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STRUCTURE IN THE UNIVERSE FROM MASSIVE NEUTRINOS

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ABSTRACT

A neutrino with hypothetical mass of order 10 eV may have an important role in the formation of large-scale structure in the Universe.

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Introduction

Physical sciences have often progressed by making a judicial simplification of the problem at hand. A prime example is modern cosmology, where we disregard all fine structure and treat the Universe as perfectly homogeneous. According to our present knowledge the history of the Universe has two crucial ingredients:

(i) expansion, which idea goes back to Friedmann,[1] who first (1922) considered non-static solutions of the cosmological equations;

(ii) radiation epoch, which was discussed originally (1927) by Lemaitre[1] and later explored by Gamov,[1] who in particular studied the synthesis of heavier nuclei from protons and neutrons in the hot soup of the early Universe.

These two ingredients of our standard cosmology are reflected in its popular name: the Hot Big Bang. For a better understanding of what follows let me recall the basic assumptions and arguments leading to the standard model. The first two steps involve the relevant physics:

(i) General Relativity is the theory appropriate to describe the selfgravitation of the Universe. The Einstein field equations are in standard notation[2]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G \, T_{\mu\nu} \tag{1}$$

and relate the curvature of the space-time metric ($g_{\mu\nu}$) on the left-hand side to the energy-momentum tensor ($T_{\mu\nu}$) on the right-hand side via the Newton gravitational coupling constant (G);

(ii) Particle and nuclear physics to provide the correct expression for the energy-momentum tensor in (1).

The following two steps in the argument introduce the crucial simplifications, which make the problem tractable:

(iii) Homogeneity and isotropy, which reduce the metric to the Robertson-Walker form, greatly simplifying the set of 10 equations in (1);

(iv) Thermal equilibrium and entropy conservation of the matter content of the Universe, which leads to dependence on the overall thermodynamic quantities only, e.g., the temperature T.

These steps combined lead to simple differential equations, which describe the evolution of the Universe, see for example Chapter 15 of Weinberg's book.[2] Radio source counts and especially observations of the cosmic background radiation (CBR) support the assumptions (iii) and (iv). This part of cosmology, which I would like to call "smooth cosmology," is in great shape. In fact, we dare to use the success of the model to constrain possible modifications of the particle physics. Specifically, the more or less correctly predicted abundances of ^2D, ^4He, ^7Li from cosmic nucleosynthesis would restrict the number (N_ν) of neutrino types (and hence the number of families) to 4 or 5 at most.[3] The reason for this is as follows: a larger number N_ν would lead to faster expansion, which leaves less time for the neutrons to decay and thus results in a higher abundance of ^4He. [It may be that for specific values of neutrino mass differences and mixing angles this upper bound on N_ν is raised[4].] Hopefully, measurement of the Z° width at SLC and LEP will confirm that N_ν is close to the presently known value of 3 and not very much larger.

Up till now I have briefly reviewed the stretching and evolution of the canvas of the Universe without any attention to the fine print on it. Nevertheless we do observe "ripples" to be present, i.e. galaxies, groups, clusters and superclusters ranging in mass from some $10^{11}M_\odot$ to $10^{16}M_\odot$. Many questions arise in what I would like to call "ripple cosmology":

(i) What determined the amplitude and shape of the primordial spectrum of the density perturbations, which grew into the presently observed structure?

(ii) Are there interrelations between the different entities, for example do galaxies arise as fragments in a larger system or do the galaxies form first and then agregate into clusters?

(iii) What sets the mass scale? Recall that the mass scale of stars arises from balancing the inward pull of gravitation with the thermal and radiation pressure of the interior of the star (Eddington in Ref. 1). We would like to have the same understanding for the typical mass of the ripples.

Furthermore, all we see is not all there is: <u>dark matter</u> seems to be 10 times more abundant than the visible matter.[5] In fact, our present estimates for the density ratio are

$$\Omega_{luminous} \sim 0.01 \qquad (2a)$$

$$\Omega_{dynamical} \sim 0.1 - 1 \quad . \qquad (2b)$$

I take the opportunity to introduce some notation; the present density ratio, critical density and the Hubble expansion parameter are[2]

$$\Omega \equiv \rho/\rho_{crit}$$

$$\rho_{crit} \equiv \frac{3H_o^2}{8\pi G} = 2.\ 10^{-29}\ h^2\ g\ cm^{-3}$$

$$H_o \equiv 100\ h\ km\ s^{-1}\ Mpc^{-1} = (10^{10}\ h^{-1}\ yr)^{-1}$$

and for Ω <1 or >1 the Universe is open or closed and will expand for ever or contract again, respectively. Observations show h to lie between approximately 1/2 and 3/4. Let us return to the density ratios in (2), which have been deduced from the study of many different systems. As an example, I take the first such study, namely Zwicky's[1)] analysis (1937) of the Coma cluster. Its mass can be determined in two ways:

(a) counting the galaxies or measuring their luminosity and then multiplying by a typical galaxy mass or mass to light ratio gives the cluster mass $M^{(a)}_{Coma}$;

(b) using the virial theorem with the observed velocities and orbits of the cluster galaxies gives the mass $M^{(b)}_{Coma}$.

Surprisingly $M^{(b)}_{Coma}$ turned out to be more than 100 times larger than $M^{(a)}_{Coma}$, or in other words, most of the matter in the Coma cluster is invisible! Adding up similar mass estimates of other systems leads to the overall mass densities reported in (2a,b). For dynamics dark matter is the most important and at best the visible galaxies may serve as tracers of the overall gravitational potential. But what is this dark matter made of? We do not know yet. Probably it is not baryonic, since the cosmic nucleosynthesis results[3)] do not allow for such a large baryon density $\Omega_b \sim 0.1 - 1$. An ideal candidate, truly dark and collisionless, would be a massive neutrino. Only the heaviest neutrino would matter cosmologically, which in the absence of mixing may be the τ-type neutrino. [Let me indulge in some numerology: if the neutrinos follow the charged lepton (e^-, μ^-, τ^-) mass ratios their masses could be in the ballpark 0.01:3:50 eV.]

Alas, there is no definitive experimental result on neutrino mass. Of course, this should not, and did not, deter astrophysicists from speculating "what if..." and in the following I will summarize the results of their wishful thinking. Henceforth, I will assume N_ν = 3 neutrino types and one dominant neutrino mass $m_{\nu 1}$.

Massive Neutrinos in Cosmology

The implications of possible massive neutrinos for "smooth cosmology" were first discussed by Gershtein and Zeldovich,[6] who derived an important bound on the sum of the masses

$$\Sigma \equiv \sum_{i=1}^{N_\nu} m_{\nu i} \lesssim 100 \text{ eV} \quad . \tag{3}$$

The argument is very simple: since the neutrinos participated in the thermal equilibrium before they decoupled at $T \sim 1$ MeV, we know their present number density ($n_{\nu i} \sim \frac{1}{3} n_{\gamma CBR}$); multiplying with the mass $m_{\nu i}$ gives the present energy density ratio $\Omega_{\nu i} = 1.36$ ($m_{\nu i}$/100 eV) and in order to avoid a prohibitively large total density of the Universe or, equivalently, a too short age of the Universe the bound (3) must hold. The same argument gives mass limits of other weakly interacting particles (gravitinos...), provided their lifetime is large enough. Later it was noted[7] that massive neutrinos could provide Zwicky's "missing mass" in the Coma cluster and others. But the real breakthrough in understanding the role of massive neutrinos for "ripple cosmology" occurred in 1980, just after the first ITEP result on non-zero neutrino mass was announced. It was realized more or less simultaneously by different collaborations, located in the three superpowers the Netherlands,[8] the US,[9] and the USSR,[10,11] that massive neutrinos have two nice properties:

(i) a damping scale of order

$$1.8\, M_p^{\,3}/m_{\nu 1}^2 \sim 3\; 10^{14} \left(\frac{100 \text{ eV}}{m_{\nu 1}}\right)^2 M_\odot \quad , \tag{4}$$

where the Planck mass is $M_p \equiv (\hbar c/G)^{1/2} = 1.22\; 10^{28}$ eV = $2.17\; 10^{-5}$ g, this would give the desired mass scale of the ripples of our Universe;

(ii) only a small initial amplitude of the primordial (adiabatic) density perturbations is needed

$$\delta_{\nu 1}^{in} \equiv \left(\frac{\rho_{\nu 1} - \bar{\rho}_{\nu 1}}{\bar{\rho}_{\nu 1}}\right)^{in} \sim \frac{1 + z_{GF}}{1 + z_{\gamma\nu}} \sim 2\; 10^{-4} \;\; \frac{100 \text{ eV}}{m_{\nu 1}} \;\; \frac{1 + z_{GF}}{6} \tag{5}$$

and hence only small residual fluctuations in the CBR at the arcminute scale are to be expected.

In (5) $z_{\gamma\nu}$ and z_{GF} are the respective redshifts of the epoch when the neutrino mass becomes dominant and when the galaxies form ($\delta_{\nu 1} = 1$).

Recall that the redshift is given by $z \equiv (\lambda_{obs} - \lambda_{em})/\lambda_{em}$ and that $1 + z = a_{obs}/a_{em}$, where a is the scale factor of the Universe and "obs" and "em" refer to the present observer and the emission epoch.

For a neutrino mass of several tens of eV the scenario would be as follows: neutrinos ν_1 collapse into large systems and shortly afterwards the baryonic gas, which was dragged along, dissipates its energy and fragments into galaxies. I will now discuss in somewhat more detail how structure could have arisen in this way. In addition to Ref. 8-11 I will only give the reference of the best calculation to date of a particular problem.

Linear Growth Period

We have no definitive idea on the origin of the primordial density fluctuations, which grow into the observed structures of the Universe, and for simplicity we will use a scale-free initial spectrum

$$|\delta_k|^2 \propto k^n \quad , \tag{6}$$

where δ_k is the Fourier transform of $\delta \equiv (\rho - \bar{\rho})/\bar{\rho}$ with $\bar{\rho}$ the average density. The Harrison-Zeldovich spectrum with $n = 1$ is a special case, in that all different wavelength perturbations have the <u>same</u> rms gravitational potential depth. Spectra with $n>1$ or <1 would lead to strong inhomogeneities on small scales (black holes) or ultimately on large scales, unless there is a cut-off in (6). For this reason most astrophysicists prefer to use $n = 1$, but the reader should keep in mind our total lack of understanding of the initial perturbations. [An inflation scenario may lead to a $n = 1$ spectrum, but its amplitude appears too large;[12] nevertheless this is an interesting hint.] In this section we consider the period when $\delta_{\nu 1} << 1$, which allows us to linearize the perturbation equations. Before we look at the neutrino perturbations, we must consider in more detail the global evolution of the Universe. A neutrino mass $m_{\nu 1}$ is irrelevant for the very early epoch ($T >> m_{\nu 1}$). It may be that the light (lefthanded) neutrino ν_L has a super heavy (righthanded) partner ν_R, but it can be shown[13] that this partner does not upset standard nucleosynthesis or baryogenesis (creation of net baryon number at temperatures of the Grand Unification scale). For the later evolution Fig. 1 gives the energy densities of the relevant particles vs. the photon temperature (now 2.7 K): first the photons (and relativistic neutrinos) dominate, but for $T < m_{\nu 1}$ the massive non-relativistic

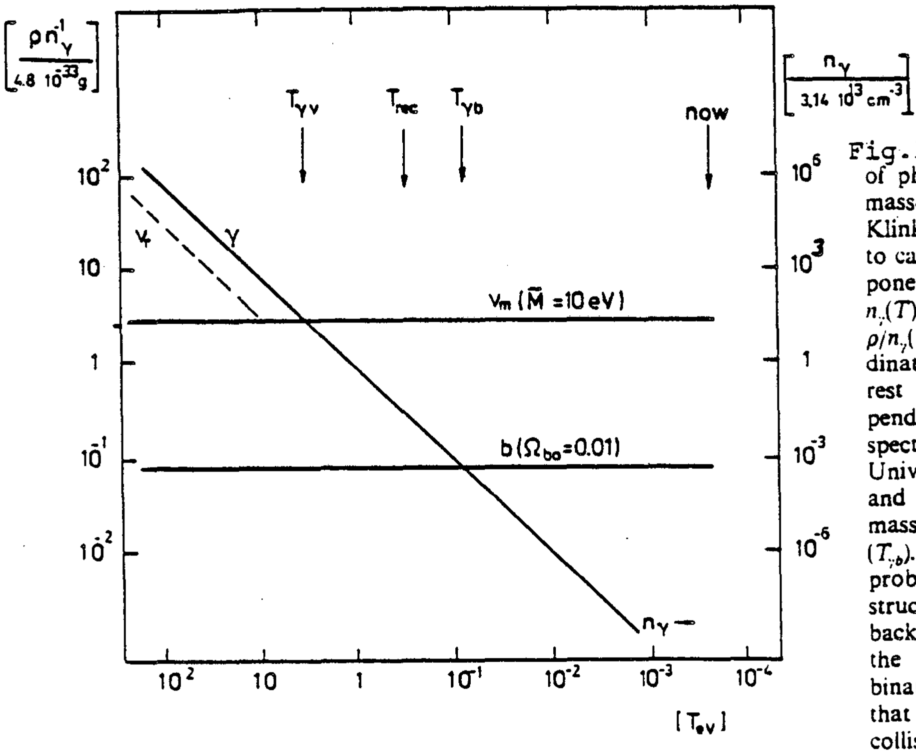

Fig.1 Thermal history of energy densities of photons, relativistic neutrinos and rest masses of neutrinos and baryons (see Klinkhamer and Norman 1981). Procedure to calculate the energy density of one component at a certain temperature: multiply $n_\gamma(T)$ from the ordinate at the right by the $\rho/n_\gamma(T)$ ratio of the component of the ordinate at the left. Note that the ρ/n_γ ratio of rest masses of neutrinos and baryons depends linearly on $\bar{M} = \sum/3$ and Ω_{b_0} respectively. We thus see that the cooling Universe first was dominated by radiation and relativistic particles and then by rest mass either from neutrinos ($T_{\gamma\nu}$) or baryons ($T_{\gamma b}$). In the latter epoch density fluctuations probably grew into the presently observed structure, leaving a trace in the cosmic background radiation which decoupled from the baryons at $T_{rec} \sim 4500$, when recombination of the ionized gas occurred. Note that for $T \lesssim 1$ MeV the neutrinos were a collisionless gas

neutrinos take over and now ($z < z_{\gamma\nu}$) the neutrino density perturbations can grow. This growth occurs earlier than would be the case for simple baryonic perturbations (in Fig. 1 $z_{\gamma b}$ is 100 times less than $z_{\gamma\nu}$). Since δ grows now as the scale factor a (if $\Omega_o < 1$ then the growth stops for $1 + z < \Omega_o^{-1}$, so that we must have $z_{GF} > \Omega_o^{-1} - 1$) the required initial amplitude to have condensation at z_{GF} is given by (5). After recombination the baryonic gas catches up rapidly with the neutrino perturbation. The small neutrino fluctuations are accompanied by fluctuations in the photons, which are free streaming after recombination of the nuclei (p^+, He^{++}) and the electrons (e^-) at T ~ 4500 K. Hence there will be small fluctuations in the CBR. Observational limits[14)] are $\Delta T/T_{CBR} < 2\ 10^{-5}$ and calculations[15)] show that small enough initial neutrino fluctuations are possible, provided

$$z_{GF} < 8.7 \quad \text{for } \Omega_o = 1,\ h = 0.75 \qquad (7)$$

$$z_{GF} < 3.5 \quad \text{for } \Omega_o = 1,\ h = 0.50$$

and Fig. 2 shows the expected $\Delta T/T$ for $\Omega_o = 1$, $h = 0.75$, $z_{GF} = 3$ (these calculations were for a $n = 1$ spectrum). Standard adiabatic perturbations without massive neutrinos require an initial amplitude larger than (5) and the resulting CBR fluctuations would violate the observational limits.

Now we turn to the emergence of a mass scale (4). In a normal collisional gas we know that for perturbations with wavelength λ larger

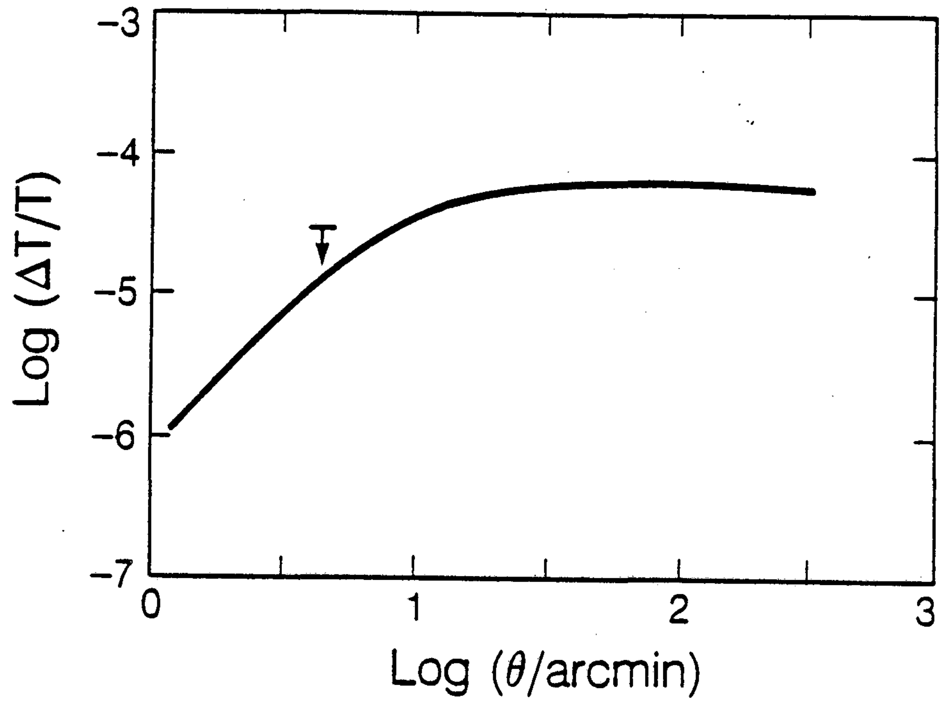

Fig. 2. Temperature fluctuations[15)] of the cosmic background radiation vs. angular scale for a massive neutrino with $\Omega = 1$, $\Omega_b = 0.03$, $h = 0.75$ and $z_{GF} = 3$ and for the same experimental setup as in Ref. 14, which gave the upper limit (95% c.l.) shown by the arrow.

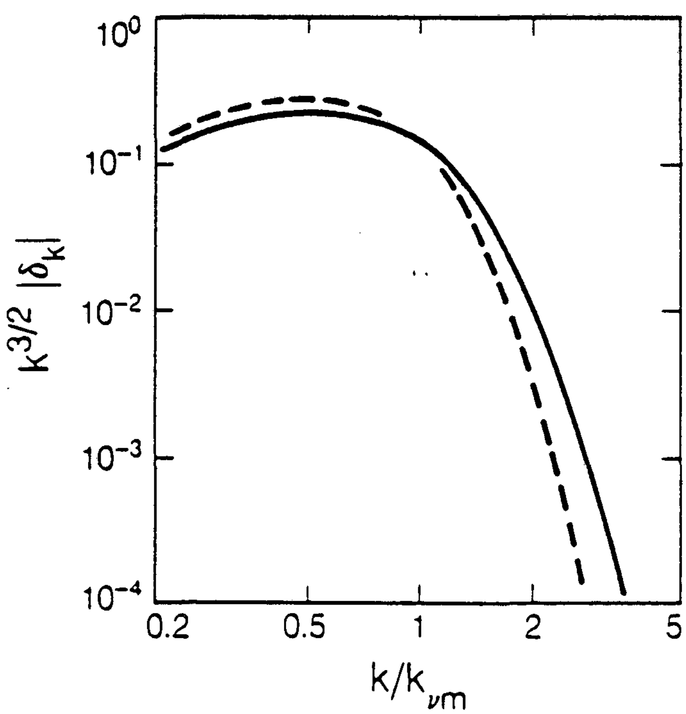

Fig. 3. Collisionless damping[16)] of neutrino perturbations (full curve: a single massive neutrino, dashed: three equally massive neutrinos). The final result (at $z \simeq 75$) for an initial spectrum (6) follows by multiplication of $(k/k_{\nu m})^{n/2}$.

than the Jeans length λ_J the amplitude grows by gravitational instability, whereas smaller wavelengths oscillate as an acoustic wave with constant amplitude, the profile being maintained by the collisions. Things are different in a collisionless gas, such as the one of our massive neutrinos. Again for large wavelengths there is growth, but now for $\lambda <$ "λ_J" there is damping arising from the freestreaming of the particles and also from directional dispersion, which occurs in $d > 1$ dimensions. The critical wavelength turns out to be (its value at the present is given)

$$\lambda_{\nu m} = 41 \quad \frac{30 \text{ eV}}{m_{\nu 1}} \quad \text{Mpc} \tag{8a}$$

and the masscale is

$$M_{\nu m} \equiv \frac{\pi}{3} \rho_{\nu 1} \lambda^3_{\nu m} = 3.2 \; 10^{15} \left(\frac{30 \text{ eV}}{m_{\nu 1}}\right)^2 M_\odot \quad . \tag{8b}$$

The transfer function which describes the small scale damping has been calculated in detail[16)] and is shown in Fig. 3.

To summarize, during the relativistic epoch perturbations with small wavelengths are damped, so that the final spectrum has a cut-off (8a) determined from the moment when the neutrinos become non-relativistic. Since the growth period is long the initial amplitude δ^{in} can be rather small and is consistent with present limits on the CBR fluctuations. We now turn to the precise structure that this spectrum gives rise to later.

Nonlinear Growth Period

We can follow analytically the growth of the neutrino perturbations as long as $\delta \ll 1$. When stronger contrasts occur the linear approximation breaks down. Presently the only way to follow the further evolution is through numerical simulations (N-body methods). There are severe restrictions by computer limitations, but nevertheless a number of interesting results have been obtained.

It is clear that the cutoff (8a) in the spectrum will have significant effects.[17)] To illustrate this we present in Figs. 4a,b and 4c,d the result of a calculation[18)] with a $n = 0$ spectrum without and with the sharp cut off. The resulting structure is very different in the two cases. When there is a cutoff a structure of filaments and cells appears, which is rather similar to that observed on the supercluster scale.[19)] In

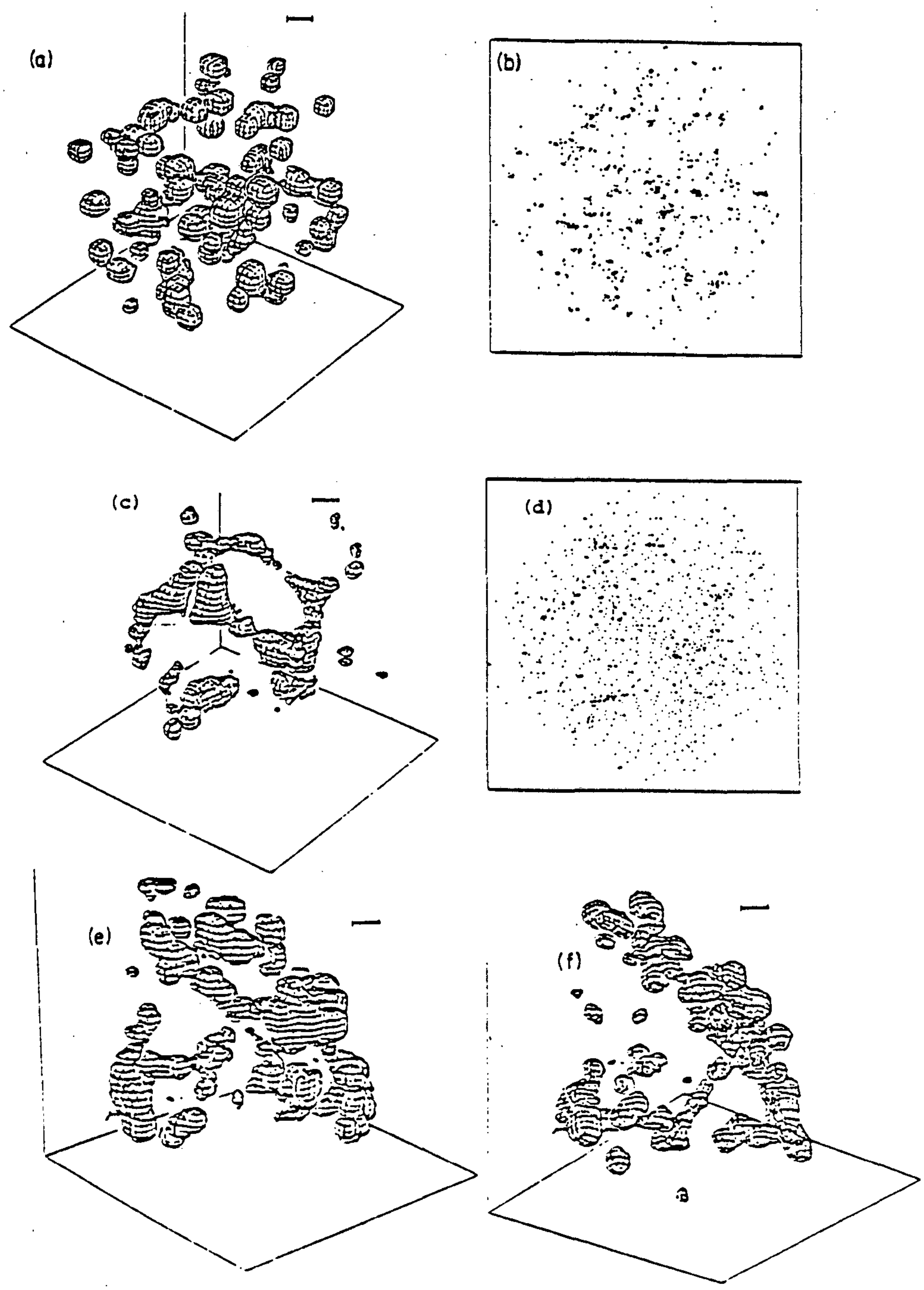

Fig. 4: Isodensity surfaces (at 4 times the mean density) and projected point distributions from massive neutrino models ($h = 0.78$, $\Omega = 1$) from Ref. 18. The bar on the right-hand upper corner represents 10 Mpc.

a,b: n = 0 spectrum without small scale cut-off

c,d: n = 0 spectrum with the cut-off (8a)

e,f: real galaxy distributions from the CfA survey (Ref. 20).

Fig. 4e,f the Center for Astrophysics redshift survey[20] of galaxies in our Northern neighborhood is plotted in the same way as the model results.

I will now discuss the interesting numerical results of White, Frenk and Davis[21] (WFD). These authors start ($a = 1$) from the $n = 1$ spectrum calculated in Ref. 16 with an amplitude of 22%. Figure 5 shows in the left frames the neutrino density pattern after expansion by a factor 4, 6 and 10. Since the neutrinos are invisible, one should try to identify the regions where the galaxies would form, but this is precisely the subject we understand least about. The (reasonable) procedure adopted by WFD is to look for volume cells of the initial grid that have gone through collapse and to tag the corresponding particles as "galaxies." The underlying idea is that the baryon gas dragged along would have shocked there, possibly triggering galaxy formation.[17] At $a = 2.9$ already 1% of the particles have had a collapsed volume element and this moment is defined to be the onset of galaxy formation (GF; note this is a different convention than $\delta = 1$ used in the previous section). Later frames can be interpreted as the present day situation if galaxy formation occurred at a redshift $z_{GF} = a/2.9 - 1$. The frames on the right of Fig. 5 show that the epoch with filamentary structure is rather ephemeral ($z_{GF} = 1.1$) and that at later epochs only strong clusters remain. In Fig. 6 the resulting correlation functions $\xi(r)$ are given. Recall that $\xi(r)$ is the excess probability over random to find at a distance r of a given galaxy another one. Observations of the sky learn that for real galaxies this function is given by

$$\xi(r) \sim (r/R_o)^{-1.8} \tag{9a}$$

$$R_o \sim 5\ h^{-1}\ \text{Mpc} \quad . \tag{9b}$$

Figure 6b shows that the lengthscale (R_o) of the candidate "galaxies" is much too large, which was to be expected from a comparison of (9b) with the neutrino scale (8a). In Fig. 7 we give on the right the final distribution of the "galaxies," which should be compared with the real galaxy distribution on the left. Qualitatively two problems of the neutrino scenario are clear:

(i) too large voids;

(ii) too large clusters.

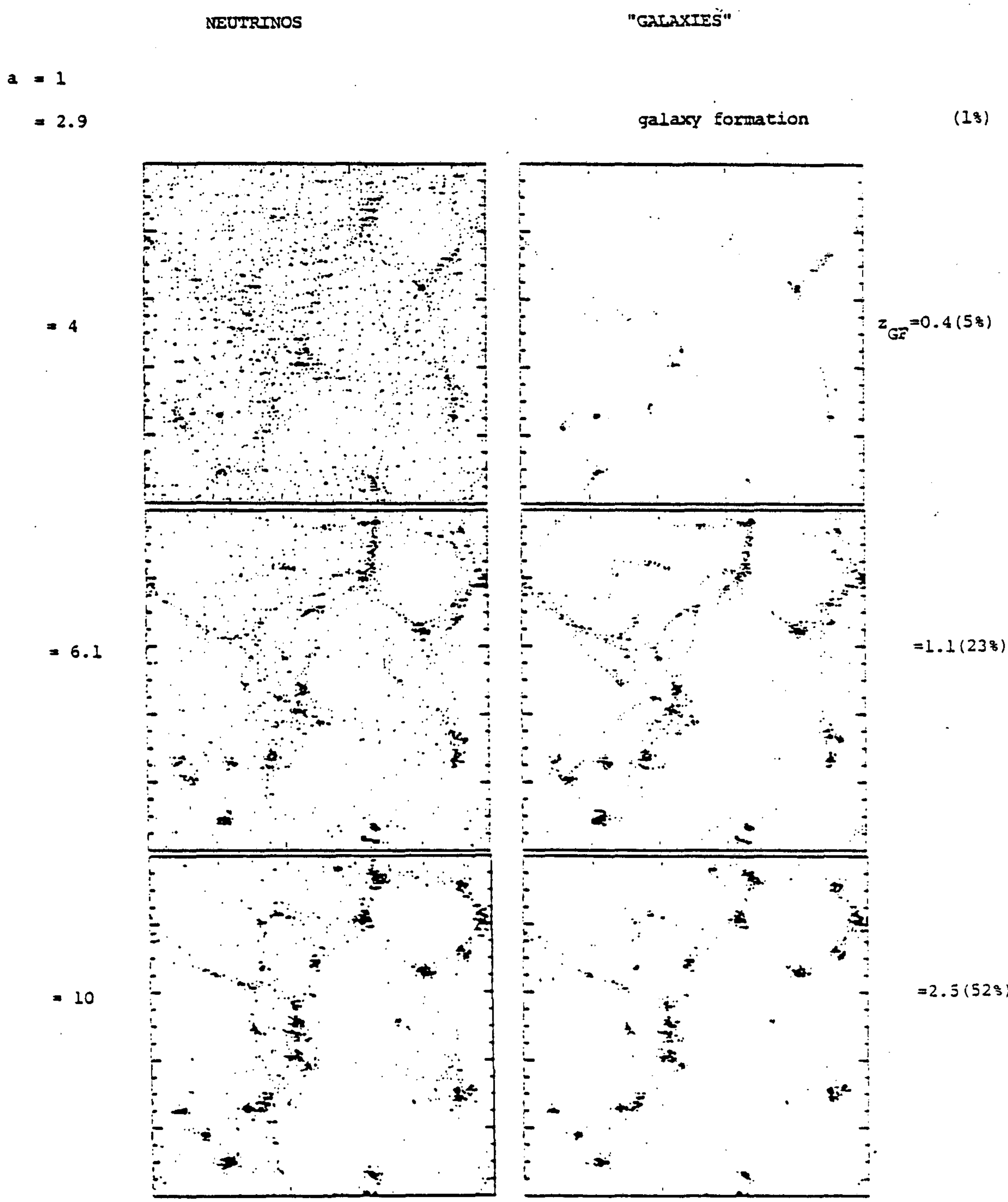

Fig. 5: Projected particle distributions in a massive neutrino model (Ω = 1) from Ref. 21. The left-hand panels show all particles, the right-hand panels those tagged as "galaxies," see text. The expansion factor a is shown on the left while for the "galaxy" picture the redshifts of galaxy formation z_{GF} and the fraction of tagged particles are shown on the right.

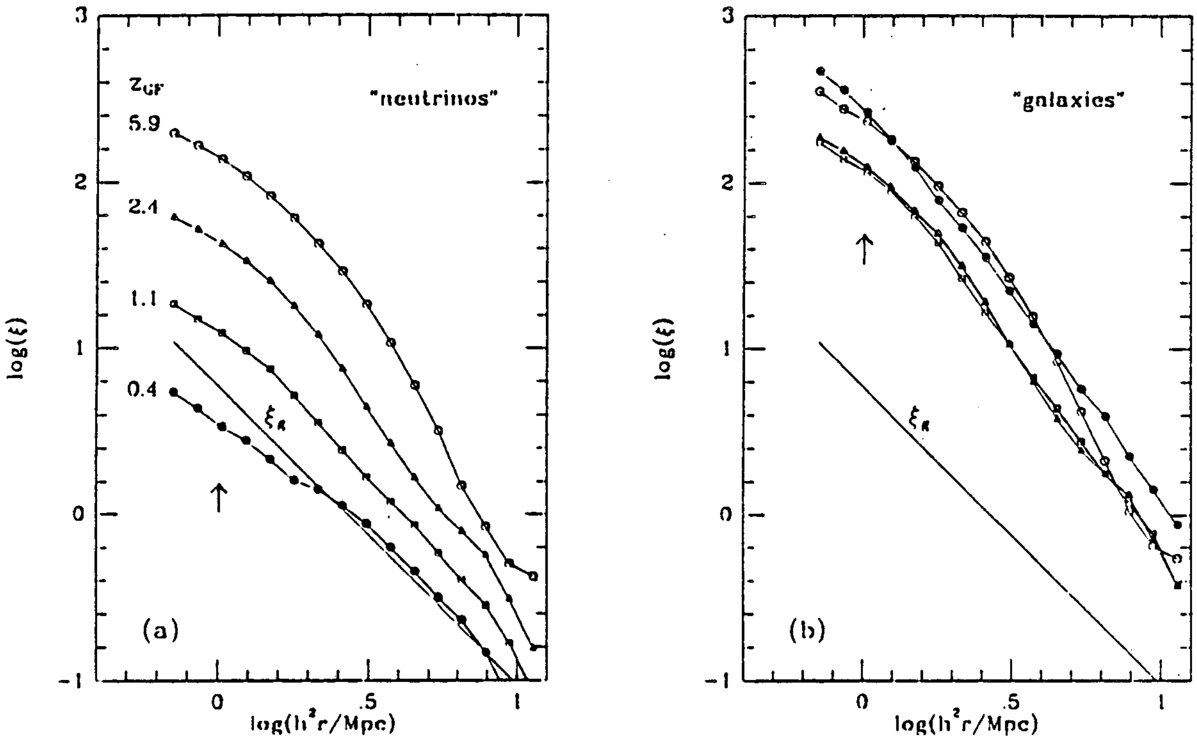

Fig. 6: Correlation functions for the models of Fig. 5. The straight line is the observed function (9).

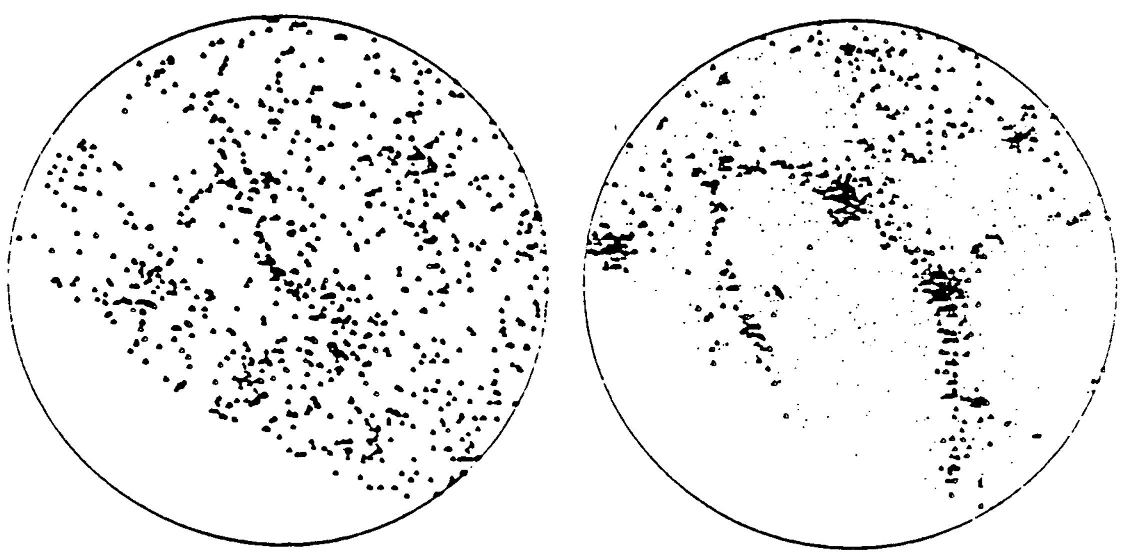

Fig. 7: On the right distribution of "galaxies" in a massive neutrino model (z_{GF} = 2.5, h = 0.54, Ω = 1), on the left the real galaxies of the CfA survey (Ref. 20).

In order to proceed more quantitatively WFD consider the clustering scale R_c, which turns out to be close to R_o, defined by

$$R_c^{-3} \int_0^{R_c} \xi(r) r^2 dr = 1 \quad . \tag{10}$$

For real galaxies $R_c \sim 5\ h^{-1}$ Mpc and with the equation (8a) for $\lambda_{\nu m}$ one then gets the relation

$$[R_c/\lambda_{\nu m}]_{model} = 0.4\ \Omega h\ \Theta^{-2} \quad , \tag{11}$$

where $\Theta \equiv T_{CBR}/2.7$ K. The numerical calculations give the number on the left-hand side of (11), which implies a value for $\Omega h \Theta^{-2}$, depending on z_{GF}, of course. Figure 8 shows the required value of $h\Theta^{-2}$ for given Ω. Other observations[2)] lead us to expect $h\Theta^{-2} \sim 0.5 - 0.75$, since Θ is close to 1. Also many quasars are observed at $z = 3$, so that we must have galaxy formation at z_{GF} of at least 3. Taking Fig. 8 at face value would rule out models with $n = 1$, $\Omega \leq 1$, possibly allowing those with $n = 4$. WFD conclude that "the conventional neutrino-dominated picture appears to be ruled out." I would agree that things do not look great, but I find their

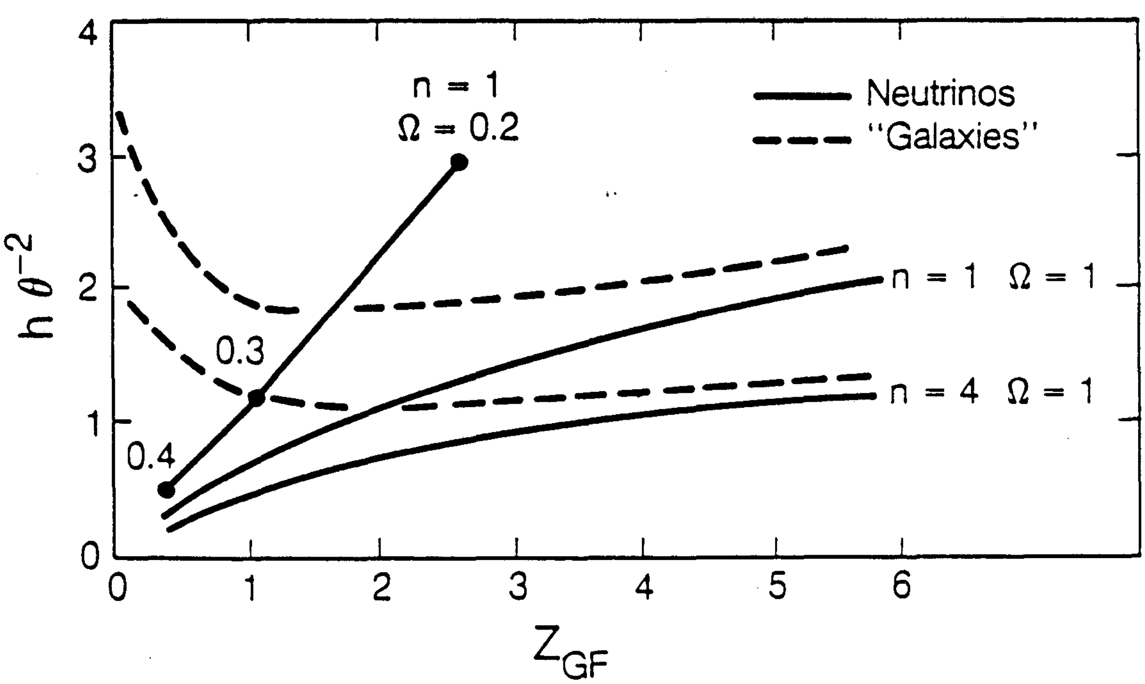

Fig. 8: Values of $h\Theta^{-2}$ implied by equation (11) for massive neutrino models (Ref.21).

conclusion too severe. Since we do not know how real galaxies form let us just look at the "neutrino" curves in Fig. 8 to get a rough idea. Then the n = 1 - 4, $\Omega = 1$ curves are in the ballpark ($h\Theta^{-2} \sim 1$, $z_{GF} \sim 3$). This may be all the accuracy we could hope for since we do not know the correct primordial perturbation spectrum, nor, as said above, where and when the galaxies visible to us form. In fact, it might be that galaxy formation is not an entirely fair tracer of the neutrino density. Still, for $\Omega_o = 1$ the age of the Universe

$$t_o = \frac{2}{3} H_o^{-1} = 6.7\ h^{-1}\ 10^9\ \text{yr} \tag{12}$$

may be uncomfortably short for $h \sim 1$, considering the ages quoted for globular clusters of at least 12 10^9 yr. [t_o could be larger than (12) if there were a nonzero cosmological constant, see Ref. 2].

Independent of the uncertainties of galaxy formation there is another problem for the massive neutrino scenario, namely the large neutrino clusters mentioned above and in Fig. 7. The following numbers[22)] (for the case n = 1, but n = 4 gives similar results) speak for themselves:

	Neutrino Clusters		Abell Clusters
	$z_{GF} = 2.5$	$z_{GF} = 6$	
mean spacing (Mpc)	65	70	50
mass fraction of Universe	0.39	0.61	<0.01
mass (10^{15} $M_\odot$)	26	54	1
X-luminosity (10^{45} erg/s)[a)]	8	30	0.5
number expected in HEAO-1	500	2000	≤ 7[b)]

a) Using $\Omega_b = 0.1$

b) Unidentified sources in the X-ray survey of the HEAO satellite.

Further details can be found in Ref. 22, but it is clear from this table that these objects are not likely to constitute such an important part of our present Universe and escape detection. On the other hand, unless the neutrino structures condense very recently ($z \sim 1$ in Fig. 5), it seems hard to avoid having such neutrino clusters.

Conclusions

The reason for the two severe problems of the massive neutrino scenario encountered in the previous section can be phrased as follows: the non-linear collapse of the neutrinos is too fast if we also want them to trigger galaxy formation at a redshift z_{GF} of at least 3. Still, galaxy formation may have some surprises in store for us theorists, so I would not dismiss massive neutrinos entirely.* There is the possibility, albeit a rather unattractive one, that galaxies originate from a different kind of seeds than the large-scale structure.

Cosmology provides an upper limit of ~100 eV for the neutrino mass and for neutrinos to be important dynamically requires a lower "limit" of several eV. It is intriguing that a neutrino mass in this narrow range could generate a mass scale (4,8) and structure (Figs. 4 c,d, and 5) corresponding to precisely the largest structures observed in the Universe. Although "ripple cosmology" would not dare to predict a neutrino mass of tens of eV, astrophysicists could probably live (and be happy) with it, were such a mass to be found in laboratory experiments.

Acknowledgments

I would like to thank Marc Davis for informative discussions over the years and to congratulate the organizers of this workshop, especially J. Tran Thanh Van and Orrin Fackler, for bringing about such a lively and interesting meeting.

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* Note that other dark matter candidates (gravitinos, axions...) also have their share of problems.

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