Title
Destroying resonance between Neptune and its resonant Kuiper Belt Objects by stochastic planetesimal scatterings

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Destroying Resonance Between Neptune and Kuiper Belt Objects By Stochastic Planetesimal Scatterings[1]

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ABSTRACT

We revisit the destruction of resonance between Neptune and Kuiper Belt Objects (KBOs) by random planetesimal scatterings, which has been studied by Murray-Clay and Chiang (2006) previously. In this work, we seriously consider the encounters between Neptune's resonant KBOs and planetesimals and the levy flight behavior of resonant KBOs corresponding to a single big kick. The analysis in this work is based on order-of-magnitude estimation.

Subject headings: celestial mechanics – diffusion – Kuiper Belt – Planets and satellites: formation – solar system: formation

1. Frequently Used Quantities

The frequently used constants in this work are presented in Table 1. Another important quantity is the maximum libration amplitude. It can be expressed as,

$$\Delta a_{\rm Nep,lib} = 2C_{lib} a_{\rm Nep} \left(\frac{M_{\rm Nep} e_{res}}{M_{\odot}} \right)^{1/2}, \tag{1}$$

where $C_{lib} \sim 3.64$ is a constant (see Murray & Dermott 1999). By plugging in all the physical quantities, it yields that $\sigma a_{\rm Nep,lib} \sim 0.7$ AU. The change semimajor axis of perturbed Neptune (or resonant KBOs), Δa_{res}, has to be smaller than $\Delta a_{\rm Nep,lib}/2 \sim 0.35$ AU in order to keeping KBOs in resonance.

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2. Encounters in Different Regimes

We first define $x \equiv a - a_{res}$ as the difference between the semimajor axis of the perturber and the resonant object of interest (which could be either Neptune or its resonant KBOs). Then we define b as the impact parameter of the encounter and u as the epicyclic (random) velocity of planetesimals (perturbers), which represents the typical relative velocity during the encounter. Encounters performance differently according to how $|x|$ and b compare with the Hill radius of resonant object of interest,

$$R_H = a_{res} \left(\frac{M_{res}}{3M_\odot} \right)^{1/3}, \tag{2}$$

and according to how u compares with the Hill velocity,

$$v_H \equiv \Omega_{res} R_H, \tag{3}$$

as well as the escape velocity,

$$v_{esc} = \left(\frac{GM_{res}}{R_{res}} \right)^{1/2}, \tag{4}$$

where a_{res}, M_{res} and R_{res} are the mass and the radii of interested resonant object, respectively.

2.1. $u < v_H$

While relative velocity is smaller than Hill velocity $u < v_H$, the encounter is dominated by Keplerian shearing. When $|x| \le 2R_H$, the encounter pulls the perturber into the Hill sphere. It accelerates in a chaotic way and exits the Hill sphere in a random direction with u of order Hill velocity, v_H (Petit & Hénon 1986). This regime has been investigated by Murray-Clay & Chiang (2006).

2.2. $v_H < u < v_{esc}$

When $u > v_H$, the perturber can enter the Hill sphere and exit while changing an order-unity rotation of the direction of the perturber's random velocity vector, and requires that impact parameter $b \le GM_{res}/u^2$. This regime has been investigated by Murray-Clay & Chiang (2006) as well.

2.3. $|x| \leq R_H$, *Horseshoes Orbit*

When $|x| < R_H$, the perturbers can occupy horseshoe orbits. Murray-Clay & Chiang (2006) has studied this regime. However, they restricted that the perturbers having sub-Hill eccentricity ($e \leq R_H/a_{res}$) because highly eccentric 1:1 horseshoe resonators are unstable.

2.4. $u > v_{esc}$

When $u > v_{esc}$, the perturbers nearly do not change their trajectory due to gravitational interaction and can encounter with the resonant object of interest with any angle[2]. This regime is actually the most important regime for the perturbers encountering with the resonant KBOs of Neptune. The regime will be explored in this work.

3. Case I: "5000 Plutos, 5 Gyrs"

In this section, we assume that all planetesimal perturbers have the same mass, which is the mass of Pluto $M_{\rm Plu}$, and this kind of planetesimal disk can exist for the whole solar system life, which is 5 Gyrs. The planetesimal disk density is fixed to minimum mass trans-Neptunian disk $\Sigma = 0.2$ g cm^{-2}. We discuss the kicks on Neptune, the big kicks on resonant KBOs and the small kicks on resonant KBOs, respectively. First, we estimate the total number of perturbers in disk. It is given by

$$N \sim \frac{\Sigma a_{\rm KBO}^2}{M_{\rm Plu}} \sim 5 \times 10^3. \tag{5}$$

3.1. Kicks on Neptune

Planetesimal scatterings on Neptune has been discussed by Murray-Clay & Chiang (2006). In this section, our aim is to address whether any single kick on Neptune can destroy the resonance. First we discuss the encounters in the regime of $v_H < u$. The strength of kick can be expressed as

$$|\Delta a_{\rm Nep}| \sim \frac{M_{\rm Plu}}{M_{\rm Nep}} a_{\rm Nep} e, \tag{6}$$

[2]Here, the angle is defined as the angle between impact parameter vector and the velocity vector of the interested resonant object.

where e is the eccentricity of the perturber. By plugging in the physical quantities, we have $|\Delta a_{\rm Nep}| \sim 0.0035e < 0.0035$ AU, which is much smaller than the resonance libration width. Thus, any single kick on Neptune cannot destroy the resonance.

Furthermore, we calculate the total number of such encounters during the whole solar system life (5 Gyrs). Such kicks occur at impact parameters $b \leq GM_p/u^2$[3] and random velocity u is greater than Keplerian shearing velocity in these cases. Thus, the number of encounters can be derived as

$$N \sim \frac{\Sigma}{M_{\rm Plu}} \frac{\Omega_{\rm Nep}}{u} u b^2 t, \tag{7}$$

where $u/\Omega_{\rm Nep}$ represents the scale height of planetesimal disk[4]. Substitute $b \sim GM_{\rm Nep}/u^2$ into equation (7), it gives that

$$N \sim \frac{\Sigma}{M_{\rm Plu}} \Omega_{\rm Nep} \left(\frac{GM_{\rm Nep}}{u^2} \right)^2 t \sim 3000/e^4 > 3000. \tag{8}$$

The strength of single big kick on Neptune is much smaller when comparing with the resonance libration width and such encounter can happen very often. Neptune unfold Brownian motion by planetesimal scatterings. Statistically, the cumulative change semimajor axis of Neptune can be calculated by

$$\Delta a_{\rm Nep}^{cum} \sim |\Delta a_{\rm Nep}| N^{1/2} \sim 0.2/e \text{ AU}, \tag{9}$$

where e is the eccentricity of perturber, which is not well known nowadays. The cumulative change semimajor axis is of order resonance libration width.

Furthermore, we examine the encounters in the regime of $u < v_H$. In this regime, the encounters are dominated by Keplerian shearing. The biggest kick occur while $|x| \leq 2R_H$. In this case, the encounter pulls the planetesimal into the planet's Hill sphere. The planetesimal runs in a chaotic way and exits the Hill sphere in a random direction with u^* of order the Hill velocity, v_H. Equation (6) is still valid but e has to be modified as Hill eccentricity, $e_H \equiv R_H/a_p \sim (M_P/M_\odot)^{1/3}$. The strength of kick can be estimated as

$$|\Delta a_{\rm Nep}| \sim \frac{M_{\rm Plu}}{M_{\rm Nep}} a_{\rm Nep} e_H \sim \frac{M_{\rm Plu}}{M_{\rm Nep}} R_H \sim 0.00013 \text{ AU}, \tag{10}$$

which is much smaller than the resonance libration width. The total number of such encounters can be estimated as

$$N \sim \frac{\Sigma}{M_{\rm Plu}} \Omega_{\rm Nep} R_H^2 t \sim 5 \times 10^8. \tag{11}$$

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[3]The relation is required for $\Delta u \sim u$.

[4]This statement is correct since we assume $e \sim i$.

By using equation (9), the cumulative change semimajor axis of Neptune is approximately $|\Delta a_{\rm Nep}| \sim 3$ AU. Thus, Neptune loses most of its resonant KBOs due to planetesimal scatterings.

3.2. Big Kicks on Resonant KBOs

We are interested in addressing whether a single big kick on Resonant KBOs can destroy its resonance with Neptune. The observed eccentricity of resonant KBOs is 0.2, on average. The relative velocity can be estimated as

$$u \sim v_k e \sim 1000 \text{ m s}^{-1}, \tag{12}$$

where v_k is the Keplerian velocity. It is greater than the escape velocity of the resonant KBOs[5]. For a super-escape encounter at impact parameter b, the interaction time can be estimated as $\Delta t \sim b/u$. By employing impulse approximation, we can estimate the change velocity of the resonant KBOs as

$$\Delta v \sim \frac{GM_{\rm Plu}}{b^2}\frac{b}{u} \sim \frac{GM_{\rm Plu}}{bu}, \tag{13}$$

where $M_{\rm Plu}$ is the mass of Pluto-mass perturber. The change of the resonant KBO's specific energy over the encounter is approximately

$$\Delta\left(-\frac{GM_\odot}{2a}\right) \sim \Delta\left(\frac{1}{2}v^2\right) + \Delta\left(-\frac{GM_\odot}{r}\right), \tag{14}$$

where v is the velocity of the resonant KBO relative to the Sun and r is the distance between the KBO and the Sun. It is reasonable to assume that r does not change during such impulsive encounter. Then the equation above can be simplified as

$$\Delta a_{\rm KBO} \sim \frac{v\Delta v}{GM_\odot} a_{\rm KBO}^2, \tag{15}$$

while we have an order-of-magnitude estimation of v given by

$$v \sim \left(\frac{GM_\odot}{a_{\rm KBO}}\right)^{1/2}. \tag{16}$$

By combining equations (13), (15) and (16) and assuming $u \sim ve$, we have

$$\Delta a_{\rm KBO} \sim \frac{\Delta v}{v} a_{\rm KBO} \sim \frac{M_{\rm Plu}}{M_\odot}\frac{a_{\rm KBO}^2}{be}. \tag{17}$$

[5]The escape velocity of Pluto is ~ 900 m s^{-1}.

According to equation (17), $\Delta a_{\rm KBO}$ is proportional to $1/b$.

Next we calculate the strongest encounter which can occur in 5 Gyrs. The minimum impact parameter b_{min} is defined as b of the closest encounter that can occur once in 5 Gyrs statistically. This statement can be expressed as

$$\frac{\Sigma}{M_{\rm Plu}}\Omega_{\rm KBO}b_{min}^2 t \sim 1. \tag{18}$$

By plugging in realistic quantities, equation (18) gives $b_{min} \sim 6 \times 10^8$ cm. Substitute b in equation (17) with b_{min}, the maximum change semimajor axis of resonant KBO yields $max|\Delta a_{\rm KBO}| \sim 0.8$ AU, which is greater than resonant libration width 0.35 AU. Therefore, a single big kick on resonant KBO is strong enough to destroy resonance between Neptune and its resonant KBOs.

3.3. Small Kicks on Resonant KBOs

The encounter with impact parameter b greater than b_{min} can occur many times in 5 Gyrs. The resonant KBO behaves Brownian motion by planetesimal scatterings in those cases. The average effect of many encounters should be $< \Delta a_{\rm KBO}^{cum} >\sim 0$. The deviation from the average effect $\Delta a_{KBO,rms}^{cum}$ is caused by the statistical fluctuation of Brownian motion. According to the statistical theory of Brownian motion and quadratic law, $\Delta a_{\rm KBO,rms}^{cum}$ can be derived as

$$\Delta a_{\rm KBO,rms}^{cum} = \left(\sum \Delta a(b)^2 N(b)\right)^{1/2}, \tag{19}$$

where $\Delta a(b)$ is the effect of encounters with specific b and $N(b)$ is the total number of encounters with specific b. While $N(b)$ can be calculated as

$$N(b) \sim \frac{\Sigma}{M_{\rm Plu}}\Omega t b \Delta b. \tag{20}$$

By combining the equations (17), (19) and (20), we get

$$\Delta a_{\rm KBO,rms}^{cum} \sim \frac{M_{\rm Plu}}{M_{\odot}}\frac{a_{\rm KBO}^2}{e}\left(\int_{b_{min}}^{b_{max}} \frac{\Sigma\Omega_{\rm KBO}t}{M_{\rm Plu}b}db\right)^{1/2} \sim \left(\ln(b_{max}/b_{min})\right)^{1/2}\ \text{AU} \sim 3\ \text{AU}, \tag{21}$$

where $b_{max} \sim a_{\rm KBO}e \sim u/\Omega_{\rm KBO} \sim 8$ AU.

4. Case II: "10 Plutos, 5 Gyrs"

Brown (2008) reviewed the 8 discoveries of largest KBO, namely, Eris, Pluto, Sedna, 2005 FY9, 2003 EL61, Quaoar, Orcus and Ixion, respectively. Based on the completeness of the current surveys, it appears that 3 more KBOs of the same size range likely still await discovery (Brown 2008). According to the current observed evidence, we assume there are only ten Pluto mass perturbers in the planetesimal disk and the disk can exist for 5 Gyrs.

4.1. Kicks on Neptune

Since there are only a few perturbers, we expect that the minimum impact parameter b_{min} would be greater than in Case I. The surface number density is approximately

$$n \sim \frac{N_{total}}{a_{\rm Nep}^2}, \tag{22}$$

where $N_{total} = 10$ is the total number of perturbers. By plugging the surface number density into equation (9), we can estimate the cumulative change semimajor axis of Neptune in the regime $u > v_H$ as

$$\Delta a_{\rm Nep}^{cum} \sim |\Delta a_{\rm Nep}| N^{1/2} \sim 0.003/e \text{ AU}. \tag{23}$$

We also calculate the cumulative change semimajor axis in the regime $u < v_H$ in the same way. It yields $\Delta a_{\rm Nep}^{cum} \sim 0.04$ AU, where we use $e \sim e_H$.

4.2. Big Kicks on Resonant KBOs

First we estimate the minimum impact parameter $b_{min} \sim 2\times10^{-3}$ AU. Then we calculate the biggest kick by using equation (17). The result yields $\Delta a_{\rm KBO} \sim 0.03$ AU. Thus, a single big kick cannot destroy the resonance between Neptune and its resonant KBOs.

4.3. Small Kicks on Resonant KBOs

Equation (21) has to be modified as

$$\Delta a_{rmKBO,rms}^{cum} \sim \frac{M_{\rm Plu}}{M_\odot}\frac{a_{\rm KBO}^2}{e}\left(\int_{b_{min}}^{b_{max}} \frac{N_{total}\Omega_{\rm KBO} t}{a_{\rm KBO}^2 b} db\right)^{1/2} \sim 0.03\times(\ln(b_{max}/b_{min}))^{1/2} \text{ AU} \sim 0.08 \text{ AU}, \tag{24}$$

where $b_{min} \sim 10^{-3}$ AU and $b_{max} \sim u/\Omega_{\rm KBO} \sim 8$ AU.

5. Case III: "Lots of comets, 1 Gyrs"

In this section, we assume that all planetesimals are 10 km size comets and the planetesimal disk can only exist in the first 1 Gyrs of solar system life. The planetesimal disk density is fixed to minimum mass trans-Neptunian disk $\Sigma = 0.2$ g cm^{-2}. After that, there were no perturbers existing. The mass of single perturber is

$$M_{per} = \frac{4}{3}\pi R_{per}^3 \rho \sim 10^{16} \text{ kg} \tag{25}$$

5.1. Kicks on Neptune

By plugging new parameters in equations (6), (7) and (9), the cumulative change semimajor axis of Neptune is estimated $\Delta a_{\rm Nep}^{cum} \sim 0.0001/e$. It is not comparable with resonance libration width. Thus, kicks on Neptune cannot destroy the resonance by gravitational interaction.

By applying equation (18), the minimum impact parameter yields $b_{min} \sim 1.5 \times 10^6$ cm. Note that b_{min} is much smaller than the radii of Neptune. Therefore, the planetesimals can hit Neptune physically. These collisions have to be considered in this case. In the following discussion, we make order-of-magnitude estimate of the change semimajor axis of Neptune and the total number of collisions.

Neptune suffering a collision with a planetesimal change $\sim M_{per}u$ of its momentum. Then the change of its velocity yields

$$\Delta v \sim \frac{M_{per}}{M_{\rm Nep}} u \sim 10^{-10} u. \tag{26}$$

By applying equation (17), we can estimate the change semimajor axis of Neptune due to a single physical collision

$$\Delta a_{\rm Nep} \sim \frac{\Delta v}{v} a_{\rm Nep} \sim \frac{10^{-10} ve}{v} a_{\rm Nep} \sim 10^{-10} e a_{\rm Nep} \sim 3 \times 10^{-9} e \text{ AU}, \tag{27}$$

where we assume $u \sim ve$.

A serious analysis of collision rate is complicate, since the collision cross section depends on the random relative velocity u. We refer the reader to Goldreich et al. (2004) for details.

When $u > v_H$, gravitational focusing enhances the collision cross section by a factor of $\sim (v_{esc}/u)^2$. Thus, the total number of collision is derived as

$$N \sim \frac{\Sigma}{M_{per}} \Omega_{\rm Nep} R_{\rm Nep}^2 \left(\frac{v_{esc}}{u}\right)^2 t \sim \frac{5 \times 10^7}{e^2}. \tag{28}$$

The cumulative change semimajor axis yields $\Delta a_{\rm Nep}^{cum} \sim 2\times10^{-5}$ AU in this regime.

When $u < v_H$, the encounters with impact parameter less than $b_{graze} \sim R_{Nep} v_{esc}/v_H$ will result in physical collision (Goldreich et al. 2004). The total number of collision then is given by

$$N \sim \frac{\Sigma}{M_{per}} \Omega_{\rm Nep} b_{graze}^2 t \sim \frac{\Sigma}{M_{per}} \Omega_{\rm Nep} R_{\rm Nep}^2 \left(\frac{v_{esc}}{v_H}\right)^2 t \sim 4\times10^{10}. \tag{29}$$

The cumulative change semimajor axis yields $\Delta a_{\rm Nep}^{cum} \sim 6\times10^{-4} e$ AU in this regime.

5.2. Big Kicks on Resonant KBOs

By applying equation (18), the minimum impact parameter yields $b_{min} \sim 1.5\times10^6$ cm. Note that b_{min} is smaller than the average radii of Neptune's resonant KBOs $R_{\rm Plu} \sim 10^7$ cm. Physical collision occur with strength as

$$\Delta a_{\rm KBO} \sim \frac{M_{per} u}{M_{\rm KBO} v} a_{\rm KBO} \sim 0.04e \sim 0.008 \text{ AU}, \tag{30}$$

where $e \sim 0.2$. It is much greater than the change semimajor axis by gravitational encounter with $b \sim R_{KBO}$, which is $\Delta a_{\rm KBO} \sim 10^{-6}$ AU.

In this case $u > v_{esc}$, gravitational focusing is negligible. Thus, the collision cross section is exactly the physical surface area of resonant KBOs. Then the total number of collisions can be derived as

$$N \sim \frac{\Sigma}{M_{per}} \Omega_{\rm KBO} R_{\rm KBO}^2 t \sim 100. \tag{31}$$

The cumulative change of semimajor axis of resonant KBOs yields $\Delta a_{\rm KBO}^{cum} \sim 0.08$ AU.

5.3. Small Kicks on Resonant KBOs

Equation (21) has to be modified as

$$\Delta a_{\rm KBO,rms}^{cum} \sim \frac{M_{per}}{M_\odot} \frac{a_{\rm KBO}^2}{e} \left(\int_{b_{min}}^{b_{max}} \frac{\Sigma \Omega_{\rm KBO} t}{M_{per} b} db\right)^{1/2} \sim 0.0005 \times (\ln(b_{max}/b_{min}))^{1/2} \text{ AU} \sim 0.003 \text{ AU}, \tag{32}$$

where $b_{min} \sim R_{\rm KBO} \sim 10^7$ cm and $b_{max} \sim u/\Omega_{\rm KBO} \sim 8$ AU.

6. Case IV: "Nice model"

In this section, we assume that all planetesimals are 100 km size objects and the planetesimal disk can only exist in the first 1 Gyrs of solar system life. The planetesimal disk density is fixed to minimum mass trans-Neptunian disk $\Sigma = 0.2$ g cm^{-2}. It is the planetesimal disk in "Nice" model. After that, there were no perturbers. The mass of single perturber is

$$M_{per} = \frac{4}{3}\pi R_{per}^3 \rho \sim 10^{19} \text{ kg} \tag{33}$$

6.1. Kicks on Neptune

First we consider the physical collision on Neptune. According to equations (26) and (28), we know a single collision strength $\Delta a_{\rm Nep}$ is proportional to the mass of perturber M_{per} and the total number of collision N is proportional to $1/M_{per}$. Then the cumulative effect of collision is $\Delta a_{\rm Nep}^{cum} \sim M_{per}^{1/2}$. Thus, the cumulative change of semimajor axis of Neptune due to physical collisions is 30 times greater than the change due to collisions in Case III. When $v_H < u < v_{esc}$, we get $\Delta a_{\rm Nep}^{cum} \sim 6 \times 10^{-4}$ AU. When $u < v_H$, we get $\Delta a_{\rm Nep}^{cum} \sim 0.02e$ AU.

Next, we consider the gravitational interaction on Neptune. By plugging new parameters in equations (6), (7) and (9), the cumulative change semimajor axis of Neptune is estimated $\Delta a_{\rm Nep}^{cum} \sim 0.003/e$. It is not comparable with resonance libration width. Thus, kicks on Neptune cannot destroy the resonance in this case.

6.2. Big Kicks on Resonant KBOs

By applying equation (18), the minimum impact parameter yields $b_{min} \sim 5 \times 10^7$ cm. It is greater than the radii of resonant KBOs $R_{\rm KBO}$. Statistically, there is no physical collision in this case. The strength of biggest kick then can be calculated by using equation (17). The resultant change semimajor axis is $\Delta a_{\rm KBO} \sim 6 \times 10^{-4}$ AU. Thus, a single big kick cannot destroy the resonance between Neptune and its resonant KBOs.

6.3. Small Kicks on Resonant KBOs

Equation (21) has to be modified as

$$\Delta a^{cum}_{\rm KBO,rms} \sim \frac{M_{per}}{M_\odot}\frac{a^2_{\rm KBO}}{e}\left(\int_{b_{min}}^{b_{max}} \frac{\Sigma\Omega_{\rm KBO}t}{M_{per}b}db\right)^{1/2} \sim 0.015\times(\ln(b_{max}/b_{min}))^{1/2}\ {\rm AU} \sim 0.06\ {\rm AU}, \tag{34}$$

where $b_{min} \sim 5\times10^7$ cm and $b_{max} \sim u/\Omega_{\rm KBO} \sim 8$ AU.

7. Case V: "Planet X"

In this case, we consider an outer planet beyond Pluto (Planet X; Lykawka & Mukai 2008 and references therein). There are three basic assumptions in this section: the mass of planet X is around $0.1M_\oplus$; its semimajor axis is ~ 100 AU; the eccentricity of its orbit is ~ 0.4.

First we estimate the minimum impact parameter b_{min} of encounters as

$$b_{min} \sim (1-e_X)a_X - (1+e_{\rm KBO})a_{\rm KBO} \sim 10\ {\rm AU}. \tag{35}$$

Given that the planet X sits on an eccentric orbit and b_{min} is quite large, the encounters dominate by the one discussed in section 3.2. Since all the equations there do not include the mass of Neptune's resonant KBOs, we do not distinguish Neptune and its resonant KBOs below. We use $a \sim 40$ AU representing the semimajor axis of Neptune and its resonant KBOs uniformly. This simplification does not change the final result much. By plugging all quantities into equation (17), we get $\Delta a \sim 10^{-4}$ AU for a single kick.

The total number of encounters is determined by the number of orbits planet X can complete during 5 Gyrs. Then we have

$$N \sim \frac{t\Omega_X}{2\pi} \sim 5\times10^6. \tag{36}$$

Thus, the cumulative change of semimajor axis can be derived as

$$\Delta a^{cum} \sim \Delta a N^{1/2} \sim 0.2\ {\rm AU}. \tag{37}$$

The change of semimajor axis derived in the above equation is indeed an upper limit, since the interaction between Neptune (or KBOs) and planet X is actually a celestial mechanics problem. The solution should be much less than the estimation from impulse approximation here.

8. Conclusion

We present the conclusion in Table 2.

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Table 1. Frequently Used Constants

Symbol	Definition	Value
G	Constant of Gravitation	6.672×10^{-8} cm^3 g^{-1} s^{-2}
AU	Astronomical Unit of Distance	$1.495978707 \times 10^{13}$ cm
$M_\odot$	Mass of Sun	1.98911×10^{33} g
$M_\oplus$	Mass of Earth	5.9736×10^{27} g
$M_{\rm Nep}$	Mass of Neptune	1.0243×10^{29} g
$R_{\rm Nep}$	Radii of Neptune	2.5225×10^{9} cm
$M_{\rm Plu}$	Mass of Pluto	1.27×10^{25} g
$a_{\rm Nep}$	Semimajor Axis of Neptune	30 AU
ρ	Planetesimal internal density	2 g cm^{-3}
a_p	Average Semimajor Axis of Resonant Planetesimals	40 AU
e_p	Average Eccentricity of Resonant Planetesimals	0.2
Σ	Disk Surface Density	0.2 g cm^{-2} (for minimum mass trans-Neptunian disk)

Table 2. Conclusion

Case No.	Encounter Type	Δa^a (AU)	Remark
I “5000 Plutos, 5 Gyrs”	Single Kick on Neptune	<0.0035	No destroying
	Cumulative Small Kicks on Neptune	3	Yes
	Single Kick on KBO	0.8	Yes
	Cumulative Small Kicks on KBO	3	Yes
II “10 Plutos, 5 Gyrs”	Single Kick on Neptune	<0.0035	No destroying
	Cumulative Small Kicks on Neptune	0.04	No destroying
	Single Kick on KBO	0.03	No destroying
	Cumulative Small Kicks on KBO	0.08	No destroying
III “Lots of Comets, 1Gyrs”	Single Kick on Neptune	too small	No destroying
	Cumulative Small Kicks on Neptune	$0.0001/e$	No destroying
	Single Kick on KBO	0.008	No destroying
	Cumulative Small Kicks on KBO	0.08	No destroying
IV “Nice model”	Single Kick on Neptune	too small	No destroying
	Cumulative Small Kicks on Neptune	$0.003/e$	No destroying
	Single Kick on KBO	6×10^{-4}	No destroying
	Cumulative Small Kicks on KBO	0.06	No destroying
V “Planet X”	Single Kick on Neptune	10^{-4}	No destroying
	Cumulative Small Kicks on Neptune	0.2	No destroying
	Single Kick on KBO	10^{-4}	No destroying
	Cumulative Small Kicks on KBO	0.2	No destroying

[a] The change semimajor axis reported in this table is the maximum value among the result in different encounter regimes.

[b] RF = Retention Fraction