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Mental Arithmetic Efficiency: Interactivity and Individual Differences

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Abstract

Thinking efficiency as a function of interactivity was examined in a mental arithmetic task. Participants carried out single-digit additions, involving either 7 or 11 numbers, as fast and as accurately as possible. They completed the sums in blocks, five from the 'easy' set first, and five from the 'hard' set second. These sets were interpolated among a series of other tasks that measured numeracy, working memory capacity, visuo-spatial processing speed and attention switching, in such a way as to permit the presentation of the sets twice, once with each of the sums presented on a piece of paper and participants placing their hands flat on the table and once with the sums presented as a set of manipulable tokens. Efficiency was measured as the ratio of performance over time invested. A significant interaction between condition and difficulty was observed: Efficiency was slightly better in the static condition for easy sums but declined substantially relative to the interactive condition for hard sums. Regression analyses revealed that in the static condition 22% of the variance in efficiency for the harder sums was explained by numeracy and working memory capacity, but 45% by numeracy, working memory capacity and attention switching skills in the interactive condition. Verbal protocols revealed that paths to solution and arithmetic strategies were substantially transformed by the opportunity to manipulate tokens.

Keywords: Mental arithmetic, interactivity, efficiency, individual differences, distributed cognition

Introduction

Mental arithmetic is clearly an important skill with many quotidian applications. It is the quintessential example of what Kahneman (2011) calls "slow thinking": "(a) deliberate, effortful, and orderly" (p. 20) mental process that can be slowed down by a working memory busy holding information about interim steps and selecting strategies to proceed closer to the result. To be sure, for very simple arithmetic problems, answers are retrieved rather than computed; but as problem complexity increases, performance is constrained by limited internal resources.

The role of working memory in mental arithmetic is clearly revealed with experiments employing a dual-task methodology: Performance is significantly impaired by concurrent tasks that tax different components of working memory (e.g., Logie, Gilhooly, & Wynn, 1994). There is a substantial body of evidence that implicates working memory deficits and poor maths performance in primary school children (e.g., McLean & Hitch, 1999). In adults, the impact of maths anxiety (Ashcraft & Kirk, 2001) and test pressure (DeCaro, Rotar, Kendra, & Beilock, 2010) is explained in terms of the rehearsal and retrieval of performance related thoughts and memories that limit the working memory resources that can be committed to solving the problem.

Interacting with External Resources

When confronted with internal resource limitations, reasoners naturally mine their surrounding physical space for additional resources. "Artifacts saturate everyday environments" (Kirsh, 2009a, p. 284) and they are routinely recruited to supplement and augment internal cognitive resources. Within such an extended cognitive system (Wilson & Clark, 2009) internal and external resources are coupled by actions, producing a dynamic distributed problem representation. As a result, performance may surpass a level of accuracy and efficiency achievable on the basis of resources internal to the reasoner alone.

Recent experiments on insight and non-insight problem solving reveal how interactivity transforms performance. For example, release from mental set in Luchins's well known volume measurement problems is significantly facilitated when participants interact with actual jars with water (Vallée-Tourangeau, Euden, & Hearn, 2011). Additionally, insight in matchstick algebra problems is substantially enhanced when participants solve these problems with actual matchstick-like objects that permit the physical re-arrangement of the problem representation (Weller, Villejoubert, & Vallée-Tourangeau, 2011). Performance is facilitated by the affordances offered by a modifiable problem representation. In the case of mental set, the physically available resources are more easily perceived as offering simpler and less costly solutions (in terms of pouring and transposing) and help defuse mental set. As for matchstick algebra, the physical movement of a matchstick transforms the presentation of the problem which anchors new mental projections of potential solutions that in turn can be reified by additional physical modification. Insight is thus better driven by a concrete and explicit project-create-project cycle (Kirsh, 2009b).

Mental Arithmetic. As for mental arithmetic, recruiting artifacts, such as pen and paper, substantially augments performance largely because working memory content is nearly completely off-loaded onto the external environment. In this case, potential working memory limitations can be compensated by externalising the algorithmic process. While measures of working memory processing capacity may well be correlated with unaided mental arithmetic performance, these correlations would likely disappear when the process is completely externalised (or indeed delegated to a computational device). Thus, examining the role of interactivity in mental arithmetic may more fruitfully proceed in a cognitive system where reasoners cannot record subtotals and remainders in arriving at a solution, but still can interact and modify a physical problem representation (Neth & Payne, 2011).

The experiment reported here examined the impact of interactivity on mental arithmetic. Participants completed simple additions involving single-digit numbers. These additions were carried out for sets of 7 or 11 numbers. Thus one of the independent variables was problem difficulty. The second independent variable was interactivity. In one condition, participants completed the sums by looking at the set of numbers with hands down on the table in front of them. In a second condition, the sums were presented as a set of movable tokens: Participants were free to manipulate and re-arrange the tokens to arrive at a solution. Engineering an extended cognitive system such as the one created through interacting with number tokens may augment performance and enhance reasoning efficiency. The shaping and re-shaping of the physical representation of the problem may encourage and cue different paths to solution and different arithmetic strategies. Limited internal resources in the absence of interactivity may constrain the manner with which participants arrive at a solution.

Measuring efficiency involves assessing the benefits accrued as a function of cost or resources invested. An index of efficiency was calculated as the ratio of performance accuracy –proportion of correct answers– over the proportion of time invested to solve the problem out of the maximum time the slowest participants required to solve the task (Hoffman & Schraw, 2010, refer to such a measure as a likelihood model). Efficiency might be improved in an interactive context because some aspect of executive control is governed, guided and constrained by the shifting physical representation of the problem, freeing internal resources to ensure arithmetic accuracy. In other words, fewer resources are devoted to rehearsing subtotals or identifying and re-identifying the numbers to be added with a dynamic configuration of the sum to complete, enabling participants to devise more creative and efficient ways to solve the problems.

Finally, individual differences in terms of skills and working memory processing capacity were measured and correlated with performance in the different experimental conditions. Patterns of correlations can help understand more precisely how coupling of internal and external resources lead to better performance. Importantly, the experiment employed a repeated-measures design. Thus the same participants completed the easy and hard sets in both the static and interactive conditions: Between-subjects variance could not explain differences in performance across the experimental conditions.

Method

Participants

Forty two university undergraduates (35 females, overall mean age = 21.8, *SD* = 6.8) received course credit for their participation**.** Three additional participants (all females, mean age 23.0) were later recruited to provide verbal protocols while they performed the easy and hard sums in both conditions.

Material and Measures

Numeracy. Numeracy was measured using the subjective numeracy scale developed by Fagerlin, Zikmund-Fisher, Ubel, Jankovic, Derry, and Smith (2007) which consists of eight questions (such as “how good are you at calculating a 15% tip”). Participants answer using a 7-point scale (1 = “not good at all” and 6 “extremely good”). An objective measure of arithmetic skill was designed by having participants complete as many simple problems (such as 11 – 9 = ?) as they could in 60 seconds.

Visuo-spatial information processing speed. The clerical checking subtest of the Beta III (Kellog & Norton, 1999) was used to measure visuo-spatial processing speed. In this test, participants must identify whether two symbols, figures or strings of digits are identical or not. The measure is the number of correct judgments out of a possible 55 in a 2-min period.

Executive function: Shifting. Attention switching skills were measured using the plus-minus task (Miyake, Friedman, Emerson, Witzki, & Howerter, 2000). Using three different series of 30 double-digit numbers, participants were instructed to add 3 to each in the first series, subtract 3 to each in the second series, and alternate between adding and subtracting 3 with the third series. The switching cost, measured in seconds, was the difference in completion time for the third series minus the average completion time for the first two.

Working memory capacity. Working memory capacity was assessed with a modified reading span test. Sentences in series ranging in number from 3 to 6 were presented on index cards to participants which they read aloud. At the end of a series they were prompted to recall the last word of each sentence in that series. There were two different series for each sequence length for a total of 36 sentences. Working memory performance was measured as the total number of words recalled.

Arithmetic Task. Participants carried out single-digit additions, involving either 7 or 11 numbers (see Fig. 1), as fast and as accurately as possible They completed the problems in blocks, five from the ‘easy’ set first, and five from the ‘hard’ set second. Performance was measured as the proportion of correct sums, the mean absolute deviation from the actual sums, the mean latency to announce a solution, and in terms of efficiency. Efficiency was measured as the ratio of addition accuracy (proportion correct sums) over time invested in the task. The latter was measured as the proportion of actual time to complete the sums divided by the maximum time needed to complete them in that condition; this maximum was determined by taking the average of the top quartile latencies. A ratio smaller than 1 meant that proportion accuracy was smaller than proportion time invested, indicating inefficient performance.

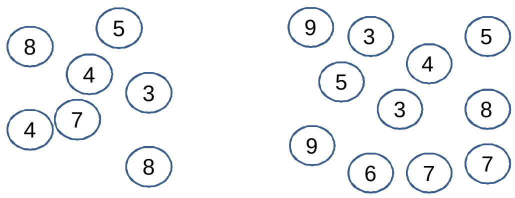

Figure 1: Examples of single-digit additions from the ‘easy’ set (7-digit additions) and the ‘hard’ set (11-digit additions). Participants performed 5 additions from both sets for a total of 10.

Procedure

Participants first completed the 8-item subjective numeracy scale, followed by the objective arithmetic test, the clerical checking subtest from the Beta III, and the plus-minus task. They were then presented with the five additions from the ‘easy’ set. After a 2-min distractor task (a word search puzzle), participants were presented with the five additions from the ‘hard’ set. These two sets of sums were presented twice to the participants. For one presentation participants performed the additions with their hands on the table facing them (the static condition) and announced their answer out loud; for the second presentation, numbered tokens (2-cm in diameter) were used, and participants were encouraged to move the tokens about in helping them add the numbers (the interactive condition); as in the static condition, participants announced the solution for each problem out loud. While the hard set always followed the easy set, the order of condition (static, interactive) was counterbalanced across participants. With 10 different problems, involving 10 unique configurations of digits, and 90 digits across the two sets, it was unlikely that participants remembered the solution to each problem when presented a second time. Still, to prevent a direct retrieval of solutions during the second presentation, the participants completed the reading span test which lasted approximately 10 minutes. After this test of working memory, participants were presented with the 10 sums again (either in the interactive or static condition depending on which they had experienced first). Thus set size (with two levels) and interactivity (with two levels) were independent variables that were manipulated within subjects in a 2x2 repeated measures design. The experimental session lasted approximately 45 minutes.

Results

The order of presentation of the interactivity conditions did not significantly influence performance on any of the dependent measures nor did set repetition: Performance on the first 10 sums was no different than performance on the second iteration of the same 10 problems within each experimental condition. Hence, order and repetition were not included in any of the analyses reported below.

Percent Correct

The mean percent correct solutions for the easy and hard sums are plotted in the top left quadrant of Figure 2. Interactivity did not influence performance for the easy sums, but substantially enhanced performance for the hard sums. In a 2x2 repeated measures analysis of variance (ANOVA), the main effect of condition was significant, $F(1, 41) = 6.58$, $p = .014$, as were the main effect of difficulty, $F(1, 41) = 20.9$, $p < .001$ and the interaction, $F(1, 41) = 12.5$, $p = .001$.

Absolute Error

Non-interactive mental addition did not lead to larger absolute deviations from the correct solution for the easy set, but did for the hard set (see top right quadrant of Fig. 2). In a 2x2 repeated measures ANOVA, the main effect of interactivity was significant, $F(1, 41) = 13.8$, $p = .001$, as was the main effect of difficulty, $F(1, 41) = 28.6$, $p < .001$; the more important pattern was the significant interaction between condition and difficulty, $F(1, 41) = 28.9$, $p < .001$.

Latency to Solution

Set size had a large impact on solution latencies (see Fig 2. bottom left quadrant). Interactivity influenced latencies in an interesting manner: For the easy sums, interactivity slowed down participants (by nearly 2.5 s), but marginally reduced latencies (by .4 s) with the hard sums. In a 2x2 repeated measures ANOVA, the main effect of interactivity was not significant, $F(1, 41) = 1.45$, $p = .236$, but the main effect of difficulty was significant, $F(1, 41) =$

182, $p < .001$, as was the interaction, $F(1, 41) = 6.64$, $p = .014$.

Efficiency

Participants were more efficient when solving the easy problems without the tokens (see bottom right quadrant of Fig. 2). Efficiency dropped marginally for the hard sums when participants could use the tokens, but dipped substantially without the tokens. In a 2x2 repeated measures ANOVA, the main effect of condition was not significant, $F < 1$, but the main effect of difficulty, $F(1, 41) = 13.3$, $p = .001$, as well as the condition by difficulty interaction, $F(1, 41) = 10.6$, $p = .002$, were significant.

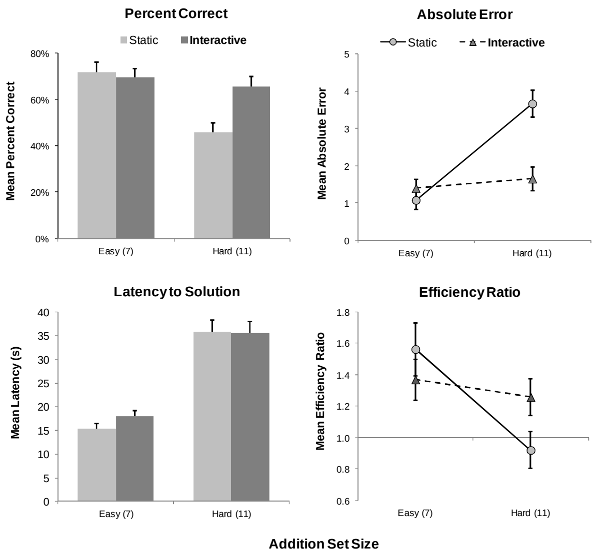

Figure 2: Mean percent correct additions in the static (light bars) and interactive (dark bars) condition (top left quadrant); mean absolute error per sum in the static (circles) and interactive (triangle) conditions (top right quadrant); mean latencies in the four conditions (bottom left quadrant); mean ratio of correct proportion over proportion of maximum time to complete problem (or efficiency ratio) in all four conditions (bottom right quadrant) as a function of set size. Error bars are standard error of the means.

Predictors of efficiency. To better understand the relative contribution of different internal resources to performance in the hard set of additions, initial analyses determined the nature of the correlations between efficiency and individual differences (see Table 1). The strongest correlations were observed in the interactive condition with objective numeracy, $r(40) = .43$, $p = .005$, and attention switching, $r(40) = -.39$, $p = .01$; in the static condition, objective numeracy was significantly correlated with efficiency, $r(40) = .32$, $p = .04$. A stepwise regression analysis for the static condition produced a significant model, $F(2, 41) = 5.40$, $p = .009$, composed of objective numeracy ($\beta = .400$) and reading span ($\beta = .350$) that explained 22% of the variance in efficiency. In the interactive condition, the analysis identified a significant model, $F(3, 41) = 10.2$, $p < .001$, that explained 45% of the efficiency variance; the model included objective numeracy ($\beta = .447$), reading span ($\beta = .426$) and attention switching ($\beta = -.398$).

Table 1: Correlation matrix involving individual differences in terms of subjective and objective numeracy, clerical checking, attention switching, reading span and the efficiency ratio in the static and interactive condition for the hard set (involving 11 single digit numbers); df = 40.

	1	2	3	4	5	6	7
	SBJ-N	OBJ-N	C-C	Att-S	Span	ER- S	ER-I
1	-	.47 **	.16	-.09	.16	.29	.38 *
2		-	.26	-.20	-.24	.32 *	.43 **
3			-	.04	-.03	.10	.16
4				-	.22	-15	-.39 *
5					-	.26	.23
6						-	.60 **
7							-

Note: * $p < .05$ ** $p < .01$. **SBJ-N** = Subjective numeracy; **OBJ-N**. = Objective numeracy (basic arithmetic skill); **C-C** = Clerical Checking; **Att-S** = Attention Switching; **Span** = Reading Span.; **ER-S** = Efficiency in the static condition; **ER-I** = Efficiency in the interactive condition.

Path to Solution and Strategies

In order to obtain a window onto the paths to solution and the strategies employed to chart these paths in both conditions, three additional participants completed the mental arithmetic tasks while verbalising their progress – the sessions were also videotaped. Inferential statistics could not be performed on data from such a small sample, but very clear differences in strategies emerged in the two conditions.

The simplest strategy, and in the static condition the one that taxes working memory the least, is to add the numbers in the order scanned, without seeking to group numbers to create more congenial sub-totals. Across the three participants, and over all problems, the sequential scan strategy was used exclusively 15 times in the static condition (or for 50% of the problems) and twice in the interactive condition. There were 26 instances of grouping numbers (mostly in pairs) on the path to solution in the static condition, but 75 instances of such groupings in the interactive condition. Congenial sub-totals (defined as Σ MOD 5 = 0) on the path to solution was observed 28 times in the static condition but 53 times in the interactive condition. Figure 3 below illustrates the paths to solution and strategies employed by participant 44 for problem A, a 7-number addition. She clearly employed a sequential scanning strategy in the static condition, but was much more creative in the interactive condition, grouping numbers to produce convenient sub-totals to arrive at the solution.

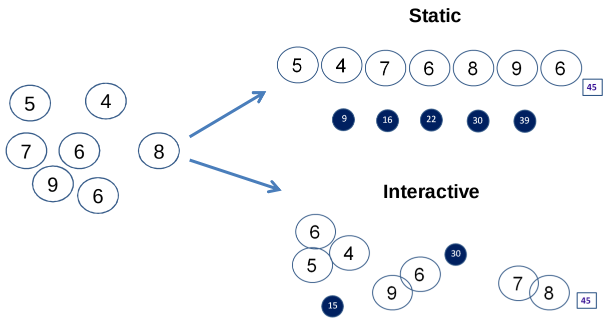

Figure 3: Path to solution and strategy employed for problem A (a 7-number problem) by participant 44 in the static and interactive condition.

Discussion

This experiment examined mental arithmetic in conditions where participants only used their internal cognitive resources to complete easy and hard sums of single digit numbers or where they could couple their cognitive resources to modifiable external resources in completing the sums. The experiment employed a repeated measures design such that the same participants completed the arithmetic problems in both conditions, thus eliminating between condition variance due to between-subjects differences. This is a particularly important feature of this experiment because it ensured that whatever benefits were conveyed in the interactive condition, these could not be attributed to better or different internal resources brought to the task by a different group of participants.

Interactivity substantially enhanced performance in terms of accuracy and efficiency with the harder sums involving 11 single-digit numbers: Participants were more accurate and the wrong answers were closer to the actual sums in the interactive condition than in the non-interactive condition. Solution latencies offered a gauge of the effort invested to solve the additions. With the hard sets the mean latencies were nearly identical between conditions (35.9 s vs. 35.5 s in the static and interactive condition, respectively) but mean percent accuracy was 20% higher in the interactive condition. Hence, reasoning efficiency was substantially enhanced by allowing participants to couple and regulate their cognitive efforts with a continuous reconfiguration of the tokens in a manner that best served their goal. With the easier sums participants performed marginally better without manipulating tokens, relying solely on their internal resources. The degree to which the design of an extended cognitive system can augment performance is clearly relative to the degree of task difficulty and the cognitive ability of the reasoner (Webb & Vallée-Tourangeau, 2009).

Interactivity offered the opportunity to deploy more creative and efficient paths to solution, which was clearly beneficial for the harder sums. The improvement in performance and the greater efficiency in the interactive condition was not simply a matter of off-loading content from working memory onto the environment. Rather, a shifting environment suggests different arithmetic paths and permits the identification of congenial interim sums that simplify the task and enhance efficiency. Thus the opportunity to interact with the tokens substantially transformed the nature of strategies employed and the paths to solution. Some of these paths might have been discovered strategically or accidentally by moving the tokens. Still, a dynamic physical presentation of the problem shouldered some of the executive functions freeing resources to better plan how to achieve the goal efficiently. These data support the conjecture that reasoners are better able to deploy arithmetic skills, and may be more receptive to learning new ones, in an environment that augments storage and processing capacity through the coupling of internal and external resources.

Individual Differences

Profiling participants in terms of cognitive skills and capacities and then correlating these measures with indices of performance help identify the cognitive factors that drive mental arithmetic. This approach has been employed with some success to identify the skills and capacities implicated in insight and non-insight problem solving (Gilhooly & Fioratou, 2009). The resulting data inform the development of process models of performance in these problems. Such process models will likely differ for tasks that are purely reliant on internal cognitive resources in comparisons with tasks that afford a tighter reciprocal influence between cognition, perception and action.

In the static condition, basic arithmetic skills and working memory capacity explained 22% of the variance in efficiency for the harder sums. In the interactive condition, nearly 50% of the variance in efficiency was explained by a model composed of arithmetic skill, working memory capacity and attention switching. These findings suggest that participants with better arithmetic skills, larger working memory capacity and swifter attention switching abilities were more likely to benefit from interacting with tokens in arriving at a solution. In other words, the coupling of internal and external resources was more effectively deployed by participants with better internal resources. This pattern of results was also observed in a recent experiment that contrasted non-interactive and interactive version of Luchins's volume measurement problems: Participants scoring higher in fluid intelligence performed better with the interactive version of the task (Vallée-Tourangeau et al., 2011). Designing an interactive version of an otherwise non-interactive static problem solving task does not benefit every reasoner in the same way. Future research should also determine whether non-intellectual factors such as anxiety or self-efficacy mediate the impact of interactivity on problem solving performance.

The measure of working memory capacity explained unique variance in performance efficiency in the non-interactive context for the hard sums. Still, the correlation between performance and working memory capacity was modest. This finding suggests one of two things. The first is that the task may not have taxed working memory that much. Certainly the degree of absolute departure from the correct answers in the non-interactive condition suggests that participants rarely miscalculated sums by a substantial margin. Future research may thus more fruitfully contrast non-interactive and interactive conditions with a more challenging arithmetic task, either by using larger single-digit sets (e.g., sums including 15 or more numbers) or by using double-digit numbers. A better window onto the role of interactivity in supplementing working memory capacity might be proffered by a task that is more reliant on working memory when it is completed without interaction. Second, the exact composition of the complex span measure of working memory should include arithmetic material and operations. There is evidence to suggest that span and outcome measures are better correlated when they share a domain (DeStefano & Lefevre, 2004).

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