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CONSIDERATIONS AFFECTING THE DESIGN OF
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APPARATUS EMPLOYING TWO-POLE STATE SELECTORS

Douglas McColm
February 7, 1966

Considerations Affecting the Design
of Atomic- And Molecular-Beam Resonance Apparatus
Employing Two-Pole State Selectors*

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February 7, 1966

ABSTRACT

Optimum design criteria for atomic- and molecular-beam apparatus are discussed, and signal strengths under a number of conditions are calculated. Signal strength and signal-to-noise ratios are shown graphically as functions of the design parameters.

INTRODUCTION

The basic principles involved in the design of atomic- and molecular-beam apparatus have been understood for many years, and are discussed in the standard treatments of the subject.^{1, 2} But an optimum design for these devices is a question that has not been settled, although several studies have been made.³⁻⁵

This article presents an approach to the optimization problem for beam devices employing state selectors of two-pole design. The variables to be studied are the sizes of slits and apertures, and the lengths of deflecting fields and drift spaces. The signal and noise are calculated in terms of these parameters. The analysis depends, of course, on whether the number of atoms leaving the oven is proportional to the area of the oven aperture or not. We distinguish two cases:

1. Efflux from the oven is independent of aperture area. This case applies to situations in which the beam intensity is limited by the speed of the pumps, so that for any given source aperture one adjusts the source pressure to give this limiting beam intensity. Gas-discharge sources are often operated this way. Furthermore, this case also applies to situations in which a limited number of atoms is available for the experiment, so that one adjusts the source pressure (or temperature) so that the beam will last a suitably long time. Experiments on radioactive isotopes and on certain enriched isotopes are of this type.

2. Efflux from the oven is proportional to the oven aperture. This condition is met in the use of discharge tube sources, where the range of temperatures over which the source can be operated and still produce the desired species is limited but the amount of material available for the

experiment is not. In this case one can double the number of atoms in the beam simply by doubling the area of the source aperture, whereas in case 1 one could not. Surprisingly, case 2 includes relatively few experiments.

Most experiments are not solely either of these cases, of course. The limit to the temperature to which one can raise an oven containing radioactive material is set by the point at which chemical reactions occur between the atoms of the source and those of the oven container; thus most experiments on radioactive atoms fall in case 1 only if the temperature is not raised too high. And similarly there is a limit, set by their cost, to the number of atoms that can be used in many experiments on stable isotopes.

The result of an optimization procedure depends also on the nature of the limiting noise present at the detector. This noise also falls into two categories.

(a) Noise whose amplitude is proportional to the beam intensity. Such is the case for a background arising from contaminants in the source, or from unwanted states of the desired species. Noise produced by photons emanating from the source, from turbulence in the source, or from fluctuations in the vacuum, is of this type also.

(b) Noise whose amplitude depends on the area of the detector aperture only. Hydrocarbon background in mass spectrometers is of this type, as is also cosmic-ray background in counters used for detection of radioactive isotopes. This source of noise is minimized only by having a sharp focus for the particle trajectories at the detector, so that the detector aperture can be made as small as possible. This requires

that the position of the focus be independent of the velocity of the particles, which in turn requires that the state selectors have a uniform gradient.

This discussion is limited to beam apparatus having two-pole state selectors; the analysis of other configurations, such as the six-pole, is enough different that it should properly be put in another report. No detailed comparison, therefore, is presented between these possible field configurations, though it should be noted that the six-pole has a larger signal intensity at the detector than the two-pole design,⁶ but its detector is of larger area and consequently noisier.

STATE SELECTORS

The state selectors can be either magnetic or electric, the choice depending on experiments considered. For a magnetic selector, the particle to be deflected must have a magnetic dipole moment, and the force is given by

$$F_z = \mu_{\text{eff}} \frac{\partial H}{\partial z},$$

where $\mu_{\text{eff}} = -(\partial W/\partial H)$, H is the magnetic field produced by the state selector, and W is the energy of the particles. For an atom in the Paschen-Bach region, $\mu_{\text{eff}} = g_J m_J \mu_0$, with g_J being the electron g value, m_J the electron magnetic quantum number, and μ_0 the Bohr magneton.

For an electric state selector, the particle to be deflected must have an electric dipole moment, and the force is given by

$$F_z = \mu_{\text{eff}} \frac{\partial E}{\partial z},$$

where $\mu_{\text{eff}} = -(\partial W/\partial E)$ and E is the electric field produced in the

state selector. For polar molecules in states having J less than five, values of μ_{eff} are given by Kusch and Hughes.⁷

The field gradients needed for deflection are produced by first selecting a two-dimensional solution of Laplace's equation having the desired properties and by machining the pole pieces to fit two of the equipotentials of that solution. Both atomic- and molecular-beam devices traditionally use two state selectors, one as polarizer and the other as analyzer for the beam particles; the position of the focus after the beam has passed through two state selectors is chosen for the particle detector. The condition that the position of this focus be independent of particle velocity can be satisfied if the field gradient is of uniform magnitude everywhere inside the state selector. Of course no solution to the Laplace equation exists for which the field gradient is constant in magnitude, and so the ideal focusing situation cannot be realized. But the suitability of a choice of field configuration depends in part on how closely this ideal is approximated.

The most popular field configuration employed is the "two-wire geometry," in which the field gradient is very uniform over the height of the beam but changes radically in value along the z direction. If we choose for the magnet aperture a square region over which the gradient drops to only one half its former value, as shown in Fig. 1, then we obtain for H and $\partial H/\partial z$ at the center of the aperture the relation

$$\frac{\partial H}{\partial z} \approx \frac{5}{6} \frac{H}{a},$$

where a is height of the aperture. A similar equation applies to the electric case, with H replaced by E .

A field configuration having a more nearly constant field gradient can be derived from the "quadrupole geometry," for which the magnitude of the

field is proportional to $(y^2 + z^2)^{1/2}$. The aperture can be chosen so that the gradient does not change by more than 10% inside it; for this case,

$$\frac{\partial H}{\partial z} \approx \frac{2}{3} \left(\frac{H}{a} \right),$$

where H and $\partial H/\partial z$ are evaluated at the center of the selector aperture. A plot of this geometry is shown in Fig. 2.

Although the field gradient is not constant inside the state selectors, for simplicity in what follows we assume that it is. It is clear that a velocity-independent focus requires that this condition be well approximated.

We can summarize the equations for these field configurations and others like them by the equation

$$F_z = \mu_{\text{eff}} \gamma(H/a),$$

where γ is a numerical factor appropriate to the geometry. It is convenient to introduce a new parameter ρ . For the magnetic case,

$$\rho^2 \equiv \frac{\gamma \mu_{\text{eff}} H}{2kT},$$

and for the electric case H is replaced by E ; T is the temperature of the source, and k is Boltzmann's constant.

In order to consider a variety of beam shapes, I will take the width of the actual apertures in the state selectors to be a fraction h^2 of that shown in the figures while the height remains as shown; $h \leq 1$. The figures therefore show apertures of maximum width.

TRAJECTORIES

A beam apparatus is of course a device with a source of particles at one end, then a state selector. Following this is a region of homogeneous static field into which an electromagnetic perturbation is applied; here the particles undergo transitions among their states. Then there is another state selector and finally a particle detector. This arrangement is shown in Fig. 3.

The basic specifications needed for the design of such an apparatus are first the line width of the transitions that must be obtained, and second the signal intensity required. The approach to be followed is to specify the first as a requirement in the calculations and then to calculate the second in terms of the parameters of the apparatus. For rf transitions the line width can be estimated from the uncertainty principle, $\delta v \approx v_0/L$, where L is the length of the transition region and $v_0 = (2kT/m)^{1/2}$. Because v_0 is not easily changed, we take L as a fixed number given by the line-width requirement, and this in turn fixes L_C , the length of the static field region.

Most experiments have a background of particles that are undeflected in the state selectors. These may be particles of another species entirely, such as an impurity present in the source, or they may be particles of the desired species but in an undesired state. In any case the situation is so general that a stop in the state selectors is assumed to be necessary to prevent the passage of undeflected particles, and so the state selectors must be sufficiently strong that they completely separate the undeflected from the deflected particles. Of course this requirement implies a cutoff in the velocity distribution, because the faster the particles go, the less they are deflected in the selectors. And because the apertures of the machine are not very much smaller than the deflections in the selectors, particles passing through different parts of the apertures will have cutoffs at different velocities.

The cutoff in velocity for certain trajectories is introduced as a parameter p . In terms of it the other trajectories reaching the detector are calculated, and the total intensity at the detector determined.

There are two kinds of defining trajectories; the first kind applies to the condition in which the state selectors are turned off and the stops removed, so that the particles travel in straight lines, passing through both oven and collimator apertures. Those particles that pass through the center of the oven aperture and just graze the edges of the collimator aperture travel on what I call the "limiting trajectory." The distribution of particles at the detector as a function of position along the z axis is a trapezoid, and this limiting trajectory ends on the middle of the slanted sides of that trapezoid. Thus it does not define the outside edges of the beam at the detector, but passes through the points where its intensity is half its maximum value.

The other kind of defining trajectories applies to the condition in which the state selectors are turned on; the trajectories are followed by particles that pass through the center of the source, collimator, and detector apertures. Of course there will be many such "central" trajectories, each for a different particle velocity. That corresponding to a velocity equal to pv_0 , is called an "inner trajectory," and that corresponding to $(1/p)v_0$ and "outer trajectory." Here $p > 1$.

The positions along the beam axis at which these central trajectories achieve their maximum deflection are called "stop positions," where stops (if needed) should be placed. The stop positions are indicated in Fig. 4.

In order to take full advantage of the state selectors, we require that the useful trajectories fill the selector aperture at the stop position. A larger aperture than this would imply a longer selector and consequently smaller detector solid angle than necessary, while a smaller aperture would imply that a smaller fraction of the velocity distribution reaches the detector. Therefore we choose the width of the stops to be the distance between the inner trajectories at the stop position, and we

choose the width of the selector aperture as the distance between the outer trajectories at the stop position. This condition causes particles on central trajectories having velocities greater than pv_0 to be eliminated by the stops, and those having velocities less than $(1/p)v_0$ to be eliminated by the edges of the selector apertures; therefore it introduces p as a parameter of the calculation.

To satisfy these two conditions for every species of particle to be studied clearly calls for alteration of the field strength (and consequently of the field gradient) in the state selectors to suit each particular case. Design is based on the species of particle most difficult to deflect and the strongest fields attainable. Thus we take both selectors to have the same field strengths but not necessarily the same lengths or apertures.

To define the collimator aperture, we choose the limiting trajectory to meet the inner trajectory at the stop position in the B selector; and we choose the detector aperture so that the B stop just blocks all straight-line trajectories coming toward it through the collimator.

In a similar fashion we choose the oven aperture so that the A stop just blocks all straight-line trajectories originating in it and passing through the collimator. Thus this choice of apertures makes use of both stops as noise-eliminating devices.

The detector plane has been placed at the exit of the second state selector; no intervening space has been assumed. The presence of such intervening space only makes the signal weaker and so should be avoided if possible. Similarly, the source plane has been placed at the entrance of the first state selector.

This approach is suitable both (a) when the resonances are observed as a decrease in the beam intensity ("flop-out"), where the inner and outer trajectories apply to particles which have not made a resonance, and (b) when resonances are observed as an increase in beam intensity ("flop-in"), where the inner and outer trajectories apply only to particles undergoing resonance.

The various apertures, the signal, and the noise are now calculated in terms of four parameters; p and ρ already introduced, and r and s :

$$r = L_{AC}/L_C,$$

$$s = L_A/L_C,$$

where L_{AC} , L_A , and L_C are defined in Fig. 3.

The values of the various apertures, calculated on the basis of the foregoing, are then as follows:

The aperture of the first state selector is

$$a_A = \frac{1}{2h} (\rho p L_C) \left[\frac{s(s+2r)}{s+r} \right].$$

The width of the stop in the first state selector is

$$Z_A = h^2 a_A / p^4$$

while its position (as measured from the oven slit) is

$$X_A = \frac{1}{2} L_C \left[\frac{s(2r+s)}{r+s} \right].$$

The aperture in the second state selector is

$$a_B = \left(\frac{1-r}{r} \right) a_A.$$

The stop size is given by

$$Z_B = h^2 a_B / p^4.$$

The position of the B stop (as measured from the detector) is

$$X_B = \left(\frac{1-r}{r} \right) X_A .$$

The collimator size is

$$a_C = \left(\frac{hpL_C}{p^3} \right) \left[\frac{s(s+2r)(r+s)(1-r)}{s(s+2r)(1+r) + 2r^2} \right] ,$$

and the detector size is

$$a_D = a_C / r ,$$

while the oven size is

$$a_0 = 2a_C \left(\frac{r}{1-r} \right) q(r, s) ,$$

where

$$q(r, s) = \left[\frac{s(s+2r) + r}{s(s+2r) + 2r^2} \right] .$$

The length of the first selector, sL_C , is a parameter of the calculation.

The length of the second selector is

$$L_B = L_C \left[\frac{s(1-r)}{r} \right] .$$

The total length of the apparatus L_T is

$$L_T = \left(1 + \frac{s}{r} \right) L_C .$$

The quantities a_0 , a_A , a_C , a_B , Z_A , Z_B , and a_D all depend on ρ , so a machine that is to be useful for more than one kind of particle must be so arranged that ρ can be kept constant. This is done by having the field variable (and consequently the field gradient also variable) and by designing around the largest possible field and the most difficult kind of particle to deflect. Of course the surfaces of the state selectors have to be equipotentials; in the magnetic case this means that no part of the iron of the pole tips may be saturated. Because the magnetic field is greatest at points above and below the aperture, where the pole tips

come close together, and has a value almost three times that at the point in the aperture used for calculation, the magnetic-field value used for design purposes must not exceed one-third the value at which the iron saturates. And in the electric case, similar arguments, based on consideration of the breakdown voltage, apply to the selection of the limiting potential that can be applied to the selector.

Plots of the trajectories in the apparatus are facilitated by the following: In the first state selector, the deflection z at any point a distance x from the source is

$$z = z_0 + \left(\frac{x}{L_C}\right) \left(\frac{z_c - z_0}{r+s}\right) + \left(\frac{\rho x}{pV^2}\right) \left[1 - \left(\frac{x}{L_C}\right) \left(\frac{s+r}{s(s+2r)}\right)\right], h = 1$$

where z_0 is the displacement in the source plane, z_c the displacement in the collimator plane, and $V = v/v_0$; v is the velocity of the particle.

In the second state selector, z is given by

$$z = z_0 + \left(\frac{s+1}{r+s}\right) (z_c - z_0) - \left(\frac{\rho L_C}{pV^2}\right) \frac{s(1-r)}{2r+s} + \left(\frac{x-s-1}{L_C}\right) \left[\frac{z_c - z_0}{r+s} - \left(\frac{\rho L_C}{pV^2}\right) \left(\frac{s}{2r+s}\right)\right] + \left(\frac{x-s-1}{L_C}\right)^2 \left[\left(\frac{\rho L_C}{pV^2}\right) \left(\frac{r}{1-r}\right) \left(\frac{s+r}{s(s+2r)}\right)\right], h = 1.$$

The fact that all the quantities listed depend on ρ , and not on ρ^2 as one would expect, is a result of the requirement that the inner and limiting trajectories meet at the B stop position.

SIGNAL INTENSITIES

The signal intensity was calculated for cases 1 and 2 (see Introduction) as a function of the five parameters ρ , L_C , p , r , and s . Best values of p , r , and s were sought. Since L_C is determined by the precision required in the experiments and ρ by the size of the magnetic moments to be measured, these two parameters were treated as fixed at the outset. Thus the dependence of the signal on p , r , and s is the primary interest.

It is important to notice that neither the signal nor the figure of merit depend on h , so this parameter need not be considered further.

For case 1 the intensity $I_{(1)}$ at the detector is given by

$$I_{(1)} = \frac{N_0}{\pi A_0} \int dA_0 \int \cos \theta \, d\Omega \int P(V) V^3 e^{-V^2} \, dV,$$

where N_0 is the number of particles of the desired species in the desired state leaving the source per second. $A_0 = a_A a_0$ is the area of the source aperture; θ is the angle made by the particle trajectory with the x axis. For all cases considered $\cos \theta \approx 1$. Also, $V = v/v_0$, and $P(V)$ is the transition probability of the particles in the homogeneous field region.

For case 2 we introduce $n_0 = N_0/A_0$. The difference between these two cases is seen to be that N_0 is a given quantity in case 1, and n_0 in case 2;

$$I_{(2)} = \frac{n_0}{\pi} \int dA_0 \int \cos \theta \, d\Omega \int P(V) V^3 e^{-V^2} \, dV.$$

Both these integrals must apply to regions of V , θ , and Z_0 in such a way that the particles pass around the stops and through all the apertures. (See Appendix.) The integration is aided by the fact that one parameter is quite small, thus allowing an expansion. For if pv_0 is the velocity of

a particle which passes through the centers of the oven and collimator apertures and just grazes the A stop, and if $p'v_0$ is the velocity of a particle which also grazes the A stop but passes through some other part of the oven and collimator apertures, then $p' - p \approx 1/p^4 \ll 1$.

With the help of this expansion, the first-order effect of the parameter p can be separated from that of r and s and be separately maximized.

The result of the integration is

$$I_{(1)} = N_0 \rho^2 F_1(p) S_1(r, s) [1 + \epsilon(p) B(r, s) + \dots],$$

$$I_{(2)} = n_0 \rho^4 L_C^2 F_2(p) s^2 S_2(r, s) [1 + \epsilon(p) B(r, s) + \dots].$$

We notice at once that the intensity $I_{(1)}$ does not depend on L_C . One can get as much intensity with a high-resolution machine (large L_C) as with one of low resolution (small L_C). The intensity also depends on ρ^2 , showing that machines designed for small moments necessarily have low intensity, as expected.

The functions $F(p)$ and $S(r, s)$ lead to more interesting conclusions: $F(p)$ was calculated for three possible forms of the transition probability $P(V)$. The first, $P_I(V) = P_0 \sin^2(\pi/2V)$, is appropriate to radio-frequency transitions at the center of the absorption curve and for approximately optimum rf power. The second, $P_{II}(V) = P_0/V$, is appropriate to optical transitions such as were used in the experiments of Perl, Rabi, and Senitzky⁸ or of Marrus and McColm.⁹ The third, $P_{III}(V) = P_0$, was included to describe a "flop-out" geometry (selector gradients in opposite directions) in which a signal is obtained with no transitions occurring in the homogeneous field. The functions $F(p)$ showed a maximum as a function of p in all these cases. The best values of p and $F(p)$ and of the second-order correction $\epsilon(p)$ are shown in Table I. It is seen that there is little difference between the best values of p regardless of the

choice of $P(V)$, but a large difference between cases 1 and 2. This is because the source aperture area goes as p^2 . We notice also that $\epsilon(p)$ is small for case 1 but not so for case 2; still $B(r, s)$ is small enough that the total correction term is still not large.

The signal function S_1 is shown in Fig. 5. As a function of s , S_1 does not show a maximum but exhibits saturation, with a knee at $s \approx 2r$. Also, S_1 increases as r decreases, so that a very small r corresponds to a large signal. However, the area of the source aperture goes as r , so that as r is made smaller and smaller, progressively higher and higher temperatures are needed in the source to maintain the desired number of particles leaving it. Thus a compromise must be reached in the choice of a value of r , one which depends on the type of source used.

The signal function S_2 is shown in Fig. 6. Note that the intensity $I_{(2)}$ depends on L_C^2 , and so increases as the homogeneous field region is lengthened. It also depends on $s^2 S_2$, and so the question arises whether it is more profitable to increase s or L_C ; an increase in L_C is usually accompanied by an increase in precision; not so with an increase in s . Thus as soon as S_2 is maximum, further increases in $I_{(2)}$ should be achieved by increasing L_C . As seen in the figure this occurs at $s \approx 2r$. We also note that the dependence on r is not strong, that a symmetric machine ($r = 1/2$) has about as strong a signal as an asymmetric one ($r < 1/2$).

The dependence of the second-order correction function $B(r, s)$ on r and s is shown in Fig. 7; and it is seen that this dependence is not sufficiently strong to materially alter these conclusions, aside from somewhat reducing the dependence of $I_{(1)}$ on r .

The figure of merit M is defined as the ratio of the signal to the square root of the noise, on the assumption that the fluctuations in the

noise will go as its square root. This is admittedly a poor assumption in some cases, fluctuations in noise sometimes being as large as the noise itself; still it is the most useful measure for most instances. For noise case (a) (see Introduction) the noise is taken as proportional to the signal $N_a = k_a I$ so that

$$M_{ja} = [I_{(j)}/k_a]^{1/2}$$

for both case -1 signals ($j=1$) and case -2 signals ($j=2$). In order to isolate the dependence of M on r and s we introduce

$$m_{ja} = S_j^{1/2}$$

so that

$$M_{1a} = \rho m_{1a} [N_0 F_1(p)(1+\epsilon B)]^{1/2} k_a^{-1/2},$$

$$M_{2a} = \rho^2 s L_C m_{2a} [n_0 F_2(p)(1+\epsilon B)]^{1/2} k_a^{-1/2}.$$

Plots of m_{1a} and m_{2a} are shown in Fig. 8. The best values of p , r , and s arrived at above are appropriate to M , since k_a is taken as independent of these parameters.

For noise case (b), the noise is taken as proportional to the detector area, $N_b = k_b A_D$, so that

$$M_{jb} = I_j / [k_b A_D]^{1/2}.$$

And since $A_D = (\rho L_C / p)^2 s^2 \beta^2 (r, s)$, where $\beta^2 = (1/2d)[(s+2r)(1-r)/r]^2$, we define m_{jb} as

$$m_{1b} = S_1 / s\beta$$

$$m_{2b} = S_2 / \beta$$

so that

$$M_{1b} = \rho (N_0 / L_C) (m_{1b}) (p F_1) (1+\epsilon B) k_b^{-1/2},$$

$$M_{2b} = \rho^3 (n_0 s L_C) (m_{2b}) (p F_2) (1+\epsilon B) k_b^{-1/2}.$$

The quantities m_{1b} and m_{2b} are shown in Fig. 9. The best values of

p are shown in Table II. It is to be noted that close to their maxima the functions pF are quite flat; for example, using transition probability P_I , the value of pF_1 at $p = 1.5$ is different from its maximum value at $p = 1.7$ by only 4% .

Note also that M_{1b} is large only for $s \approx r$, while $M_{2b} \propto sm_{2b}$ increases with s . And since M_{2b} is proportional to L_C , the question arises whether to increase s or L_C in order to improve M . From Fig. 9 one sees that beyond $s \approx 2r$ increases in M are best made by increasing L_C . The best value of r is $r \approx 1/2$ (symmetric machine).

I conclude that in order to have an apparatus operate well in the presence of both types of noise, in case 1 one should choose $s \approx 2r$, and make r as small as is consistent with possible source temperatures (asymmetric machine), and that one should have $p \approx 1.5$. The apparatus should be kept short, for while its length does not affect the strength of the signal, the area of the detector increases as the machine is lengthened, and consequently the detector noise increases. In case 2, where the signal increases both with s and with L_C , $s \approx 1$ is the best choice; further increases in signal are better made by lengthening the apparatus (increasing L_C). However, as the length is increased, the total quantity of material in the source also must be increased, so that an extremely long device might be inconvenient to operate. Finally, in this case the optimum choices are $r \approx 1/2$ (symmetric machine) and $p \approx 1.3$.

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APPENDIX

The integrals $I_{(1)}$ and $I_{(2)}$ are transformed by setting $d\Omega = a_A dZ_c / (r+s)^2 L_C^2$. The values of Z_c , the displacement at the collimator, and of Z_0 , the displacement at the oven, are restricted by the condition that the particles pass through the collimator,

$$-\frac{a_c}{2} \leq Z_c \leq \frac{a_c}{2},$$

and through the detector aperture,

$$-\frac{a_c}{2} + (1-r)Z_0 \leq Z_c \leq \frac{a_c}{2} + (1-r)Z_0.$$

For the particles to pass the A selector, $V = v/v_0$ is limited by

$$W_A/p \leq V \leq pW_A,$$

while for them to pass the B selector,

$$W_B/p \leq V \leq pW_B.$$

The quantities W_k are given by

$$W_k^2 = \frac{1}{1 + \chi_k},$$

where $k = A$ or B ,

$$\chi_k = a_c(k)Z_c + a_0(k)Z_0.$$

Here,

$$\begin{aligned} a_c(A) &= -2/[p\rho(r+s)] & a_0(A) &= Da_c(A)/[s(s+2r)], \\ a_c(B) &= da_1(A)/[s(s+2r)(1-r)], & a_0(B) &= -a_0(A), \\ d &= s(s+2r)(1+r) + 2r^2, & D &= s(s+2r) + 2r^2. \end{aligned}$$

Note, $\chi \approx p^{-4}$. Therefore, we do a Taylor expansion as a function of χ .

Thus f_{AB} , defined by

$$f_{AB}(p, z_c, z_0) = \int_{W_{B/P}}^{PW_A} 2V^3 P(V) e^{-V^2} dV,$$

is expanded,

$$f_{AB} = f(p) + \chi_B g(1/P) - \chi_A g(p) + \dots,$$

where

$$f(p) = \int_{1/P}^P 2V^3 P(V) e^{-V^2} dV,$$

$$g(p) = p^4 P(p) e^{-P^2}.$$

Therefore, the intensity function $I_{(1)}$, which with the restrictions listed above has the form

$$I_{(1)} = \frac{N_0 a_A}{2\pi a_0 (r+s)^2} \left\{ \int_{-a_c/2}^{a_c/2} dz_c \int_{-a_0/2}^{z_c/\Gamma} dz_0 f_{AB} + \int_{-a_c/2}^{a_c/2} dz_c \int_{z_c/\Gamma}^{a_0/2} dz_0 f_{BA} - \int_{qa_c/2}^{a_c/2} dz_c \int_{-a_0/2}^{t_1} dz f_{AB} - \int_{-a_c/2}^{-qa_c/2} dz_c \int_{t_2}^{a_0/2} dz_0 f_{BA} \right\},$$

becomes

$$I_{(1)} = \frac{N_0}{2\pi} \Omega_C f(p) \left\{ 1 - \frac{r}{2} q(r, s) + \frac{3}{4} \epsilon(p) r Q(r, s) + \dots \right\}.$$

Here

$$\Gamma = \frac{1-r}{r} q(r, s), \quad t_1 = [(-a_c/2) + z_c]/(1-r),$$

$$t_2 = [(a_c/2) + z_c]/(1-r), \quad \Omega_C = a_C a_A / (r+s)^2,$$

$$Q(r, s) = [s(s+2r) + r]/d.$$

With the definitions

$$S_1 = \frac{1}{4\pi} \left[\frac{s^2 (s+2r)^2 (1-r)}{(r+s)^2 d} \right] \left[1 - \frac{1}{2} r q(r, s) \right] ,$$

$$S_2 = \frac{1}{4\pi} \left[\frac{s^2 (s+2r)^4 r (1-r)}{(r+s)^2 d^2} \right] q(r, s) \left[1 - \frac{1}{2} r q(r, s) \right] ,$$

$$B = \left[\frac{3}{4} r Q(r, s) \right] / \left[1 - \frac{1}{2} r q(r, s) \right] ,$$

$$F_1(p) = p^{-2} f(p) ,$$

$$F_2(p) = p^{-4} f(p) ,$$

we obtain the equations given in the text.

FOOTNOTES AND REFERENCES

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Table I. Dependence of the intensity on the parameter p .

Case	Transition probability	Best value of p	$F(p)/P_0$	$\epsilon(p)$
1	I	1.5	0.24	0.27
1	II	1.5	0.24	0.26
1	III	1.6	0.26	0.22
2	I	1.3	0.13	0.80
2	II	1.3	0.13	0.69
2	III	1.3	0.13	0.85

Table II. Dependence of the figure of merit on the parameter p for noise case (b).

Case	Transition probability	Best value of p	pF/P_0
1	I	1.7	0.36
1	II	1.8	0.40
1	III	1.9	0.44
2	I	1.3	0.17
2	II	1.4	0.17
2	III	1.4	0.18

FIGURE CAPTIONS

- Fig. 1. State selector employing two wire equipotentials. The two equipotentials suitable for surfaces of a state selector and the selector aperture, a square of side a , are indicated. The relative gradient, a quantity defined for any point in the aperture as the ratio of the field gradient at that point to that at the center, is shown as a function of z/a .
- Fig. 2. State selector employing quadrupole equipotentials. The two equipotentials suitable for the surfaces of a state selector are shown and the aperture, again a square of side a , is indicated. The hyperbolic surface is well approximated by a circular one of radius $2.5 a$ having a center a distance $3 a$ to the right of the center of the aperture. The relative gradient is shown as a function of z/a . Note that the gradient is very inhomogeneous near the 90-deg angle, and a screen with the aperture cut in it is needed in the selector.
- Fig. 3. Beam apparatus. The source, collimator, and detector apertures a_S , a_C , and a_D , respectively, are shown, as are parts of the two state selectors. The state selector nearest the source is of length L_A ; that nearest the detector, L_B . L_C is the distance between the two state selectors, while L_{AC} is the distance between the collimator and the end of the first state selector. No drift space between the source and the first selector or between the detector and the second state selector is assumed. Distances in the z direction have been exaggerated for clarity.

Fig. 4. Trajectories. The limiting trajectories, LT, the outer trajectories, OT, and the inner trajectories, IT, are shown. S is the source slit; C, the collimator slit; and D, the detector slit. A is the limiting aperture in the first selector, placed at the stop position, and B is the limiting aperture in the second selector, also placed at the stop position. The stops are not shown. Distances in the z direction have been exaggerated for clarity.

Fig. 5. Signal function for case 1 (where source efflux is independent of source aperture). Terms s and r are defined mathematically in the text.

Fig. 6. Signal function for case 2 (where source efflux is proportional to source aperture area). Terms s and r are defined mathematically in the text.

Fig. 7. Correction term $B(r, s)$ as a function of r and s.

Fig. 8. Dependence of the figure of merit on r and s for situations included in noise case (a) (fluctuation in the noise proportional to the square root of the signal); m_{1a} and m_{2a} are defined in the text.

Fig. 9. Dependence of the figure of merit on r and s for situations included in noise case (b) (fluctuations in the noise proportional to the square root of the detector area); m_{1b} and m_{2b} are defined in the text.

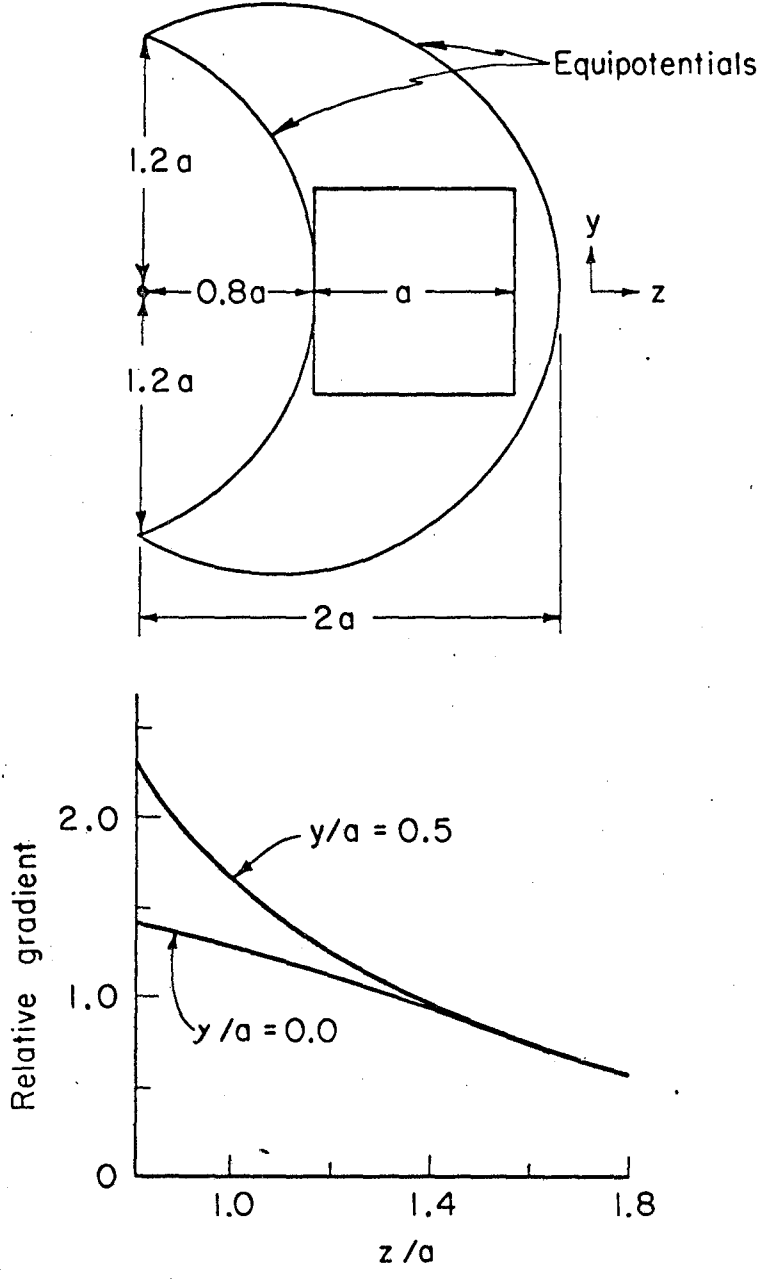
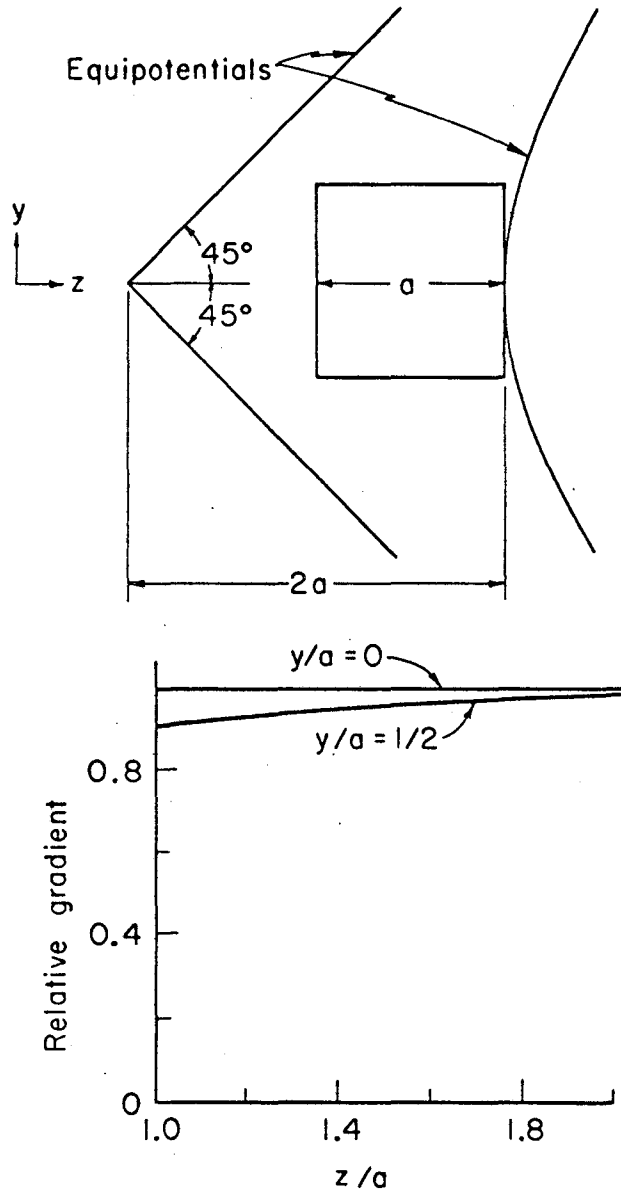


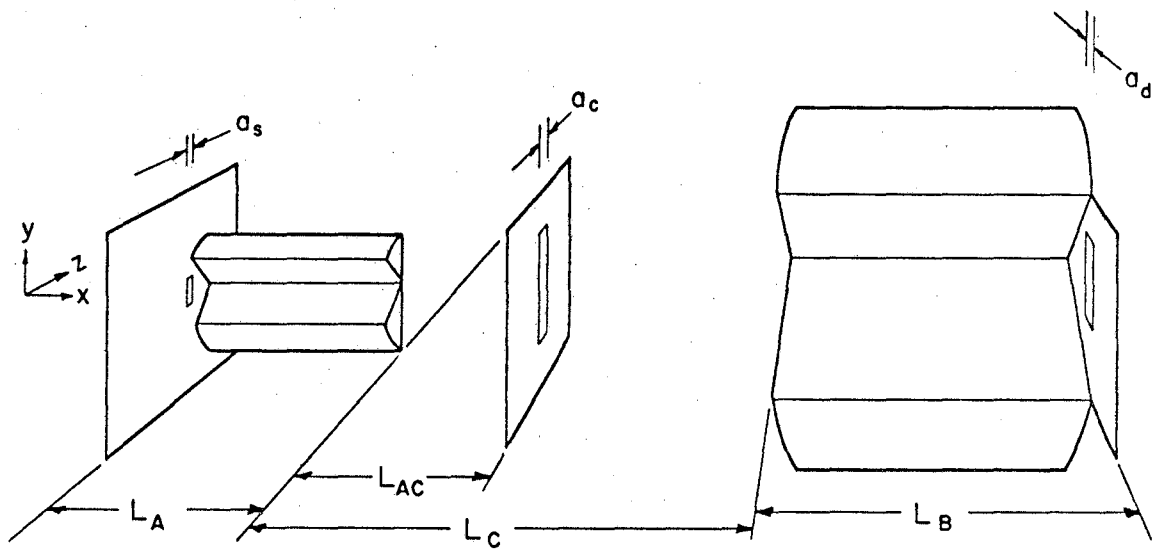
Fig. 1

MUB-4248



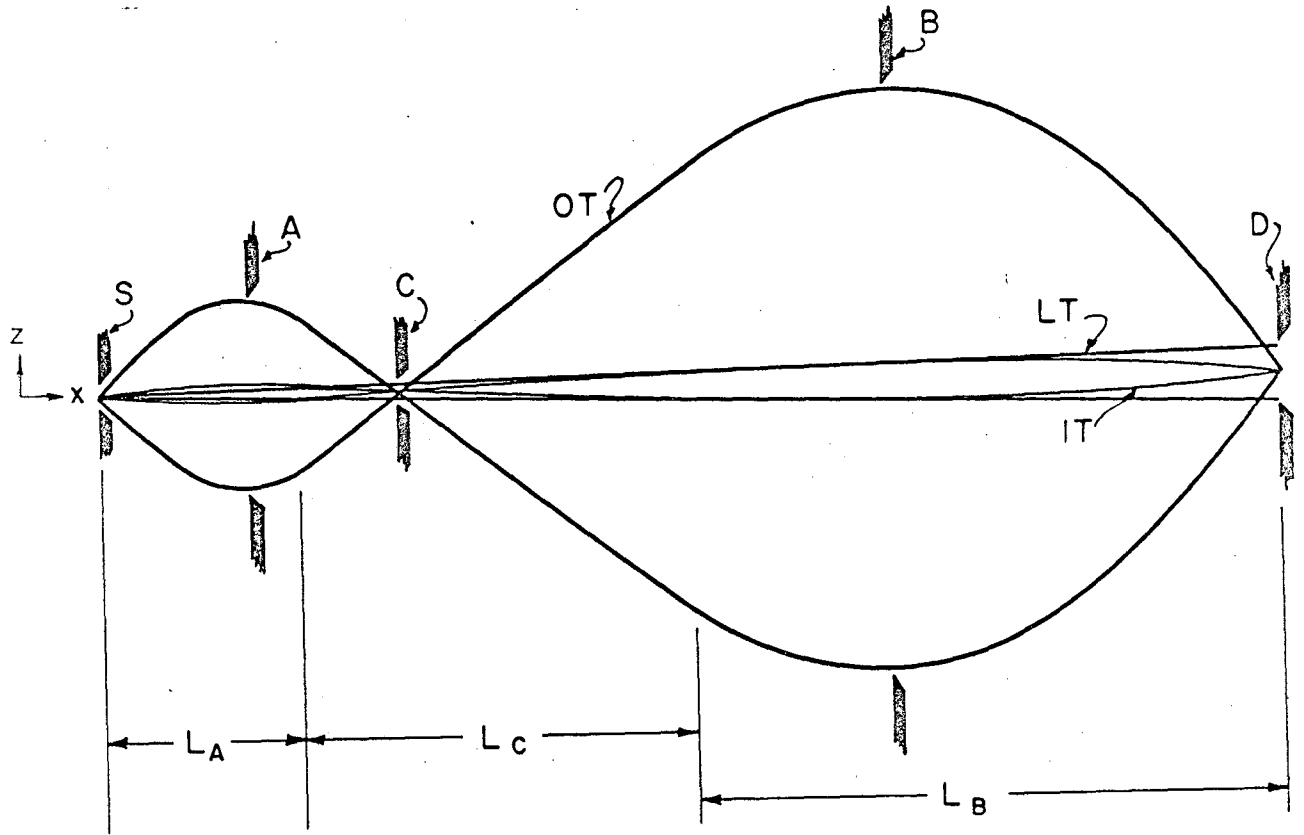
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Fig. 2



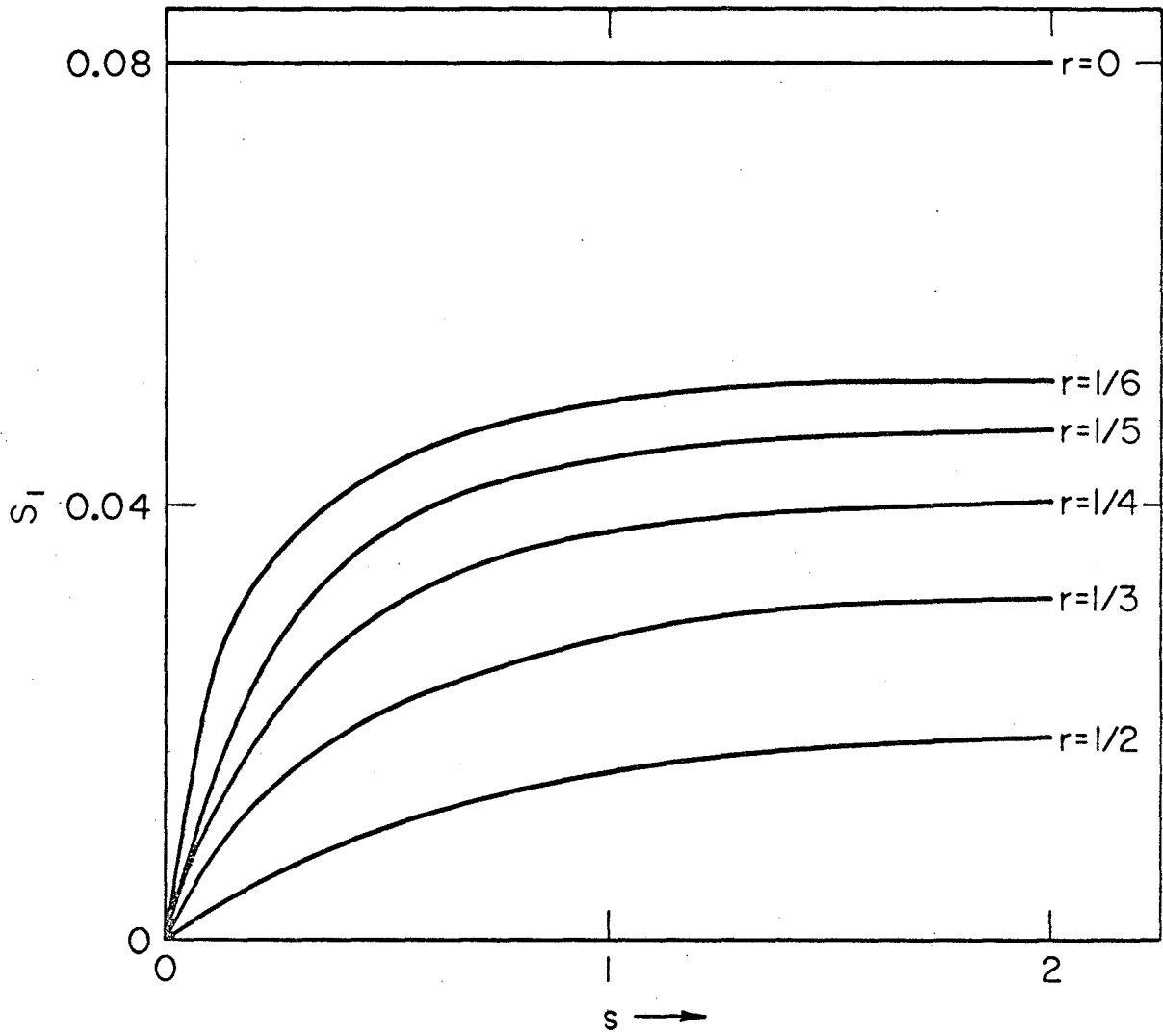
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Fig. 3



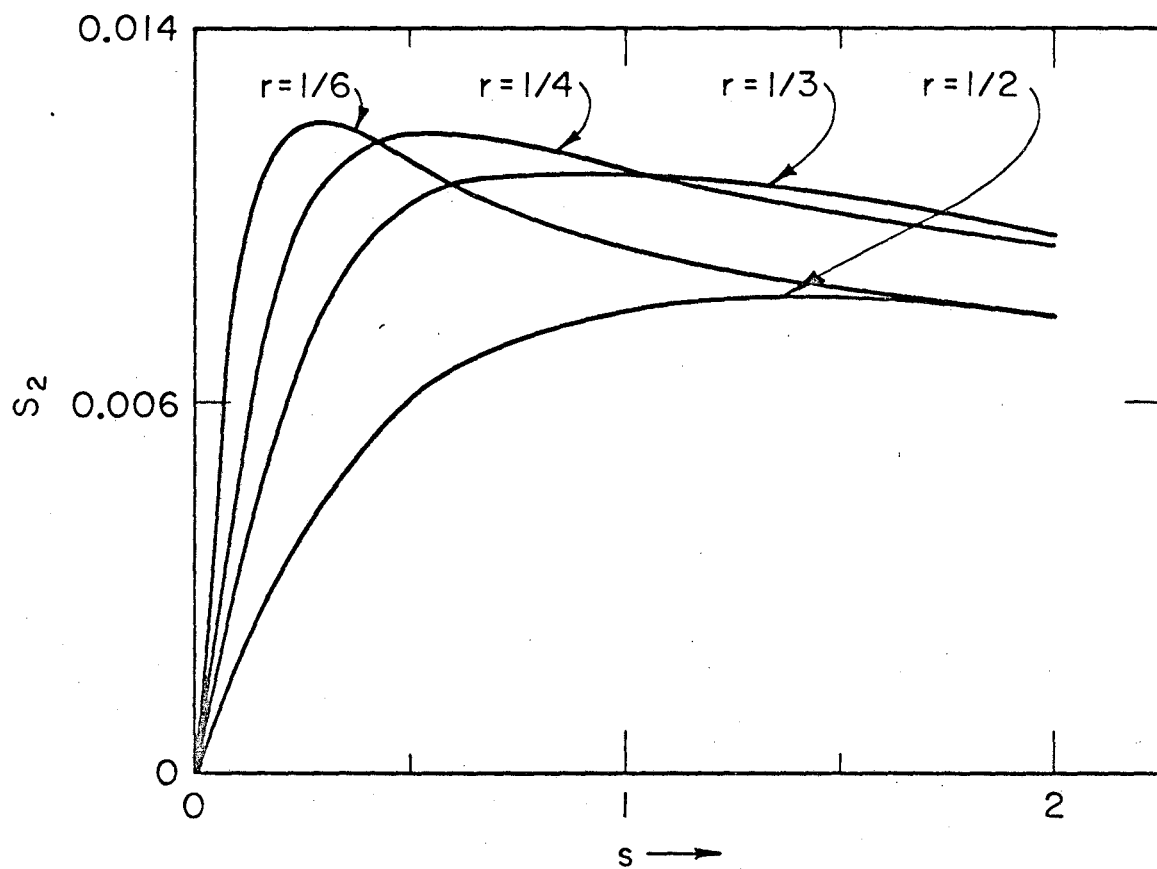
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Fig. 4



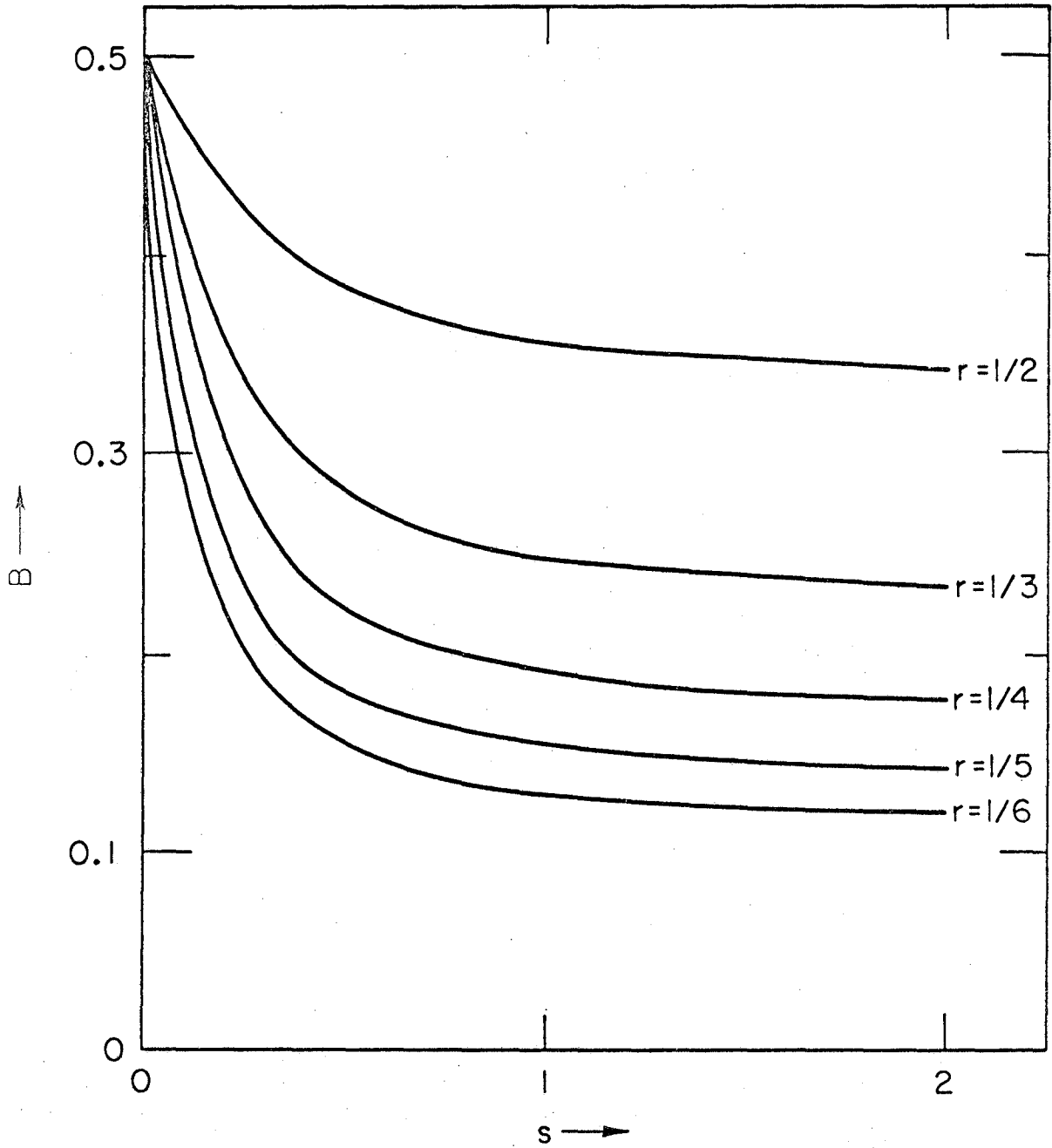
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Fig. 5



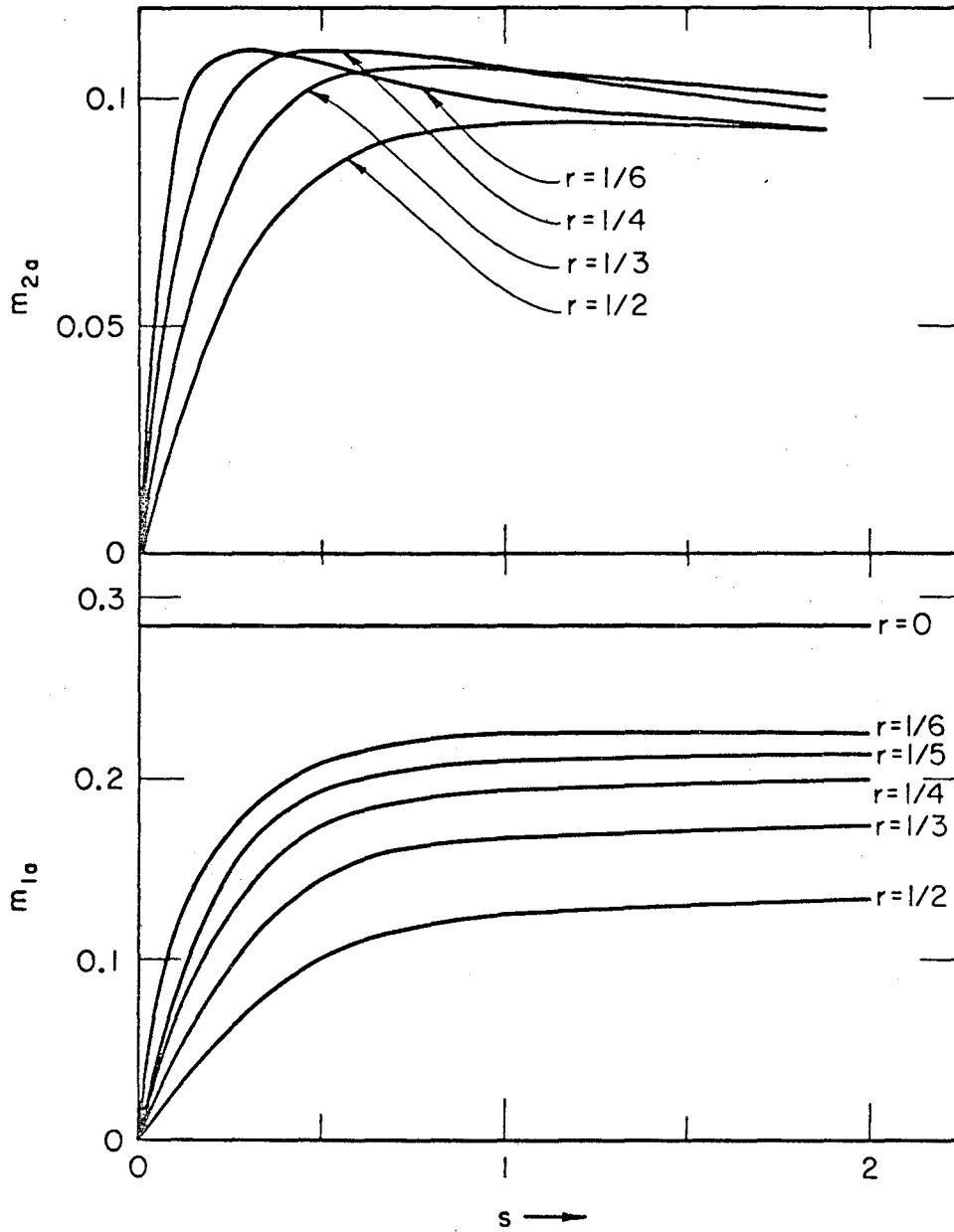
MUB-9637

Fig. 6



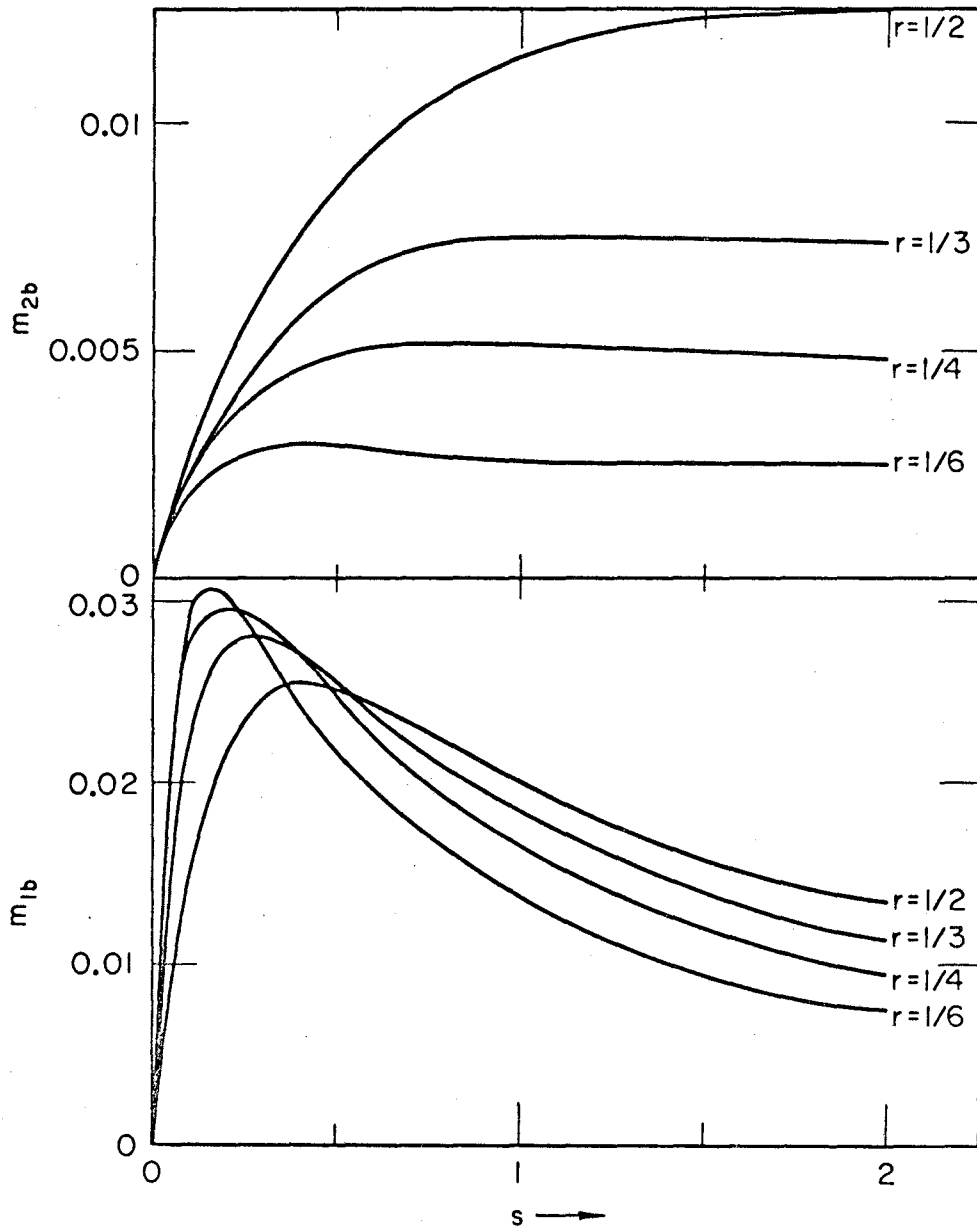
MUB-9638

Fig. 7



MUB-9639

Fig. 8



MUB-9640

Fig. 9

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