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Abstract

The ability to choose problem solving strategies flexibly and adaptively is an important part of proficiency. However, it is unclear how simple forms of problem solving practice as well as feedback affect this ability. On the one hand, as demonstrated by the Einstellung and Stroop effect, practice can decrease adaptivity. On the other hand, practice helps to associate problem types with effective solution strategies what can increase adaptivity.

In a microgenetic design with 48 trials of a mathematical problem solving task, we found that the adaptivity of strategy choices increased linearly during practice without feedback in a group of ninth-graders. Instructional support to stimulate insight sped up this process in a second experimental group. The results are interpreted in terms of cognitive models of strategy choices. They demonstrate the adaptive nature of human problem solving with minimal informational input.**Keywords:** mathematical problem solving, flexibility

Theoretical Background

Strategies are step-by-step procedures for solving a problem. From among available strategies adaptive problem solvers choose the strategy which allows them to solve the problem most efficiently, particularly in terms of accuracy and speed. Adaptive strategy choices are relevant in many everyday situations, for example, when we go to office by car or by bike depending on the weather or when we choose the type of knife we use based on what we want to cut. The adaptivity of strategy choices has been recognized as an important outcome of competence development (Siegler, 2007; Siegler & Lemaire, 1997), as a central component of being proficient (Hatano & Inagaki, 1986; Star & Newton, 2009), and as a favorable educational outcome (Baroody & Dowker, 2003; Star & Rittle-Johnson, 2008).

A person chooses a strategy adaptively when he or she chooses the most efficient strategy given the specific problem, the problem solver, and the socio-cultural context (Verschaffel, Luwel, Torbeyns, & Van Dooren, 2007). In the current study, we focus on strategy adaption to task characteristics. In line with many studies (e.g., Lemaire & Siegler, 1995; Torbeyns, Ghesquiére, & Verschaffel, 2009), we see a strategy as more efficient than another one when it leads to higher accuracy and to a shorter solution time than the alternative strategy. Adaptivity is closely related to, but should not be confused with, accuracy and speed of the problem solutions, how many strategies a problem solver knows, and how efficiently he switches between strategies (Star & Newton, 2009). For instance, a person can know many strategies, flexibly switch between them, carry them out quickly and accurately and still not adapt the strategy choices to task characteristics. This could happen, for example, if the problem solver did not know the relative efficiency of a strategy for solving a task with particular characteristics and therefore selected a less efficient strategy (Lemaire & Siegler, 1995; Selter, 2009; Shrager & Siegler, 1998; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009).

Possible Negative Effects of Practice on Adaptivity

A high degree of practice in a domain can make problem solvers more accurate but less flexible. This has been suggested in educational studies on mathematical problem solving strategies (De Smedt, Torbeyns, Strassens, Ghesquière, & Verschaffel, 2010; Selter, 2009; Star & Newton, 2009; Torbeyns, Ghesquiére et al., 2009; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009) and in in studies about costs of expertise (Baroody, 2003; Bilalić, McLeod, & Gobet, 2008, 2010; Frensch & Sternberg, 1989; Hesketh, 1997; Sternberg & Frensch, 1992; Zeitz, 1997). For example, Sternberg (1996, p. 347) writes “There are costs as well as benefits to expertise. One such cost is increased rigidity: The expert can become so entrenched in a point of view or a way of doing things that it becomes hard to see things differently.” A similar idea is demonstrated by research on the Einstellung effect, that is, participants’ preference for familiar strategies instead of more effective but less familiar strategies (Luchins & Luchins, 1959, 1987; McKelvie, 1990).

A negative effect of practice on adaptive, flexible problem solving is usually explained in terms of problem solving routines (Bilalić et al., 2008; Johnson, 2003). Practice leads to increasingly automated problem solving routines. These routines are quick, require minimal cognitive resources, and appear to be beyond conscious control (Anderson, 1982; Johnson, 2003; Logan, 1988). This is an advantage when solving routine problems, but it can work against the problem solver who is faced with new or changing problem types that require flexible behavior (Hesketh, 1997).

Possible Positive Effects of Practice on Adaptivity

Several other authors suggested a positive influence of practice on the development of adaptive strategy choices. Experts have more perceptual chunks, more ways of representing a problem, and more solution approaches available than novices. In theory this allows to better adapt behavior to situational constraints (De Groot, 1978; Ericsson, Krampe, & Tesch-Römer, 1993; Gobet & Waters, 2003; Star & Newton, 2009).

In the literature on strategy change, Robert S. Siegler's cognitive models (Shrager & Siegler, 1998; Siegler & Araya, 2005; Siegler & Shipley, 1995) suggest a positive impact of problem solving practice alone on the development of adaptive strategy choices. While Siegler's original model has been extended several times, one of the remaining basic assumptions is that individuals acquire information about each strategy's effectiveness, in terms of accuracy and speed, for solving the encountered problem types. This information subsequently helps learners to choose the strategies that maximize the efficiency of the problem solution.

However, learners do not know the actual efficiency of the strategies; instead, they must infer this from their experiences. This is a gradual and probabilistic process as accurate and efficient strategies can sometimes lead to low solution times and solution rates if a superficial error is made in how the strategy is executed. The more often a learner uses a set of strategies on a set of problems, the more accurate the learner's implicit knowledge about each strategy's efficiency on different problem types becomes. As a result, the more practice a learner has with a given set of problems, the better the learner knows the relative efficiency of each strategy for each problem type, which leads to more adaptive strategy choices.

Shrager and Siegler (e.g., 1998, p. 407) label this process as "associative learning" and explicitly distinguish it from meta-cognitive learning; that is, they see the acquisition of strategy adaptivity as a low-level cognitive process. Simple problem solving practice should suffice to acquire strategy adaptivity -- at least to the extent to which the learners are able to judge the accuracy and speed of their own problem solutions.

Possible Effects of Feedback

Feedback is widely seen as an important facilitator in learning and performance (for reviews see Kluger & DeNisi, 1996; Kulhavy & Stock, 1989; Mory, 2004) however most conclusions drawn from empirical studies on feedback are general with the specific mechanisms how to relate feedback to learning unknown (Shute, 2008). Shute (2008) defines formative feedback as information intended to modify a learners thinking/ behavior to improve learning. She sees different cognitive mechanisms through which feedback can take a facilitative effect on learning: Feedback can reveal a gap between the current level of performance and the aspired level of performance what can on its part lead to a higher level of motivation and effort as a learner naturally tries to resolve this gap, feedback can reduce cognitive load (e.g., Paas, Renkl, & Sweller, 2003) or feedback can facilitate the correction of inappropriate task strategies or procedural errors (e.g. Mory, 2004; Narciss & Huth, 2004).

According to a meta-analysis by Hattie (1999) feedback is among the top of the highest influences on achievement along with direct instruction, reciprocal teaching and students' prior cognitive ability, but effect sizes for feedback show considerable variability, indicating that some types of feedback are more powerful and – maybe surprisingly – quite a few studies have reported either no effect or negative effects of feedback on learning (see Kluger & DeNisi, 1996; Mory, 2004). According to a meta-analysis by Bangert-Drowns et al. (1991) negative effects of feedback on learning is found in about one third of studies included in their analyses. There are many explanations why feedback is not necessarily facilitative[1]. One aim of intervention studies therefore is to investigate how to maximize the positive effect of feedback on learning as feedback does not necessarily have a positive influence on learning.

The Current Study

Hence, prior research has firmly established that practice influences the adaptivity of strategy choices in at least two ways: a) Practice leads to problem solving routines, which can decrease adaptivity, and b) at the same time, practice also leads to more information about how effectively alternative strategies solve a given type of problem, which increases adaptivity. The relative strength of the two opposing mechanisms is unclear. In their literature review, Bilalić and colleagues (2008, p.77) conclude: "… (in)flexibility has frequently been discussed in the scientific literature (e.g., Ericsson, 1998, 2003; Feltovich, Spiro, & Coulson, 1997; Hesketh, 1997; Krems, 1995; Sternberg, 1996; Zeitz, 1997), the empirical evidence for either possibility is sparse and unconvincing." This presents an open empirical question: does practice without feedback ultimately a) increase the adaptivity of strategy choices, b) decrease it, or c) do the two opposing influences cancel each other out and leave adaptivity unchanged?

In the present study the formative feedback aims to influence strategy use, specifically adaptivity. We therefore conceptualize feedback as information provided by an agent

[1] It can be accepted or rejected, be perceived differently than intended (Kulhavy, 1977) or interrupting the cognitive processes (Baron, 1993. Kulhavy, White, Topp, Chan, and Adams (1985) found negative effects of feedback complexity, that is, the more complex the feedback the least positive outcome effect whereas Bangert-Drowns et al. (1991) found that the more information feedback provides the more effective it is (e.g. correct–incorrect-feedback being less efficient than if feedback provides the correct answer).

(here computer program) regarding aspects of one's understanding (here strategy choice) or performance (Hattie et al., 2007) with which a learner can confirm or restructure preexisting knowledge, e.g. domain knowledge or cognitive tactics (Butler & Winne, 1995).

The current study uses a microgenetic design, that is, a trial-by-trial assessment of strategy choices on 48 trials of a mathematical problem solving task. One experimental group practiced problem solving without feedback. A second experimental group received feedback designed to stimulate high-level cognition and insight into the advantages of adaptive problem solving. This design helped to investigate the following three questions. First, does practice without feedback increase or decrease the adaptivity of strategy choices over the course of unsupervised practice? Second, how is this developmental pattern affected by instructional support designed to stimulate high-level cognition about adaptivity? Third, do increases in adaptivity in the two groups tend to be abrupt (pointing to insightful learning; Davidson, 1995) or gradual (pointing to associative learning; Gluck & Thompson, 1987)?

Method

Participants

The sample consisted of 77 Swiss ninth-graders (age: *M* = 14.9 years, *SD* = 0.8; 71% girls) who participated voluntarily during their free time for a small monetary compensation. They were recruited from, the highest track of the Swiss educational system, the 'Gymnasium' – attended by approximately 20% of students. Our sample therefore consists of high performing students.

Procedure

Each student took part in a three-hour-session comprising an instructional phase and a microgenetic session.

During instructional phase, each student was shown ten slides on a computer explaining three different strategies for solving the task – an equation system with two equations and two unknowns. Most students stated they knew these strategies from school, which was in line with the formal curricula of their schools. At the end of the instructional phase students had to demonstrate their ability to correctly name and use all three strategies before they could proceed to the microgenetic session and otherwise they received additional explanations. Students were neither instructed on when to use which strategy nor was the concept of adaptive strategy choice introduced.

In the microgenetic session, students were presented with 16 blocks of 3 trials. Each block comprised one of each problem typ. For each student problems had been individually and randomly selected from a pool of tasks. Problems of the same type varied in surface structure (i.e. the order of the two equations, the order of the terms of an equation, etc.).

For each trial the equation system was presented on the computer, students were instructed to copy it into a booklet, solve the problem in a written form, and finally, enter the solutions on the computer, in order to recorded solution accuracy and speed. A trained rater coded strategy use according to participants' solution steps in the booklet. A second trained rater coded 100 randomly selected trials and agreed with the first rater in 99 of the 100 trials.

Students were randomly assigned to one of two groups: A no-feedback group (*n* = 39) practicing problem solving without any instructional support and a feedback group (*n* = 38). Students of the feedback group had to indicate which of the alternative strategies they had used after each trial by checking one of three buttons. They then got feedback about the adaptivity and effectiveness of their strategy with either wording: "You have chosen the most efficient strategy, which saved you from unnecessary work" or "You have chosen a less efficient strategy, which created unnecessary work". This feedback condition was designed to stimulate high-level cognition. Naming and checking the strategy chosen could have fostered greater awareness of strategy use, independent from the feedback and in addition the feedback reminded them of what they were expected to do (i.e. choose a strategy adaptively) and why (i.e. to save unnecessary work).

Material

Participants had to solve systems of two algebraic equations with two unknowns, that is, to derive the numerical values of the unknowns. There were three problem types: Addition problems, equating problems, and substitution problems (see Table 1) and there were the three problem solving strategies they could choose from: the addition strategy, the equating strategy, and the substitution strategy (see Table 2). Each problem could be solved by all three strategies. However, without prior transformations of the presented equations, addition problems could only be solved by the addition strategy. Therefore we hypothesize that the addition strategy is the most efficient strategy to solve the addition problems. The same applies to equation problems and the equation strategy and to substitution problems and the substitution strategy. In the following, we label strategy use in line with this pattern as adaptive.

Table 1
Six of the forty-eight problems used in the study.

Addition problems	Equating problems	Substitution problems
4x - 13y = 10	10y + 4x = (-10)	4 = 13y - 12x
4x - 11y = 14	(-2)y = 10y + 4x	x = 6y - 20
(-11) = 4y - 3x	3y = 6 + 12x	y = (-10)x - 19
(-9) = (-9)y + 3x	6 + 12x = (-5)y - 16	5x + 18y = 8

.Results

As expected, compared to other trials, trials with adaptive strategy choices showed higher solution rates (*M* = 68 %, *SD* = 47, vs. *M* = 53 %, *SD* = 50, Mann-Whitney *U* = 558692, $p < .001$), lower solution times (*M* = 126 sec, *SD* = 61, vs. *M* = 171 sec, *SD* = 75, Mann-Whitney *U* = 133239, *p*

< .001), and less solution steps according to the written solution path in the booklet (*M* = 4.4 lines, *SD* = 1.6, vs. *M* = 6.0 lines, *SD* = 2.0, Mann-Whitney *U* = 756456, *p* < .001). This justifies our assumption that matching strategies to problem types is adaptive behavior. In the feedback group, the student-reported strategies and the rater-coded strategies matched in 92% of all trials. Table 3 displays mean solution rates, mean solution times, and mean adaptivity rates for the two experimental groups together with error probabilities *p* and Cohen's *d* from *t*-tests for repeated measures. In both experimental groups, solution rates do not significantly increase over time; solution times decrease strongly; and adaptivity increases.

Table 2
The three strategies investigated in the study applied to the same problem (not used in the study).

Addition strategy		Equating strategy		Substitution strategy	
4x - y = 27 y + 2x = 21	Original equations	4x - y = 27 y + 2x = 21	Original equations	4x - y = 27 y + 2x = 21	Original equations
4x - y = 27 2x + y = 21	Align corresponding terms of the two equations.	y = 4x - 27 y = 21 - 2x	In both equations, bring x on one side, y on the other. One side has to be the same in both equations.	4x - y = 27 y = 21 - 2x	Isolate one variable in one equation.
6x = 48	Add (or subtract) the equations so that only one variable remains.	4x - 27 = 21 - 2x	Equate the differing sides of the equations so that only one variable remains.	4x - (21 - 2x) = 27	Substitute this variable in the other equation so that only one variable remains.
x = 8	Compute the value of the variable.	x = 8	Compute the value of the variable.	x = 8	Compute the value of the variable.
y + 2 * 8 = 21	In one of the original equations, substitute one variable by its value.	y + 2 * 8 = 21	In one of the original equations, substitute one variable by its value.	y + 2 * 8 = 21	In one of the original equations, substitute one variable by its value.
y = 5	Compute the value of the remaining variable.	y = 5	Compute the value of the remaining variable.	y = 5	Compute the value of the remaining variable.

The increase in adaptivity is roughly linear and fit well by a linear regression function in the no-feedback group. In the feedback group, the fit of the linear function is less good and adaptivity increases strongly during the first four trials and subsequently stays at a high level. Figure 1 displays changes in adaptivity over time.

Discussion

The literature suggests positive as well as negative effects of practice without feedback on the adaptivity of strategy choices. This raises the question whether adaptivity ultimately increases or decreases with practice. Our results show that the positive effects of practice without feedback on adaptivity far outweigh the negative effects, that is, participants get more adaptive. Adaptivity increased significantly from 78% to 90% over 48 trials of practice in the no-feedback group. In the feedback group – receiving feedback designed to foster insightful learning –, less surprisingly, and in accordance with the literature on strategy learning, adaptivity also increased.

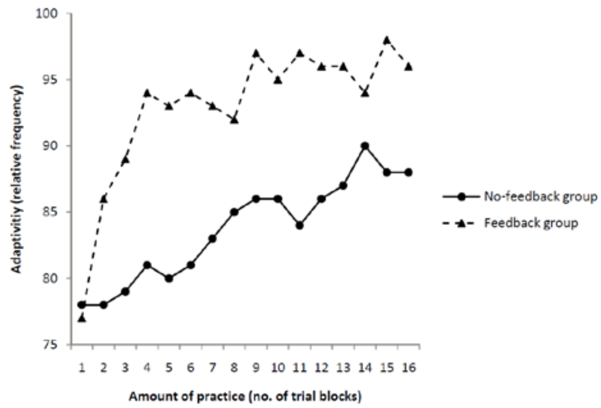

Figure 1
Microgenetic development of the relative frequency of adaptive strategy choices over the 16 blocks of tasks averaged over all participants in a group.

The strong positive net effect of practice without feedback on the adaptivity of strategy choices is remarkable because students were in a complex learning situation and received minimal information. They had to solve equation problems while simultaneously learning how to increase adaptivity, a combination placing high demands on working memory (Mayer & Moreno, 2003; Renkl, 2005). The students were neither instructed to focus on strategy use nor to increase the adaptivity of their solution approaches. They also did not know beforehand that there would be three different problem types, each corresponding to one strategy. Further, each of the three problem types was presented with four different surface structures to prevent superficial learning. The students received neither feedback on the correctness of their answers nor on the efficiency of the strategies chosen. Reaching higher adaptivity therefore included the following steps: a) identifying the three problem types, (2) evaluating the relative efficiencies of the nine combinations of strategies and problem types, and (3) choosing the adaptive strategy.

Two empirical findings point to cognitive mechanisms for increasing adaptivity of strategy choices. First, in the feedback group adaptivity increased quickly and stayed high. In contrast, in the no-feedback group adaptivity increased linearly throughout the course of practice. The more abrupt learning curve in the feedback condition points to insightful learning (Blöte, van der Burg, & Klein, 2001; Rittle-Johnson & Star, 2009),and, the more gradual learning curve in the no-feedback condition to associative learning (cf. Davidson, 1995). As proposed by Siegler's models of strategy changes (Shrager & Siegler, 1998; Siegler & Araya, 2005; Siegler & Shipley, 1995), over the course of unsupervised practice, the participants learned to associate each combination of problem type and strategy with its respective efficiency and then based their strategy choices on this information. Second, adaptive strategy choice (i.e. matching strategy choice to problem type e.g., the use of the addition strategy for an addition problem) led to a) shorter written solution paths, b) higher solution rates c) lower solution times. Thus, matching strategy choices to problem characteristics was clearly adaptive because it saved time and mental effort during problem solving. Since the students received no external rewards for choosing their strategies adaptively, they must have perceived the reduced time and mental effort required to derive a solution as inherently rewarding.

Compared to other studies (cf. Torbeyns, De Smedt et al., 2009), the relative frequency of adaptive strategy choices was already high (> 75%) at the beginning of our study and increased further during learning. This demonstrates the relatively high prior knowledge of our participants: They knew all three strategies prior to the microgenetic session– from school instruction and through a brush up instructional phase prior to assessment – and came from the highest track of the Swiss educational system. Future studies will have to test whether less knowledgeable students are able to use the learning mechanisms for adaptivity in a similar way as the high performing students in our sample. Still, the present results provide a strong case for the inherently adaptive nature of human problem solving (Anderson, 1990; Siegler, 1996) which shapes behavior even in complex learning situations with minimal information input.

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Appendix

Table 3
Differences between the two experimental groups.

	Feedback group		No-feedback group	
	Parameter	SD	Parameter	SD
Solution rate (%)				
Mean at T1	53	51	50	51
Mean at T48	66	48	74	44
p	.257		.070	
d	.189		.316	
Solution time (s)				
Mean at T1	276	133	251	93
Mean at T48	98	36	105	48
p	.000		.000	
d	1.862		1.457	
Adaptivity (%)				
Mean at T1	74	45	78	42
Mean at T48	97	17	90	30
p	.017		.023	
d	.438		.440	
Linear regression				
β	.762		.952	
p	.001		.000	
R^2	.762		.899	