Title

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Publication Date

1956-04-19

UCRL 3383

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(Thesis)

April 19, 1956

Printed for the U. S. Atomic Energy Commission

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Contents

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William L. Imhof

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ABSTRACT

The relative yields of π^+ mesons from H, Li, Be, B^{10}, B^{11}, C, Al, Cu, Ag, and Pb produced by 335-Mev bremsstrahlung and 340-Mev protons were measured at 135° to the beam line. The pions were identified in a plastic scintillator telescope by the π-μ decay method.

At the Berkeley synchrotron the relative π^+ yields for these elements were measured at seven different pion energies, ranging from 12 Mev to 65 Mev. The results showed little variation of the A dependence with pion energy. By the model of Brueckner, Serber, and Watson the mean free path for mesons in nuclear matter is less than $3r_0$ at all meson energies.

The yields of 36-Mev and 63-Mev mesons were measured at the 184-inch cyclotron. The pion-production efficiency dropped off faster with increasing A under proton bombardment than under gamma bombardment. These results compared well with the expressions given by Gasiorowicz for the effect of the energy degeneration of the incident protons in nuclear matter.

I. INTRODUCTION

The purpose of studying pion production from complex nuclei is not to learn about the basic production process, which can best be learned by using hydrogen and deuterium as targets. It is more to learn about the behavior of nucleons in nuclei and about scattering and absorption of mesons in nuclei.

The first work on the Z dependence of pion production was done by Mozley.[1] Under 317-Mev bremsstrahlung bombardment he measured the relative yields of 42 ± 7-Mev and 76 ± 6-Mev positive pions at $90^{\circ} \pm 8^{\circ}$ to the beam line for the following elements: H, Li, Be, C, Al, Cu, Sn, and Pb. He showed that the efficiency of photoproduction of positive pions from a proton in a nucleus goes roughly as $1/A^{1/3}$.

Then in 1952 Littauer and Walker,[2] using the Cornell 310-Mev synchrotron, measured the yield of π^+ and π^- mesons in the energy band 50 to 80 Mev at 135° from several nuclei. By plotting the $(\pi^+ + \pi^-)$ yield versus A they got a better fit to an $A^{2/3}$ law than by plotting either the π^+ or the π^- yield alone. But even this better fit yielded more points off the curve than would have been warranted by the statistics. Also their π^-/π^+ ratios yielded rather prominent deviations from the values expected on the basis of the number of nucleons of each type. These ratios exhibited a striking correlation with the binding energy of the struck nucleus.

The yield of neutral photomesons above 85 Mev was shown by Panofsky, Steinberger, and Steller[3] to obey an $A^{2/3}$ dependence.

More recently, at the Stanford Linear accelerator Motz, Crowe, and Friedman[4] measured π^-/π^+ ratios for 53 ± 10-Mev mesons at 75° to the bremsstrahlung beam. For bremsstrahlung energies on the order of 300 Mev their results agreed fairly well with those of Littauer and Walker. At high energies (500 Mev) the π^-/π^+ ratio seemed to tend toward the neutron-proton ratio.[5]

The first work on meson yields as a function of atomic number under proton bombardment was done by Clark.[6] He measured the relative differential cross sections for production of 40-Mev π^+ and

π^- mesons by 240-Mev protons on Be, C, Al, Cu, Ag, W, and Pb. The results showed no simple correlation of the cross sections with the mass number A, as observed in the photomeson case. He found that $\sigma(A)/A^{2/3}$ gave a maximum at Al for π^+ and a maximum at Cu for π^-.

Using nuclear emulsions and a 381-Mev proton beam, Block, Passman, and Havens[7] studied the production of charged pions from H, D, C, Cu, and Pb at 90°. They found that the negative-pion production cross section increases approximately as $A^{2/3}$, whereas the positive-pion production increases with a dependence slower than $A^{2/3}$.

Sagane and Dudziak[8, 9] observed the production of 12.5-, 27-, and 33-Mev pions at 90° from Be, C, Al, Cu, Ag, and Pb by 340-Mev protons. They found that the relative yield of negative pions is nearly proportional to the number of neutrons in the nucleus. On the other hand the yield of positive pions from the heavy nuclei relative to the light ones dropped off with decreasing pion energy. This has been attributed at least partially to effects of the Coulomb barrier.[10, 11]

Hales[12] studied the relative yield of neutral mesons, whose energy spectrum was peaked at 40 Mev, from various elements placed in a 340-Mev proton beam. For the elements from hydrogen to sodium he found the yield to be proportional to N, whereas from aluminum to lead it was proportional to $N/A^{1/3}$. He interpreted these results as being compatible with a π^0 mean free path in nuclear mater of $10r_0$.

The yield of π^+ mesons at 0° from various nuclei bombarded by 340-Mev protons has been measured by Merritt and Hamlin.[13] Their results were explained by assuming a proton mean free path in nuclear matter of 4 to 6 x 10^{-13} cm and a meson mean free path that decreases from 7 x 10^{-13} cm at 55 Mev to 1.1 x 10^{-13} cm at 145 Mev.

Table I

Summary of experiments on Z dependence of pion production

Experimenters	Incident beam	Type of pions	Obs. Angle	Pion Energy (Mev)	Conclusions
Mozley	317-Mev x-rays	π^+	90°	42±7 76±6	$\sigma(\pi^+) \sim Z/A^{1/3}$
Littauer Walker	310-Mev x-rays	π^+, π^-	135°	65±15	$\sigma(\pi^+ + \pi^-) \sim A^{2/3}$ π^-/π^+ deviated from N/Z
Panofsky Steinberger Steller	330-Mev x-rays	π^0	45°	> 85	$\sigma(\pi^0) \sim A^{2/3}$
Motz Crowe Friedman	300-Mev x-rays 500-Mev x-rays	π^+, π^-	75°	53±10	π^-/π^+ agreed with Littauer and Walker $\pi^-/\pi^+ \rightarrow N/Z$
Clark	240-Mev protons	π^+ π^-	140° 40°	40	$\sigma(\pi^+)/A^{2/3}$ gave max. at Al $\sigma(\pi^-)/A^{2/3}$ gave max. at Cu
Block Passman Havens	381-Mev protons	π^+, π^-	90°	Energy spectrum	$\sigma(\pi^-) \sim A^{2/3}$ $\sigma(\pi^+)$ increased slower than $A^{2/3}$
Sagane Dudziak	340-Mev protons	π^+, π^-	90°	12.5±2 27±1.5 33±3	$\sigma(\pi^-)$ gave better fit to N than $N^{2/3}$ σ(33 Mev π^+) gave better fit to $Z^{2/3}$ than Z. σ(12.5, 27 Mev π^+) increased slower than $Z^{2/3}$.
Hales	340-Mev protons	π^0	135°	Detection peaked at 40	$\sigma(\pi^0) \sim N$ from H to Na. $\sigma(\pi^0) \sim N/A^{1/3}$ from Al to Pb. $\lambda_{\pi^0} \sim 10 r_0$.
Merritt Hamlin	340-Mev protons	π^+	0°	Energy spectrum	$\lambda_{proton} = 4\text{-}6 \times 10^{-13}$ cm. $\lambda_{\pi^+} = 7 \times 10^{-13}$ cm at 55 Mev $= 1.1 \times 10^{-13}$ cm at 145 Mev

In the case of gamma ray bombardment the experiments by Mozley and by Littauer and Walker have indicated that the efficiency of production of positive pions from a proton in a nucleus goes roughly as $1/A^{1/3}$. Binding energy, internal momentum, and exclusion-principle effects tend to lower the production efficiency from complex nuclei as compared to that from free nucleons. But it would be difficult to explain the observed uniform variation with A by such effects only. One should further consider the interaction of the produced meson with the rest of the nucleus. Using the optical model,[14] Brueckner, Serber, and Watson have given an expression for the probability of a meson's leaving the nucleus as a function of the mean free path for absorption of mesons in nuclear matter.[15]

$$\text{Probability of escape} = 3[1/2x - 1/x^3 + (1/x^3)(1+x)e^{-x}], \ldots\ldots \quad (1)$$

where $x=2R/\lambda$, R=nuclear radius, and λ=mean free path of mesons in nuclear matter. For $\lambda \sim 1 r_0$, where r_0=(nuclear radius)/$A^{1/3}$, this gives a $1/A^{1/3}$ dependence and hence would explain the photoproduction data.

This mean free path for mesons in nuclear matter can be measured by other means. It can be calculated from meson-nucleon scattering data. Or it may be obtained by bombarding various nuclei with pions and observing the elastic scattering and the total attenuation of the mesons, as done by Stork.[16] He indicates that the mean free path for pions of energy approximately 70 Mev and higher is probably 1 to 3 r_0, but it should increase with decreasing pion energy, becoming long enough for 20- to 30-Mev pions to give practically a constant production efficiency for all nuclei.

The photoproduction efficiency as a function of A has previously been measured only for pions of energy greater than 42 Mev. It was therefore felt desirable to measure the A dependence for production of lower-energy pions. The A dependence was measured for positive pions of energy 12±6 Mev, 16.5±4.5 Mev, 22.5±6 Mev, 30±4.5 Mev, 36±4 Mev, 46±4 Mev, and 65±4 Mev.

The production of mesons by proton bombardment is further complicated by the interaction of the incident protons in nuclear matter In order to learn more about this process it was desirable to measure the A dependence for production of the same energy mesons under gamma bombardment and under proton bombardment. Mesons of energy 36±4 Mev and 63±4 Mev were observed at the cyclotron with the same equipment and at the same angle as used in obtaining the synchrotron data.

II. EXPERIMENTAL APPARATUS AND PROCEDURE

A. General Description of the Experiment

Li, Be, B^{10}, B^{11}, C, CH_2, Al, Cu, Ag, and Pb targets were bombarded at the Berkeley 335-Mev synchrotron and 340-Mev synchrocyclotron, and the positive pions coming off at 135° to the beam line were counted.

1. Synchrotron Arrangement I

The experimental arrangement is shown in Fig. 1. The spread-out bremsstrahlung beam was used so that the duration of each pulse was about 4 milliseconds. The beam first passed through a thin-walled ionization chamber, then through two lead collimators, a sweeping magnet, another lead collimator, the target, and finally into a thick-walled copper ionization chamber of the type used at Cornell. Identification of the π^+'s was made by means of their characteristic π-μ decay. In order to be counted the meson had to decay in the third scintillator, after it had come to rest. The energy of the pions detected was thereby determined by the amount of absorber placed in front of the telescope. Two sets of scintillation counters were used in order to measure two energy bands simultaneously.

2. Synchrotron Arrangement II

This experimental arrangement, shown in Fig. 2, was identical to that described above, except the mesons were deflected by a magnetic field before they reached the detector. The magnet served to reduce the electron background so as to permit detection of low-energy mesons.

3. Cyclotron Arrangement

The external scattered proton beam was used as shown in Fig. 3. The protons were scattered into the magnetic deflection channel and then into an evacuated tube in which they traveled through the focusing magnet and into the cave. The beam passed through an ionization chamber and then the target. The detector arrangement was similar to that described in Synchrotron Arrangement I except that only one set of counters was used.

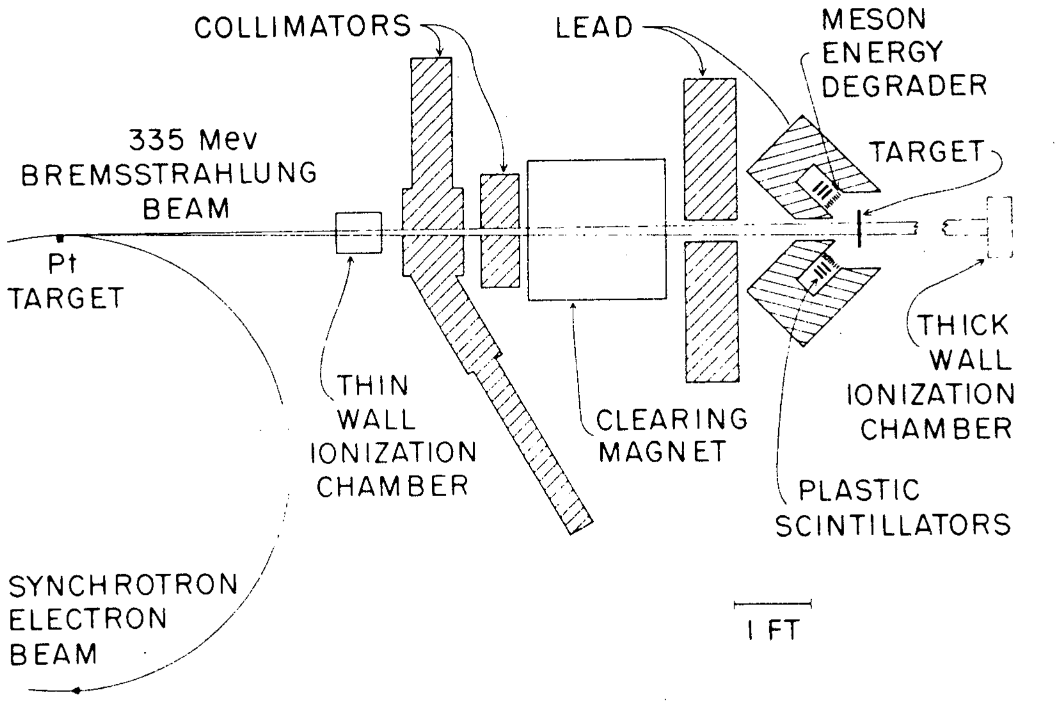

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Fig. 1. Synchrotron Arrangement I.

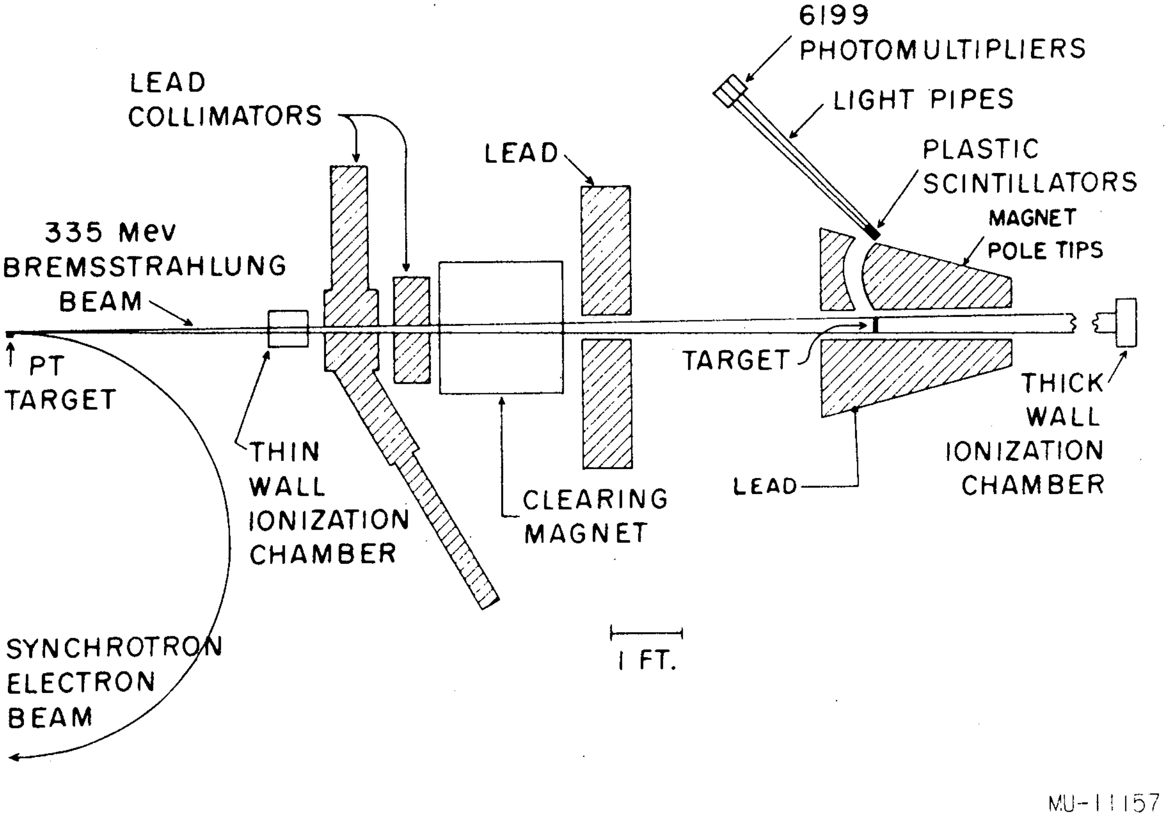


Fig. 2. Synchrotron Arrangement II.

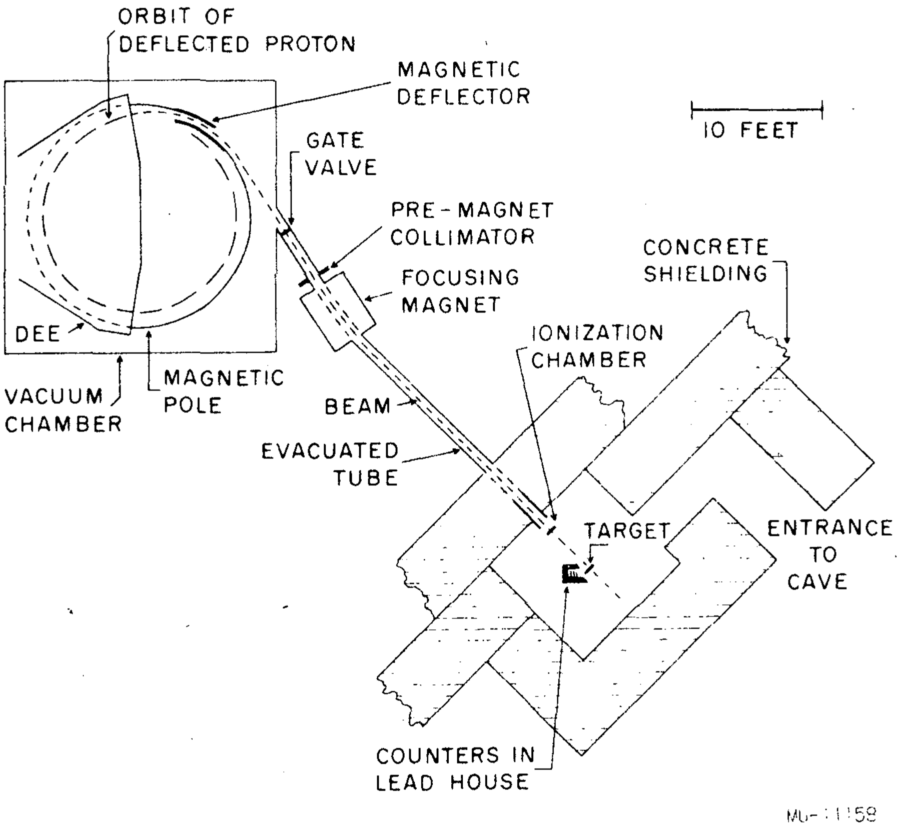


Fig. 3. Cyclotron Arrangement.

B. Pion Detection

1. Electronics

Detection of positive pions on the basis of their π^+-μ^+ decay was first demonstrated by Jakobson, Schulz, and Steinberger.[17] Identification of the pions was based on two characteristic properties of the decay scheme. The muon receives an energy of 4.1 Mev[18] and the mean life of the pion is 2.54×10^{-8} second.[17] The detection scheme required a coincidence between the muon signal and a properly delayed signal from the pion. A similar detection system was used for this experiment.

A block diagram of the electronics is shown in Fig. 4. Whenever a charged particle of sufficient dE/dx passes through the front two scintillators a coincidence is made which triggers a gate generator. This gate is 6×10^{-8} sec long and is made to arrive at another coincidence circuit, "coinc. 1," with a 3×10^{-8}-sec delay. Then particles going through all three scintillators give no count in "coinc. 1." But for a π^+ meson that comes to rest in the third crystal and decays into a μ^+ meson at the same time the gate is on, a coincidence is recorded. In order to take proper account of accidental delayed coincidences an identical gate is sent to "coinc. 2" 11×10^{-8} sec later. At this long delay virtually no mesons are counted. Therefore, this gate serves as a suitable monitor of the accidentals.

The distributed amplifiers and coincidence circuit used in conjunction with scintillators No. 1 and No. 2 are shown in Fig. 5. They were built by Jakobson, Schulz, Hamlin, and Merritt, patterned after the original design of Wiegand.[19] The distributed amplifiers have a gain of about 40 and a bandwidth of about 175 megacycles. With input signals of 0.3 volt the output of the distributed coincidence circuit is 6 volts. This signal is then inverted by passing it through a pulse transformer (type PCA 111-0.1) and is sent to the gate generator shown in Fig. 6.

The gate generator was designed and built in cooperation with V. Perez-Mendez and R. Kalibjian. The rise time of the gate is about 5×10^{-9} second and its amplitude is 5 volts. Its duration is determined by the length of the clipping delay line and was set at 6×10^{-8} second.

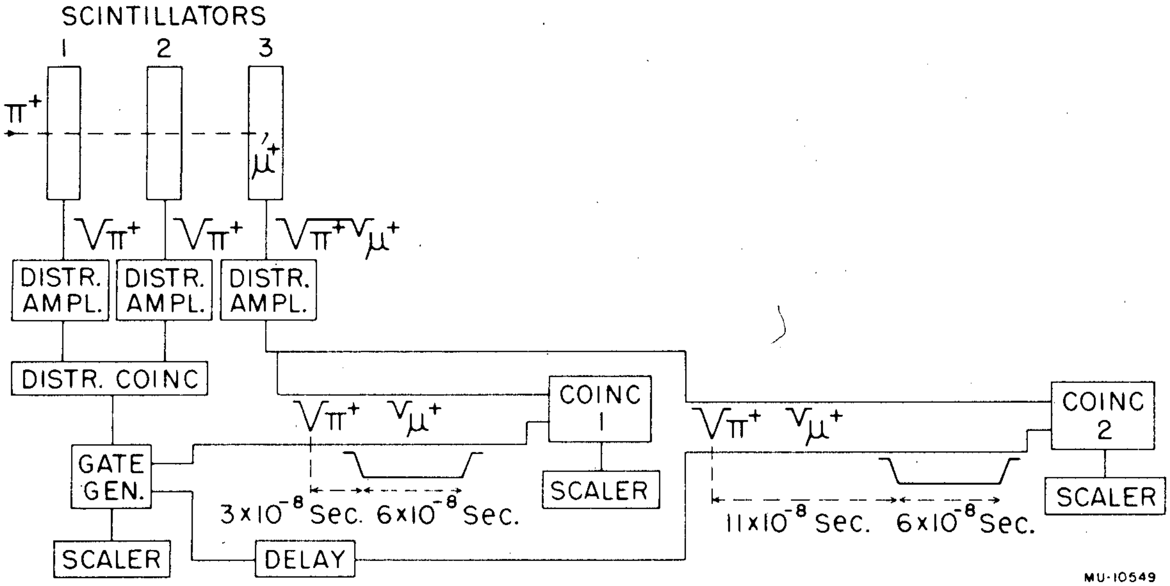
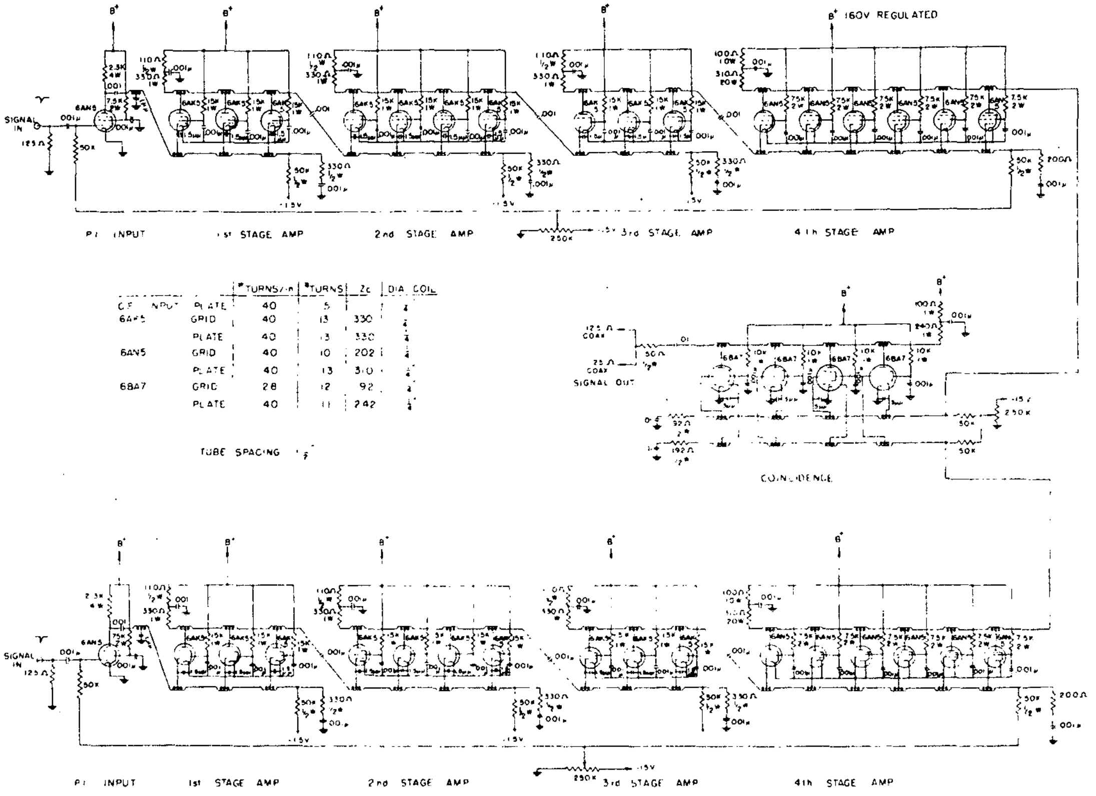


Fig. 4. Block diagram of the electronics.



		# TURNS/in	# TURNS	Zc	DIA. COIL
C.F. INPUT	PLATE	40	5		¼
6AK5	GRID	40	13	330	¼
	PLATE	40	13	330	¼
6AN5	GRID	40	10	202	¼
	PLATE	40	13	310	¼
6BA7	GRID	28	12	92	¼
	PLATE	40	11	242	¼

TUBE SPACING ½"

Fig. 5. Circuit diagram for distributed amplifiers and coincidence circuit.

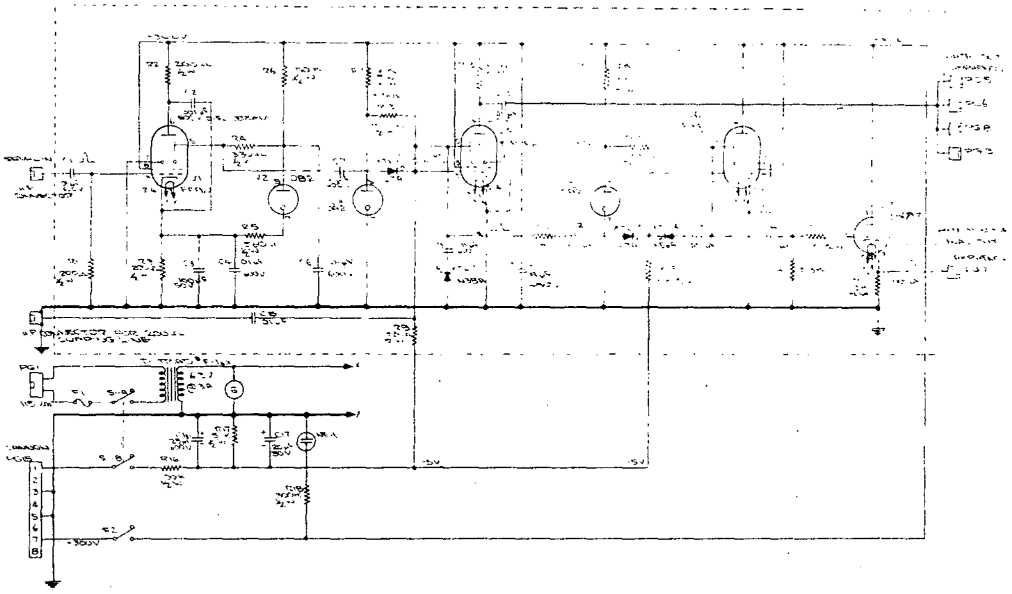


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Fig. 6. Circuit diagram for gate generator.

This gate is then sent to two different coincidence circuits through separate RG 63/U cables of the appropriate length. This arrangement is an improvement over the delayed-coincidence equipment of Jakobson, Schulz, Hamilin, and Merritt[20] in that identical gates are sent to the two coincidence circuits, as contrasted with the earlier practice of having separate gate generators.

Signals from the third scintillator pass through a Hewlett-Packard distributed amplifier type 460A. These amplifiers have a maximum gain of 10 and a bandwidth of 200 megacycles. The muon signals are thereby amplified from about 0.4 volt to about 4 volts and are sent to one input of each of the two coincidence circuits.

The coincidence circuits, shown in Fig. 7, are similar to the crystal diode coincidence circuit described by Madey, Bandtel, and Frank.[21] Optimum operation was obtained for input signals of the order of 2 to 5 volts, where the singles-to-doubles ratio was about 12. Owing to the limiting action of the distributed amplifiers, singles were never able to feed through. The resolving time as normally used was measured as 4 or 5×10^{-9} second. The discrimination level was varied by adjusting one of the grid biases of a 5687 as shown in Fig. 7.

The discrimination levels of the two coincidence circuits were balanced by feeding pulses from a 100-kilocycle pulser into the gate generator and placing a Ru^{106}-Rh^{106} source near the third scintillator. At the cyclotron this balance was then checked under normal operating conditions by increasing the beam intensity by about a factor of 10, in which case the accidentals far outnumbered the meson counts. "Coinc. No. 1" and "coinc. No. 2" should then have the same counting rate except for a negligible number of mesons. At the synchrotron it was in general not possible to increase the beam intensity over the normal running level. Balance of the coincidence circuits was checked by using a thicker target to get more accidentals or by using the 100-kc pulser to generate an abundance of gates that would occur at a random time relative to the beam and hence should yield only accidental counts in "coinc. No. 1" and "coinc. No. 2."

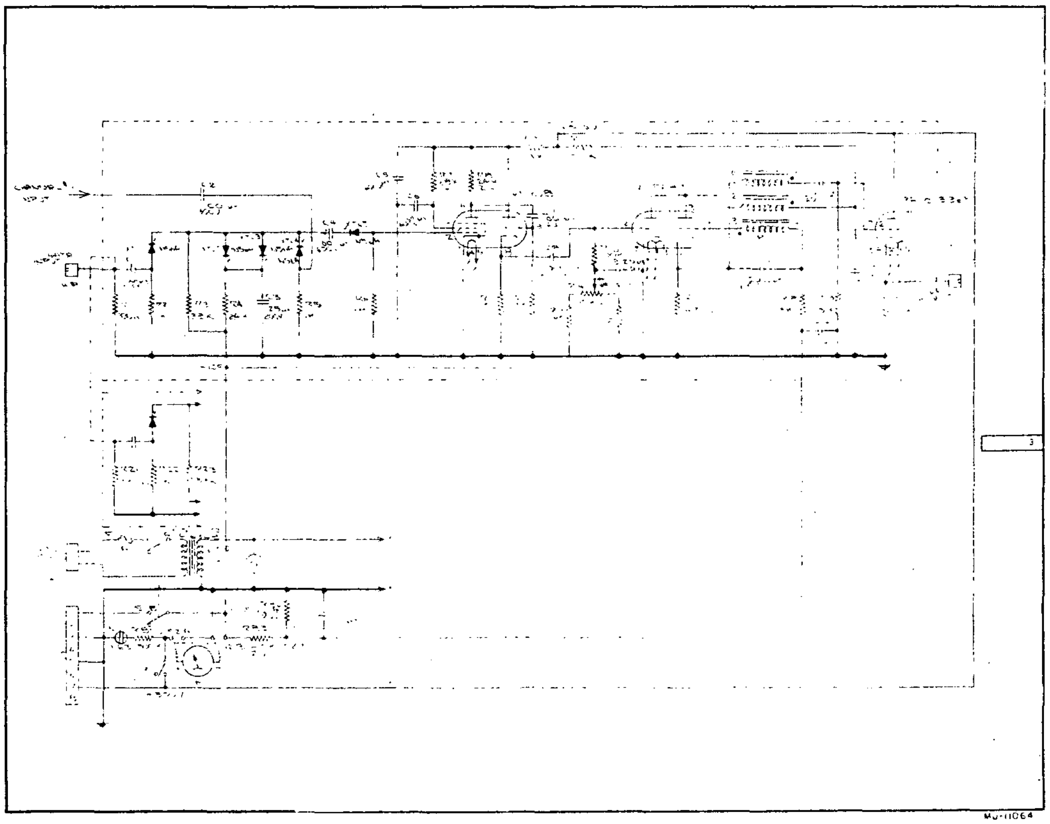

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Fig. 7. Circuit diagram for crystal diode coincidence circuits.

Drift in the relative counting rates of the two coincidence circuits generally amounted to less than 5% over a period of weeks. Such long-term stability was necessary at the synchrotron where runs typically lasted for several weeks. Drift in the absolute counting rate of the coincidence circuits over a period of weeks amounted to less than the change in counting rate brought on by raising one of the photomultiplier voltages about 25 volts. This change in voltage was small compared to the width of the plateaus.

2. Scintillation Counters

One telescope consisted of three plastic scintillators measuring 3 by 3 by 0.5 inches. Each scintillator was viewed by two 1P21 photomultiplier tubes at the end of 1-inch-long plexiglas light pipes placed on opposite sides of the scintillaor.

Another telescope consisted of four plastic scintillators, all 3 by 3 inches. The front two were each 0.25 in. thick, while each of the rear ones, in which the π-μ decays were detected, was 0.5 in. thick. Each scintillator was viewed by one 6199 photomultiplier tube located at the end of a 2-in.-long plexiglas light pipe.

The third set of counters was designed for use in a magnetic field. The front two scintillators, measuring 3 by 3 by 0.25 in., and the third scintillator, measuring 3 by 3 by 0.5 in., were all viewed by 6199 photomultipliers at the ends of plexiglas light pipes 30 in. long. In order to get the meson yield at 16.5 ± 4.5 Mev, π-μ decays were observed in the second scintillator. For the 12 ± 6-Mev point, only the first scintillator was used both to trigger the gate and to make π-μ coincidences.

3. Validity of Detection

The following evidence is presented to indicate that the counters were detecting positive pions properly:

(a) A plateau was observed for the meson counting rate as a function of voltage on the photomultiplier of the last scintillator, as shown in Fig. 8. This plateau corresponded to an energy loss of about 4 Mev in the scintillator as calibrated with the 3.55-Mev beta spectrum from a Ru^{106}-Rh^{106} source.

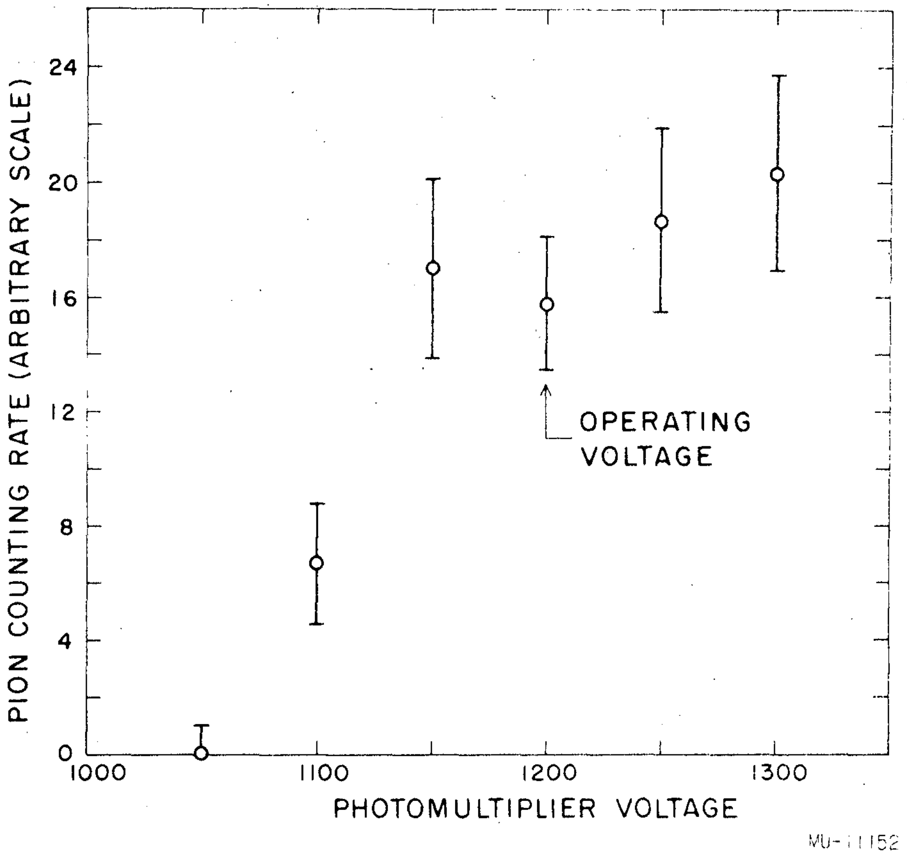

Fig. 8. Voltage plateau for photomultiplier on third scintillator.

(b) A plateau was observed for the meson counting rate as a function of voltage on the photomultipliers of the front two scintillators. This plateau, as seen in Fig. 9, started at a voltage lower than that required to count minimum-ionizing cosmic rays, corresponding to π^+ mesons stopping in the third scintillator.

(c) The measured mean life, shown in Fig. 10, agreed within statistics with the value of 2.54×10^{-8} second measured by Jakobson, Schulz, and Steinberger.[17]

(d) The maximum energy of the synchrotron was lowered below meson threshold and there was no significant meson counting rate.

(e) When the magnetic field was reversed in polarity no mesons were counted.

(f) The absolute cross section for π^+ photomeson production from H at 135° agreed within the experimental accuracy with the value given by White, Jakobson, and Schulz.[22] The absolute cross section for π^+ production at 90° from 340-Mev protons on carbon agreed within the accuracy of the experiment with the values of Dudziak.[10]

C. Targets

The targets were all considerably larger in cross-sectional area than the x-ray or proton beam, which were both about 1.5 in. in diameter at the target. For meson energies above 30 Mev the targets were of thickness between 0.5 g/cm^2 and 1.75 g/cm^2 in the direction of the beam. For the lower-energy points the targets were all less than 1 g/cm^2 thick.

The boron targets were in the form of compressed powder held in place by aluminum foils 0.002 in. in thickness. A similar container was necessary for the lithium, to protect it from the air. Duplicate empty containers were made to get the background counting rate. For the other targets no containers were necessary.

Spectroscopic and chemical analysis of the targets revealed that none had impurities amounting to more than 1%. However, the sample of boron enriched in B^{10} contained about 19% B^{11}.

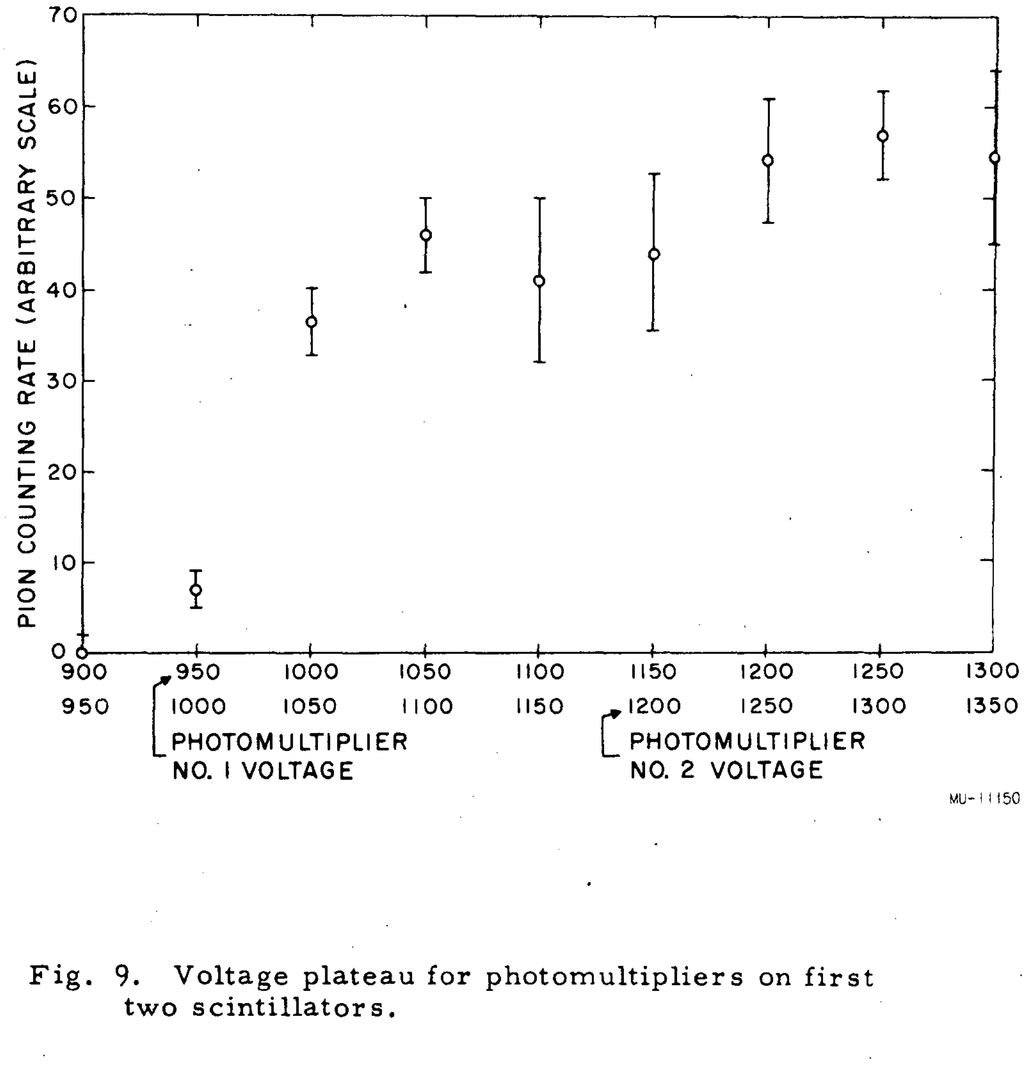

Fig. 9. Voltage plateau for photomultipliers on first two scintillators.

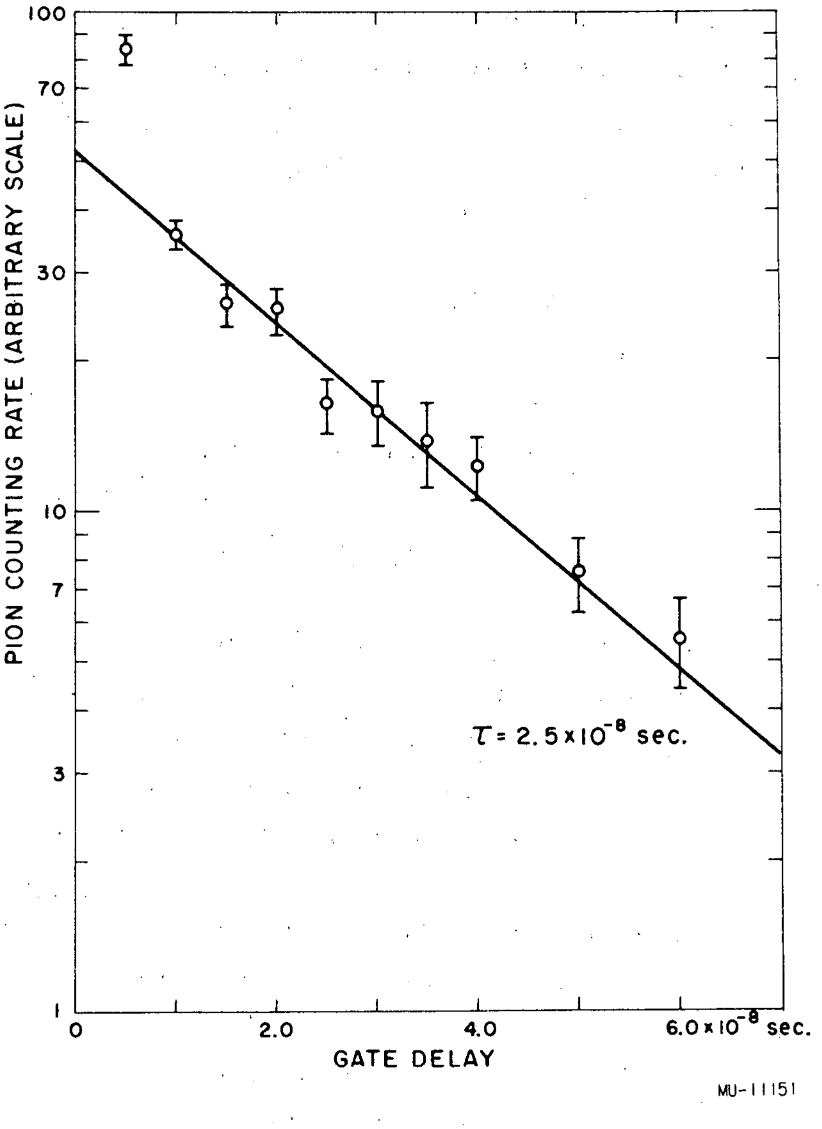

Fig. 10. Pion mean life curve.

Meson production from hydrogen was obtained by using a CH_2-C subtraction technique.

D. Channels

1. Channel with No Magnetic Field

This channel was nothing more than a lead house surrounding the counters with an opening on one side. The angle of acceptance, of total half width 7°, was determined solely by the dectectors.

2. Channel with Magnetic Field

The channel used in conjunction with a magnetic field was designed to accept mesons coming from the target at $135^{\circ} \pm 18^{\circ}$ to the beam line. The channel had a 10-in. radius and the field was set at a value to allow mesons of the proper energy to stop in the detector. The energy resolution of the channel was much wider than that of the counters. This fact was checked experimentally by measuring the variation of meson counting rate with field strength for a given energy of acceptance of the telescope.

E. Collimators

1. At the Synchrotron

The primary collimator was a lead piece 9.5 in. long with a tapered hole 0.75 in. in diameter at the small end toward the synchrotron. This collimator was located 56 in. from the synchrotron target. The two additional collimators had holes somewhat larger in diameter than the beam. At the target the beam was about 1.5 in. in diameter.

2. At the Cyclotron

After leaving the magnetic deflector the protons passed through the premagnet collimator. Further collimation was carried out in a 48-in.-long brass collimator leading to the cave. The beam size at the target was approximately 1 by 1.5 in.

F. Beam Monitors

1. At the Synchrotron

The beam was monitored with a thick-walled ionization chamber of the type used at Cornell. A thin-walled chamber was also used in front of the primary collimator, but did not track very well with the thick chamber, as the front monitor apparently was quite sensitive to low-energy electrons and hence depended considerably on the operating conditions of the synchrotron. The absolute calibration of the Cornell-type chamber was checked by the method of Blocker, Kenney, and Panofsky[23] and was also checked at Illinois with a calorimeter.[24] All the methods agreed within 10% and, since only relative cross sections were measured in this experiment, the absolute calibration of the beam monitor introduced no error.

2. At the Cyclotron

A parallel-plate ionization chamber filled with one atmosphere of argon was used to monitor the beam. Here again the absolute calibration of the monitor introduced no error.

G. Experimental Procedure

The procedure at the beginning of a run was to first run plateaus and select the optimum voltages. The counting rates at these voltages with a Ru^{106}-Rh^{106} source in a certain geometry were measured for further checking throughout the run. The targets were then cycled, each one remaining in the beam from 30 to 60 minutes at a time. Generally, for a given meson energy of detection, cycling of the whole set of targets was repeated 10 to 30 times.

III. RESULTS OF THE EXPERIMENT

A. Corrections

Only corrections involving the targets were considered, since other effects would merely change the detection efficiency by the same factor for all targets, and only the A dependence at each meson energy was of interest. The following effects were considered and corrections were made for the first four:

1. Interaction of incident gammas or protons in target.
2. Nuclear interaction of emitted pions in target.
3. Variation with target of energy resolution of detector.
4. Variation with target of average energy of detection.
5. Other sources of error.

1. Interaction of Incident Gammas or Protons in Target

The bremsstrahlung beam was monitored with ionization chambers placed both in front of and behind the target. Although the front monitor was not too reliable it did give some indication of the incident gamma flux. The average amount of flux in the target could then easily be estimated from the known attenuation of 150- to 335-Mev gammas.[25] The corrections were small enough, never exceeding 6%, so that it was unnecessary to know the excitation function very well.

The nuclear absorption of the proton beam in each target was estimated to be about 1.5%;[26] the difference between the targets was considerably less. The energy degeneration of the protons in passing through the targets was between 2.5 Mev and 4.5 Mev. On the basis of the excitation function for C at 90° given by Hamlin[27] there would be a maximum difference of 1.3% in the pion-production efficiency due to this effect.

2. Nuclear Interaction of Emitted Pions in the Target

Measurements[16] on attenuation cross sections indicate that the total attenuation cross section is geometric or less. A geometrical cross section gives an average loss of mesons in the targets of 1.9% or less.

3. Variation with Target of the Energy Resolution of the Detector

The energy resolution of the detector is determined by the minimum and maximum range a meson may have and still be counted. The width of the energy band of detection depends somewhat on the target material the mesons must pass through. This gives a maximum difference in the detection efficiency for the various targets of 5.2% for all meson energies except 12 ± 6 Mev, in which the maximum difference is 7.5%.

4. Variation with Target of the Average Energy of Detection

The average energy of detection also depends on the target material the mesons must pass through. The maximum difference between targets in the average energy of detection amounts to 3.4 Mev or less. This energy difference was converted to a difference in detection efficiency by using the known pion energy spectrum from carbon at this angle.[28] This effect gives a maximum correction factor of 5.2% for all energies except 12 ± 6 Mev, which requires a maximum correction of 9.9%.

5. Other Sources of Error

The sources of error in addition to those inherent in the statistics were:

(a) Drift in the counting efficiency. The sensitivity of the electronics was regularly checked with radioactive sources. On the basis of these checks it is estimated that the meson-detection efficiency did not vary by more than 5% during a run. The actual error introduced by this drift was probably much less than 5%, since for each meson energy the targets were cycled from ten to thirty times.

(b) Fluctuations in the synchrotron beam. The maximum energy and the relative fallout time of the circulating electrons in the synchrotron were subject to drift. If the reaction being studied was produced only by a narrow energy band of gammas it would have merely been necessary to monitor gammas of such energy with a pair spectrometer, and then these drifts would be of no consequence. But pions of a particular energy may be produced from a complex nucleus by a very wide energy band of x-rays. For example, a kinematical calculation made on the assumption of a maximum internal proton momentum of 195 Mev/c shows that 46-Mev mesons were produced at 135° by gammas of energy between 200 Mev and 335 Mev. An ideal monitor would therefore have had this same region of sensitivity. The "Cornell" chamber was calibrated at Cornell and found to have a sensitivity of 3.74×10^{18} Mev/coulomb for 315-Mev bremsstrahlung, and was calibrated at California Institute of Technology and found to be 4.13×10^{18} Mev/coulomb for 500-Mev bremsstrahlung.[29, 30] The Cornell chamber with a sensitivity of 5.3% per 100 Mev was a much better approximation to an ideal monitor for this experiment than a pair spectrometer. The error introduced by fluctuations in the beam was further reduced by cycling the targets many times and was believed to be less than 5%.

(c) Chemical purity of the targets. All the targets were analyzed spectroscopically, and some of them chemically, for impurities. The total amount of impurities in any target never exceeded 1%.

(d) Measurement of target thickness. The thickness of the targets was measured within 1% or 2%.

(e) Differences in effective target shape and position. Since the targets did not all have the same linear thickness their effective shapes were different. In all cases, however, the centers of the targets were placed in as nearly the same position as possible. These differences in the geometry of the various targets could have given a maximum difference in counting rate of 2%.

B. Final Results

1. Summary of Data

Table II gives the final data after the corrections were applied to the raw data. The errors shown are the standard deviations due to the counting statistics. These results are shown graphically in Figs. 11-16.

2. Apparent Mean Free Path

In Fig. 11 curves for the A dependence of the meson production efficiency as given by Brueckner, Serber, and Watson[15] are superimposed on the synchrotron data. The curves are plots of the

Table II

Relative Yield of Positive Pions per Proton in the Nucleus at 135°

Incid. Beam	335-Mev Bremsstrahlung							340-Mev Protons	
T_π / Target	12 ±6 Mev	16.5 ±4.5 Mev	22.5 ±6.0 Mev	30 ±4.5 Mev	36 ±4 Mev	46 ±4 Mev	65 ±4 Mev	36 ±4 Mev	63 ±4 Mev
H	2.607 ±.758	2.457 ±.356	1.255 ±.268		1.731 ±.359	1.922 ±.156	1.952 ±.419		
Li						1.210 ±.114		0.858 ±.077	0.569 ±.272
Be						1.144 ±.060	1.174 ±.102	0.892 ±.039	0.568 ±.089
B^{10}						1.144 ±.054	1.210 ±.108	0.971 ±.048	0.826 ±.112
B^{11}						1.204 ±.054	1.138 ±.108	1.034 ±.051	1.061 ±.118
C	1.000 ±.091	1.000 ±.064	1.000 ±.038	1.000 ±.046	1.000 ±.025	1.000 ±.037	1.000 ±.031	1.000 ±.031	1.000 ±.104
Al	0.766 ±.115	0.977 ±.087	0.774 ±.038	0.916 ±.047	0.890 ±.025	0.891 ±.037	0.858 ±.029	0.950 ±.054	
Cu	0.678 ±.103	0.761 ±.077	0.666 ±.038	0.790 ±.045	0.739 ±.025	0.760 ±.035	0.640 ±.028	0.745 ±.043	
Ag	0.447 ±.077	0.445 ±.053	0.512 ±.059	0.604 ±.060	0.663 ±.032	0.649 ±.037	0.611 ±.029	0.453 ±.033	
Pb	0.310 ±.062	0.281 ±.042	0.297 ±.037	0.311 ±.092	0.431 ±.061	0.486 ±.034	0.424 ±.036	0.321 ±.027	

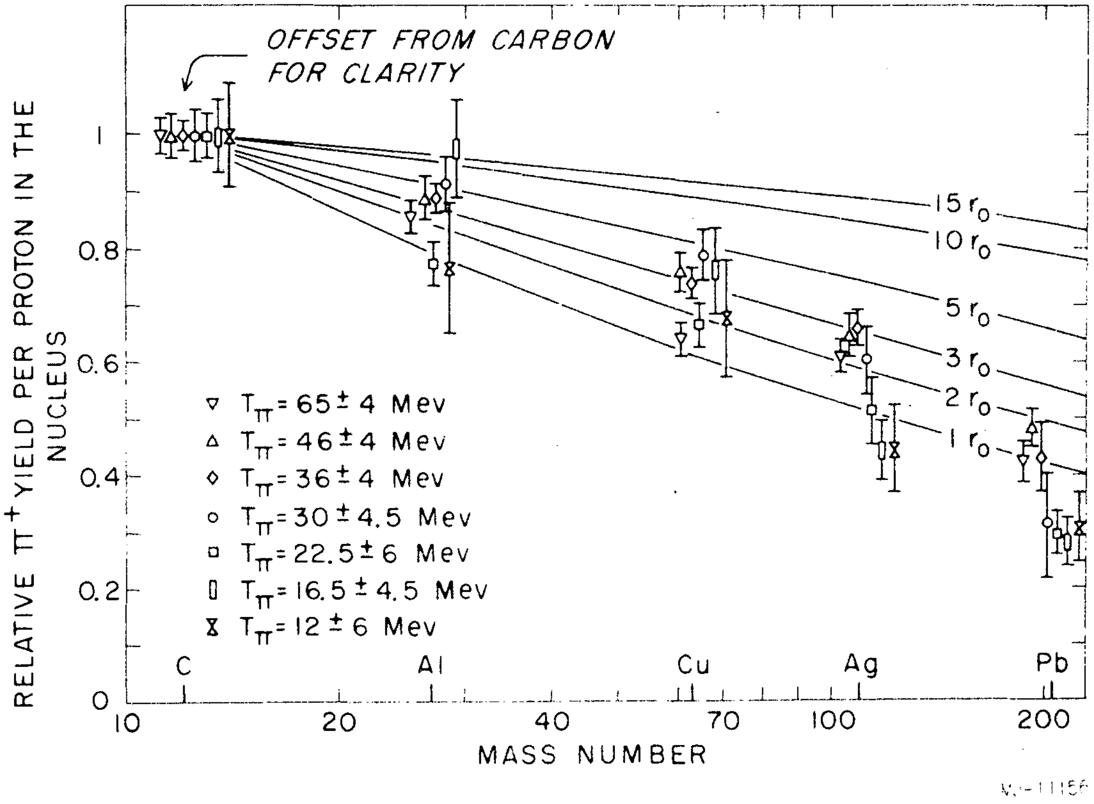

Fig. 11. Relative π^+ yield per proton in the nucleus for 335-Mev bremsstrahlung bombardment superimposed on the curves of Brueckner, Serber, and Watson. At each pion energy the relative yields are normalized to a C value of 1.00.

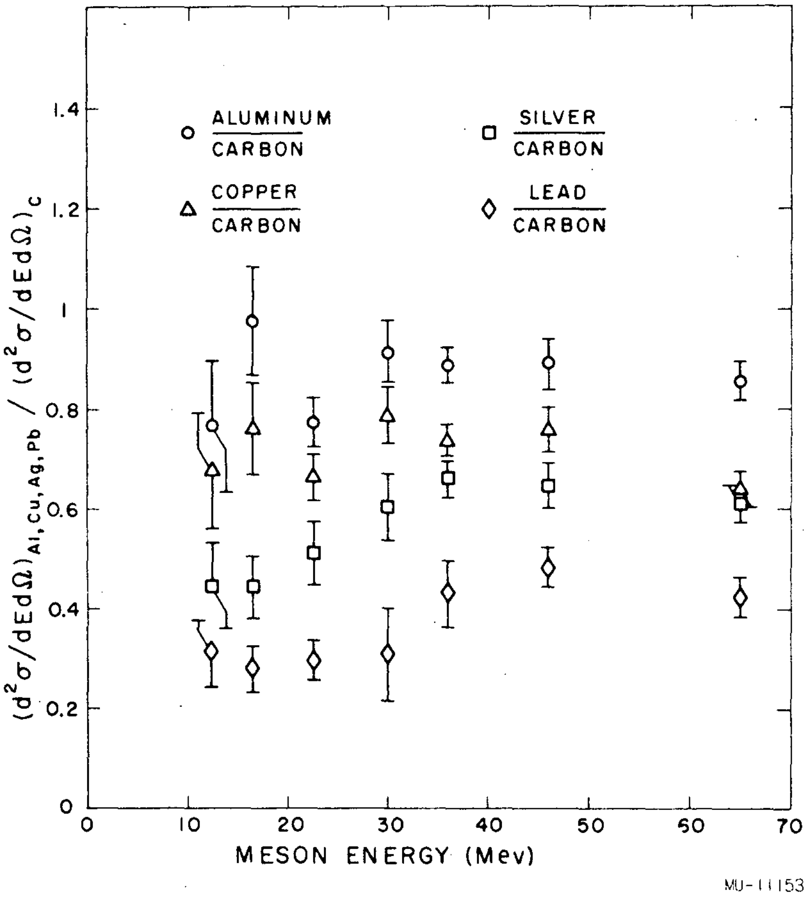

Fig. 12. Relative π^+ photomeson yield per proton in the nucleus from Al, Cu, Ag, and Pb relative to C as a function of pion energy.

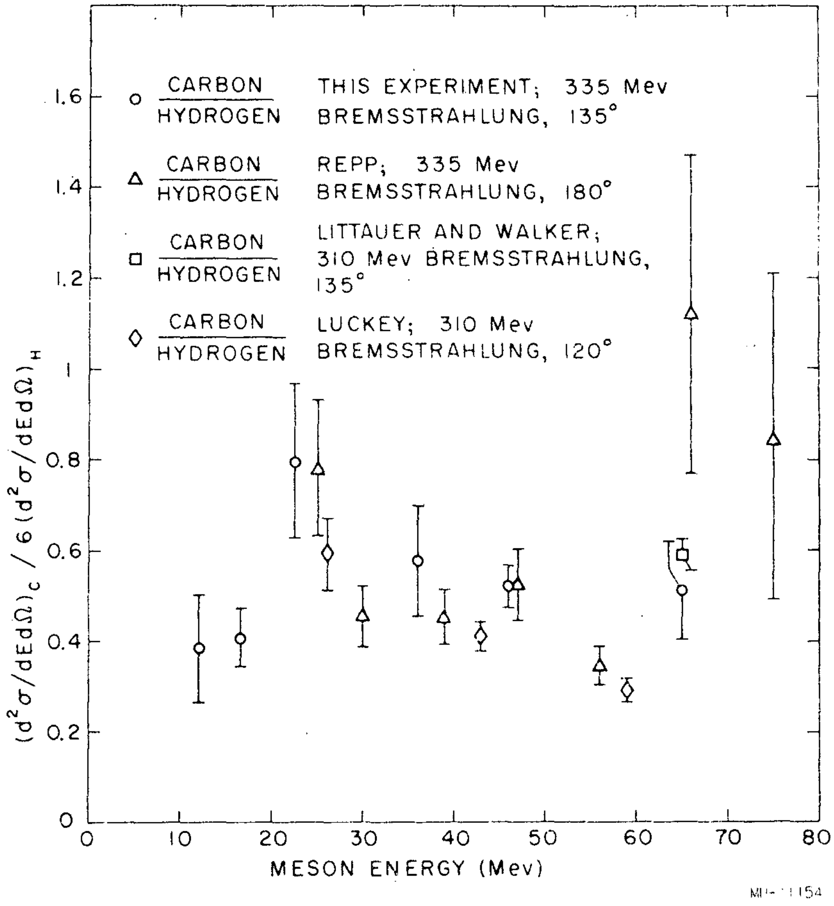

Fig. 13. Ratios of π^+ photomeson production efficiency from carbon relative to hydrogen.

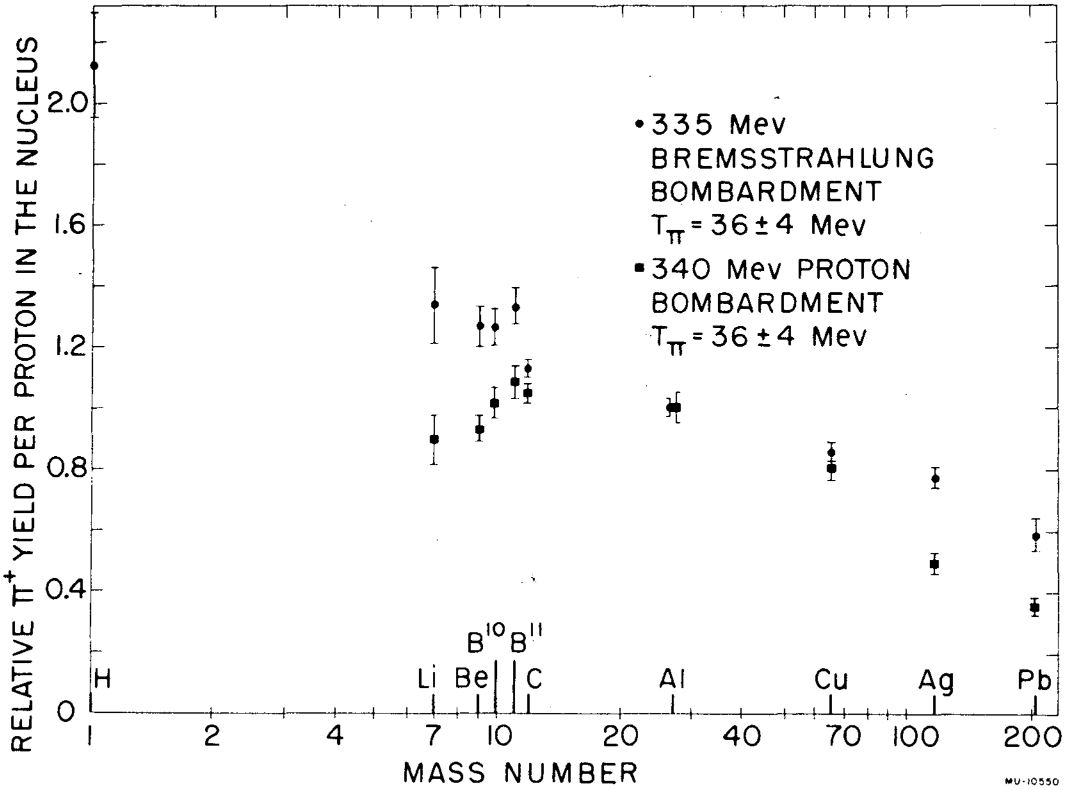

Fig. 14. Relative π^+ yield at 135° produced by 335-Mev bremsstrahlung and 340-Mev protons. In both cases the kinetic energy of the mesons is 36 ± 4 Mev, except for the pions produced from H through B^{11} by 335-Mev x-rays. Their energy is 46 ± 4 Mev.

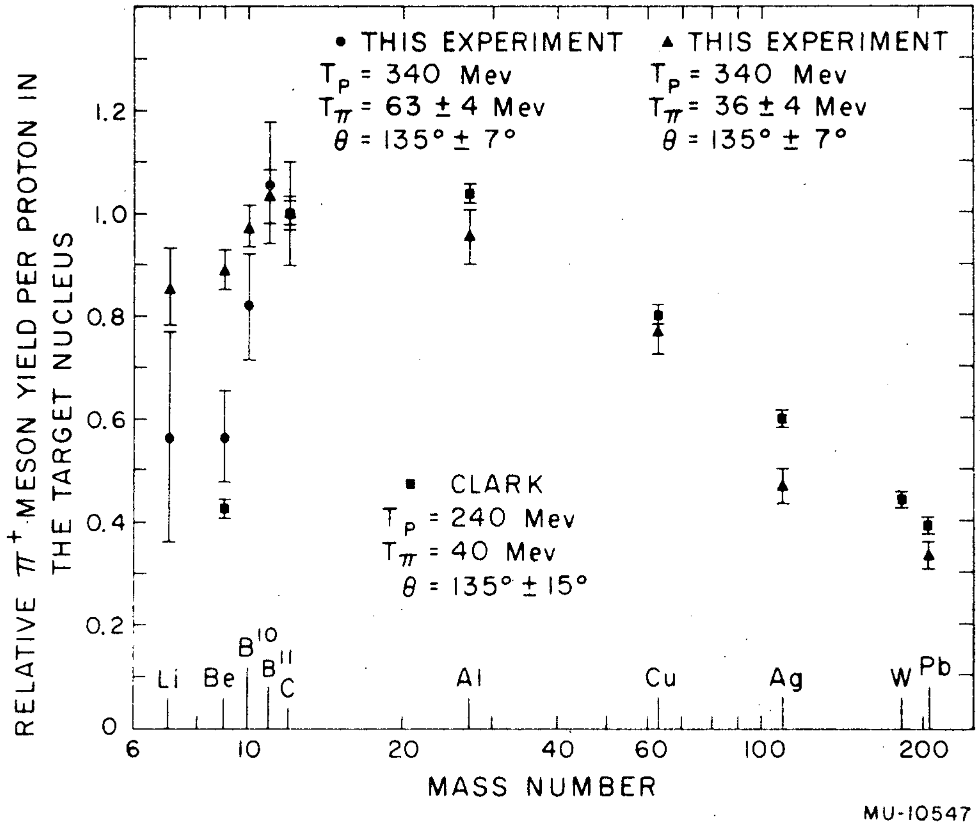

Fig. 15. Relative π^+ meson yield per proton in the target nucleus versus mass number. The data of this experiment at two different pion energies and the data of Clark are shown.

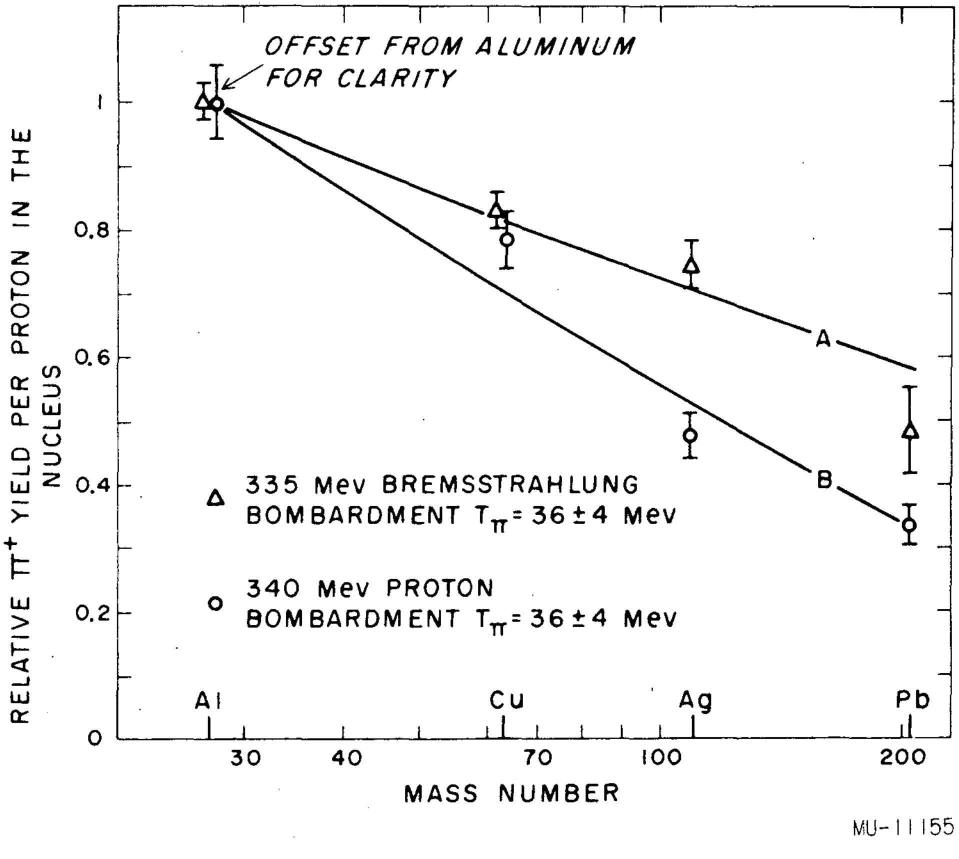

Fig. 16. Relative π^+ yield per proton in the nucleus at 135° produced by 335-Mev bremsstrahlung and 340-Mev protons. In both cases the kinetic energy of the mesons is 36 ± 4 Mev. Superimposed on the experimental points, which are normalized to an Al value of 1.00, are two theoretical curves: curve A = expression by Brueckner, Serber, and Watson for $\lambda_\pi = 2r_0$; curve B = expression by Gasiorowicz for $\lambda_\pi = 2r_0$.

equation

$$f_a = 3\,[1/2x - 1/x^3 + (1/x^3)(1+x)e^{-x}],$$

where $x = 2R/\lambda$, $R=r_0A^{1/3}$=radius of nucleus, and λ=mean free path for absorption of mesons in nuclear matter. One can see that the apparent mean free path for positive pions as given by this model is less than $3r_0$ at all energies. Such as interpretation neglects the possibility of other effects discussed more fully in Section IV.

3. Interaction of Incident Protons

Comparison of the synchrotron and cyclotron data for 36-Mev mesons at 135° indicates that the pion-production efficiency drops off faster with increasing A under proton bombardment than under gamma bombardment. It has been pointed out by Gasiorowicz[11] that in the former case meson production may be possible only in the "front" of the struck nucleus, owing to the energy degeneration of the incident protons in nuclear matter. Assuming only protons in the target nucleus are effective in producing positive mesons,[31] Gasiorowicz obtains the following expression for the production cross section for 340-Mev protons:

$$\sigma = \sigma_0(Z_1 + 1/2\, Z_2),$$

where $Z_1(\lambda_s)/Z=(R/P)^3\,\{1/2(P/R)^2\,[x/R+\lambda_s/R - 1] + 1/2(x/R)[(\lambda_s/R)-1]$

$$+\, 1/2(P/R)^3 - 1/4(x/R)^2(\lambda_s/R)\,\},$$

and $\quad (Z_1+Z_2)/Z=Z_1(2\lambda_s)/Z,$

where $(x/R) = [(P/R)^2 - (1-\lambda_s/R)^2](R/2\lambda_s)$.

Here R=nuclear radius, P=radius of "charge distribution"=0.90R, and λ_s=proton mean free path for elastic scattering. To get the yield this expression should be multiplied by e^{-d/λ_a}, where λ_a=mean free path for absorption of mesons, and d=average distance traversed by the mesons in the nucleus, taken to be (1/3)R in this case. Figure 16 shows a plot of this expression for $\lambda_a=2r_0$. The presence of the Coulomb barrier has been neglected in this calculation. On the same graph is shown the yield curve predicted by Brueckner, Serber, and Watson for $\lambda_a=2r_0$, as well as the pertinent data.

IV. DISCUSSION

A. Internal Momentum

A striking difference between gamma and proton bombardment occurs for elements below aluminum, as seen in Fig. 14. Here the two curves slope in opposite directions, presumably an effect of internal momentum. With proton bombardment the production of mesons from hydrogen at this angle and energy is kinematically impossible. As one goes to heavier nuclei the internal momentum makes it possible to have such production. However, at the synchrotron such meson production from hydrogen is quite possible kinematically. Then, going to complex nuclei, the internal momentum and exclusion principle cause a drop-off in the production efficiency, as shown by Lax and Feshbach.[32]

Evidence is now given to show that with proton bombardment, under the kinematical conditions of this experiment, the meson yield should be quite sensitive to the internal proton momentum distribution. Henley[33] has shown that for an incident beam of 340 Mev protons the meson spectrum from a complex nucleus at 90° depends strongly on the nucleon momentum distribution. This dependence becomes more pronounced at the high-energy end of the spectrum. At 0°, however, he found the dependence on the internal momentum distribution to be much less.

An analysis at 180° by Leonard[34] showed a considerable difference between the pion spectrum predicted by a Gaussian momentum distribution and that predicted by a Chew-Goldberger momentum distribution, the disagreement between the two models becoming quite pronounced at meson energies above 30 Mev for 340-Mev incident protons. It seems reasonable, then, that the meson production studied in this experiment depended considerably on the distribution of momentum of protons in the target nuclei.

This prediction can be made plausible by the following argument. At 0° the meson spectrum in proton-proton collisions is cut off at about 70 Mev by energy-momentum conservation. On the other hand, at 135° the cutoff energy is about 3 Mev. The momentum distribution is needed to broaden the spectrum to the pion energy values observed.

In Table III, opposite each experimental condition, is given the minimum momentum required by the struck nucleon, P_{min}, in order to make the reaction possible. It was assumed that in the proton-proton collision the final nucleons form a deuteron. Such a deuteron-forming reaction accounts for about 70% of the π^+'s produced in P-P collisions.[35] The analysis was carried out for two different models of the kinematic situation. For the "impulse approximation," only the two interacting nucleons were considered in balancing momentum and energy. For the model used by Neher,[44] the residual nucleus was assumed to carry off kinetic energy. Here 20 Mev had to be supplied to cover 8 Mev binding energy for the removed proton and 12 Mev kinetic energy for the residual nucleus. The kinetic energy of the struck nucleon did not contribute to the energy of the final products.

Table III

Minimum momentum P_{min} required by struck nucleon for different experimental conditions.

Case	Angle	Pion Energy (Mev)	Proton Energy (Mev)	Impulse-Approximation P_{min} (Mev/c)	Neher P_{min} (Mev/c)
I	135°	36	340	114	184
II	135°	63	340	186	291
III	140°	40	240	216	391

From Fig. 15 it is seen that the slopes of the production efficiency below carbon increase progressively from case I to case III. Likewise the minimum internal momentum required for the reaction increases in going from case I to case III. In other words the higher the value of P_{min} the greater is the variation of meson yield with atomic number. This suggests an increase with atomic number of the occurrence of high-momentum components in the momentum distribution. The data seem to qualitatively indicate that the protons in B^{11} have a larger momentum than in B^{10}.

Wilcox and Moyer[36] found, by bombarding light nuclei with 340-Mev protons and detecting two emerging protons in coincidence, that the protons in beryllium have a larger momentum than the protons in lithium. Beryllium and boron were seen to be rather similar.

B. Mean-Free-Path Interpretation

Figure 11 shows that there is little variation of the A dependence with pion energy. If the simple picture of meson absorption given by Brueckner, Serber, and Watson[15] is sufficient, then the mean free path for mesons in nuclear matter is less than $3r_0$ at all meson energies. Such a mean free path is inconsistent with estimates based on meson-nucleon and meson-nucleus scattering.[16, 37]

The expression of Brueckner, Serber, and Watson, Eq. (1), assumes that all the meson interactions are of an absorption nature and that the pions are not scattered. Scattering would increase the average path length of the pions before leaving the nucleus and thus alter the apparent mean free path for interaction as obtained from Eq. (1).

The mean free path for meson interaction, including scattering, has been estimated by Stork[16] and be Tenney and Tinlot.[37] Stork obtained the mean free paths by three different means. He analyzed the pion-nucleus interaction data by a partial wave analysis and the optical model. Multiple-scattering theory was used to determine the mean free paths from pion-nucleon scattering phase shifts. The optical model gave the shortest mean free paths, partly because it neglects reflection and refraction at the nuclear boundary. The minimum values of the mean free path so obtained were greater than $10r_0$ for pions of energy less than about 33 Mev. This result is also in agreement with the values of Tenney and Tinlot.

For $\lambda=10r_0$ Eq. (1) predicts that 84.6% of the produced pions escape from a carbon nucleus and 66.8% leave a lead nucleus without any interaction. If one assumes that the mesons are produced uniformly throughout the nucleus and that they are emitted with an isotropic angular distribution, then less than 15.4% of the mesons observed from carbon were scattered in the nucleus. This maximum scattering would be increased to 33.2% for lead.

Therefore, if the mean free path is assumed to be $10r_0$ at 33 Mev, scattering could have no greater effect than to distort the ratio of lead to carbon production efficiency predicted by Eq. (1) from 0.790 to a minimum of 0.668. This latter ratio is still in disagreement with the experimental results. Therefore, under the above assumptions, scattering within the nucleus will not account for the discrepancy in predicted mean free paths.

However, the assumption that mesons are produced in the nucleus with an isotropic angular distribution may be a poor approximation. Although the angular distribution of photomesons from H at these energies is isotropic within about a factor of two,[42, 43] McVoy[38] estimates that in a nucleus, under the conditions of this experiment, production in the forward hemisphere may be much more likely because of momentum-space requirements. He then estimates that about half the mesons observed in this experiment at 135° and 65 Mev were initially produced at a higher energy in the forward direction and scattered backward with loss of energy before leaving the nucleus. He believes this effect is somewhat smaller at lower meson energies.

The production of low-energy mesons from heavy nuclei is affected by the Coulomb barrier.[10] The transmission of mesons through this barrier was calculated on the basis of the WKB approximation[39] for S-wave mesons. This calculation predicted, for the production from lead, a reduction of 8% for 22.5 ± 6-Mev mesons, a reduction of 23% for 16.5 ± 4.5-Mev mesons, and a reduction of about 60% for 12 ± 6-Mev mesons. Even after such corrections are made the apparent mean free path for pions is still no larger than $3r_0$ for mesons as low in energy as 16.5 Mev.

The theory has been postulated by Butler that photomesons are produced preferentially on the surface of nuclei, meson production in the nuclear core being suppressed. A mechanism for such suppression, in which the emitted meson is reabsorbed by the parent nucleon and its nearest neighbor, was proposed by Wilson.[40] The surface production model predicts roughly a $1/A^{1/3}$ production efficiency, regardless of what the mean free path for meson absorption actually is. Such a behavior is certainly consistent with all of the data presented here as well as well as the previous work by Mozley and by Littauer and Walker.

Further evidence supporting Butler's model and contradicting the optical model has recently been given by George.[41] He observed pions produced in emulsions at 90° to the 315-Mev bremsstrahlung beam of the Cornell synchrotron. The ratio of stars to mesons was 11.4:1. Predictions for this ratio were made on the basis of the optical model, in which the formula of Brueckner, Serber, and Watson given in Section I was used, as well as the Butler model. On the Butler model the nucleus was divided into two parts: a core region in which photon absorption in the energy interval 150 to 315 Mev always results in a nuclear disintegration with no free-meson production, and a surface region in which the nucleons may be treated as if they are free. The ratio of the number of nucleons in the core to those in the surface was taken as 4:1. The values of the ratio of stars to mesons based on the Butler model were close to the observed value, whereas the results based on the optical model were an order of magnitude lower.

V. ACKNOWLEDGMENTS

I wish to express my deep appreciation to Professor A. C. Helmholz for his stimulation and guidance throughout the course of my graduate work. To Dr. Victor Perez-Mendez I am highly indebted for his great help in the planning and carrying out of this experiment. Sincere thanks are given to Dr. H. T. Easterday and Mr. Edward Knapp for their contributions to the experiment.

I also wish to thank Dr. Robert Kenney, Mr. Rudin Johnson, and the synchrotron crew for their help and cooperation during the synchrotron bombardments. The efforts of Mr. James Vale and the cyclotron crew are greatly appreciated.

This work was done under the auspices of the U. S. Atomic Energy Commission.

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