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The Smallest $SU(N)$ Hadrons

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ABSTRACT: If new physics contains new, heavy strongly-interacting particles belonging to irreducible representations of $SU(3)$ different from the adjoint or the (anti)fundamental, it is a non-trivial question to calculate what is the minimum number of quarks/antiquarks/gluons needed to form a color-singlet bound state (“hadron”) with the new particle. Here, I prove that for an $SU(3)$ irreducible representation with Dynkin label (p, q) , the minimal number of quarks needed to form a product that includes the $(0,0)$ representation is $2p + q$. I generalize this result to $SU(N)$, with $N > 3$. I also calculate the minimal total number of quarks/antiquarks/gluons that, bound to a new particle in the (p, q) representation, give a color-singlet state: $n_g = \lfloor (2p + q)/3 \rfloor$ gluons, $n_{\bar{q}} = \lfloor (2p + q - 3n_g)/2 \rfloor$ antiquarks, and $n_q = 2p + q - 3n_g - 2n_{\bar{q}}$ quarks (with the exception of the $\bar{\mathbf{6}} \sim (0,2)$ and of the $\bar{\mathbf{10}} \sim (0,3)$, for which 2 and 3 quarks, respectively, are needed to form the most minimal color-less bound state). Finally, I show that the possible values of the electric charge Q_H of the smallest hadron H containing a new particle X in the (p, q) representation of $SU(3)$ and with electric charge Q_X are $-(2p + q)/3 \leq Q_H - Q_X \leq 2(2p + q)/3$.

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1 Introduction

In Quantum Chromo-Dynamics (QCD), a gauge theory with gauge group $SU(3)$ that describes the strong nuclear force in the Standard Model of particle physics, color confinement is the phenomenon that color-charged particles cannot be isolated, i.e. cannot subsist as stand-alone asymptotic states. From a group-theoretical standpoint, quarks belong to the fundamental representation of $SU(3)$, antiquarks to the antifundamental representation, and the force mediators, gluons, to the adjoint representation¹. Color-confinement can thus be stated in group-theoretic language as the phenomenon that asymptotic, physical states must belong to the singlet (trivial) representation of $SU(3)$, which I indicate below as $\mathbf{1} \sim (0, 0)$. For instance, in real life physical states of strongly-interacting particles include mesons, which are quark-antiquark states, belonging to the singlet representation resulting from $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$; and baryons, which are three-quark states, belonging to the singlet representation resulting from $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$. In addition, glueballs, bound states of two gluons, could also exist [4], since $\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{10} \oplus \bar{\mathbf{10}} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$.

Here, I am interested in which bound states would form around a hypothetical new particle X charged under $SU(3)$ and belonging to some irreducible representation of $SU(3)$ with Dynkin label (p, q) . Specifically, I address two questions: the first, simple question is how many “quarks” would be needed to form a color-less bound state, i.e. what is the minimal number of copies of the fundamental representation such that the direct product of those copies and of the (p, q) contains the trivial representation $(0, 0)$. The answer is $2p + q$: I prove this in two different ways below. I then generalize this result to $SU(N)$. Secondly, I pose the slightly less trivial question of what is the minimal number of “elementary constituents”, i.e. quarks, antiquarks and gluons, needed to form a colorless

¹In what follows, I will both use the notation \mathbf{d} to indicate a representation of dimension d and $\bar{\mathbf{d}}$ to indicate the corresponding conjugate representation, and the notation (p, q) , with p and q non-negative integers; I use the convention that the bar corresponds to representations where $q > p$. The dimension of representation (p, q) is $d = (p + 1)(q + 1)(p + q + 2)/2$. For instance, quarks belong to the irreducible representation $\mathbf{3} \sim (1, 0)$, antiquarks to $\bar{\mathbf{3}} \sim (0, 1)$, and gluons to $\mathbf{8} \sim (1, 1)$ (for details see [1, 2]; for an exhaustive review on Lie groups see e.g. [3])

bound state with the new particle X . I list the results for all $SU(3)$ representations with dimension smaller than 100. As a corollary, I show that if the new particle X , of label (p, q) , has electric charge Q_X , the “smallest hadron” H containing X has electric charge $-(2p + q)/3 \leq Q_H - Q_X \leq 2(2p + q)/3$. Also as a corollary, assuming $Q_X = 0$, I list all possible (p, q) irreducible representation such that the “smallest” hadron can be electrically neutral.

The reasons why the questions above are interesting include the fact that, at least in the real world, the “smallest” hadrons (protons, neutrons, pions) are also the lightest ones in the spectrum, and there are good reasons to believe that the same could be true for a new exotic heavy state. Limits on new strongly-interacting states imply that the mass of the X be much higher than the QCD scale Λ_{QCD} [5–7]. Thus any state containing more than one X , such as for instance the color-singlet $\bar{X}X$, would be significantly heavier than any bound state of X with quarks, antiquarks or gluons. Additionally, such exotic hadrons could be stable, and under some circumstances could even be the dark matter, or a part thereof (see e.g. [8–10]). However, charge neutrality would restrict, through the arguments made here, which irreducible representations the X could belong to.

The remainder of the paper is organized as follows: in the next section 2 I provide two proofs that the minimal product of fundamental representations of $SU(3)$ is $2p + q$ and generalize the result to $SU(N)$; in the following section 3 I calculate the composition of the smallest hadron in $SU(3)$; the final sec. 4 concludes.

2 The minimal direct product of fundamental representations of $SU(N)$ containing the trivial representation

Irreducible representations of $SU(N)$ are conveniently displayed with Young tableaux via the following rules (for more details, see e.g. [11–14]):

- (i) The fundamental representation is represented by a single box;
- (ii) Young tableaux for $SU(N)$ are left-justified $N - 1$ rows of boxes such that any row is not longer than the row above it;
- (iii) Any column with N boxes can be crossed out as it corresponds to the trivial (singlet) representation.

Any irreducible representation can be obtained from direct products of the fundamental representation; the direct product of two representations proceeds via the following rules:

- (i) Label the rows of the second representation’s tableau with indices a, b, c, \dots , e.g.
- (ii) Attach all boxes from the second to the first tableau, one at a time, following the order a, b, c, \dots , in all possible way; the resulting Young tableaux is admissible if it obeys the rules above, and if there are no more than one a, b, c, \dots in every column;
- (iii) Two tableaux with the same shape should be kept only if they have different labeling;

a	a	a	a
b	b		
c			

- (iv) A sequence of indices a, b, c, \dots is admissible if at any point in the sequence at least as many a 's have occurred as b 's, at least as many b 's have occurred as c 's, etc.; all tableaux with indices in any row, from right to left, arranged in a non-admissible sequence must be eliminated.

The direct product of k fundamentals is especially simple, since it entails repeated attachment of one additional box up to k new boxes to any row, if that operation produces an admissible tableau (for instance, one cannot attach a box to a row containing as many boxes as the row above).

In the case of $SU(3)$, Young tableaux have only two rows, and can be labeled with the Dynkin indices (p, q) , where q is the number of boxes in the second row, and p the number of additional boxes in the first row with respect to the second (thus, the first row has $p + q$ boxes). The dimensionality of the representation is given by

$$\dim = \frac{1}{2} (p + 1) (q + 1) (p + q + 2); \quad (2.1)$$

similar formulae exist for $N > 3$.

The direct product of the fundamental and a generic irreducible representation (p, q) generally includes

$$(p, q) \otimes (1, 0) = (p + 1, q) + (p - 1, q + 1) + (p, q - 1) \quad (2.2)$$

where the last two representations only exist if $p \geq 1$ and $q \geq 1$, respectively. As a result, to obtain the singlet representation $(0, 0)$ from (p, q) we need exactly p copies of the fundamental to bring $p \rightarrow 0$ (visually, by adding the extra boxes all to the second row); these will bring us to the representation $(0, q + p)$; at that point, we attach $q + p$ boxes to the third row (i.e. multiply by an additional $q + p$ fundamentals) to obtain the singlet representation.

The operational sequence outlined above is also the most economical, since, as Eq. (2.2) shows, p can only decrease by one unit for each additional fundamental representation factor, but doing so costs an increment of one unit to q ; similarly, q can also only decrease by one unit at a time, thus *the minimal number k of fundamental representations needed to obtain a representation that includes the singlet representation from the direct product of a given representation (p, q) and k copies of the fundamental representation is $k = 2p + q$.*

Visually, one simply needs to fill the Young tableaux of the representation (p, q) to a rectangle of $3 \times (p + q)$ boxes; this requires $3p + 3q - (2q + p) = 2p + q$ additional boxes, or copies of the fundamental representation, as shown in fig. 1.

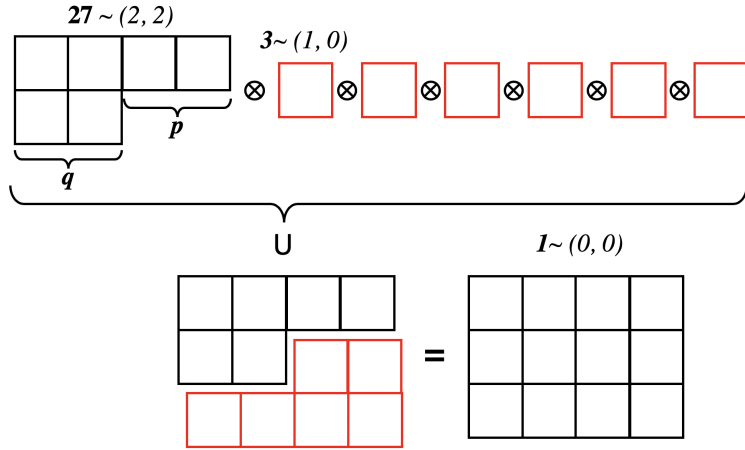


Figure 1: A schematic representation of how the minimal number of direct products of the fundamental representation (here, for $\mathbf{27} \sim (2, 2)$, that number is $2p + q = 6$) produces a Young tableau containing the trivial representation.

This result is easily generalized, by the same argument, to $SU(N)$, where irreducible representations are labeled by $(p_1, p_2, \dots, p_{N-1})$, and the number of fundamental representations is given by

$$k_N = p_{N-1} + 2p_{N-2} + \dots + (N-2)p_2 + (N-1)p_1. \quad (2.3)$$

A more formal proof of the statement above can be obtained from the Schur-Weyl duality² [15]: the direct product of k copies of the fundamental representation \mathbf{N} of $SU(N)$ decomposes into a direct sum over of irreducible representations labeled by all ordered partitions $\lambda_1 \geq \lambda_2 \dots \geq \lambda_i$ of k with $i \leq N$. The question of whether, given a representation X , the representation $X \otimes \mathbf{N}^{\otimes k}$ contains the trivial representation is equivalent to asking whether $\overline{\mathbf{N}}$ is contained in the Schur-Weyl duality sum. But given that for a representation $X \sim (p_1, p_2, \dots, p_{N-1})$ the conjugate representation $\overline{\mathbf{X}} \sim (p_{N-1}, p_{N-2}, \dots, p_2, p_1)$, whose Young tableaux contains exactly $k_N = p_{N-1} + 2p_{N-2} + \dots + (N-2)p_2 + (N-1)p_1$ boxes, the $\overline{\mathbf{X}}$ certainly belongs to the Schur-Weyl duality decomposition; this also proves that k_N is the smallest possible number k such that $X \otimes \mathbf{N}^{\otimes k}$ contains the trivial representation, since $k_N - 1$ would not have a sufficient number of Young tableaux to produce $\overline{\mathbf{X}}$ in the Schur-Weyl duality decomposition.

3 The minimal number of gluons, quarks, antiquarks

Here I will prove that, with two exceptions, one can always substitute the product of two $SU(3)$ fundamentals $\mathbf{3} \sim (1, 0)^{\otimes 2}$ for one antifundamental $\overline{\mathbf{3}} \sim (0, 1)$, and of three fundamentals $(1, 0)^{\otimes 3}$ for one adjoint $\mathbf{8} \sim (1, 1)$. This procedure yields the minimal number of quarks/antiquarks/gluons from the results of the previous section. The proof is as follows: in the previous section I have proved that for a representation $\mathbf{X} \sim (p, q)$, the minimal

²I am grateful to Martin Weissman for pointing this out to me.

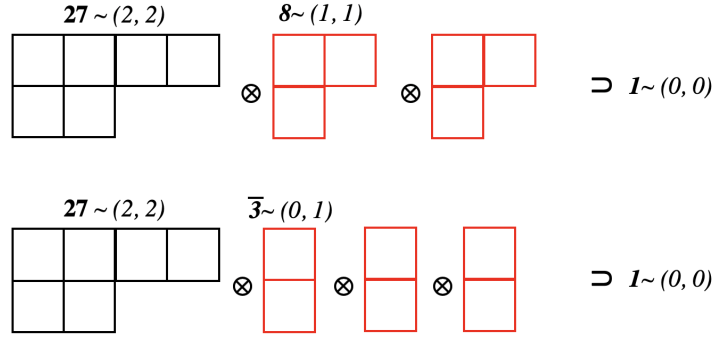


Figure 2: With two exceptions, one is always allowed to trade 2 copies of the fundamental representation for one antifundamental, and 3 copies for one adjoint.

number of fundamentals one needs to multiply to obtain a representation that contains the singlet representation is $2p + q$, i.e.

$$\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q)} = \mathbf{Y} + \mathbf{1}, \quad (3.1)$$

where \mathbf{Y} is a generic direct sum of irreducible representations. Since $\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}}$, I can write:

$$\mathbf{X} \otimes (\mathbf{6} \oplus \bar{\mathbf{3}}) \otimes \mathbf{3}^{\otimes(2p+q-2)} = \left[(\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q-2)}) \otimes \mathbf{6} \right] \oplus \left[\mathbf{X} \otimes \bar{\mathbf{3}} \otimes \mathbf{3}^{\otimes(2p+q-2)} \right] = (3.2)$$

$$\left[\mathbf{Z} \otimes \mathbf{6} \right] \oplus \left[\mathbf{X} \otimes \bar{\mathbf{3}} \otimes \mathbf{3}^{\otimes(2p+q-2)} \right] = \mathbf{Y} + \mathbf{1}. \quad (3.3)$$

I will prove that for any representation \mathbf{Z} , the product $\mathbf{Z} \otimes \mathbf{6}$ contains the singlet representation if and only if $\mathbf{Z} = \bar{\mathbf{6}} \sim (0, 2)$. Therefore, with the exception of $\mathbf{Z} = \bar{\mathbf{6}}$, the singlet representation must be contained in the direct product $\mathbf{X} \otimes \bar{\mathbf{3}} \otimes \mathbf{3}^{\otimes(2p+q-2)}$, and one can eliminate two fundamentals in favor of one antifundamental (see fig. 2).

Let $\mathbf{Z} \sim (p, q)$. The representation $\mathbf{Z} \otimes \mathbf{6}$ is obtained by adding two boxes to any of the three rows, which gives the following possibilities (I indicate with $[x, y, z]$ x boxes added to the first row, y to the second row, z to the third row):

- $[2, 0, 0]$: $(p + 2, q)$
- $[1, 1, 0]$: $(p, q + 1)$
- $[1, 0, 1]$: $(p + 1, q - 1)$, only possible if $q \geq 1$
- $[0, 2, 0]$: $(p - 2, q + 2)$, only possible if $p \geq 2$
- $[0, 1, 1]$: $(p - 1, q)$, only possible if $p \geq 1$
- $[0, 0, 2]$: $(p, q - 2)$, only possible if $q \geq 2$

Since p, q are non-negative integers, the only resulting representations that could correspond to the singlet representation are $(p - 1, q)$ and $(p, q - 2)$, for $\mathbf{Z} = \mathbf{3} \sim (1, 0)$ and $\bar{\mathbf{6}} \sim (0, 2)$,

respectively. Since $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$, the $(1, 0)$ allows for the substitution of two fundamentals for one antifundamental; however, since $\bar{\mathbf{6}} \otimes \bar{\mathbf{3}} = \mathbf{15} \oplus \mathbf{3}$, the $\bar{\mathbf{6}} \sim (0, 2)$ representation *does not* allow for the substitution of two fundamentals for one antifundamentals, and the minimal number of elementary constituents to produce a color-neutral hadron is two quarks.

The proof for the substitution of three fundamentals for one adjoint follows along similar lines. Here, note that

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}, \quad (3.4)$$

so I can write, similarly to what done above,

$$\begin{aligned} \mathbf{X} \otimes (\mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}) \otimes \mathbf{3}^{\otimes(2p+q-3)} &= (\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q-3)}) \otimes \mathbf{10} \\ \oplus (\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q-3)}) \otimes \mathbf{8} \oplus (\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q-3)}) \otimes \mathbf{8} \oplus (\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q-3)}) \end{aligned} \quad (3.5)$$

$$(3.6)$$

By the proof in the preceding section, $\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q-3)}$ cannot contain the singlet representation (since the minimal number of products of the fundamental is $2p + q$, not $2p + q - 3$); therefore, unless $\mathbf{X} \otimes \mathbf{3}^{\otimes(2p+q-3)} \otimes \mathbf{10}$ contains the singlet representation, the substitution of three fundamentals for one adjoint is allowed.

As above, I consider the product $\mathbf{Z} \otimes \mathbf{10}$, for a generic representation $\mathbf{Z} \sim (p, q)$. Using the same notation as above, multiplication by the $\mathbf{10}$ is obtained by adding three boxes to any of the three rows, which gives the following possibilities:

- $[3, 0, 0]: (p + 3, q)$
- $[2, 1, 0]: (p + 1, q + 1)$
- $[2, 0, 1]: (p + 2, q - 1)$, only possible if $q \geq 1$
- $[1, 2, 0]: (p - 1, q + 2)$, only possible if $p \geq 1$
- $[1, 1, 1]: (p, q)$
- $[1, 0, 2]: (p + 1, q - 2)$, only possible if $q \geq 2$
- $[0, 3, 0]: (p - 3, q + 3)$, only possible if $p \geq 3$
- $[0, 2, 1]: (p - 2, q + 1)$, only possible if $p \geq 2$
- $[0, 1, 2]: (p - 1, q - 1)$, only possible if $p \geq 1$ and $q \geq 1$
- $[0, 0, 3]: (p, q - 3)$, only possible if $q \geq 3$

Inspection of the cases above, noting again that p and q are non-negative integers, indicates that the only candidate representations to give a singlet when multiplied by a $\mathbf{10}$ are $\mathbf{1} \sim (0, 0)$, $\mathbf{8} \sim (1, 1)$ and $\bar{\mathbf{10}} \sim (0, 3)$. Now, since $\mathbf{1} \otimes \mathbf{10} = \mathbf{10}$ and $\mathbf{8} \otimes \mathbf{10} = \mathbf{35} \oplus \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{8}$, we conclude again that the only representation for which the substitution of the product of three fundamentals for one adjoint is the $\bar{\mathbf{10}}$, for which the minimal number of elementary

constituents is three quarks (a quark-antiquark pair would not suffice since $\mathbf{10} \otimes \overline{\mathbf{3}} \otimes \mathbf{3} = \mathbf{35} \oplus \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{8}$).

With the results outlined above, assuming the electric charge Q_X of a new hypothetical strongly-interacting particle X belonging to a representation $\mathbf{X} \sim (p, q)$ is known, it is possible to calculate both the electric charge of the “smallest” hadron Q_H , and, generally, of any hadron containing X . Given the number of quarks n_q , antiquarks $n_{\bar{q}}$ and gluons n_g listed in Tab. 1, the possible values of the charge of the smallest hadron H are the following:

$$-\frac{1}{3}n_q - \frac{2}{3}n_{\bar{q}} - n_g \leq Q_H - Q_X \leq \frac{2}{3}n_q + \frac{1}{3}n_{\bar{q}} + 2n_g. \quad (3.7)$$

This can be equivalently expressed in terms of the Dynkin label (p, q) as

$$-\frac{2p+q}{3} \leq Q_H - Q_X \leq \frac{2(2p+q)}{3}. \quad (3.8)$$

Any other hadron H' could only have electric charge $Q_{H'} = Q_H + k$ for integer k .

Notice that all and only the representations of the form $\mathbf{X} \sim (k+3n, k+3m)$ (with the exception of the one case $k=0, n=0, m=1$, i.e. $(0, 3)$) exclusively contain gluons in their “smallest hadron” (equivalently, the direct product $\mathbf{X} \otimes \mathbf{8}^{\otimes(2n+m+k)}$ contains the trivial representation). Thus, it is only those representations (*including* $(0, 3)$) that will yield hadronic bound states with integer charge if the “new physics particle” is neutral or of integer charge. I indicate those representations in green in Tab. 1. Notice that this set of representations includes *all* real (self-adjoint) representations (p, p) .

4 Conclusions

I proved that the smallest number of copies k of the fundamental representation $(1, 0)$ of $SU(3)$ such that the direct product of irreducible representation $(p, q) \otimes (1, 0)^{\otimes k}$ contains the trivial representation $(0, 0)$ is $k = 2p + q$; I generalized this result to $SU(N)$, where for irreducible representation $(p_1, p_2, \dots, p_{N-1})$, $k_N = p_{N-1} + 2p_{N-2} + \dots + (N-2)p_2 + (N-1)p_1$. I showed that in $SU(3)$ one can “trade” any two fundamentals in the direct product of $2p+q$ fundamentals for one antifundamental (with the exception of representation $\overline{\mathbf{6}} \sim (0, 2)$, for which such substitution is not allowed) and any three fundamentals for one adjoint (with the exception of representation $\overline{\mathbf{10}} \sim (0, 3)$, for which such substitution is not allowed); finally, I showed that if a new strongly interacting particle X in the (p, q) representation of $SU(3)$ has electric charge Q_X , the possible values of the electric charge of its “smallest hadron” H are $-(2p+q)/3 \leq Q_H - Q_X \leq 2(2p+q)/3$, and those of any other hadron H' is $Q_{H'} = Q_H + k$ for integer k .

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p	q	dim	$2p + q$	n_g	$n_{\bar{q}}$	n_q
0	0	1	0	0	0	0
0	1	$\overline{\mathbf{3}}$	1	0	0	1
1	0	3	2	0	1	0
0	2	$\overline{\mathbf{6}}$	2	0	0	2
2	0	6	4	1	0	1
1	1	8	3	1	0	0
0	3	$\overline{\mathbf{10}}$	3	0	0	3
3	0	10	6	2	0	0
0	4	$\overline{\mathbf{15}}$	4	1	0	1
1	2	$\overline{\mathbf{15}}$	4	1	0	1
2	1	15	5	1	1	0
4	0	15	8	2	1	0
0	5	$\overline{\mathbf{21}}$	5	1	1	0
5	0	21	10	3	0	1
1	3	$\overline{\mathbf{24}}$	5	1	1	0
3	1	24	7	2	0	1
2	2	27	6	2	0	0
0	6	$\overline{\mathbf{28}}$	6	2	0	0
6	0	28	12	4	0	0
1	4	$\overline{\mathbf{35}}$	6	2	0	0
4	1	35	9	3	0	0
0	7	$\overline{\mathbf{36}}$	7	2	0	1
7	0	36	14	4	1	0
2	3	$\overline{\mathbf{42}}$	7	2	0	1
3	2	42	8	2	1	0
0	8	$\overline{\mathbf{45}}$	8	2	1	0
8	0	45	16	5	0	1
1	5	$\overline{\mathbf{48}}$	7	2	0	1
5	1	48	11	3	1	0
0	9	$\overline{\mathbf{55}}$	9	3	0	0
9	0	55	18	6	0	0
2	4	$\overline{\mathbf{60}}$	8	2	1	0
4	2	60	10	3	0	1
1	6	$\overline{\mathbf{63}}$	8	2	1	0
6	1	63	13	4	0	1
3	3	64	9	3	0	0
0	10	$\overline{\mathbf{66}}$	10	3	0	1
10	0	66	20	6	1	0
0	11	$\overline{\mathbf{78}}$	11	3	1	0
11	0	78	22	7	0	1
1	7	$\overline{\mathbf{80}}$	9	3	0	0
7	1	80	15	5	0	0
2	5	$\overline{\mathbf{81}}$	9	3	0	0
5	2	81	12	4	0	0
3	4	$\overline{\mathbf{90}}$	10	3	0	1
4	3	90	11	3	1	0
0	12	$\overline{\mathbf{91}}$	12	4	0	0
12	0	91	24	8	0	0
1	8	$\overline{\mathbf{99}}$	10	3	0	1
8	1	99	17	5	1	0

Table 1: List of all irreducible representations of SU(3) with dimension smaller than 100, with the minimal number of gluons, antiquarks and quarks needed to form a color-singlet hadron. The smallest hadrons for representations in green only contain gluons, and, if the “new physics particle” belonging to that representation is electrically neutral, would also be electrically neutral.

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