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BOVINE AERODYNAMICS IN TERMS OF FUZZY LAGRANGIAN DYNAMICS

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Bovine Aerodynamics In Terms Of Fuzzy Lagrangian Dynamics

Physicists often work in the realm of smooth surfaces, massless ropes, and spherical cows. When asked to tackle a problem, physicists usually make assumptions that simplify the math, and more often than not, the theoretical result is close to the experimental one. We assume that the wind resistance of a falling object is negligible, that the inertia of a rope won't noticeably affect the motion of a pulley, or that a cow's uneven weight distribution won't affect its trajectory when launched from a catapult. These assumptions work well for simple systems, so we can apply Newton's Second Law to find the equations of motion. However, these simplified models start to fail for complex systems. For more complicated systems, we can apply a new set of tools called Fuzzy Lagrangian Dynamics, which not only allow physicists to tackle complicated physical systems, but also allows scientists in other disciplines to analyze nonphysical systems as well. These new tools may be the basis for a plethora of possible future applications, ranging from the dynamics of a catapult to the dynamics of an economy.

Sir Isaac Newton gets a lot of credit for his equation governing motion, reproduced below, where F is the net force on an object, p-dot is the time derivative of its momentum, m is the mass of the object, and a is its acceleration:

$$\overrightarrow{F} = \dot{\vec{p}} = m\vec{a}$$

Unfortunately, this equation has one major shortcoming. It works fine in standard "x, y, z", or Cartesian, coordinates, but the math gets messy if you introduce rotations. While not necessarily impossible, it can get cumbersome and tedious to make the necessary transformations between Cartesian and spherical coordinates, like carving a turkey with a pocket knife.

There have been several attempts to reformulate Newtonian mechanics to more easily accommodate spinning, and one of the more famous attempts was that of Joseph-Louis Lagrange, an Italian mathematician who lived in Berlin in the late 18^{th} century. Lagrange formulated a new equation, reproduced below, that can accept any coordinate system without requiring transformations, making it much more adaptable, but also requires more complicated math. In the equations below, q is the general coordinate, q-dot is that coordinate's velocity, and L is a quantity called the Lagrangian, given by the kinetic energy T minus the potential energy V:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$L = T - V$$

Referring to the elegance of Lagrangian Mechanics, Dr. Robert Adair, in his book *The Great Design: Particles, Fields, and Creation*, asserted that "Indeed, at the present time it appears that we can describe all the fundamental forces in terms of a Lagrangian. The search for Nature's One Equation, which rules all of the universe, has been largely a search for an adequate Lagrangian." In other words, Adair is arguing that this equation allows us to describe complex motion in relatively simple terms, given the system's energy. An adequate Lagrangian that describes the energy of the entire universe, once found, would allow us to derive the behavior of the universe, and all things in it, in terms of whatever coordinate system is convenient.

How do Lagrangian mechanics actually work, though? Lagrange's equations work based on an idea called the principle of least action. It works a little like this: Imagine that you want to go to a friend's house. There are infinitely many paths that lead from your house to theirs. You could walk there directly. You could walk down the street and take a bus. You could drive down the street and then turn onto theirs. You could ride in a wagon pulled by a bear around the city a few times before heading to their house. If you ask your GPS for directions, though, it will suggest the path that takes the least amount of time. Lagrange's equation works similarly. There are infinitely many ways that a system could have a specific energy, but Lagrange's equation finds the most efficient path, which minimizes a quantity called the "action". This means that, knowing the initial conditions of a system, we can accurately predict what it will do.

However, precisely understanding the starting parameters is the sticking point in most experiments. If you were to model a cow catapulted off of a building, you could easily predict where the cow will land. Now suppose that you actually conduct the experiment with 100 cows. After you finish explaining to the authorities why you thought this was a good idea, you'll notice that the cows didn't all land in precisely the same spot. There are many unknowns acting on the system that you, the experimenter, have little to no control over. Each time you launch a cow, something will be different: each cow varies in mass and shape, the catapult isn't always compressed the exact same amount, the catapult might shift a little between firings, the wind may blow sporadically, and the cow could tumble in mid-flight, all of which would affect the trajectory of the cow. This means that there's a certain amount of ambiguity in exactly how much we can understand about the behavior of a system. This kind of ambiguity creates a problem for scientists, since we have to find different ways to accurately model a system's behavior.

One of the ways we can tackle ambiguity is with something called a fuzzy set. Fuzzy sets were introduced in 1965 by Dr. Lofti A. Zadeh, a mathematician from UC Berkeley, to help better understand systems that we only have an approximate, or 'fuzzy', grasp on. Instead of containing a specific set of points, they describe a relationship between points, given by a relationship function. It's like two different ways to give someone an address. You can give the accurate street address, or you can say their destination is between two landmarks. While the latter isn't as precise as the former, you may not have the knowledge to give the precise answer, and the person will eventually find their destination either way.

Dr. Uziel Sandler, a professor at the Jerusalem College of Technology in Israel, recently published a paper merging the concepts of Lagrangian Mechanics and Fuzzy Set Theory. According to Sandler, when we do not precisely understand a system's initial parameters, it makes sense to instead describe the results as a domain. Applying it to our cow example from earlier, instead of saying that we think the cow will land on a specific point, we can accurately say that the cow will land somewhere in a specific region. Taking this a step further, instead of saying that the cow is at a position x at some time t, we say that there is a probability it is near position x at time t. We can apply the same thing to the velocity, and now we have physical quantities as probabilities and relationships. Using Dr. Sandler's equations, we can predict the flight of the cow knowing only imprecise parameters about the cow's initial condition.

Launching cows is one thing, but warfare has long since surpassed the need for controlled aerial heifers. Instead, the military favors missiles, mortar shells, and more recently, rail-guns. While launching an explosive piece of metal at someone we don't like is an art that has been well-explored, more advanced technology means we can create smarter, more complicated weapons. Some anti-tank missiles, for example, launch several smaller munitions after the main payload has been shot, and then each individual payload drops molten copper onto the targets below. This system exhibits a complicated chaotic behavior, and Dr. Sandler's equations could help model such behavior, allowing weapon designers to create more efficient payloads that minimize collateral damage and civilian casualties.

The beauty of these equations, though, is that they are generalized; they are not bound to the realm of corporeal matter. We are not restricted only to farm animals launched in increasingly complicated ways. Consider a stock market, an abstract system which at first has little resemblance to either a hypothetically propelled heifer or the realities of aerial warfare. A stock market has position in terms of money the way a concrete, physical system has position in terms of meters. An economy is a system with quantifiable characteristics and identifiable trends that affect long term behavior, so why not model it the same way you'd model a physical system with forces acting on it? The field of econophysics came about for this very reason, although it had traditionally suffered from ambiguity in how much you can truly understand about not only what the current state of a market is, but what its future state will be. This is where Fuzzy Dynamics comes in, lowering its shades as it calmly says "Stand back, I've got this." The ambiguous nature of economics lends itself perfectly to Sandler's probabilistic model, where we can now account for wide range of initial and future conditions.

Why limit ourselves to economics, though? In politics, for example, we say that candidates "race" to win votes for an election, the same way that runners race across physical distances during a competition. If we can find a runner's velocity during a race, then we could also model the rate at which candidates acquire votes. This analogy only works, however, if we assume that the runners are constantly hurling banana peels at each other. The tumultuous nature of political machinations does not lend itself to the simple analysis we'd use to model the relatively constant motion of someone running in a straight line, but does lend itself to the probabilistic model used in Fuzzy Dynamics. We can use historical data to create a model of the "motion" of a campaign based on the rate at which money is spent and the quality of the people running it, like a NASCAR race that runs on lies instead of gasoline.

There is, however, a minor caveat to all of this. The probabilistic nature of Fuzzy Sets means that Fuzzy Dynamics produces something called a differential inclusion, which is a strange type of solution that we have little understanding of, like a mathematical platypus. The only solutions we can meaningfully obtain usually come from numeric methods, which is a fancy term for having a computer crunch numbers. The newness of Fuzzy Dynamics also means that its membership functions will need to be derived from experimentation, or analysis of historical data.

Although these details will need to be addressed, the framework of Fuzzy Dynamics is still a strong foundation. A foundation for what, though, is still yet to be discovered. It may become the new industry standard for weapon engineering firms. It may be the basis for all future stock market analysis. It could be used to model the spread of a disease, and save millions of lives during major outbreaks. It could even be used as an excuse to launch cows out of catapults. The possibilities are truly endless.

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