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## ABSTRACT

The 323-Mev hardened bremsstrahlung beam from the Berkeley synchrotoron was used to produce electron-positron pairs and triplets in a 4-in.-diam liquid hydrogen bubble chamber. It was found that the experimental triplet cross sections for detectable recoils (momentum greater than 0.27 Mev/c) and for recoils with momentum greater than mc rise logarithmically with photon energy to 100 Mev, then level off at approximately 2.8 mb and 1.5 mb, respectively. The total triplet cross section agrees with that of Borsellino above 20-Mev photon energy. No contribution due to exchange terms was found. The positron energy distribution agrees with that of Wheeler and Lamb. The recoil momentum distribution agrees substantially with that of Suh and Bethe. Approximately one event due to multiple pair production was expected. None was found.

# ELECTRON TRIPLET PRODUCTION BY HIGH-ENERGY PHOTONS IN HYDROGEN

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## I. INTRODUCTION

Electron pair production by a photon in the Coulomb field of an electron, commonly called triplet production, is one of the major electromagnetic processes contributing to the absorption of energetic photons in light elements.

Bethe and Heitler[1] originally developed the theory of pair production in the nuclear Coulomb field, taking into account screening of this field by atomic electrons through use of the Fermi-Thomas model of the atom. Their method considered only the static nuclear Coulomb field and thereby neglected the effect of retardation on the nuclear Coulomb potential (due to nuclear recoil), which is negligible in the cases of hydrogen and heavier nuclei.

Perrin[2] was the first to point out the possibility of triplet production. He showed that in the laboratory system the threshold energy is $k = 4mc^2$, twice that for pair production, and estimated the cross section to be the same as that for a nucleus with Z = 1. After Perrin, many authors have contributed to this work, making a variety of approximations. Table I summarizes some of the details and Fig. 1 shows their results for the total triplet cross section as a function of photon energy.

Wheeler and Lamb developed triplet theory for high-energy photons along the lines of the Bethe-Heitler pair theory, properly taking Coulomb field screening into account. For hydrogen, they calculated the pair and triplet cross sections, using exact atomic wave functions for the screening effect, and their result supersedes that of Bethe and Heitler for that case. They neglected (in the triplet theory)

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the effect of retardation, the $\gamma$-e interaction, and the exchange effect (Pauli exclusion principle affecting the two negatons in the final state). But since these effects are important only for large momentum transfer (high-momentum recoils) where screening is unimportant, the Wheeler-Lamb screening correction can be applied to other results that treat the large recoils properly while ignoring screening. The screening correction for hydrogen given in Eq. (1) is a simple difference between the unscreened Bethe-Heitler result for hydrogen and the properly screened Wheeler-Lamb hydrogen triplet cross section.

Borsellino[4] was first to consider the effect of retardation on the triplet cross section. Retardation effects, a result of the motion of the recoiling "target" electron, become important at relativistic velocities. In the high energy limit his result, which does not contain screening, approaches the unscreened Bethe-Heitler cross section. Although he neglected the $\gamma$-e interaction and exchange effect, his retardation correction to the Bethe-Heitler result is of the same order as these effects (estimated by Joseph and Rohrlich) and suggests that they are probably negligible in the extreme relativistic limit also. His work served as the basis for a later calculation of the recoil distribution by Suh and Bethe.[5]

Votruba[6] was the first to formulate the theory exactly (in Born approximation; no screening). However, owing to the complexity of the equations he was forced at high energies to make an approximation limiting the validity of his results to the region of low-momentum recoils. Later, Joseph and Rohrlich[7] improved the accuracy of Votruba's results and showed that the cross section for low-momentum recoils is the same as that for pair production. It follows from comparing this with the results of Suh and Bethe that the effect of retardation, the $\gamma$-e interaction, and exchange terms are negligible for low-momentum recoils.

At moderately high energies, the total cross section $\Phi_t$ is expected to lie between the results of Borsellino and Votruba (after correction for screening);

$$\Phi_t^{(V)} - \text{(screening correction)} < \Phi_t < \Phi_t^{(B)} - \text{(screening correction)} \qquad (1)$$

where

$$\text{(screening correction)} = \Phi_p^{(\text{BHunscr})}(z=1) - \Phi_t^{(\text{WL})} ,$$

V = Votruba, B = Borsellino, BHunscr = Bethe-Heitler, Unscreened, and WL = Wheeler-Lamb. Upper and lower limits on $\Phi_t$ are expected to be valid in the range $k > 40$ Mev. Below 40 Mev the relativistic approximation is known to break down for pair production, and the same is probably true for triplet production. The upper limit is believed to be valid because there is reason to believe that the net effect of the $\gamma$-e interaction and exchange terms is to reduce the cross section.[7] The lower limit is believed to be valid because it is the cross section for recoils with $P_r \leq mc$; the neglected terms have negligible effect in this region.

The work of Suh and Bethe[5] shows that in the extreme relativistic limit $\Phi_t$ may be expected to approach the upper limit, which becomes $\Phi_t^{(\text{WL})}$. At lower energies, where the exchange effect cannot be neglected, $\Phi_t$ is expected to be nearer the lower limit.[7] Unscreened cross sections appearing in Eq. (1) are also given in Ref. 7.

Unfortunately, no one has yet calculated the recoil angular distribution, or made a quantitative estimate of the effect of the $\gamma$-e interaction and exchange terms on the high-momentum recoil part of the differential and total cross sections for the energy range of this experiment.

The total cross section for triplet production in hydrogen has been measured at several energies by Anderson et al.,[8] Moffatt et al.,[9] and Malamud,[10] using an absorption method.[11] In this method, the total cross section for all absorptive processes is measured and the triplet cross section is obtained by subtraction of the known experimental or theoretical cross sections for the Compton effect, pair production, etc. The method is not suitable for obtaining differential cross sections. Those results in the energy range of this experiment have been plotted in Fig. 1 along with the theoretical upper and lower limits, and the Bethe-Heitler pair cross section. They seem to indicate that the triplet cross section does indeed lie between the limits suggested earlier.

However, Hart et al.[12] have made a direct cloud chamber measurement of the cross section, which they found to be near to the Wheeler-Lamb result at low energies where they see almost all of the recoils. Their low-energy results are also plotted in Fig. 1. At higher energies, where the shortest recoils are not detectable, their cross sections for triplets with visible recoils are in reasonable agreement with theory of Suh and Bethe.

It was intended that this experiment should more precisely determine the triplet total cross section, and determine the differential angular and momentum distributions of the recoil electron. Also, the $\gamma$-e interaction and exchange terms were not expected to be completely negligible in this energy range; some indication of the magnitude of these terms was expected.

Hydrogen is an ideal target because the ratio of pairs to triplets is a minimum, the atomic binding of the electron has little effect on the process and is negligible below 100 Mev, and the effect of atomic screening has been accurately calculated by Wheeler and Lamb.

## II. APPARATUS AND PROCEDURE

Figure 2 shows the arrangement of apparatus used to produce a clean "hardened" 323-Mev photon beam incident on the 4-inch bubble chamber. Details of apparatus and the bremsstrahlung spectrum are presented elsewhere.[13, 14]

A Cornell-type thick-wall ionization chamber was placed behind the bubble chamber in the position shown in Fig. 2 and was used as the primary monitor for the experiment. Its calibration was corrected for the hardened spectral shape. The average photon flux through the bubble chamber was 124-equivalent quanta (323-Mev) per picture.

## III. MEASUREMENT OF PHOTOGRAPHIC TRACKS

More than 16,000 stereoscopic pictures were taken during the bubble chamber run. All pictures were scanned for events at least twice. All events were identified by at least two persons, and the measured scanning efficiency was greater than 99%.

Sufficient measurements were performed on most events to allow calculation of the photon energy, pair member and recoil energies, recoil angle, and the position of the event in the chamber. Particle track curvatures in the 7.4-kG pulsed magnetic field were measured by visually fitting the optically projected track photographs to a set of curve templates with curvatures at 10% intervals. Energies of the shortest tracks were determined from their range. Data were processed on an IBM 650 computer. Detailed disucssion of technique is presented elsewhere.[14]

## IV. RESULTS AND DISCUSSION

### A. Cross Section for Visible Recoils

#### 1. Relative Cross Section

The ratio of the experimental triplet and pair cross sections is plotted versus photon energy in Fig. 3(a).

The ratio of the two cross sections is of particular interest because it is independent of several factors that contribute as sources of error for the cross sections individually, e.g., the shape of the spectrum, the monitor calibration, the density of hydrogen, etc.

#### 2. Absolute Cross Section

The experimental cross section $\Phi_t^{exp}$ for detectable recoils, and for recoils with momentum greater than mc, are presented in Fig. 3(b). The "average" minimum detectable momentum was determined from a delta-ray count to be 0.27 Mev/c.

Leveling off of the triplet cross section above 100 Mev explains the decrease in the triplet-to-pair ratio in the same region. It may be understood qualitatively on the basis that as photon energy increases, interactions at larger impact parameters (which produce smaller recoil momenta) contribute increasingly to the cross section. A point is finally reached where increase in the cross section is primarily due to the impact parameters, which result in undetectable low-momentum recoils. Then the observed part of the cross section is seen to level off.

## B. Total Cross Sections

### 1. Pair Plus Triplet Cross Section

The total triplet cross section $\Phi_t(k)$ could not be obtained directly from this experiment, of course, because the minimum recoil momentum in triplet production, $P_{min} = 2(mc^2)^2/k$ Mev/c, was much smaller than the lower limit of momenta detectable in the chamber, 0.27 Mev/c. Some triplets therefore appeared to be pairs. The sum of the experimental pair and triplet cross sections $\Phi_{p+t}^{exp}(k)$ is given in Fig. 3(c). Also shown for comparison are two corresponding theoretical curves obtained by adding the theoretical pair cross section $\Phi_p^{(WL)}$ to the theoretical upper and lower limits on the triplet cross section. The theoretical limits are uncertain below 40 Mev because of the relativistic approximation in the triplet cross section. It is seen that they are in good agreement with this experiment.

### 2. Triplet Cross Section

The total triplet cross section for this experiment was obtained from the relation

$$\Phi_t \doteq \Phi_{p+t}^{exp} - \overline{\Phi}_p^{(WL)}, \qquad (2)$$

where $\overline{\Phi}_p^{(WL)}$ was the screened Wheeler-Lamb pair cross section, averaged over the photon energy interval. The Wheeler Lamb pair cross section was chosen because its screening correction is based upon exact atomic wave functions rather than the Fermi-Thomas model. The results are given in Fig. 3(d).

The magnitude of the γ-e interaction and exchange terms was found to be negligible within our statistical errors by fitting the 18 data points above 20 Mev to a cross section of the form

$$\Phi_t = \Phi_t^{(B)} - \left[\Phi_p^{(BHunscr)}(Z = 1) - \Phi_t^{(WL)}\right] - (B \alpha r_o^2 mc^2/k) \ln (2k/mc^2), \qquad (3)$$

where B is an adjustable parameter giving the magnitude of the γ-e and exchange terms. The first two terms on the right-hand side of Eq. (3) are just the theoretical limit to $\Phi_t$, i.e., Borsellino's cross section corrected for atomic screening. The last term is the approximate functional form due to the γ-e interaction and exchange terms, which Borsellino neglected.[7] A least-squares analysis gave B = 2.4 ± 7.4, with a value of 1.4 for $\chi^2$ divided by the number of degrees of freedom. The curve given by Eqs. (3) and (4) is shown in Fig. 3(d) along with the triplet cross section obtained from Eq. (2), and $\Phi_t^{(WL)}$. This experiment is therefore in agreement with the theory of Borsellino,[4] for energies greater than 20 Mev.

## C. Partition Function

The positron energy distribution, or partition function, $d\Phi/df_+$, is a measure of the relative energy sharing between the member particles of an event. The partition variable $f_+$ is defined by

$$f_+ = (E_+ - mc^2) / (k - 2\,mc^2), \qquad (5)$$

where $0 \leq f_+ \leq 1$ for all k and $E_+$, and $f_+$ is equal to the fraction of the available kinetic energy that is carried away by the positron.

The pair and triplet partition functions averaged over all photon energies k are shown in Fig. 4. The apparent asymmetry in the pair partition function was an instrumental effect due to track distortion caused by turbulence in the bubble chamber. For this reason, rather than investigate the triplet partition function alone, it was more meaningful to calculate the triplet-to-pair ratio $(d\Phi_t/df_+)/(d\Phi_p/df_+)$. This ratio is expected to be independent of systematic measurement errors.

The only theoretical partition function available for comparison is that of Wheeler and Lamb,[3] expected to be correct in the extreme relativistic limit. The Wheeler-Lamb triplet partition function is nearly equal to that for pair production in the energy range of this hydrogen experiment because of the near absence of atomic screening. Thus, the experimental results are to be compared with a theoretical ratio very near to unity for all $f$ and $k$. The experimental triplet-to-pair partition-function ratio was calculated for six intervals of photon energy, and for each interval the results were found consistent with a uniform distribution. The experimental ratio, averaged over all photon energies, is shown in Fig. 5. A comparison of the results with a uniform distribution (unity) gave a value of 1.3 for $\chi^2$ divided by the number of degrees of freedom. It was concluded that, within the accuracy of this experiment, the triplet partition function is not significantly different from the pair partition function, except for a multiplicative constant.

### D. Recoil Momentum Distribution

The observed differential recoil momentum distribution $d\Phi_t(k)/dP_r$, for six ranges of photon energy is given in Fig. 6. Because the shape of the recoil momentum distribution was practically independent of the photon energy, it was possible in Fig. 7 to combine all energies and include an additional group of events of which only the recoils were measured. At each energy, and in the combined results, it was necessary to take into account the fact that the low-energy photons could not contribute to the bins having larger recoil momentum.

The theoretical recoil momentum distribution of Suh and Bethe is valid for photon energies greater than 100 Mev.[5] Their curve for $k = 100$ Mev is shown in in each figure along with the experimental values. The theoretical curve has been arbitrarily normalized to fit the experimental results at $P_r = 1$ Mev/c.

The two bins of lowest recoil momentum are not expected to agree with the theory because of the uncertain detection efficiency below 0.32 Mev/c.

Agreement between experiment and theory is very good in the photon energy ranges above 92 Mev and in the combined results, while below that energy the theoretical values can be seen to be slightly too high in the region of large recoil momentum. The thoery must be considered to be approximate in the latter region.

### E. Recoil Angular Distribution

The angular distribution $d\Phi_t/d\theta_r$ of events in 5-deg intervals of recoil angle $\theta_r$ is given in Fig. 8. All photon energies have been combined because the angular distribution was nearly independent of that quantity.

The small recoil angles corresponded to large momenta and the large recoil angles corresonded to small momenta. At the position of the peak in the angular distribution, it was found that the principal contribution was made by recoils of approximately 1 Mev/c momentum.

The position of the peak, and the shape of the distribution, is believed to have been strongly influenced by the value of the minimum detectable momentum, especially at the larger angles. Unfortunately, there is no theoretical angular distribution with which the results may be compared.

### F. Multiple Pair Production

Since there are two extra electromagnetic vertices in the lowest-order Feynman diagrams for double-pair production, one would expect the cross section to be reduced by a factor of $(1/137)^2$, relative to the sum of the pair and triplet cross sections. There was a total of approximately 24,000 pairs and triplets in the film scanned for this experiment. Thus, one would "expect" to find one double-pair event in this experiment.

This experiment contained no conclusive evidence for the existence of double-pair production in hydrogen.

## V. Summary

Electron-positron pairs and triplets were produced by a 323-Mev hardened bremsstrahlung beam in a 4-inch liquid hydrogen bubble chamber. Measurements and analyses were performed on 5417 triplets and 4019 pairs of the approximately 24,000 events photographed. The results may be summarized as follows:

(a) The experimental triplet-to-pair ratio was 0.291 ± 0.0097. It was approximately constant below 100 Mev, but decreased above that value. The decrease was due to a leveling off of the observed fraction of the triplet cross section (for which $P_r > 0.27$ Mev/c).

(b) The total pair-plus-triplet cross section was consistent with the upper and lower limits expected from the theory. The results are in best agreement with the theory of Borsellino.[4]

(c) No exchange effect was observed; if the contribution to the Borsellino triplet cross section of the exchange terms is taken to be of the form $-(B\, \alpha\, r_o^2\, mc^2/k)\ln(2k/mc^2)$, then $B = 2.4 \pm 7.4$.

(d) The partition function agreed with that of Wheeler and Lamb.[3]

(e) The recoil momentum distribution agreed substantially with that of Suh and Bethe.[5] However, the theoretical values are slightly too large in the region of large recoil momentum.

(f) The recoil angular distribution was roughly triangular in shape, with a peak at approximately 50 deg. Large recoil momenta (greater than 1 Mev/c) were predominantly on the small-angle side of the peak.

(g) No event was found positively identifiable as a double pair. Approximately one event was expected.

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## TABLE I

Results for total triplet cross section as a function of photon energy.

| Authors | Includes | Neglects |
|---|---|---|
| Bethe-Heitler[1] | Nuclear pair cross section:<br>(a) partition function;<br>(b) total cross section.<br>Born approximation.<br>Screening, Fermi-Thomas model<br>(unscreened result also given). | Retardation.<br>γ-p interaction |
| Wheeler-Lamb[3] | Triplet and nuclear pair cross sections:<br>(a) Partition function;<br>(b) Total cross section.<br>Born approximation<br>Screening:<br>(a) Hydrogen pairs, triplets, both by exact atomic wave functions;<br>(b) $Z > 1$ by Fermi-Thomas model. | Retardation.<br>γ-e, γ-p interaction<br>Exchange. |
| Watson[16] | Triplet total cross section,<br>Born approximation.<br>Exchange. | Screening.<br>Retardation.<br>γ-e interaction. |
| Borsellino[4] | Triplet total cross section.<br>Born approximation.<br>Retardation. | Screening.<br>γ-e interaction.<br>Exchange. |

Table I (continued)

| Authors | Includes | Neglects |
|---|---|---|
| Votruba[6] | Triplet total cross section.<br>Born approximation.<br>Retardation, γ-e interaction, Exchange } for low-energy photons only ($k \ll 3$ Mev).<br>At high photon energy, considers only low-momentum recoils. | Screening.<br>At high photon energy, neglects high-momentum recoils (for this approximation retardation, γ-e interaction, and exchange are negligible). |
| Joseph and Rohrlich[7] | Good review article.<br>Integrated Votruba expression exactly to find low-momentum recoil spectrum. | |
| Suh and Bethe[5] | Triplet total cross section.<br>Triplet recoil momentum spectrum.<br>Born approximation.<br>Retardation.<br>Shows Joseph and Rohrlich recoil spectrum is identical to nuclear recoil spectrum. | Screening.<br>γ-e interaction.<br>Exchange. |

## FIGURE LEGENDS

Fig. 1. Results of previous measurements of the total triplet cross section, $\Phi_t(k)$, in the energy range of this experiment. The experimental points are those of Anderson et al.,[8] Hart et al.,[12] and Moffatt et al.[9] Also shown are the theoretical cross sections of Watson[16] (Curve D), Borsellino[4] (Curve B), and Votruba[6] (Curve C), corrected to include screening; and the Wheeler-Lamb triplet and Bethe-Heitler[15] pair cross sections for hydrogen (Curves A and E).

Fig. 2. Schematic diagram of experimental arrangement used during bubble-chamber runs. Peak energy of the bremsstrahlung spectrum was 323±3 Mev. There was 1.36 radiation length of beam-hardening material between the Pt target and the bubble chamber. Helmholtz coils on the bubble chamber were pulsed to a peak field of 7.4 kG.

Fig. 3. (a) The ratio of the experimental triplet and pair cross sections, $\Phi_t^{\exp}(k)/\Phi_p^{\exp}(k)$. There is a ±2.3% std. error in the normalization, in addition to the counting statistics shown.

(b) The upper and lower curves shown were calculated from the theory of Sah and Bethe[5] for minimum recoil momenta greater than 0.27 Mev/c and mc, respectively.

(c) The experimental triplet-plus-pair cross section, $\Phi_{p+t}^{\exp}(k)$. There is a ±3.3% std. error in the normalization, in addition to the counting statistics shown. Also shown are the sum of the Bethe-Heitler[15] and Borsellino[4] cross sections (Curve A) and the sum of the Bethe-Heitler and Votruba[6] cross sections (Curve B), i.e., theoretical upper and lower limits, respectively. The Wheeler-Lamb screening correction[3] is included in the curves.

(d) The total triplet cross section $\Phi_t(k)$, obtained from the subtraction of the Wheeler-Lamb screened pair cross section from $\Phi_{p+t}^{\exp}(k)$. The statistics shown include all sources of error. Also shown are the Wheeler-Lamb triplet cross section (Curve A) and the Borsellino cross section--corrected to include screening--minus the term $(2.4 \pm 7.4)(\alpha\, r_0^{2})(mc^2/k)\ln(2k/mc^2)$, wherein the numerical coefficient was determined by a least-squares fit to the data (Curve B).

Fig. 4. (a) The distribution $d\Phi_t/df_+$ of 4874 triplets according to the fraction $f_+$ of available kinetic energy received by the positron, $(E_+ - mc^2)/(k - 2mc^2)$. The unweighted average of the points has been normalized to unity.

(b) The normalized distribution $d\Phi_p/df_+$ of 4019 pairs. The observed asymmetry was due to an instrumental effect. All photon energies have been combined.

Fig. 5. The ratio $(d\Phi_t/df_+)/(d\Phi_p/df_+)$ calculated to eliminate instrumental effects. All photon energies have been combined.

Fig. 6. The recoil momentum distribution $d\Phi_t(k)/dP_r$ plotted versus the momentum of the recoil electron, $P_r$, for six intervals of photon energy. The curve shown is that calculated by Suh and Bethe for a photon energy of 100 Mev. It has been arbitrarily normalized to agree with the data in the region of 1.0 Mev/c. The two lowest-momentum points are not expected to agree with the theory owing to a decrease in the detection efficiency in the region below 0.32 Mev/c.

Fig. 7. The recoil momentum distribution $d\Phi_t/dP_r)$ of 5417 triplets. All photon energies have been combined. The curve is that calculated by Suh and Bethe for 100 Mev, normalized to fit the data at 1.0 Mev/c. The two lowest-momentum points are not expected to agree with the theory because the scanning efficiency rapidly becomes small for $P_r < 0.32$ Mev/c.

Fig. 8. The angular distribution of the recoil electron, $d\Phi_t/d\theta_r$, versus the polar angle of recoil, $\theta_r$, showing the number of events per 5-deg interval. All photon energies have been combined. Recoil momenta greater than 1.0 Mev/c predominate on the small-angle side of the peak.

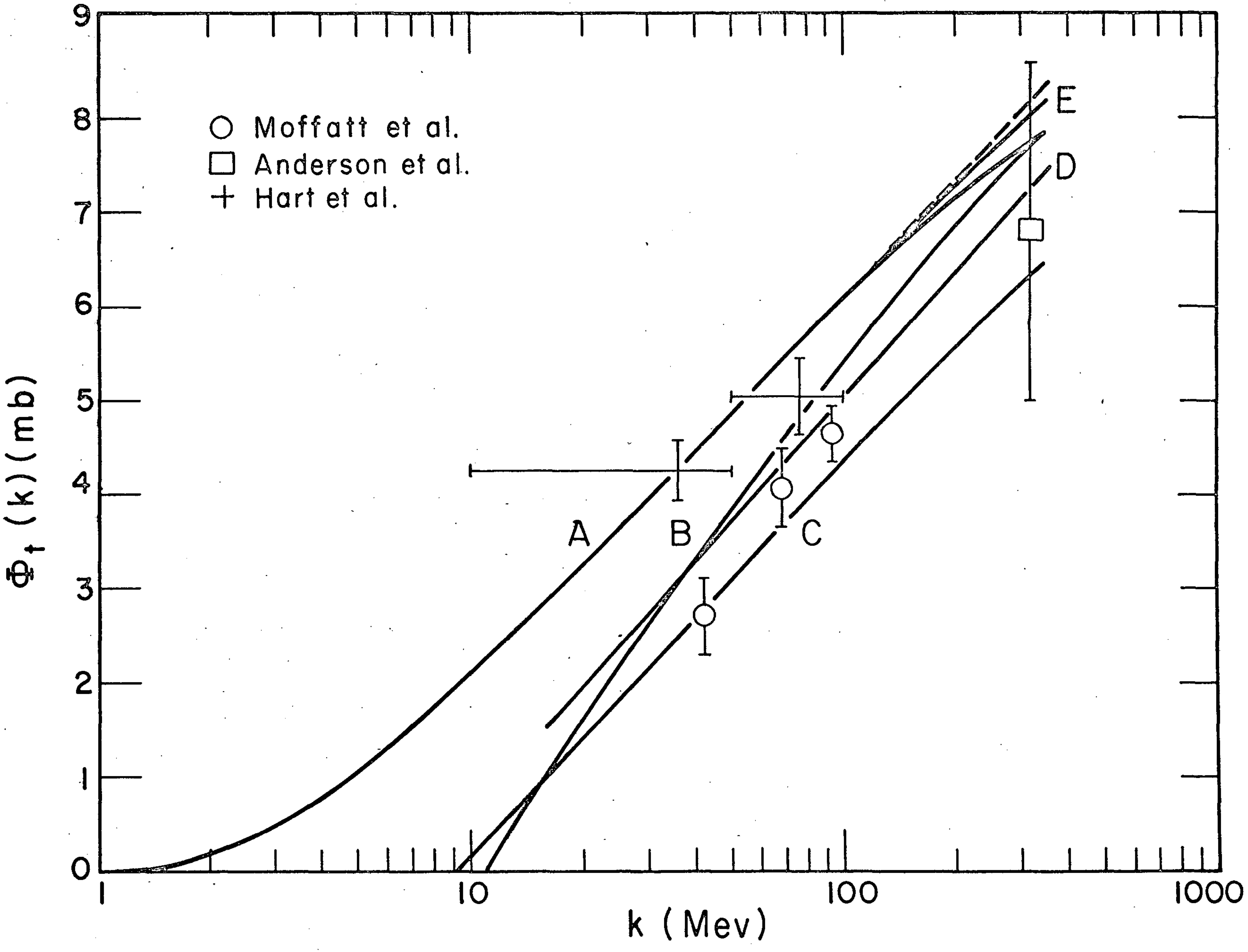
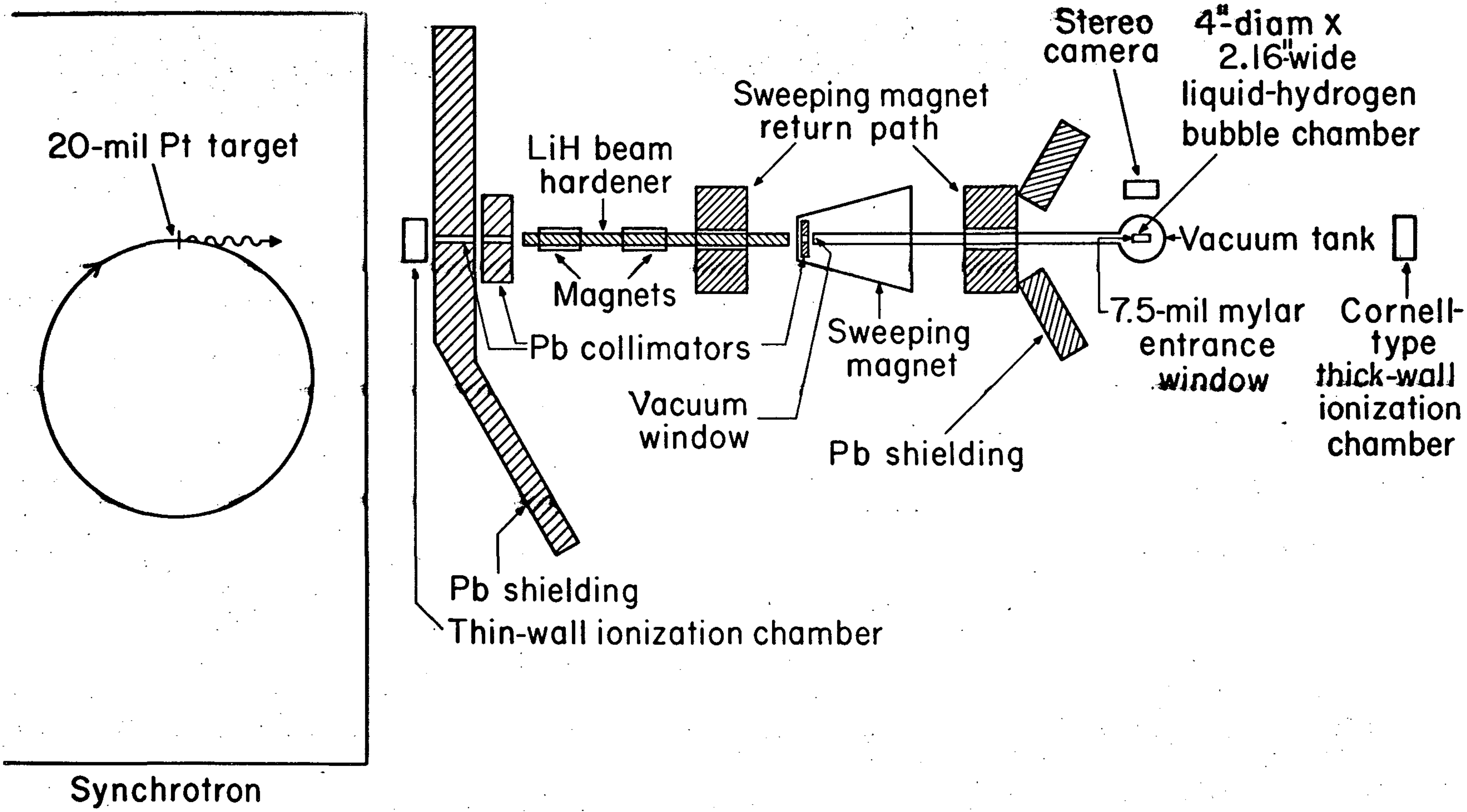


Fig. 1.

MU-21587



Mu-21780

Fig. 2.

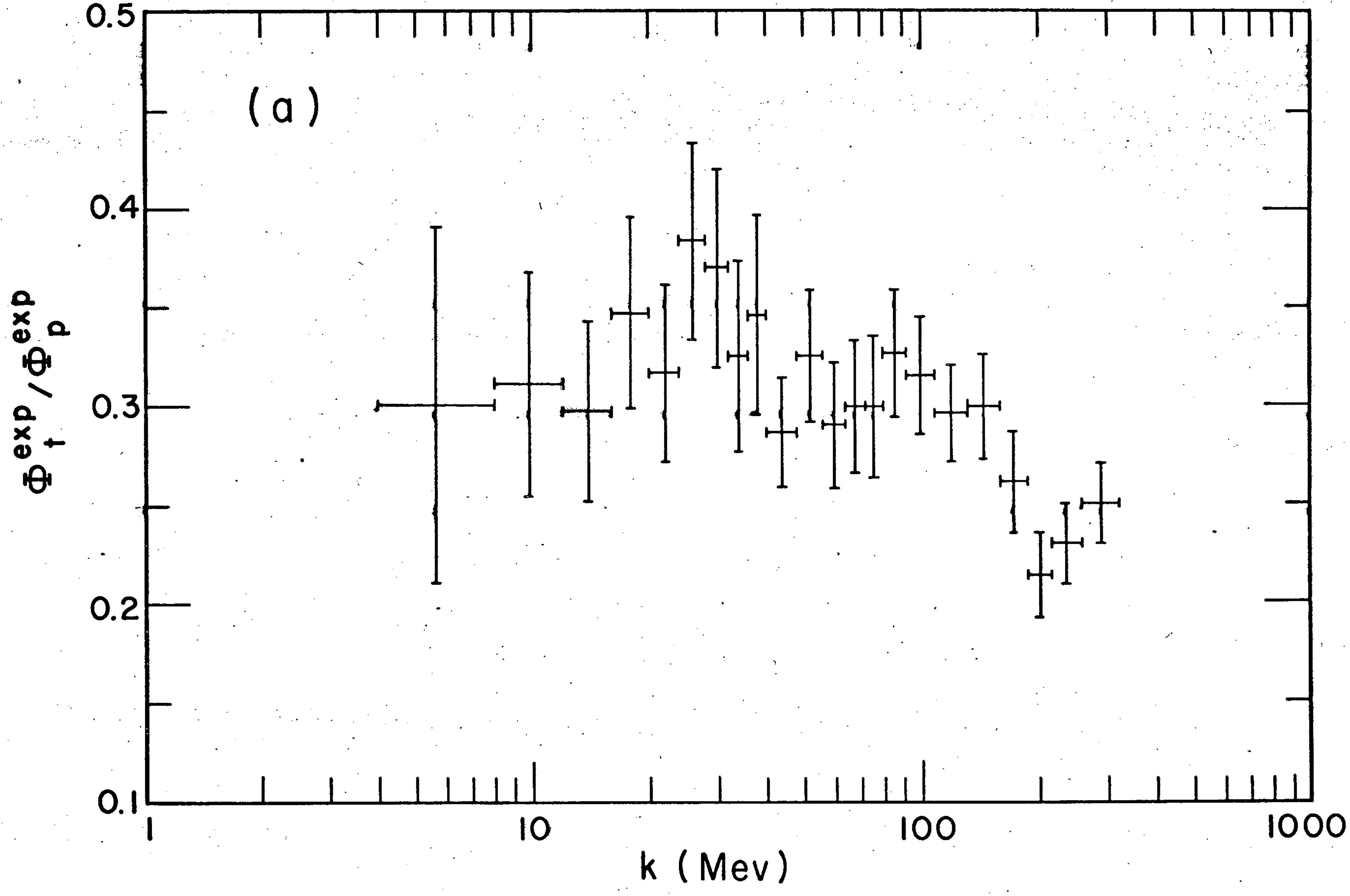


Fig. 3. (a)

Mu-21594

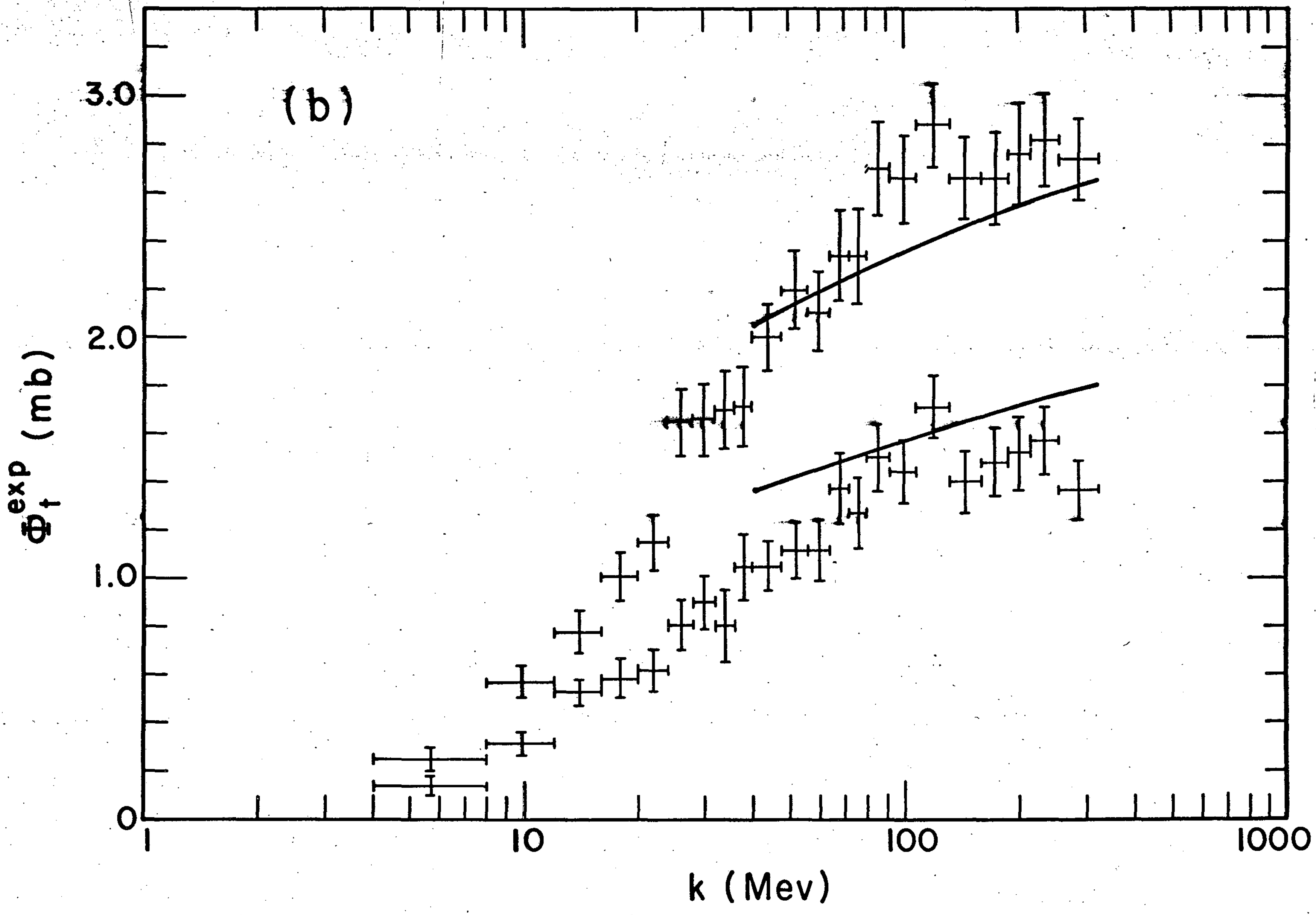


Fig. 3. (b)

Mu-21595

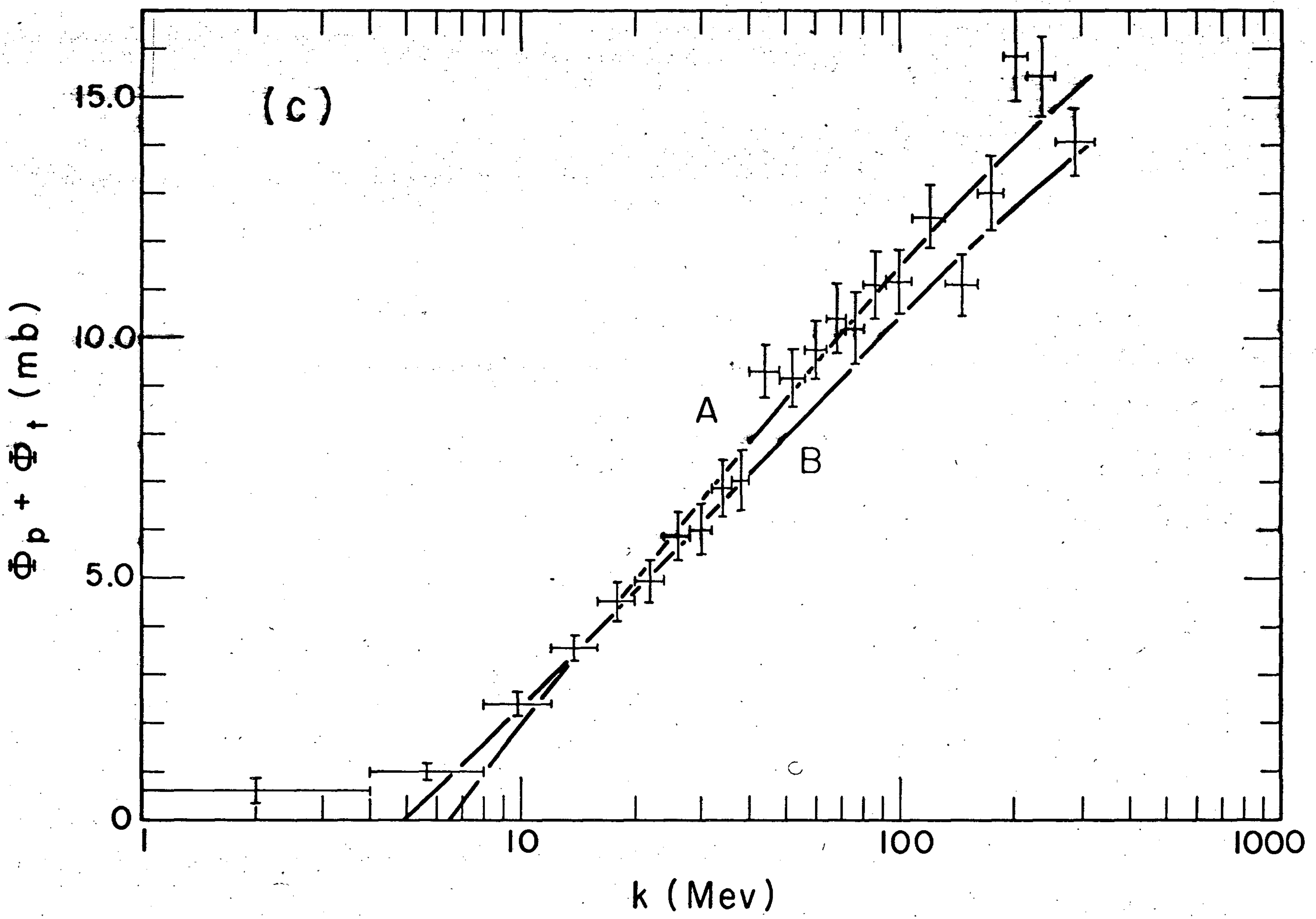


Fig. 3. (c)

Mu-21596

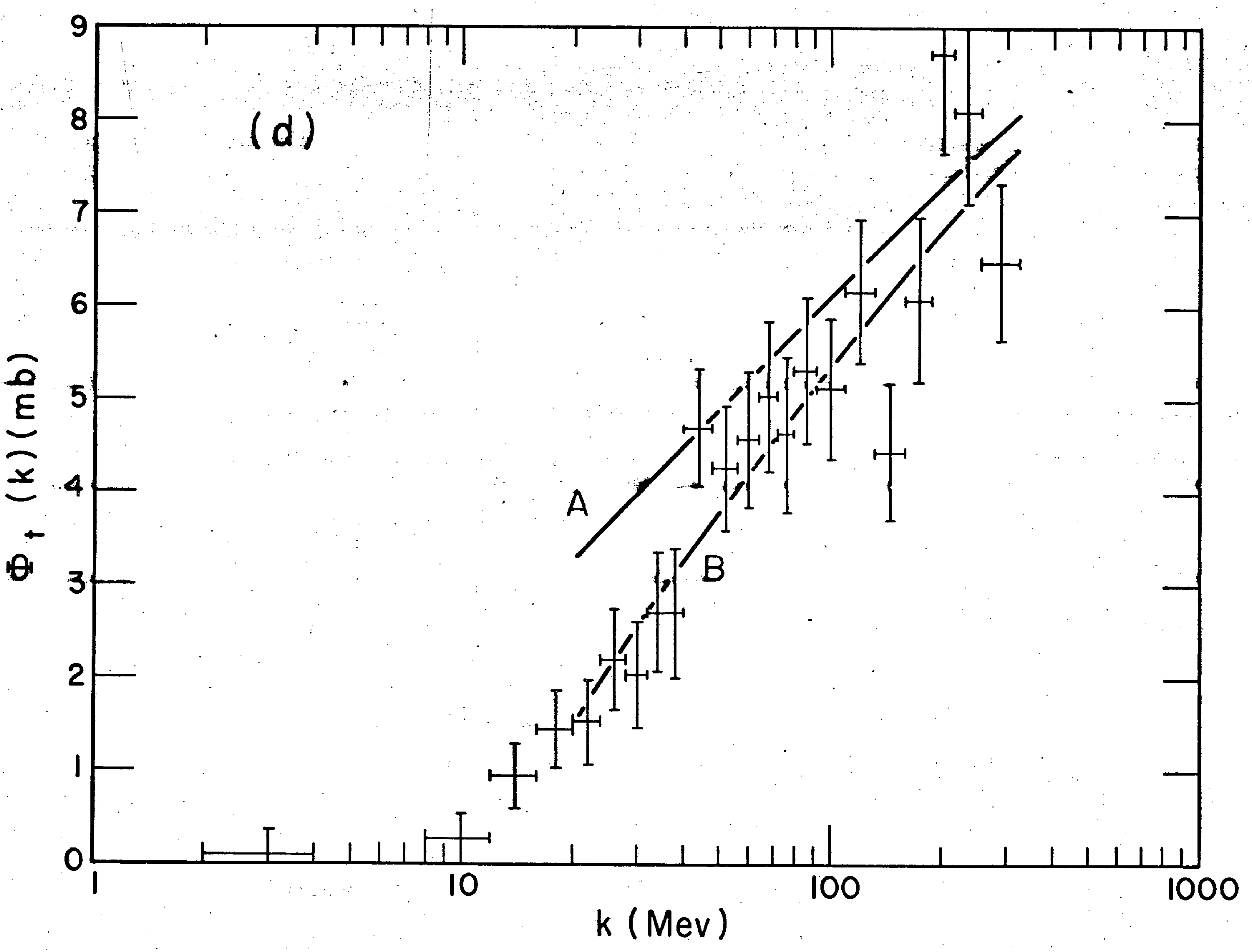


Fig. 3. (d)

Mu-21597

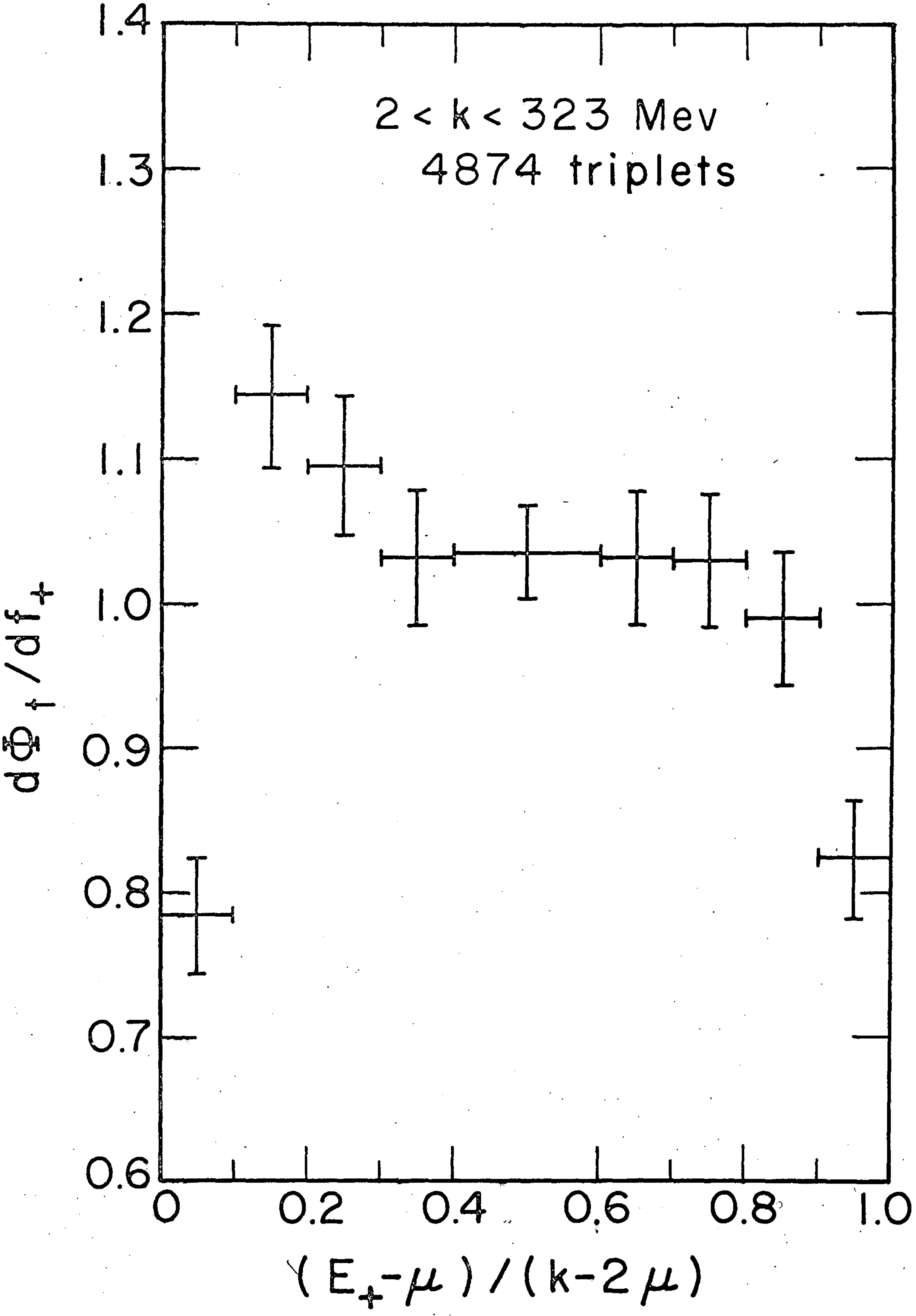


Fig. 4. (a)

Mu-21598

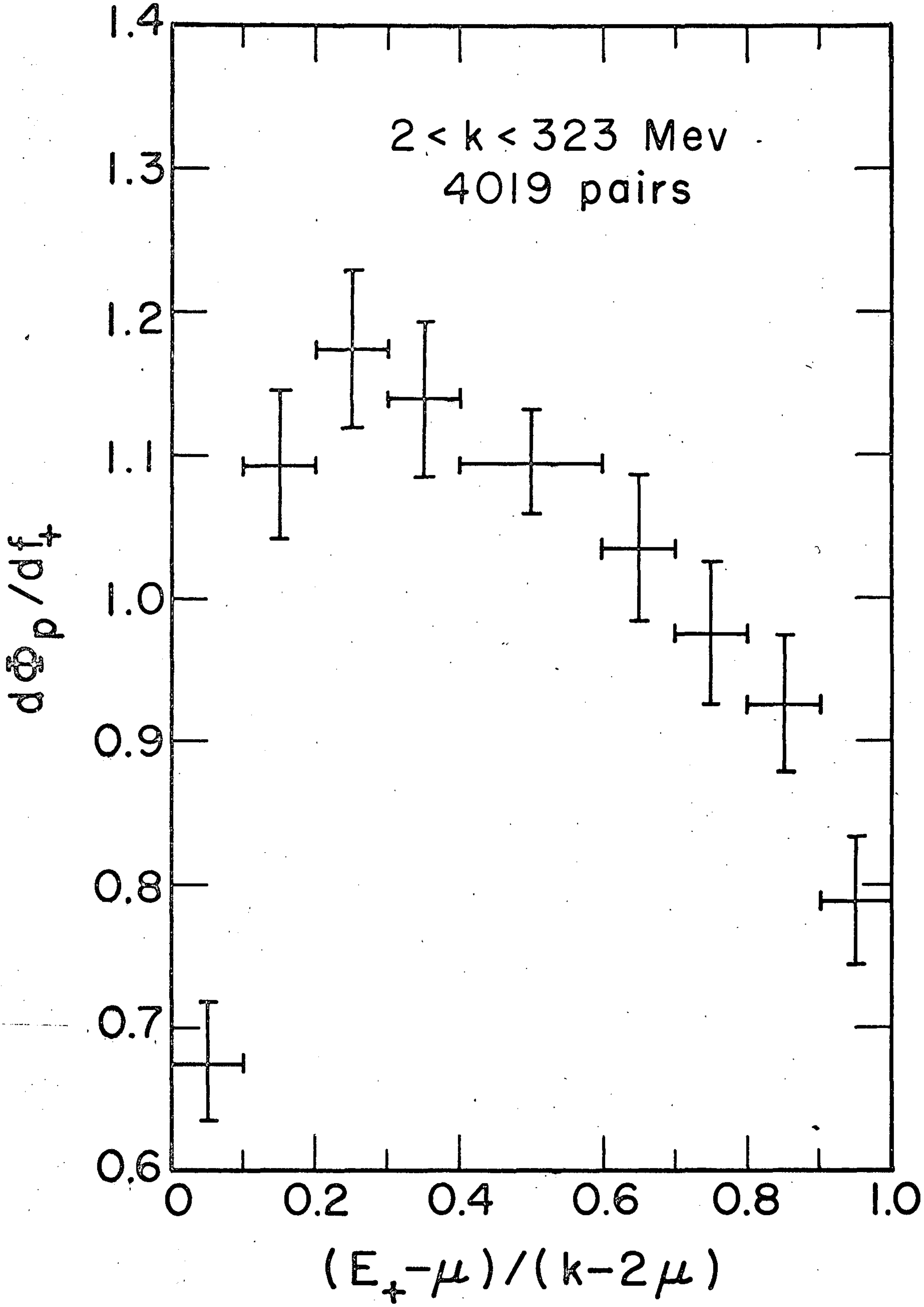


Fig. 4. (b)

Mu-21599

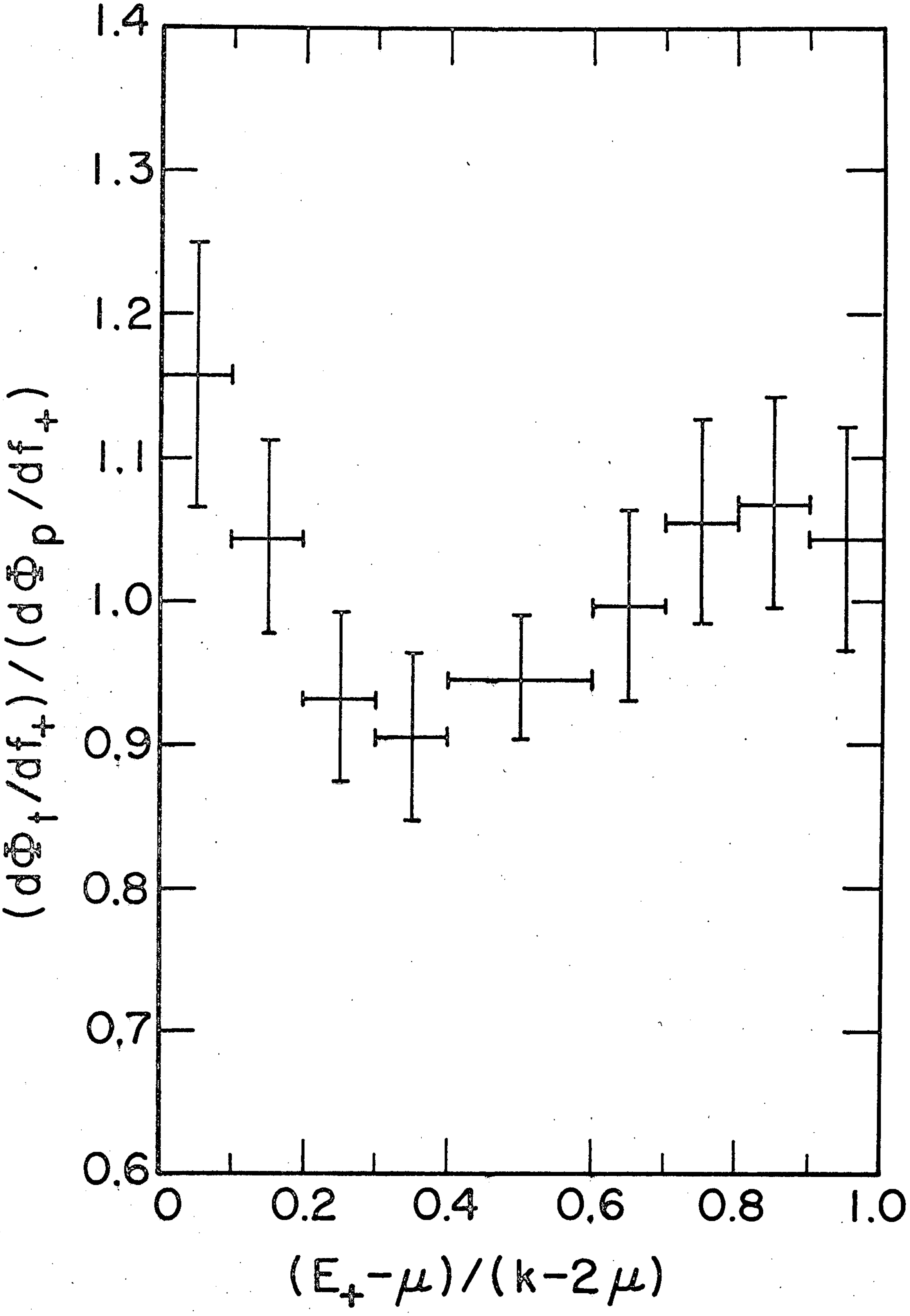

1.4
1.3
1.2
1.1
1.0
0.9
0.8
0,7
0.6
0
0.2
0.4
0,6
0.8
1.0
$(d\Phi_t/df_+)/(d\Phi_p/df_+)$
$(E_+-\mu)/(k-2\mu)$

Fig. 5.

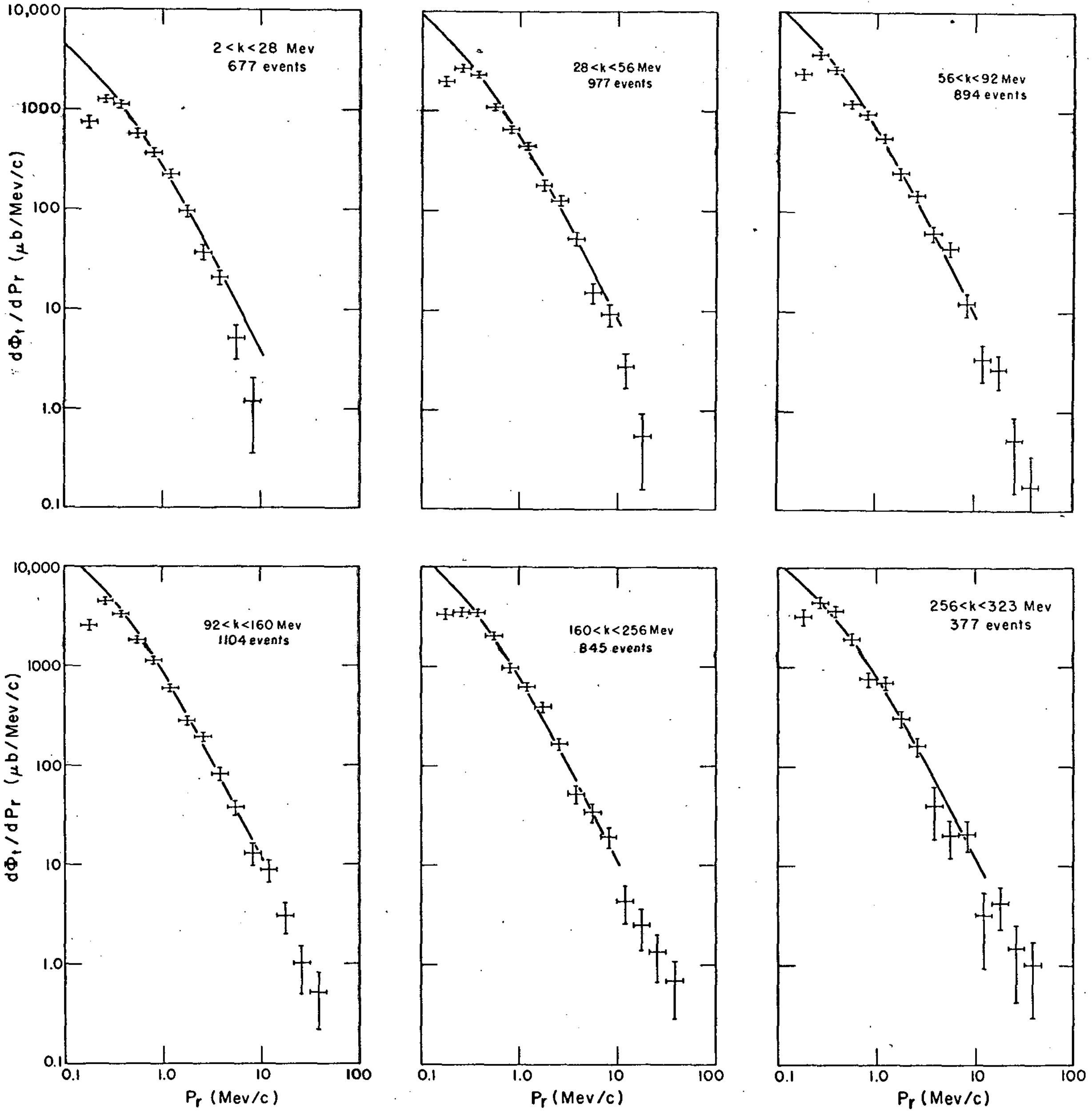

10,000
1000
100
10
1.0
0.1
$d\Phi_t / dP_r$ ($\mu$b/Mev/c)
2<k<28 Mev
677 events
28<k<56 Mev
977 events
56<k<92 Mev
894 events
92<k<160 Mev
1104 events
160<k<256 Mev
845 events
256<k<323 Mev
377 events
0.1
1.0
10
100
$P_r$ (Mev/c)

Fig. 6.

Mub-521

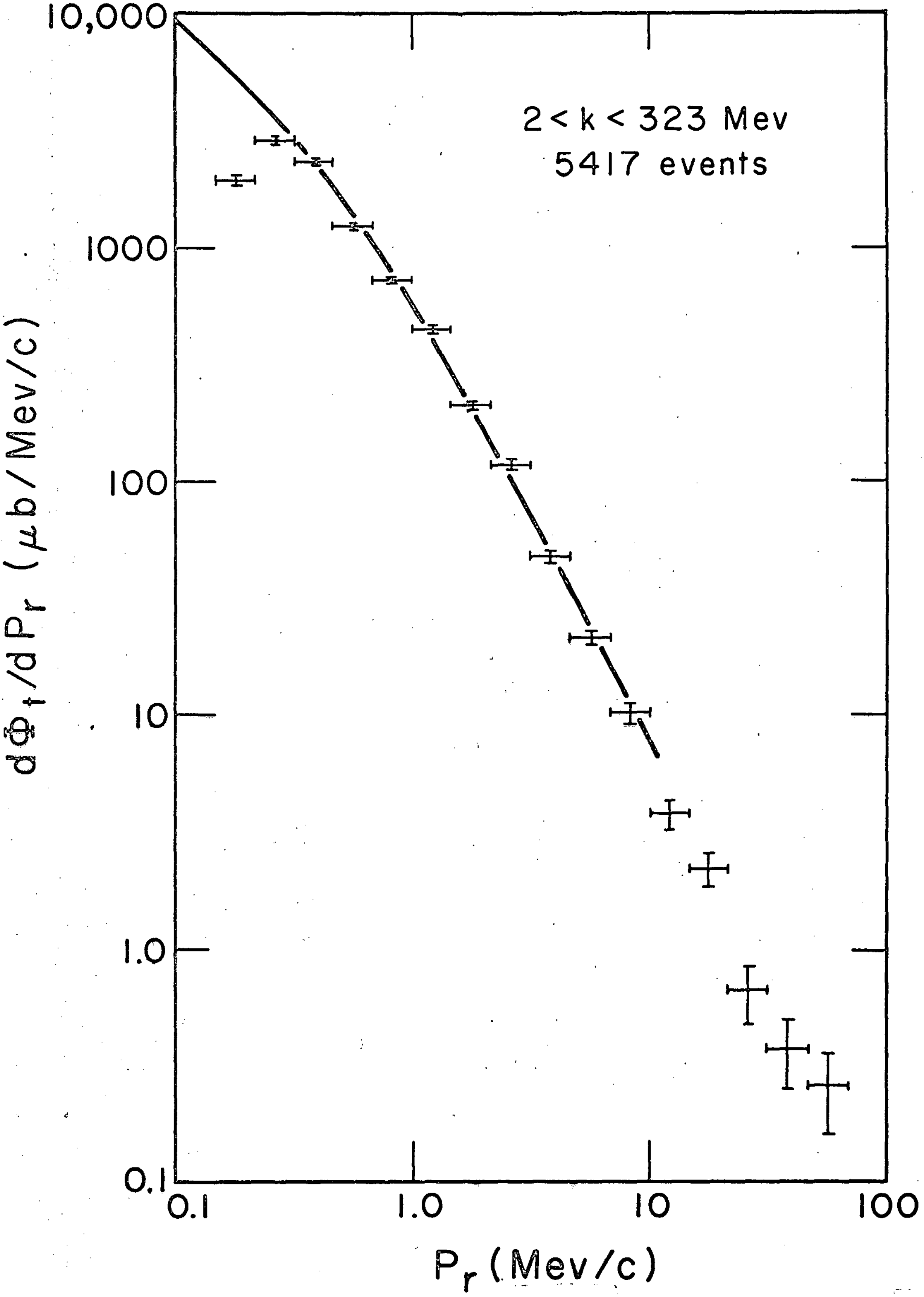


Fig. 7.

Mu-21613

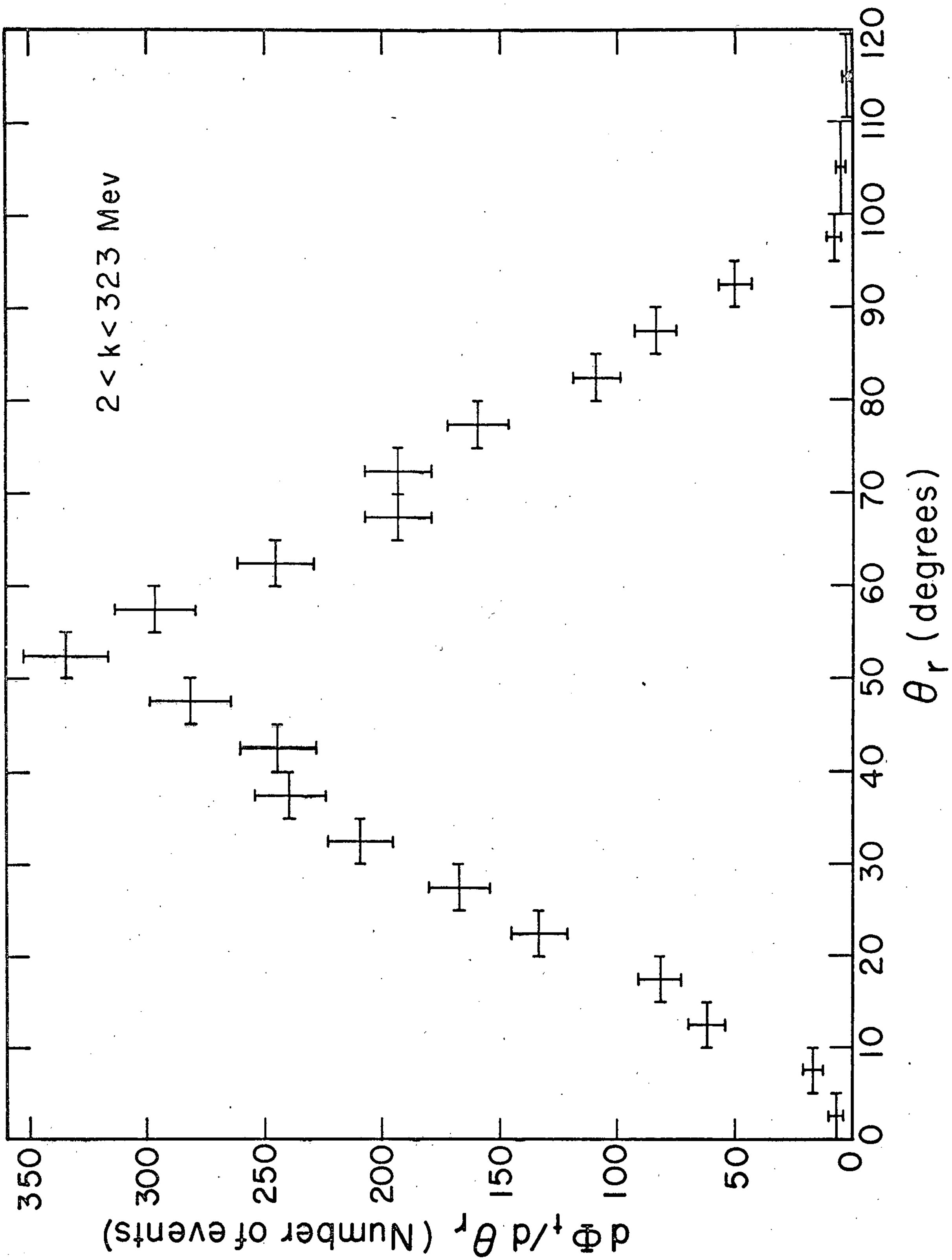


Fig. 8.

Mu-21614