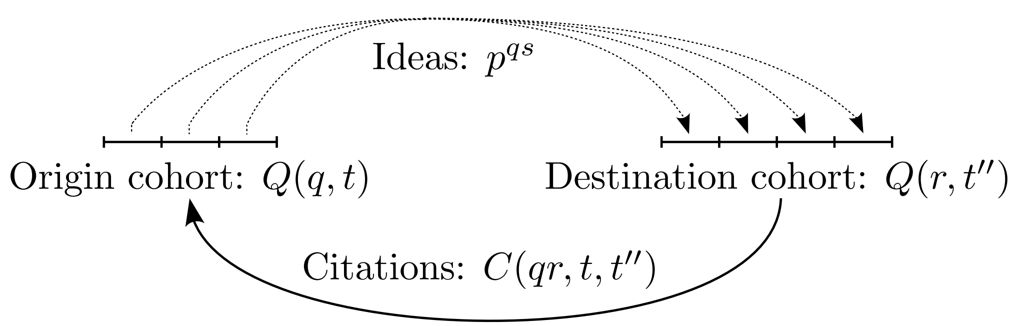


Figure 2.9: Cohort approach to estimating the $\{p^{qs}\}$ matrix

that are separated by some constant time interval Δt I obtain the MLE estimator for p^{qr}

$$p^{qr} = \frac{\sum_t C(qr, t, t + \Delta t)}{\sum_t Q(q, t)Q(r, t + \Delta t)}, \tag{2.35}$$

[13]The subscript in p_{EGK} stands for the initials of the authors of [11].

where $C(\dots)$'s and $Q(\dots)$'s are directly observable. When implementing this method I consider all patent registered within one year in an origin sector q as the origin cohort and patents registered in a destination sector r within 2-11 years after the origin cohort – as the destination cohort. Using pairs of cohorts separated by the same time interval I disregard some citation data, yet, it allows me to exclude the impact of time patterns of citations arrivals on the estimates.

In the second approach, which I name a **"sequence approach"**, I use the information on the order in which patents were issued (it follows immediately from the patent numbers). Let's consider a patent ι originating in sector q. Denote the number of patents in a destination sector r that were issued after ι as $N_\iota(r)$, out of which $K_\iota(r)$ actually cited ι. One can treat the issuance of each of the $N_\iota(r)$ patents as a trial in which a positive outcome that ι is cited has probability p^{qr}. Then the total number of citations received by ι is a random variable $K_\iota(r) \sim\ Poisson(p^{qr} N_\iota(r))$. The number of total citations received by all patents in sector q by patents in sector r is $\sum_\iota K_\iota(r) \sim\ Poisson(\sum_\iota p^{qr} N_\iota(r))$ from where the MLE estimator for p^{qr} is equal to

$$p^{qr} = \frac{\sum_\iota K_\iota(r)}{\sum_\iota N_\iota(r)}, \tag{2.36}$$

where the sums are computed across all patents ι that have ever been registered in sector q. The sequence approach uses all the available data on citations, but is likely to give lower estimates than cohort method because the cohort method considers only the part of patent life-cycle in which patents receive citations at highest rates. Yet, what matters for the optimal policy exercise is the relative size between spillover probabilities and not their absolute values, so that the absolute values of $\{p^{qs}\}$ can be scaled either upwards or downwards.

For calibrating the probabilities of spillovers I used the US patent data for patents issued in 1976–2006. Out of the whole pool of patents I consider the ones that excel the 50% threshold of citations for patents of a given age (which each patent had in 2006) and from a given sector of origin. Although this truncation shifts absolute estimates of spillover probabilities upwards, it allows to consider a pool of more homogeneous patents in terms of their significance. For the estimation procedure the data is aggregated into 93 sectors following the BLS-NAICS classification. For matching the international patent categories (IPC) to NAICS codes I use the probabilistic concordance matrices from [19]. One obvious downside of the existing concordance schemes is that they allow to match patents to the fields of economic activity that employ roughly 30-40% of labor. Namely, concordances exist for manufacturing, agriculture, utilities, mining and construction, but not for retail and wholesale trade, transportation and all kinds of services.

As Figure (2.10) shows, both above described methods for estimating $\{p^{qs}\}$ produce very similar results – the fitted line (black) is very close to the 45-degree line (red). As an

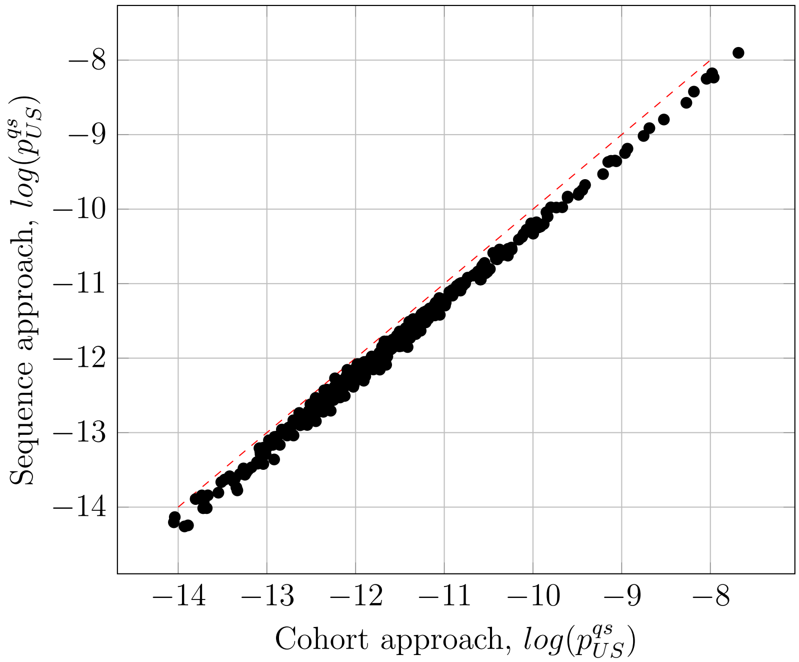


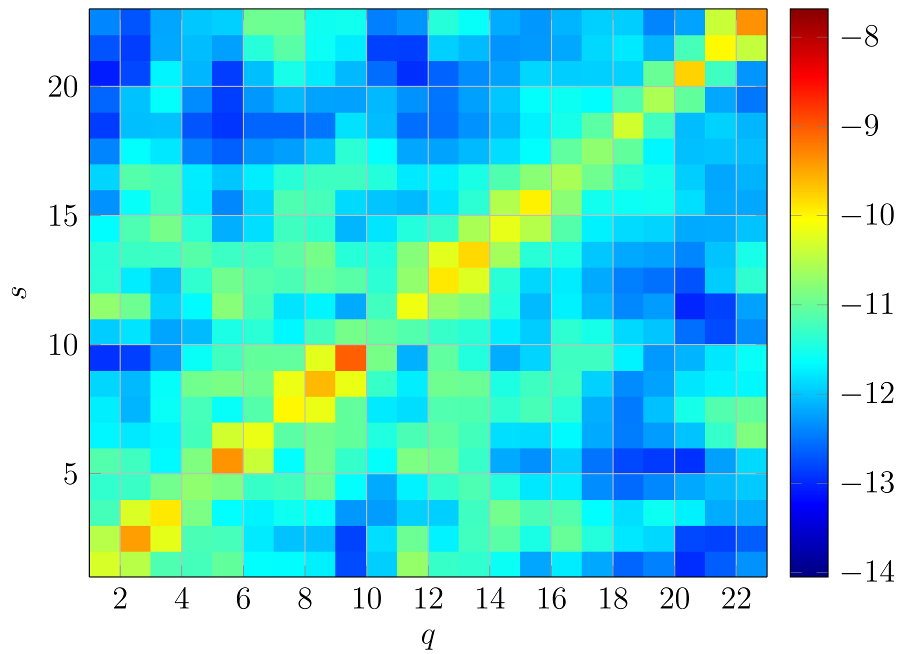
Figure 2.10: Logs of estimated spillover probabilities under cohort and sequence approaches.

additional check, I compare the estimated log-probabilities of spillovers for the USA to those of Japan. Figures (2.11a) and (2.11b) depict the estimates for the two countries. Visually, the heat-maps look similarly, though, for Japan absolute values of estimates are on average lower. Figure (2.12) confirms both the high correlation between the estimates and the difference in the average values (red line is, again, a the 45-degree line, while the black one is a fitted line). The difference in absolute values might be attributed to differences in the procedures of patenting and citing across the two countries.

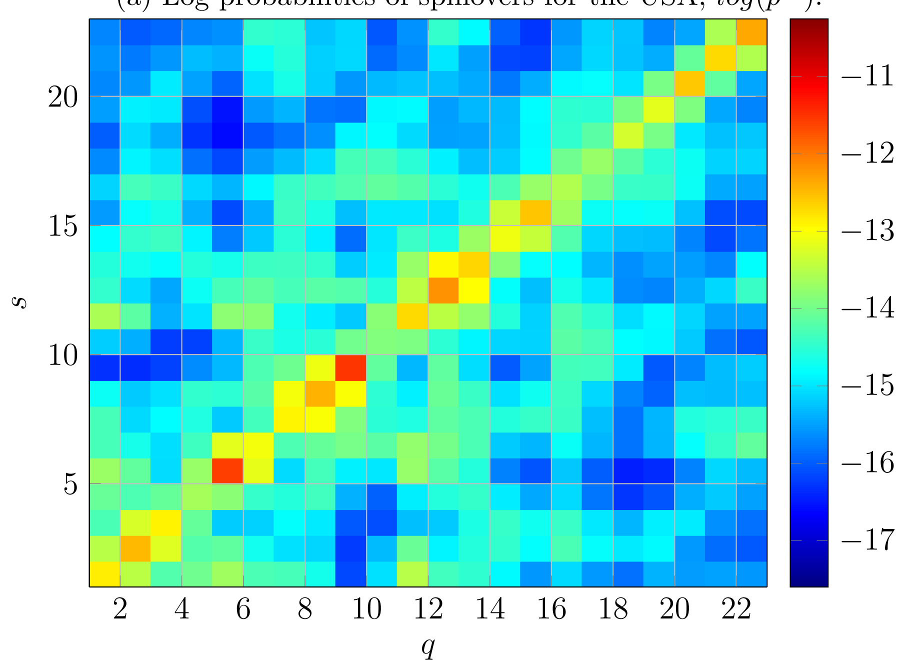
In the model what matters for spillovers is the rate at which a unit of labor in sector q generates technologies for sector s, i.e. ϕp^{qs}. In the data this rate may differ across sectors both due to variation in the probabilities $\{p^{qs}\}$ and in intensity of idea-generating process across sectors, $\{\phi^q\}$. To address this issue I calibrate the vector of intensities $\{\phi^q\}$ (ϕ^q – a number of patents generated in sector q per 1 million hours of working time) and use it to normalize the matrix of spillovers. Namely, the correspondence between parameters in the model and the data is $\phi p^{qs} = \frac{\phi^q}{\max_{r \in \mathcal{S}}\{\phi^r\}} \hat{p}^{qs}$, where $\hat{p}^{qs}$ are the above described estimates of spillover probabilities.

The second set of parameters that is required for the optimal policy exercise are the expenditure shares, $\{\alpha^q\}_{q \in \mathcal{S}}$. I calibrate these shares using the BLS input-output tables:

$$\alpha^s = \frac{X_i^s}{\sum_{q \in \mathcal{S}} X_i^q}, \tag{2.37}$$



(a) Log-probabilities of spillovers for the USA, $log(p^{qs})$.



(b) Log-probabilities of spillovers for Japan, $log(p^{qs})$.

Figure 2.11: Estimated log-probabilities of spillovers

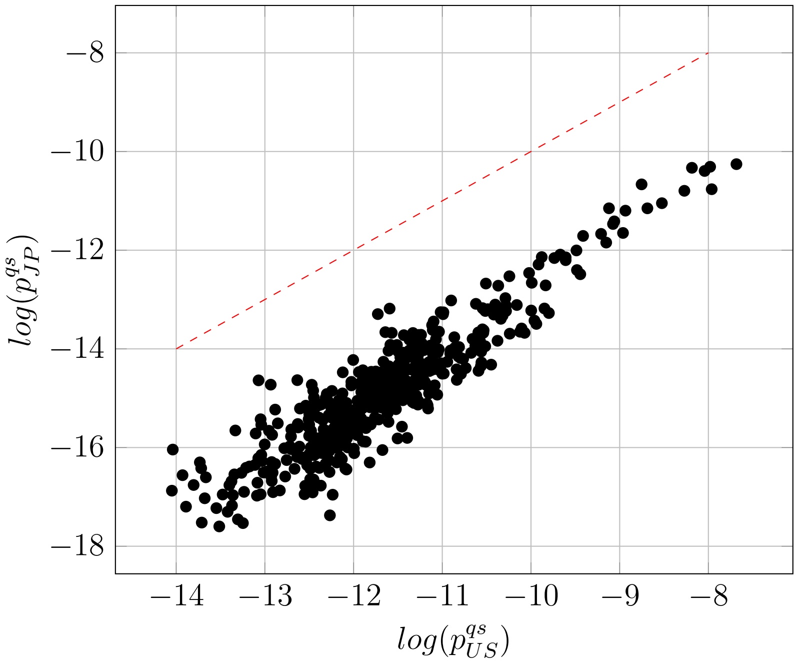


Figure 2.12: Estimated log-probabilities of spillovers for the USA and Japan

where X_i^s stands for total expenditures of country i for output of sector s. For calibrating the distribution of technologies I use the estimate of $\theta = 8$ which is in line with the estimates suggested in [9]. Finally, for computing the optimal policy in the open economy one needs the vector of labor force distribution across countries, $\{L_i\}_{i\in\mathcal{N}}$. I calibrate the latter using the data on economically active population provided by International Labour Organization (ILO).

2.5 Optimal policy

In this section I describe the policy that maximizes country's welfare on a balanced growth path. The first exercise uses the calibrated model and computes the vector of optimal labor allocation for the autarky. For the ease of computation and interpretation I aggregate the data to 24 sectors. The first 23 sectors, results on which I report, correspond to those for which the IPC-NAICS concordances exist. The 24th sector is a composite of services for which there are no such concordances, hence, it is treated as a stagnant one. To solve the optimization problem (2.27) I use the first order conditions (2.29). Taxes that result in the optimal labor allocation are computed using Equation (2.25). The second exercise considers a regime of frictionless trade between two countries of equal size, one of which chooses optimal policy, while the other doesn't implement any policy.

The results of these exercises are presented in Table (2.1). The first column of the table shows the actual shares of non-service labor force allocated across non-service sectors computed for the US in 1990-2006, l^q. The second column describes the allocation of labor across non-service sectors in the BGP – according to Proposition 1, allocation of labor would be proportional to shares of sectors in consumption, α^q. Columns 3 and 4 contain the optimal allocation of labor chosen by the optimizing country to maximize BGP utility in autarky and frictionless trade correspondingly – l^q_{aut} and l^q_{open}. Finally, columns 5 and 6 provide us with sector-specific taxes that the optimizing country should impose to achieve the optimal labor allocation – τ^q_{aut} and τ^q_{open}.[14] The optimizing social planner would allocate more labor

Table 2.1: Free-market and socially optimal labor allocation under autarky and frictionless trade.

Sector	l^q	α^q	l^q_{aut}	l^q_{open}	τ^q_{aut}	τ^q_{open}
Agriculture,fishing and hunting	8.37%	3.93%	3.66%	3.56%	0.956	0.998
Mining	2.27%	8.63%	8.59%	9.03%	0.893	0.911
Utilities	2.34%	5.38%	5.34%	5.60%	0.895	0.909
Construction	27.06%	15.77%	14.37%	12.41%	0.975	1.032
Food manufacturing	5.43%	6.62%	6.29%	6.41%	0.935	0.955
Beverage and tobacco	0.72%	1.98%	1.98%	2.09%	0.888	0.902
Textile and leather products	4.43%	3.20%	3.28%	3.42%	0.868	0.898
Wood products	2.11%	1.57%	1.45%	1.32%	0.967	1.010
Paper products	2.21%	2.32%	2.24%	2.31%	0.921	0.937
Printing and related activities	2.79%	1.44%	1.31%	1.19%	0.976	1.051
Petroleum and coal products	0.49%	6.17%	6.44%	6.82%	0.851	0.877
Chemical manufacturing	3.56%	7.62%	8.78%	8.91%	0.772	0.839
Plastic and rubber products	3.13%	2.62%	2.54%	2.60%	0.919	0.947
Nonmetallic mineral products	1.93%	1.57%	1.66%	1.72%	0.840	0.882
Primary metal	2.22%	3.13%	3.04%	3.11%	0.916	0.933
Fabricated metal products	5.90%	3.96%	3.82%	3.88%	0.921	0.954
Machinery	4.99%	3.85%	3.95%	4.13%	0.865	0.896
Computers and electronics	5.92%	5.21%	6.33%	6.11%	0.733	0.843
Electrical equipment	2.00%	1.84%	1.87%	1.94%	0.876	0.915
Transportation equipment	7.27%	9.72%	9.52%	9.84%	0.907	0.917
Furniture and related products	2.27%	1.23%	1.18%	1.19%	0.928	0.953
Medical equipment and supplies	1.06%	0.81%	0.95%	0.95%	0.764	0.847
Other manufacturing	1.54%	1.40%	1.40%	1.45%	0.893	0.920

l^q – actual labor allocation, α^q – free market BGP shares of labor, l^q_{aut} – closed economy optimal labor allocation, l^q_{open} – open economy optimal labor allocation, τ^q_{aut} – closed economy optimal taxes, τ^q_{open} – open economy optimal taxes.

to such core sectors as "Computers and electronics" (+19.3% in autarky and +15.8% in

[14] The taxes imposed on the service sector – which is not shown in the table – are normalized to 1.

open economy to the share of sector employment under no policy intervention), "Medical equipment and supplies" (+15.1% and +15.0%), "Chemical manufacturing" (+14.1% and +15.6%). Among sectors that generate technologies for others at the lowest rates are "Construction" (-9.2% and -23.9%), "Printing and related support activities" (-9.4% and -18.9%) and "Wood products" (-8.4% and -17.1%) and "Agriculture" (-7.1% and -10.1%).

The described exercises also allow me to consider the welfare implications of different labor allocations:

$$\log\left(\frac{U}{L}\right)^{*} - \log\left(\frac{U}{L}\right) = \sum_{q} \alpha^{q} \log\left(\frac{l^{q*}}{l^{q}}\right) + \sum_{n} \frac{\alpha^{n}}{\theta} \left(\frac{\sum_{q} p^{qn} l^{q*}}{\sum_{q} p^{qn} l^{q}}\right) \tag{2.38}$$

According to this formula, shifting the employment structure from the actual to the autarky-optimal one can raise productivity in the US in the BGP by 3.5% (the second summand). The whole increase in welfare is estimated at 15.5%. This number is large, yet, 12.5% out of it comes from the assumption that the labor allocation and production defines the consumption structure. Re-allocation of labor from free market BGP to autarky-optimal structure increases welfare by 0.3%. A more sizable is the impact of industrial policy in an open economy. The optimizing country can increase its BGP utility by 2.2% which, unsurprisingly, comes at the cost of non-optimizing country – the latter loses 3.7% in welfare.

2.6 Conclusion

In this chapter I build a dynamic trade model with technological spillovers and show that under general conditions it is characterized by a unique balanced growth path. The model provides a framework for predicting the long-run consequences of trade and industrial policies. I derive the conditions that allow to identify the core sectors and design the optimal industrial policy in autarky and open economy. For quantifying the probabilities of spillovers I suggest and use two approaches consistent with the modeled mechanism of technology-generating process. I use the calibrated model for computing the optimal vector of labor allocation and show that in the balanced growth path such policy can provide a 3.5% increase in productivity of the whole economy.

There are several immediate extensions for all three parts of the chapter – model, data and the optimal policy. The first extension to the model can be a possibility of international technological spillovers. In my intuition such spillovers will strengthen the centripetal forces of comparative advantage and will lead to a faster convergence towards the BGP. A simple example that supports this intuition is outlines at the end of Section 2. Presence of input-output linkages and positive trade costs are among other interesting theoretical extensions.

The calibration and optimal policy parts will benefit a lot from new concordance schemes that allow to quantify the rates of flows of ideas across all fields of economic activity, including services. Precise estimates of absolute levels of such rates would allow to talk about the speed of convergence and take the transition paths into account when designing the optimal policy. Finally, the strategic interaction between countries in the game of subsidizing the core sectors brings in the question of the existence of the Nash equilibrium in this game and its optimality for the global welfare.

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Appendix A

Log-linearization of the price setting problem

One can write down the expression for profits of producers in sector j at period t as

$$\pi_{t,j} = \left[\left(\check{P}_{t,j}\right)^{1-\theta} - \psi_{t,j}\left(\check{P}_{t,j}\right)^{-\theta}\right]\left(P_{t,j}\right)^{\theta}\left(P_t\right)^{-1} Y_{t,j} =$$

$$= \underbrace{\left(\bar{\check{P}}_j\right)^{1-\theta} \bar{Y}_j \left(\bar{P}_j\right)^{\theta} \prod_{i=1}^{J}\left(\bar{P}_i\right)^{-\varepsilon_i}}_{\equiv \bar{A}_j} \underbrace{e^{(1-\theta)\check{p}_{t,j}+y_{t,j}+\theta p_{t,j}-\sum_{i=1}^{J}\varepsilon_i p_{t,i}}}_{\equiv e_1\left(\check{p}_{t,j},p_{t,j},\{p_{t,i}\}_{i\neq j},y_{t,j}\right)} -$$

$$- \underbrace{\Omega^j \left(\bar{\check{P}}_j\right)^{-\theta} \bar{Y}_j \left(\bar{P}_j\right)^{\theta} \prod_{i=1}^{J}\left(\bar{P}_i\right)^{\xi_i^j(1-s_j)-\varepsilon_i}}_{\equiv \bar{B}_j} \underbrace{e^{-\theta\check{p}_{t,j}+y_{t,j}+\theta p_{t,j}-W_{t,j}+\sum_{i=1}^{J}\left(\xi_i^j(1-s_j)\varepsilon_i p_{t,i}\right)}}_{\equiv e_2\left(\check{p}_{t,j},p_{t,j},\{p_{t,i}\}_{i\neq j},y_{t,j},W_{t,j}\right)}$$

Capital letter denote variables in levels, $\bar{X}$ denotes the steady state value of variable X, lower case variables denote log-deviations from the steady state: $x_t = \ln X_t - \ln \bar{X}$. $\check{p}_{t,j}$ denotes the log-deviation of the price that is actually set by producers in sector j, while $\{p_{t,i}\}_{i=1}^{J}$ denote the log-deviations of sectoral price levels from their steady states. Note, that at the steady state $\bar{\pi}_j = \bar{A}_j - \bar{B}_j$, while both functions $e_1(\cdot)$ and $e_2(\cdot)$ at the steady state are equal to 1.

Using the second-order Taylor approximation around the steady state, an approximated profit of a producer in sector j at period t that is setting price $\check{p}_{t,j}$ can be written down as:

$$\hat{\pi}_{t,j}\left(\check{p}_{t,j}, p_{t,j}, \{p_{t,i}\}_{i\neq j}, y_{t,j}, W_{t,j}\right) \approx \bar{\pi}_j + \bar{\pi}_{1,j}\check{p}_{t,j} + \frac{1}{2}\bar{\pi}_{11,j}\left(\check{p}_{t,j}\right)^2 + \bar{\pi}_{12,j}\check{p}_{t,j}p_{t,j} +$$

$$+ \sum_{i\neq j}\bar{\pi}_{13(i),j}\check{p}_{t,j}p_{t,i} + \bar{\pi}_{14,j}\check{p}_{t,j}y_{t,j} + \bar{\pi}_{15,j}\check{p}_{t,j}W_{t,j} + \Theta^j$$

where

$$\begin{aligned}
\bar{\pi}_{1,j} &= (1-\theta)\bar{A}_j + \theta\bar{B}_j \equiv 0 \\
\bar{\pi}_{11,j} &= (1-\theta)^2\bar{A}_j - \theta^2\bar{B}_j = -\theta\bar{B}_j \\
\bar{\pi}_{12,j} &= \xi_j^j(1-s_j)\theta\bar{B}_j \\
\bar{\pi}_{13(i),j} &= \xi_i^j(1-s_j)\theta\bar{B}_j \\
\bar{\pi}_{14,j} &= 0 \\
\bar{\pi}_{15,j} &= (1-\theta)\bar{A}_j
\end{aligned}$$

Finally, using the above derive expressions, one can show that approximated one-period profit loss from setting a suboptimal price $\check{p}_{t,j}$ when the optimal one is equal to ${p^*}_{t,j}$ can be expressed as:

$$\hat{\pi}_{t,j}\left({p^*}_{t,j}, p_{t,j}, \{p_{t,i}\}_{i\neq j}, y_{t,j}, W_{t,j}\right) - \hat{\pi}_{t,j}\left(\check{p}_{t,j}, p_{t,j}, \{p_{t,i}\}_{i\neq j}, y_{t,j}, W_{t,j}\right) \approx \frac{|\bar{\pi}_{11,j}|}{2}(\check{p}_{t,j} - {p^*}_{t,j})^2,$$

where I also make use of the log-linearized expression for the optimal price ${p^*}_{t,j} = -W_{t,j} + \sum_{i=1}^{J}\xi_i^j(1-s_j)p_{t,i}$.

Appendix B

Solution of the forward-looking system

The price-setting mechanism in the considered economy is described with the following system of log-linearized equations:

$$
\begin{aligned}
{p^*}_{t,j} &= -W_{t,j} + \sum_{i=1}^{J} \xi_i^j (1-s_j) p_{t,i} && \text{(B.1)}\\
\check{p}_{t,j} &= (1-\delta_j\beta) \sum_{s=1}^{\infty} (\delta_j\beta)^{\tau} E_t \left({p^*}_{t+s,j}\right) && \text{(B.2)}\\
p_{t,j} &= (1-\delta_j)\check{p}_{t,j} + \delta_j p_{t-1,j} && \text{(B.3)}\\
W_{t,j} &= W_{t-1,j} + u_{t,j},\ u_{t,j} \overset{iid}{\sim} N(0,\sigma_u^j) && \text{(B.4)}
\end{aligned}
$$

Equation $(B.4)$ can be rewritten in iterative form as

$$\check{p}_{t,j} = (1-\delta_j\beta){p^*}_{t,j} + \delta_j\beta E_t \check{p}_{t+1,j} \tag{B.5}$$

Combining Equations $(C.1)$ and $(B.5)$ one can write down:

$$\check{p}_{t,j} = \delta_j\beta E_t \check{p}_{t+1,j} + (1-\delta_j\beta)\left(-W_{t,j} + \sum_{i=1}^{J} \xi_i^j (1-s_j) p_{t,i}\right) \tag{B.6}$$

For every industry j out of J we have a system of 3 equations – $(C.2)$, $(B.3)$ and $(B.6)$ – $3J$ equations in total. In the matrix form this system of $3J$ equations can be written down as:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boldsymbol{\alpha} & \boldsymbol{\beta} & \boldsymbol{\gamma} \end{bmatrix}}_{\equiv A} \begin{bmatrix} W_t \\ p_t \\ E_t \check{p}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \boldsymbol{\Delta} & \boldsymbol{\varphi} \\ 0 & 0 & 1 \end{bmatrix}}_{\equiv B} \begin{bmatrix} W_{t-1} \\ p_{t-1} \\ \check{p}_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [u_t], \tag{B.7}$$

where p_t, W_t, $\check{p}_t$ and u_t are $J \times 1$ vectors, 0 denotes $J \times J$ matrix of zeros, 1 denotes $J \times J$ identity matrix. The other elements are:

$$\boldsymbol{\Delta} = \begin{bmatrix} \delta_1 & 0 & \dots & 0 \\ 0 & \delta_2 & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & \delta_J \end{bmatrix}, \ \boldsymbol{\alpha} = \begin{bmatrix} (\delta_1\beta - 1) & 0 & \dots & 0 \\ 0 & (\delta_2\beta - 1) & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & (\delta_J\beta - 1) \end{bmatrix},$$

$$\boldsymbol{\beta} = \begin{bmatrix} (1-\delta_1\beta)(1-s_1)\xi_1^1 & (1-\delta_1\beta)(1-s_1)\xi_2^1 & \dots & (1-\delta_1\beta)(1-s_1)\xi_J^1 \\ (1-\delta_2\beta)(1-s_2)\xi_1^2 & (1-\delta_2\beta)(1-s_2)\xi_2^2 & \dots & (1-\delta_2\beta)(1-s_2)\xi_J^2 \\ \dots & \dots & \ddots & \dots \\ (1-\delta_J\beta)(1-s_J)\xi_1^J & (1-\delta_J\beta)(1-s_J)\xi_2^J & \dots & (1-\delta_J\beta)(1-s_J)\xi_J^J \end{bmatrix},$$

$$\boldsymbol{\gamma} = \begin{bmatrix} \delta_1\beta & 0 & \dots & 0 \\ 0 & \delta_2\beta & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & \delta_J\beta \end{bmatrix}, \ \boldsymbol{\varphi} = \begin{bmatrix} (1-\delta_1) & 0 & \dots & 0 \\ 0 & (1-\delta_2) & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & (1-\delta_J) \end{bmatrix}$$

The following steps make use of the solution approach for a forward looking system with rational expectation that was first described in Blanchard and Kahn (1980). Pre-multiplying both sides of Equation (B.7) by A^{-1} one can re-write it as

$$\begin{bmatrix} W_t \\ p_t \\ E_t\check{p}_{t+1} \end{bmatrix} = A^{-1}B \begin{bmatrix} W_{t-1} \\ p_{t-1} \\ \check{p}_t \end{bmatrix} + A^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [u_t], \tag{B.8}$$

Using the eigen-decomposition we can write $A^{-1} \cdot B = V^{-1} \cdot L \cdot V$, where L is a diagonal matrix of eigenvalues of $A^{-1} \cdot B$, eigenvalues are ordered by their absolute values in ascending order (smallest in the top left corner, largest – in the bottom right corner), V^{-1} is a matrix of corresponding eigenvectors (order of eigenvectors in V^{-1} correspond to the order of eigenvalues in L). Pre-multiplying both sides of Equation (B.8) by V we obtain

$$\begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \begin{bmatrix} W_t \\ p_t \\ E_t\check{p}_{t+1} \end{bmatrix} = \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix} \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \begin{bmatrix} W_{t-1} \\ p_{t-1} \\ \check{p}_t \end{bmatrix} + \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \cdot A^{-1} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [u_t]$$

Now, introducing new variables with $\tilde{\cdot}$ as

$$\begin{bmatrix} \tilde{W}_t \\ \tilde{p}_t \\ E_t\tilde{\check{p}}_{t+1} \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \begin{bmatrix} W_t \\ p_t \\ E_t\check{p}_{t+1} \end{bmatrix}$$

one can re-write the system above as

$$\begin{bmatrix} \tilde{W}_t \\ \tilde{p}_t \\ E_t\tilde{\check{p}}_{t+1} \end{bmatrix} = \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix} \begin{bmatrix} \tilde{W}_{t-1} \\ \tilde{p}_{t-1} \\ \tilde{\check{p}}_t \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} [u_t] \tag{B.9}$$

From the third equation, iterating it forward, one can write down

$$\tilde{\check{p}}_t = -L_3^{-1}C_3u_t + L_3^{-1}E_t\tilde{\check{p}}_{t+1} = -L_3^{-1}C_3u_t + \lim_{n\to\infty} L_3^{-n}E_t\tilde{\check{p}}_{t+n} = -L_3^{-1}C_3u_t,$$

where I make use of $E_t(u_{t+\tau}) = 0\ \forall\tau > 0$ and from Blanchard-Kahn conditions we have that all elements of L_3 are in absolute values greater than 1, thus, $\lim_{n\to\infty} L_3^{-n}E_t\tilde{\check{p}}_{t+n} = 0$.

Now, using the substitution formula for $\tilde{\check{p}}_t$ we can write down

$$\check{p}_t = -V_{33}^{-1}L_3^{-1}C_3u_t - V_{33}^{-1}V_{31}W_{t-1} - V_{33}^{-1}V_{32}p_{t-1} \tag{B.10}$$

Combining this expression with $p_t = \boldsymbol{\Delta}\cdot p_{t-1} + \boldsymbol{\varphi}\cdot\check{p}_t$ and $W_t = W_{t-1} + u_t$ from Equation $(B.7)$ we can express prices p_t and productivity levels W_t in terms of lagged prices p_{t-1} and productivity W_{t-1} and current productivity innovations u_t. Again, in matrix form it can be written down as

$$\underbrace{\begin{bmatrix} p_t \\ W_t \end{bmatrix}}_{\equiv P_t} = \underbrace{\begin{bmatrix} \boldsymbol{\Delta} - \boldsymbol{\varphi}V_{33}^{-1}V_{32} & -\boldsymbol{\varphi}V_{33}^{-1}V_{31} \\ 0 & 1 \end{bmatrix}}_{\equiv R} \underbrace{\begin{bmatrix} p_{t-1} \\ W_{t-1} \end{bmatrix}}_{\equiv P_{t-1}} + \underbrace{\begin{bmatrix} -\boldsymbol{\varphi}V_{33}^{-1}L_3^{-1}C_3 \\ 1 \end{bmatrix}}_{\equiv Q} u_t \tag{B.11}$$

If at period t economy is at the steady state, then $p_t = 0$ and $u_t = 0$, so

$$\begin{aligned} P_{t+1} &= Qu_{t+1} \\ P_{t+1} &= RQu_{t+1} + Qu_{t+2} \\ &\vdots \\ P_{t+s} &= \sum_{\tau=1}^{s} R^{s-\tau}Qu_{t+\tau} \end{aligned}$$

It follows immediately from the equations above that if the economy is at the steady state at period t, then $E_tp_{t+s} = 0\ \forall s > 0$. Since $u_{t+\tau}$ are iid, the formula for variance of P_{t+s} is:

$$var(P_{t+s}) = \sum_{\tau=1}^{s} R^{s-\tau}Qvar(u) \tag{B.12}$$

Finally, from Equation $(C.1)$ the vector of optimal flexible prices p_t^* can be expressed in terms of vector P_t:

$$p_t^* = \underbrace{\begin{bmatrix} \Sigma & -1 \end{bmatrix}}_{\equiv M} \underbrace{\begin{bmatrix} p_t \\ W_t \end{bmatrix}}_{\equiv P_t}, \tag{B.13}$$

where

$$\Sigma = \begin{bmatrix} (1-s_1)\xi_1^1 & (1-s_1)\xi_2^1 & \dots & (1-s_1)\xi_J^1 \\ \vdots & \vdots & \ddots & \vdots \\ (1-s_J)\xi 1^J & (1-s_J)\xi_2^J & \dots & (1-s_J)\xi_J^J \end{bmatrix}$$

Variance of optimal prices for different sectors can be expressed as

$$var(p^*_{t+s}) = var(M \cdot P_{t+s}) = M \cdot var(P_{t+s}) \cdot M', \tag{B.14}$$

where $var(P_{t+s})$ can be expressed in terms of $var(u)$ using Equation $(B.12)$.

Appendix C

Frequency and magnitude of price adjustments

As the variables in system are expressed in logs, then the percentage change of price for a cohort k (those that adjusted their prices last time k periods ago) in sector j will be just $\check{p}_{t,j} - \check{p}_{t-k,j}$. The average percentage change of prices in sector j will be a weighted average percentage change across all the cohorts in sector j:

$$PA_{t,j} = (1-\delta_j)[\check{p}_{t,j} - \check{p}_{t-1,j}] + (1-\delta_j)\delta_j[\check{p}_{t,j} - \check{p}_{t-2,j}] + (1-\delta_j)\delta_j^2[\check{p}_{t,j} - \check{p}_{t-3,j}] + \dots \quad \text{(C.1)}$$

Equation $(C.1)$ can be rewritten in terms of period-by-period percentage changes in adjusted prices:

$$PA_{t,j} = \delta_j[\check{p}_{t,j} - \check{p}_{t-1,j}] + \delta_j[\check{p}_{t-1,j} - \check{p}_{t-2,j}] + \delta_j^2[\check{p}_{t-2,j} - \check{p}_{t-3,j}] \dots \quad \text{(C.2)}$$

Now we need to obtain an expression for $[\check{p}_{t+s,j} - \check{p}_{t+s-1,j}]$ $\forall s$. From the solution of the model we can express current values $\check{p}_t$, p_t and W_t (all are $J \times 1$ vectors) using their values at $t-1$ and current vector of shocks u_t:

$$\underbrace{\begin{bmatrix} \check{p}_t \\ p_t \\ W_t \end{bmatrix}}_{P_t} = \underbrace{\begin{bmatrix} 0 & -V_{33}^{-1}V_{32} & -V_{33}^{-1}V_{31} \\ 0 & \Delta - \varphi V_{33}^{-1}V_{32} & -\varphi V_{33}^{-1}V_{31} \\ 0 & 0 & 1 \end{bmatrix}}_{\equiv Z} \underbrace{\begin{bmatrix} \check{p}_{t-1} \\ p_{t-1} \\ W_{t-1} \end{bmatrix}}_{P_{t-1}} + \underbrace{\begin{bmatrix} -V_{33}^{-1}L_3^{-1}C_3 \\ -\varphi V_{33}^{-1}L_3^{-1}C_3 \\ 1 \end{bmatrix}}_{\equiv D} [u_t] \quad \text{(C.3)}$$

or in short notations $P_t = Z \cdot P_{t-1} + D \cdot u_t$ from where

$$P_t - P_{t-1} = \underbrace{(Z - I)}_{\equiv G} P_{t-1} + Du_t \quad \text{(C.4)}$$

Equation $(C.2)$ can be written down as

$$PA_{t+s} = (GP_{t+s-1} + Du_{t+s}) + \delta(GP_{t+s-2} + Du_{t+s-1}) + \dots + \delta^{s-1}(GP_{t+1} + Du_t), \quad \text{(C.5)}$$

where δ is $3J \times 3J$ matrix with the upper left quadrant equal to

$$\begin{bmatrix} \delta_1 & 0 & \dots & 0 \\ 0 & \delta_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \delta_J \end{bmatrix} \tag{C.6}$$

and zeros everywhere else. So far, PA_{t+s} can be expressed in terms of $\{P_{t+\tau}\}_{\tau=0}^{s}$ and, if the system is at the steady state at time t, we can express average percentage price adjustments in terms of sequence of TFP shocks $\{u_{t+\tau}\}_{\tau=0}^{s}$ using the formula $P_{t+s} = \sum_{\tau=1}^{s} R^{s-\tau} Q u_{t+\tau}$ from Appendix B:

$$\begin{aligned} GP_{t+s-1} + Du_{t+s} &= Du_{t+s} + GDu_{t+s-1} + GCDu_{t+s-2} + GC^2 Du_{t+s-3} + \dots + GC^{s-2} Du_{t+1} \\ \delta(GP_{t+s-2} + Du_{t+s-1}) &= \delta Du_{t+s-1} + \delta GDu_{t+s-2} + \delta GCDu_{t+s-3} + \dots + \delta GC^{s-3} Du_{t+1} \\ \vdots &= \vdots \\ \delta(GP_t + Du_{t+1}) &= \delta^{s-1} Du_{t+1} \end{aligned}$$

Finally, one can express variance of PA_{t+s} through variances of $\{u_{t+\tau}\}_{\tau=0}^{s}$.

Appendix D

Proof of Proposition 1

Let's start with proving **uniqueness** – under no isolated clusters of sectors there exists a unique BGP with positive amounts of labor allocated to each sector in each country in which sector allocation of labor is identical across countries: $l_i^s = l_j^s > 0\ \forall i \in \mathcal{N},\ \forall s \in \mathcal{S}$, where $\mathcal{N} \equiv \{1, \ldots, N\}$ and $\mathcal{S} \equiv \{1, \ldots, S\}$. In what follows we'll consider only $\mathbb{R}_{++}^{N \times S}$, i.e. only cases in which $l_i^s > 0\ \forall i, s$. Let's start with a system of equations that describes the sector labor allocation and the ratio of BGP productivities across sectors:

$$\frac{l_i^s}{l_i^r}\frac{T_i^r}{T_i^s} = \frac{l_j^s}{l_j^r}\frac{T_j^r}{T_j^s}\ \forall i, j \in \mathcal{N},\ \forall s, r \in \mathcal{S}$$
$$\sum_q l_i^q = 1\ \forall i \in \mathcal{N},\ \forall q \in \mathcal{S}$$
$$\frac{T_i^r}{T_i^s} = \frac{\sum_q p^{qr} l_i^q}{\sum_q p^{qs} l_i^q}\ \forall i \in \mathcal{N},\ \forall q \in \mathcal{S}$$

Plugging the last equation into the left hand side and right hand side parts of the first one, taking logs of it and combining with the log of the second equation we obtain the following vector-valued function:

$$F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{S-1} \\ F_S \end{bmatrix} = \begin{bmatrix} \log l^1 - \log l^2 + \log \sum_q p^{q2} l^q - \log \sum_q p^{q1} l^q \\ \log l^2 - \log l^3 + \log \sum_q p^{q3} l^q - \log \sum_q p^{q2} l^q \\ \vdots \\ \log l^{S-1} - \log l^S + \log \sum_q p^{q\,S} l^q - \log \sum_q p^{q\,S-1} l^q \\ \log \sum_q l^q \end{bmatrix}$$

For simplicity of the following steps we'll treat F as a vector-valued function of $\{\log l_i^q\}$ – since $l_i^s > 0\ \forall i, s$ the function is well-defined and differentiable at each point of its domain[1]. In equilibrium F has the same value for each country: $F(\{\log l_i^q\}) = F(\{\log l_j^q\})$. If F is

[1]Function F has the same expression for each country i, thus, country subscript i can be omitted

an injective function then $F(\{\log l_i^q\}) = F(\{\log l_j^q\})$ implies that the equilibrium vectors $\{l_i^q\}$ should be equalized across countries[2]. Thus, showing the conditions under which F is injective we show the conditions under which the equilibrium has $l_i^q = l_j^q\ \forall i, q$.

To show injectivity of F we use the sufficient condition of injectivity stated in [5], namely, that a differentiable function $F:\ G \to R^M$ on an open convex subset of $G \in R^M$ is injective if a convex hull of $\{\nabla F(x) : x \in G\}$ contains only non-singular matrices. Using for brevity new notation $\bar{t^s} = \sum_q p^{qs} l^q$ the Jacobian ∇F (derivatives w.r.t $\log l^q$) can be written down as

$$\nabla F = \begin{bmatrix} 1 + \frac{p^{12}l^1}{\bar{t^2}} - \frac{p^{11}l^1}{\bar{t^1}} & -1 + \frac{p^{22}l^2}{\bar{t^2}} - \frac{p^{21}l^2}{\bar{t^1}} & \dots & \frac{p^{S2}l^S}{\bar{t^2}} - \frac{p^{S1}l^S}{\bar{t^1}} \\ \frac{p^{13}l^1}{\bar{t^3}} - \frac{p^{12}l^1}{\bar{t^2}} & 1 + \frac{p^{23}l^2}{\bar{t^3}} - \frac{p^{22}l^2}{\bar{t^2}} & \dots & \frac{p^{S3}l^S}{\bar{t^3}} - \frac{p^{S2}l^S}{\bar{t^2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{\bar{t^S}} - \frac{p^{1S-1}l^1}{\bar{t^{S-1}}} & \frac{p^{2S}l^2}{\bar{t^S}} - \frac{p^{2S-1}l^2}{\bar{t^{S-1}}} & \dots & -1 + \frac{p^{SS}l^S}{\bar{t^S}} - \frac{p^{S\,S-1}l^S}{\bar{t^{S-1}}} \\ l^1 & l^2 & \dots & l^S \end{bmatrix},$$

where the condition $\sum_q l_i^q = 1$ was used for simplifying the expression in the last row. Considering the value of Jacobian at two different points $\{\log l_i^q\}_{i,q}$ and $\{\log l_i^{q\prime}\}_{i,q}$ and taking any value $z = z \in [0;1],\ z' \equiv (1-z)$ we can write down a convex combination of Jacobians at these two arbitrary points as $z\nabla F + z'\nabla F'$. Next, we'll describe the applied matrix operations with mentioning if the operations can be applied to the corresponding convex combination and if any claim about ∇F is valid for the convex combination as well. First, notice that sum of elements by rows in ∇F is equal to zero in each row except the last one in which it is 1. Thus, by adding all columns to the last one and using Laplace expansion we can claim that determinant of the matrix obtained from ∇F by deleting the last row and the last column is the same as the determinant of ∇F. The same is true for the combination of ∇F and $\nabla F'$. The next operation that allows us to remove -1 from the second diagonal of the obtained $(S-1)\times(S-1)$ matrix is the addition of rows: $(S-1)$th to $(S-2)$th, the resulting $(S-2)$th – to $(S-3)$th and so on. The resulting matrix J will again have the same determinant as ∇F. The same holds for $z\nabla F + z'\nabla F'$. J:

$$\underset{(S-1)\times(S-1)}{J} = \begin{bmatrix} 1 + \frac{p^{1S}l^1}{\bar{t^S}} - \frac{p^{11}l^1}{\bar{t^1}} & \frac{p^{2S}l^2}{\bar{t^S}} - \frac{p^{21}l^2}{\bar{t^1}} & \dots & \frac{p^{S-1,S}l^{S-1}}{\bar{t^S}} - \frac{p^{S-1,1}l^{S-1}}{\bar{t^1}} \\ \frac{p^{1S}l^1}{\bar{t^S}} - \frac{p^{12}l^1}{\bar{t^2}} & 1 + \frac{p^{2S}l^2}{\bar{t^S}} - \frac{p^{22}l^2}{\bar{t^2}} & \dots & \frac{p^{S-1,S}l^{S-1}}{\bar{t^S}} - \frac{p^{S-1,2}l^{S-1}}{\bar{t^2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{\bar{t^S}} - \frac{p^{1S-1}l^1}{\bar{t^{S-1}}} & \frac{p^{2S}l^2}{\bar{t^S}} - \frac{p^{2S-1}l^2}{\bar{t^{S-1}}} & \dots & 1 + \frac{p^{S-1,S}l^{S-1}}{\bar{t^S}} - \frac{p^{S-1,S-1}l^{S-1}}{\bar{t^{S-1}}} \end{bmatrix}$$

For the convex combination $z\nabla F + z'\nabla F'$ each entry $\frac{p^{qS}l^q}{\bar{t^S}} - \frac{p^{qi}l^q}{\bar{t^i}}$ in J should be replaced with a corresponding convex combination $z\left(\frac{p^{qS}l^q}{\bar{t^S}} - \frac{p^{qi}l^q}{\bar{t^i}}\right) + z'\left(\frac{p^{qS}l^{q\prime}}{\bar{t^S}'} - \frac{p^{qi}l^{q\prime}}{\bar{t^i}'}\right)$. Next, augment J

[2] Here I use the property of injective functions that if $f \circ g$ is injective then g is injective, so if F is injective the the original system (before using logs) is also injective.

(and it's convex combination counterpart) to a new matrix K of size $S \times S$ by attaching one column from the right and one row from the bottom so that the attached row and column comply with the general pattern of entries in J. Namely,

$$\underset{S\times S}{K} = \begin{bmatrix} 1 + \frac{p^{1S}l^1}{t^S} - \frac{p^{11}l^1}{t^1} & \frac{p^{2S}l^2}{t^S} - \frac{p^{21}l^2}{t^1} & \dots & \frac{p^{S,S}l^S}{t^S} - \frac{p^{S,1}l^S}{t^1} \\ \frac{p^{1S}l^1}{t^S} - \frac{p^{12}l^1}{t^2} & 1 + \frac{p^{2S}l^2}{t^S} - \frac{p^{22}l^2}{t^2} & \dots & \frac{p^{S,S}l^S}{t^S} - \frac{p^{S,2}l^S}{t^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{t^S} - \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} - \frac{p^{2S}l^2}{t^S} & \dots & 1 + \frac{p^{S,S}l^S}{t^S} - \frac{p^{S,S}l^S}{t^S} \end{bmatrix}$$

Determinant of K will be the same as that of J: to see this notice that each element of the attached row is 0 except the last one which is 1, thus, using again Laplace expansion we can see that $det(K) = det(J)$. Next, we represent matrix K as a sum of three matrices:

$$\underset{S\times S}{K} = \underset{S\times S}{I} + \underbrace{\begin{bmatrix} \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \\ \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \end{bmatrix}}_{\equiv A} - \underbrace{\begin{bmatrix} \frac{p^{11}l^1}{t^1} & \frac{p^{21}l^2}{t^1} & \dots & \frac{p^{S,1}l^S}{t^1} \\ \frac{p^{12}l^1}{t^2} & \frac{p^{22}l^2}{t^2} & \dots & \frac{p^{S,2}l^S}{t^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \end{bmatrix}}_{\equiv B}.$$

Again, for the convex combination counterpart of K each entry $\frac{p^{ij}l^i}{t^j}$ should be replaced with $z\left(\frac{p^{ij}l^i}{t^j}\right) + z'\left(\frac{p^{ij}l^{i'}}{t^{j'}}\right)$. Now we notice several regularities about matrices A and B. First, all rows of A are the same and equal to the last row of B. Second, rows in both A and B sum up to 1 and each entry of them $\in [0;1]$ so that both matrices can be considered stochastic transition matrices. The last observation allows us to use the well established fact that spectral radius of both matrices is equal to 1. Third, since $l_i^q \in (0;1)$ then every entry (q,r) of B can take a zero value only if the corresponding $p^{qr} = 0$. Finally, eigenvalues of $A - B$ are equal to $\{0(= \lambda_{A,1} - \lambda_{B,1}),\ -\lambda_{B,2},\ -\lambda_{B,3}, \dots, -\lambda_{B,S}\}$, where $\lambda_{B,1} = 1,\ \lambda_{B,2},\ \dots,\ \lambda_{B,S} \in [-1;1]$ are eigenvalues of B and $\lambda_{A,1} = 1\ \lambda_{A,2} = \dots = \lambda_{A,S} = 0$ – eigenvalues of A.

The last claim is least obvious, so here are the details. Matrix A has rank 1 so it can have at most 1 non-zero eigenvalue, besides $tr(A) = 1$, so A has eigenvalues $\lambda_{A,1} = 1$ of multiplicity 1 and $\lambda_{A,2} = \dots = \lambda_{A,S} = 0$ of multiplicity $S-1$. For matrix B – as a stochastic transition matrix – we know from Perron-Frobenius theorem that all its eigenvalues belong to the interval $[-1;1]$ and at least one is equal to 1. Now, we derive eigenvalues of $A - B$. The first obvious eigenvalue is 0 with corresponding eigenvector $(1,1,\dots,1)^T$ (this follows immediately from equal row sums in both A and B). The other $S-1$ eigenvalues of $(A-B)$ are $\{-\lambda_{B,2},\ -\lambda_{B,3}, \dots, -\lambda_{B,S}\}$. To see this let's consider the matrix equation $(A - B - (-\lambda_B I))X = (A + (I\lambda_B - B))X$, where λ_B is one of eigenvalues $\{\lambda_{B,2},\ \lambda_{B,3}, \dots, \lambda_{B,S}\}$ of B matrix. To show that $-\lambda_B$ is an eigenvalue of $A - B$ it suffices to show that $\det(A + (I\lambda_B - B)) = 0$ or – which is equivalent – that $rank(A + (I\lambda_B - B)) \leq S - 1$. Here I use the result from [22]: if C_1, C_2 are column spaces of matrices U and V and R_1 and R_2 are row spaces, $c = dim(C_1 \cap C_2)$, $d = dim(R_1 \cap R_2)$ then $rank(U+V) \leq rank(U) + rank(V) - max(c,d)$. Before proceeding, one key observation – if each row of a matrix sums up to the same number then vector $l \equiv (1,1,\ldots,1)^T$ belongs to the column space of this matrix (indeed, adding up all columns and dividing the resultant vector by the row sum of the matrix we obtain vector l). Now back to $A + (I\lambda_B - B)$: 1) A has identical rows that sum up to the same number, so its column space contains vector l – in fact, the whole column space of A consists only of vectors collinear to l; 2) $rank(A) = 1$; 3) $I\lambda_B - B$ is singular, so $rank(I\lambda_B - B) \leq S - 1$; 4) each row of $I\lambda_B - B$ sums up to $1 - \lambda_B$, so the columns space of this matrix also contains l. From 1) and 4) it follows that $c = dim(C_1 \cap C_2) \geq 1$, thus $rank(A + I\lambda_B - B) \leq rank(A) + rank(I\lambda_B - B) - max(c,d) \leq S - 1$ which proves that $\det(A + (I\lambda_B - B)) = 0$ and $\{-\lambda_{B,2},\ -\lambda_{B,3},\ldots,-\lambda_{B,S}\}$ are the other $S-1$ eigenvalues of $A - B$ except the initially mentioned 0. So, $\{0, -\lambda_{B,2},\ -\lambda_{B,3},\ldots,-\lambda_{B,S}\}$ are the eigenvalues of $A - B$.

Before proceeding to the next step, let's state another key observation that will be used shortly: if a stochastic transition matrix P describes an irreducible Markov chain then it has a unique stationary distribution vector $\underset{1\times S}{X}$ that corresponds to a unique eigenvalue equal to 1: $XP = X$ or $P^T X^T = X^T$. On the contrary, if the chain is not irreducible (has several closed sets of states) then there exists multiple stationary distributions and matrix P has eigenvalue 1 with multiplicity > 1. A detailed explanation of this statement can be found in [13] on p.229. In our case matrix B is the analogy of the above mentioned Markov chain transition matrix P – and presence of closed sets of states in P corresponds to presence of isolated clusters of sectors in B. In essence it means that under no isolated clusters of sectors B has only one eigenvalue equal to 1, while if there are isolated clusters then $\lambda_B = 1$ has multiplicity > 1. Now, returning to matrix $K = I + (A - B)$. Under no isolated clusters of sectors all eigenvalues of $(A - B)$ $\{0, -\lambda_{B,2},\ -\lambda_{B,3},\ldots,-\lambda_{B,S}\}$ belong to $(-1;1)$ ($\lambda_{B,1} = 1$ cancels out with $\lambda_{A,1} = 1$), hence, all eigenvalues of $K = I + A - B$ belong to $(0;2)$ and $det(K) > 0$. On the other hand, if matrix of spillovers has isolated clusters then $\lambda_{B,1} = 1$ has multiplicity greater than 1, thus, at least one eigenvalue of $(A - B)$ – $\{0, -\lambda_{B,2},\ -\lambda_{B,3},\ldots,-\lambda_{B,S}\}$ – is equal to -1, which will turn into a 0-eigenvalue for K and, hence, $det(K) = 0$. Thus, absence of isolated clusters in the matrix of spillovers is a necessary and sufficient condition for non-singularity on ∇F.

Clearly, all the above mentioned arguments are applicable to a convex combination analogy of matrix K since the counterparts of matrices A and B have the same properties as A and B themselves (including the property that matrix B has zero entries only if the underlying matrix of spillovers $\{p^{rs}\}$ has corresponding entries equal to 0). Thus, non-singularity of ∇F at any point of $\mathbb{R}_+^{N\times S}$ is equivalent to non-singularity of $z\nabla F + z'\nabla F'$ between any two points in $\mathbb{R}_+^{N\times S}$ and, hence, injectivity of F. The opposite is also true – if ∇F is singular at each point of the domain of F then it means that at each point there exists a non-zero vector ΔX such that $\nabla F \Delta X = 0$ – so moving along such a sequence of vectors ΔX we obtain a sequence of different points characterized by the same value of function F, hence, F is non-injective.

Summing up: if there are no isolated clusters of sectors then determinant of Jacobian of function F , ∇F, is positive at each point of $\mathbb{R}_+^{N\times S}$, so is the determinant of convex hull $z\nabla F + z'\nabla F'$, which means that F is injective on this subspace. Injectivity of F, in turn, implies that if there exists a BGP in terms of $\{l_i^s\}_{i\in\langle N\rangle, s\in\langle S\rangle}$ on $\mathbb{R}_+^{N\times S}$ then it is symmetric across countries by sectors: $l_i^s = l_j^s$ $\forall i,j \in \langle N\rangle$ $\forall s \in \langle S\rangle$. Translating the latter into the language of Economics: under no isolated clusters of sectors if there exists an equilibrium in which each country has non-zero productivity in each sector it is the equilibrium in which the same share of labor is allocated to each sector across countries.

Having derived this important characteristic of an interior equilibrium – $l_i^s = l_j^s$ $\forall i,j,s$ – we can show now that there exists only one such equilibrium. First, ratio of productivities for a pair of countries i, j and any sector s is the same and equal to the ratio of country labor supply:

$$\frac{T_j^s}{T_i^s} = \frac{\phi L_j \sum_q p^{qs} l_j^q}{\phi L_i \sum_q p^{qs} l_i^q} = \{l_i^q = l_j^q\} = \frac{L_j}{L_i}.$$

From the last equality it follows immediately that shares of expenditures on coutry i's products is the same in each sector: $\pi_i^s = \frac{T_i^s (w_i)^{-\theta}}{\sum_j T_j^s (w_j)^{-\theta}} = \pi_i^r$ $\forall r,s$. Considering ratio of sector labor demand we obtain that actual equilibrium labor allocation is $l_i^s = \alpha^s$ $\forall i,s$: $\frac{w_i L_i^q}{w_i L_i^r} = \frac{l_i^q}{l_i^r} = \frac{\alpha^q \pi_i^q}{\alpha^r \pi_i^r} = \frac{\alpha^q}{\alpha^r}$. Clearly, there exists only one BGP along which labor in each country is allocated across sectors proportional to consumption shares α's. Finally, the considered interior equilibrium is characterized by equal wages across countries – from sector labor demand for the same sector and different countries:

$$\frac{L_i^q w_i}{L_j^q w_j} = \frac{\pi_i^q \sum_k L_k w_k}{\pi_j^q \sum_k L_k w_k} = \frac{L_i w_i}{L_j w_j} = \frac{T_i^q (w_i)^{-\theta}}{T_j^q (w_j)^{-\theta}} \Rightarrow \frac{w_i}{w_j} = \left(\frac{w_i}{w_j}\right)^{-\theta} \Rightarrow \frac{w_i}{w_j} = 1$$

Now we proceed to demonstrate **stability** of the above derived unique interior equilibrium. An easy way to demonstrate it is to refer to Figure (2.1). As it was proved above, the bounded curve $\phi_i^s(t)$ intersects $gt_i^s(t)$ in the region with $t_i^s > 0$ at only one point. On Figure (2.1) this case corresponds to the solid upward sloping curve $\phi_i^s(t)$ and intersection at point A, which should be a stable equilibrium. For the sake of rigorousness let's mention that Proposition 1 makes a statement only about the uniqueness of the interior equilibrium, yet, there may exist multiple boundary equilibria even under the conditions of Proposition 1 (no isolated clusters). Figure (2.1) admits a possibility of another unstable equilibrium on the boundary – if $\phi \min_r(p^{rs}) = 0$ (sector s doesn't receive any technologies from sector r), $t_i^s = 0$ (sector s starts with zero productivity) and all labor at the beginning is allocated to such sector r, then an upward sloping curve $\phi_i^s(t)$ intersects line $gt_i^s(t)$ both at 0 and at A. To elaborate this intuition on stability let's replicate the same argument that was used for the 2×2 case and show that all eigenvalues of Jacobian of $G : \dot{t}_i^s = G(t) = \phi \sum_q p^{qs} l_i^q - gt_i^s,\ t \equiv \{t_i^s\}_{\substack{i \in \langle N \rangle \\ s \in \langle S \rangle}}$ are negative at the considered interior equilibrium. The Jacobian can be written down in a matrix form as:

$$G(t) = \begin{bmatrix} G_1^1(t) \\ G_1^2(t) \\ \vdots \\ G_1^S(t) \\ G_2^1(t) \\ \vdots \\ G_S^S(t) \end{bmatrix} \Rightarrow \nabla G = \underset{NS \times NS}{\left[\left\{ \frac{\partial G_i^s}{\partial t_j^r} \right\} \right]} = \phi \left[\underset{N \times N}{I} \otimes \underset{S \times S}{P^T} \right] \underset{NS \times NS}{\left[\left\{ \frac{\partial l_i^q}{t_j^r} \right\} \right]} - g \underset{NS \times NS}{I},$$

where P^T is a transposed matrix of cross-sectoral spillovers (reminder: in P sectors in rows are donors and in columns – recipients of ideas). For each pair of countries (i, j) and sectors (s, r) it can be re-written as

$$\frac{\partial G_i^s}{\partial t_j^r} = \phi \sum_q p^{qs} \frac{\partial l_i^q}{\partial t_j^r} - g \frac{\partial t_i^s}{\partial t_j^r}.$$

The set of derivatives $\left\{ \frac{\partial G_i^s}{\partial t_j^r} \right\}$ need to be calculated at the equilibrium. As a reminder, in equilibrium we have $\pi_i^q = \frac{L_i}{\bar{L}} \equiv l_i\ \forall q \in \langle S \rangle$, where $\bar{L} = \sum_j L_j$ is total population in all countries; $w_i = 1\ \forall i \in \langle N \rangle$, $l_i^q = \frac{L_i^q}{L_i} = \alpha^q\ \forall i \in \langle N \rangle;\ q \in \langle S \rangle;\ t_i^s = \frac{\phi}{g} \sum_q p^{qs} \alpha^q = t^s\ \forall i \in \langle N \rangle$. The difficulty is that each l_i^s is a non-linear function of the whole vector t and can not be expressed explicitly.

To obtain the derivative of labor shares employed w.r.t. the level of technology – $\frac{\partial l_i^s}{\partial t_j^r}$ – we use the equations for sector labor demand.

$$F_i^s :\ l_i^s w_i \left[\sum_k t_k^s \frac{L_k}{L_i} (w_k)^{-\theta} \right] - \alpha^s t_i^s (w_i^{-\theta}) \left[\sum_k \frac{L_k}{L_i} w_k \right] = 0$$

and the implicit differentiation to obtain $\frac{\partial l_i^s}{\partial t_j^r} = -\frac{\frac{\partial F_i^s}{\partial t_j^r}}{\frac{\partial F_i^s}{\partial l_i^s}}$. While the expression for denominator can be obtained immediately as $\frac{\partial F_i^s}{\partial l_i^s} = \frac{t^s}{L_i} \bar{L}$, the expression for the numerator requires some additional steps since it also contains the derivatives of wages w.r.t. the level of technology, $\frac{\partial w.}{\partial t_j^r}$. In the considered version of the model wages are defined as a solution to the system of trade balance equations

$$B_i: \ L_i w_i - \sum_q \pi_i^q \alpha^q \left(\sum_k L_k w_k \right) = 0 \ \forall i \in \langle N \rangle.$$

Treating w's as variables and t's as parameters we differentiate each of these equations w.r.t. some t_j^r – $\sum_k \frac{\partial B_i}{\partial w_k} \frac{\partial w_k}{\partial t_j^r} + \frac{\partial B_i}{\partial t_j^r} = 0$ – we obtain the system which can be solved for $\frac{\partial w_\cdot}{\partial t_j^r}$:

$$\begin{bmatrix} \frac{\partial w_1}{\partial t_j^r} \\ \vdots \\ \frac{\partial w_{N-1}}{\partial t_j^r} \end{bmatrix} = - \begin{bmatrix} \frac{\partial B_1}{\partial w_1} & \dots & \frac{\partial B_1}{\partial w_{N-1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial B_{N-1}}{\partial w_1} & \dots & \frac{\partial B_{N-1}}{\partial w_{N-1}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial B_1}{\partial t_j^r} \\ \vdots \\ \frac{\partial B_{N-1}}{\partial t_j^r} \end{bmatrix}$$

Since the model has only $N-1$ independent wages, so we normalized $w_N \equiv 1$. Here we will skip some algebra, but mention that for finding the inverse of matrix with $\frac{\partial B_\cdot}{w_\cdot}$ we used the result from [23]: if matrices G and $G+E$ are non-singular and E is of rank one then $(G+E)^{-1} = G^{-1} - \frac{1}{1+g} G^{-1} E G^{-1}$, where $g = tr(EG^{-1})$. To summarize this part:

$$\frac{\partial w_i}{\partial t_j^r} = \begin{cases} 0, & \text{if } i \neq j < N \text{ or } \ i = j = N \\ \frac{\alpha^r}{(1+\theta)t^r}, & \text{if } i = j < N \\ \frac{-\alpha^r}{(1+\theta)t^r}, & \text{if } i \neq j = N \end{cases},$$

where, again, w_N is normalized to 1, thus, remains constant.

Now we can return to the expressions for $\frac{\partial F_i^s}{\partial t_j^r}$ and $\frac{\partial l_i^s}{\partial t_j^r}$. Again, skipping some tedious, yet, uninvolved algebra, we'll report the derived expressions for $\frac{\partial l_i^s}{\partial t_j^r}$:

$$\frac{\partial l_i^s}{\partial t_j^r} = \begin{cases} \frac{\alpha^s(1-\alpha^s)}{t^r}(1-l_i), & \text{if } i = j, \ r = s \\ \frac{-\alpha^s \alpha^r}{t^r}(1-l_i), & \text{if } i = j, \ r \neq s \\ \frac{-\alpha^s(1-\alpha^s)}{t^r} l_j, & \text{if } i \neq j, \ r = s \\ \frac{\alpha^s \alpha^r}{t^r} l_j, & \text{if } i \neq j, \ r = s \end{cases},$$

Next, we can write down the matrix of partial derivatives of labor shares l_i^s w.r.t. the level of technology t_j^r using Kronecker product as

$$\left[\left\{ \frac{\partial l_i^s}{\partial t_j^r} \right\} \right] = (\mathbb{1}_N l - I_N) \otimes \left((\alpha \mathbb{1}_S - I_S) \alpha t^{-1} \right),$$

where I_S is the identity matrix of dimensionality $S \times S$ and

$$\mathbb{1}_S = \underbrace{\begin{bmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{bmatrix}}_{S \times S}, \ \alpha = \underbrace{\begin{bmatrix} \alpha^1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \alpha^S \end{bmatrix}}_{S \times S}, \ t = \underbrace{\begin{bmatrix} t^1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & t^S \end{bmatrix}}_{S \times S}, \ l = \underbrace{\begin{bmatrix} l_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & l_N \end{bmatrix}}_{N \times N}.$$

Finally, we can assemble the matrix for the Jacobian of the non-linear dynamic system:

$$\nabla G = g\left[\mathbb{1}_N l - I_N\right] \otimes \left[P^T(\alpha \mathbb{1}_S - I_S)\alpha\tau^{-1}\right] - gI_{NS},$$

where $\underset{S\times S}{\tau} = \frac{g}{\phi}t$. We need to show that all eigenvalues of ∇G are negative under the condition of no isolated clusters. Let's consider the first term of Kronecker product in the expression above. One can easily see that eigenvalues of $\mathbb{1}_N l$ – matrix all repeated rows that sum up to 1 – are 0 and 1 with multiplicity $N-1$ and 1 correspondingly. Thus, eigenvalues of $\mathbb{1}_N l - I_N$ are correspondingly -1 and 0 with multiplicity $N-1$ and 1. Now, to the second term – using simple matrix operations one can obtain the following expression for it $\left[P^T(\alpha \mathbb{1}_S - I_S)\alpha\tau^{-1}\right] =$

$$= \begin{bmatrix} \tau^1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \tau^S \end{bmatrix} \left[\underbrace{\begin{matrix} \alpha^1 & \alpha^2 & \dots & \alpha^S \\ \alpha^1 & \alpha^2 & \dots & \alpha^S \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^1 & \alpha^2 & \dots & \alpha^S \end{matrix}}_{\equiv M} - \underbrace{\begin{matrix} \frac{p^{11}\alpha^1}{\tau^1} & \frac{p^{21}\alpha^2}{\tau^1} & \dots & \frac{p^{S1}\alpha^S}{\tau^1} \\ \frac{p^{12}\alpha^1}{\tau^2} & \frac{p^{22}\alpha^2}{\tau^2} & \dots & \frac{p^{S2}\alpha^S}{\tau^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}\alpha^1}{\tau^S} & \frac{p^{2S}\alpha^2}{\tau^S} & \dots & \frac{p^{SS}\alpha^S}{\tau^S} \end{matrix}}_{\equiv N} \right] \begin{bmatrix} \frac{1}{\tau^1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{\tau^S} \end{bmatrix}$$

The last expression has a form $\tau(M-N)\tau^{-1}$, where τ is an invertible matrix, thus, $\tau(M-N)\tau^{-1}$ is similar to $(M-N)$ and has same eigenvalues as $(M-N)$. Now if we recall that $\tau^s = \sum_q p^{qs}\alpha^q$ we can see that $(M-N)$ has exactly same properties as matrix $(A-B)$ in the part of the proof devoted to uniqueness: M as well as A consists of repeated rows that sum up to 1 while matrix N sums up to 1 by rows, yet, has different rows and can be treated as a stochastic transition matrix. In exactly the same manner as for A – elements of N can be zero only it the corresponding elements of P-matrix are zero. So, in the same way as for $A-B$ above, we can claim that eigenvalues of $M-N$ are $\{0, -\lambda_{N,1}, \dots, -\lambda_{N,S-1}\}$. If matrix P is characterized by absent isolated clusters, all eigenvalues of $M-N$ (and of $\left[P^T(\alpha \mathbb{1}_S - I_S)\alpha\tau^{-1}\right]$) are in $(-1,1)$ interval (the eigenvalue equal to 1 that corresponds to the only stationary distribution of Markov chain represented by N is canceled out with eigenvalue equal to 1 of M). Same is true for the eigenvalues of $[\mathbb{1}_N l - I_N] \otimes \left[P^T(\alpha \mathbb{1}_S - I_S)\alpha\tau^{-1}\right]$ (since those are cross products of eigenvalues of $M-N$ and eigenvalue of $[\mathbb{1}_N l - I_N]$ which are $\{0,-1\}$) and, hence, eigenvalues of ∇G are in $(-2g; 0)$ interval – strictly negative – so, the considered internal equilibrium is locally stable! On the contrary, if P has isolated clusters then N has eigenvalue 1 of multiplicity larger than 1, thus, at least one of eigenvalues of $M-N$ is equal to -1 and at least one of eigenvalues of ∇G is equal to zero – in this case stability of equilibrium can not be guaranteed.

Appendix E

Extension of Proposition 1 to the case of positive trade costs

This appendix outlines a proof of Proposition 1 for a 2×2 economy with trade costs and a diagonal matrix of spillovers, i.e. under intra-sector but not inter-sector spillovers: $p^{AB}=0$ if $A\neq B$ and $p^{AB}>0$ if $A=B$. In this case it is easy to see that $\frac{T_i^A}{T_i^B}=\frac{p^{AA}L_i^A}{p^{BB}L_i^B}$ and, hence, $\frac{L_i^A T_i^B}{T_i^A L_i^B}=\frac{L_j^A T_j^B}{T_j^A L_j^B}$. Plugging the expressions for labor demand $w_i L_i^A=\alpha^A\sum_k \pi_{ik}^A L_k w_k$ (and similar for sector B and country j) into this ratio and substituting the the corresponding expressions for π's, one obtains

$$\frac{L_i w_i\left(T_i^A(dw_i)^{-\theta}+T_j^A(w_j)^{-\theta}\right)+L_j w_j\left(T_i^A(w_i)^{-\theta}+T_j^A(dw_j)^{-\theta}\right)d^{-\theta}}{L_i w_i\left(T_i^B(dw_i)^{-\theta}+T_j^B(w_j)^{-\theta}\right)+L_j w_j\left(T_i^B(w_i)^{-\theta}+T_j^B(dw_j)^{-\theta}\right)d^{-\theta}}=$$
$$\frac{L_i w_i\left(T_i^A(dw_i)^{-\theta}+T_j^A(w_j)^{-\theta}\right)d^{-\theta}+L_j w_j\left(T_i^A(w_i)^{-\theta}+T_j^A(dw_j)^{-\theta}\right)}{L_i w_i\left(T_i^B(dw_i)^{-\theta}+T_j^B(w_j)^{-\theta}\right)d^{-\theta}+L_j w_j\left(T_i^B(w_i)^{-\theta}+T_j^B(dw_j)^{-\theta}\right)}$$

from where, taking into account $d>1$, one can derive $T_i^A T_j^B(w_i w_j)^{-\theta}=T_i^B T_j^A(w_i w_j)^{-\theta}$ and, as a result, $\frac{T_i^A}{T_j^A}=\frac{T_i^B}{T_j^B}$. Combining the last expression with the above mentioned $\frac{L_i^A T_i^B}{T_i^A L_i^B}=\frac{L_j^A T_j^B}{T_j^A L_j^B}$ one obtains $\frac{L_i^A}{L_i^B}=\frac{L_j^A}{L_j^B}$ which means that the interior BGP (with positive output in each country-sector) is characterized by the same sectoral labor allocation within each country. From $\frac{T_i^A}{T_j^A}=\frac{T_i^B}{T_j^B}$ it follows that $\pi_{ii}^A=\pi_{ii}^B$, $\pi_{ij}^A=\pi_{ij}^B$ and same for country j from where $\frac{L_i^A}{l_i^B}=\frac{L_j^A}{l_j^B}=\frac{\alpha^A}{\alpha^B}$, hence, unique interior BGP.