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DISCLAIMER

INCLUSION OF THE PSEUDOSCALAR MESON OCTET IN THE ABFST MULTIPERIPHERAL MODEL*

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May 17, 1971

ABSTRACT

The multiperipheral model of ABFST is modified to include the exchange of the complete pseudoscalar meson octet. Assuming only conservation of isospin and hypercharge--i.e., invariance under $SU(2)_I \otimes U(1)_Y$ --one finds that the exchange of kaons and etas increases only slightly the $t = 0$ intercept of the leading output Regge pole. Alternative ways of increasing the strength of the kernel are presented.

I. INTRODUCTION

It has been known for some time that the Amati-Bertocchi-Fubini-Stanghellini-Tonin multiperipheral model[1] is well suited for a qualitative description of high energy phenomena. The model predicts Regge asymptotic behavior; slow increase of the average multiplicity with the energy (as $\ell n\ s$); mean transverse momenta of secondaries small, and independent of the energy of incident and produced particles, in the pionization region. These are all features that are in good agreement with experiment.

There remains, however, a quantitative serious discrepancy, namely, the insufficient strength of the kernel that is two to four times too weak[2,3] to generate an output Regge pole with intercept approximately equal to one in the channel with the vacuum quantum numbers. Reference 2 quotes, for example, $\alpha_P(0) = 0.30$ and $\alpha_\rho(0) = 0.16$. In the approximation of setting the pion mass equal to zero CRS[2] obtained $\alpha_P(0) = 0.45$ and $\alpha_\rho(0) = 0.23$, although, as they point out, it is a good approximation only if $\alpha_P(0) \gtrsim 1/2$.

Several reasons have been put forward to explain the insufficient strength of the kernel, and many of them have been investigated.[4] In this paper we study the effect on the kernel of including the exchange of K's and η's in the model. Although the motivation for including those particles on an equal footing with the pion relies on the SU(3) symmetry assumption, we have proceeded further[5] and have studied the problem in the case of totally broken SU(3) symmetry using a nonperturbative formalism. That is to say we have considered that the only conserved quantum numbers are isospin (I) and hypercharge (Y), and correspondingly that the dynamics is invariant under the product $SU(2)_I \otimes U(1)_Y$. In this approach we find that the model predicts an almost negligible change in the position of the output Regge poles.

We begin in Sec. II by deriving the modified integral equation for the absorptive part of the elastic $\pi\pi$ amplitude. As expected, one obtains a set of coupled integral equations.

In Sec. III we consider the trivial complication of incorporating hypercharge to the model and outline the procedure for obtaining the corresponding crossing matrices.

In Sec. IV we discuss the parametrization of the resonance region of the meson-meson amplitudes we used as input. We state our results and discuss them in Sec. V.

II. INTEGRAL EQUATION FOR THE ABSORPTIVE PART

We begin this section by recalling the basic assumption of ABFST. That is that the amplitude for two pions going to 2n pions is given by

$$T_{2n}(p_1,\cdots,p_{2n};\ p_a,\ p_b) = \frac{1}{(q_1^2-\mu^2)(q_2^2-\mu^2)\cdots(q_{n-1}^2-\mu^2)}$$

$$\times\, T_2(p_1,p_2;\ p_a,q_1)\cdot T_2(p_3,p_4;-q_1,q_2)\cdots T_2(p_{2n-1},p_{2n};-q_{n-1},p_b), \tag{II.1}$$

where (see Fig. 1) $q_i = \sum_{j=1}^{2i} p_j - p_a$, μ is the pion mass, and $T_2(p_{2i+1},\ p_{2j};\ -q_i,\ q_j)$ is the off-shell 2-to-2 $\pi\pi$ scattering amplitude.

Nof if we let the exchanged particles be of different types but all with masses of the same order of the pion mass, then under exactly the same assumptions of ABFST we can write for the amplitude for producing a particular set of 2n final particles

$$T_{2n}(p_1,\cdots,p_{2n};\ p_a,p_b) = \frac{1}{(q_1^2-\mu_1^2)(q_2^2-\mu_2^2)\cdots(q_{n-1}^2-\mu_{n-1}^2)}$$

$$\times\ T_{a1}(p_1,p_2;\ p_a,q_1)\ T_{12}(p_3,p_4;\ -q_1,q_2)\cdots$$

$$\times\ T_{(n-1)b}(p_{2n-1},p_{2n};\ -q_{n-1},p_b)\ ,$$

where μ_i, $i = 1,2,\cdots,(n-1)$ are the masses of the particles being exchanged at the ith link of the multiperipheral chain.

In the process of ordering the particles along the chain, we associate one of the initial particles with the target and the other with the projectile. Correspondingly, we order the produced particles according to their longitudinal momenta, the fastest particle associated with (or "near") the projectile, the slowest with the target.

With this in mind we can write immediately the modulus squared of the amplitude for producing 2n final particles,

$$|T_{2n}(p_1,\cdots,p_{2n};\ p_a,p_b)|^2 = \sum_{\substack{\mu_1,\mu_2,\\ \cdots,\mu_{n-1}}} |T_{a1}(p_1,p_2;\ p_a,p_b)$$

$$\times\ \frac{1}{(t_1-\mu_1^2)}\ T_{12}(p_3,p_4;\ -q_1,q_2)\ \frac{1}{(t_2-\mu_2^2)}\cdots\frac{1}{(t_{n-1}-\mu_{n-1}^2)}$$

$$\times\ T_{(n-1)b}(p_{2n-1},p_{2n};\ -q_{n-1},p_b)\ |^2\ , \qquad \text{(II.2)}$$

where $t_i = q_i^2$, and $T_{ij}(p_{2i+1},p_{2j};\ -q_i,q_j)$ is the off-shell scattering amplitude for mesons i and j, and the sum extends over the different types of particles that can be exchanged at the first, second, etc. links. In our case $\mu_i = \mu_\pi,\ \mu_\eta,\ \mu_K$; $i = 1,\ 2,\cdots,(n-1)$.

A diagram displaying one term of the sum in Eq. (II.2) is shown in Fig. 2.

In assuming the form (II.2) for the amplitude squared we are neglecting "interference" terms arising from the exchange of identical particles from different blobs along the chain, thus the diagram of Fig. 3 is not contained in our formalism. However, it has been shown[6] that the effect on the position of the output vacuum pole of such kind of diagrams is negligible at least in the original ABFST model, so we expect--and hope--such terms will not be important here either.

As usual the cross section for producing 2n particles is given by

$$\sigma_{2n}(s) = \frac{(2\pi)^{4-6n}}{2\lambda(s,\mu_a^2,\mu_b^2)}\int d^4p_1\ \delta^{(+)}(p_1^2-\mu_a^2)d^4p_2\ \delta^{(+)}(p_2^2-\mu_1^2)\cdots$$

$$\times\ d^4p_{2n}\ \delta^{(+)}(p_{2n}^2-\mu_b^2)\ \delta^{(4)}(p_a+p_b-\sum_{i=1}^{2n} p_i)$$

$$\times\ |T_{2n}(p_1,\cdots,p_{2n};\ p_a,p_b)|^2 \qquad \text{(II.3)}$$

where

$$\lambda(a,b,c) = [a^2+b^2+c^2-2ab-2ac-2bc]^{1/2}\ . \qquad \text{(II.4)}$$

The total cross section is

$$\sigma_{tot}(s) = \sum_{n=1}^{\infty} \sigma_{2n}(s)\ . \qquad \text{(II.5)}$$

The optical theorem gives

$$A_{\mu_a,\mu_b}(s) = \lambda(s,\ \mu_a^2,\ \mu_b^2)\ \sigma_{tot}(s)\ , \qquad \text{(II.6)}$$

where $A_{\mu_a,\mu_b}(s)$ is the forward absorptive part of the amplitude for elastic scattering of particles a and b of masses μ_a and μ_b respectively.

Following ABFST it is possible to derive an integral equation for $A_{\mu_a,\mu_b}(s)$. However as our main interest is the position of the output Regge poles we examine the diagonalized equation instead. In our case it can be written[7]

$$F^J_{\mu\mu'}(t,t') = G^J_{\mu\mu'}(t,t') + \sum_{\mu''} \int_{-\infty}^{0} dt'' \, F^J_{\mu\mu''}(t,t'') K^J_{\mu''\mu'}(t'',t'), \tag{II.7}$$

where

$$F^J_{\mu\mu'}(t,t') = \frac{2}{\pi} \int_{(\mu+\mu')^2}^{\infty} ds \, \frac{e^{-(J+1)\eta(s,t,t')}}{(J+1)} A_{\mu\mu'}(s,t,t'), \tag{II.8}$$

$$K^J_{\mu''\mu'}(t'',t') = G^J_{\mu''\mu'}(t'',t') \, (t'' - \mu''^2)^{-2}/(32\pi^2) , \tag{II.9}$$

and

$$G^J_{\mu''\mu'}(t'',t') = \frac{2}{\pi} \int_{(\mu''+\mu')^2}^{\infty} ds \, \frac{e^{-(J+1)\eta(s,t'',t')}}{(J+1)} A^R_{\mu''\mu'}(s,t'',t'), \tag{II.10}$$

with

$$\cosh \eta(s,t,t') = (s - t - t')/(4tt')^{1/2} .$$

In (II.10) $A^R_{\mu''\mu'}(s,t'',t')$ is the off-shell continuation of $A^R_{\mu''\mu'}(s)$, where

$$A^R_{\mu''\mu'}(s) = \lambda(s, \mu''^2, \mu'^2)\sigma_{e\ell}^{\mu''\mu'}(s) , \tag{II.11}$$

$\sigma_{e\ell}^{\mu''\mu'}(s)$ being the elastic cross section for scattering of particles μ' and μ'', and $\lambda(a,b,c)$ is given by (II.6).

Figure 4 shows Eq. (II.7) in diagrammatic form.

We regain the elastic absorptive part by computing $A_{\mu\mu'}(s,t,t')$ on mass shell, that is on $t = \mu^2$; $t' = \mu'^2$.

Let us remark that the inverse of (II.8) is

$$A_{\mu\mu'}(s,t,t') = \frac{1}{4\pi i(tt')^{1/2}} \int_{c-i\infty}^{c+i\infty} dJ \, \frac{e^{+(J+1)\eta(s,t,t')}}{\sinh \eta(s,t,t')} (J+1) F^J_{\mu\mu'}(t,t'), \tag{II.12}$$

the contour of integration running to the right of the rightmost singularity in J of $F^J_{\mu\mu'}(t,t')$.

The important point to notice here is that Eq. (II.7) is actually a system of integral equations. The amplitude for scattering of particles μ and μ' is coupled to the amplitude for scattering of particles μ and μ'' with μ'' running over the different types of particles being exchanged.

It will be shown in the next section that after incorporating isospin and hypercharge in the scheme we get three integral equations for the output vacuum pole and two for the ρ pole.

III. GENERALIZATION TO INCLUDE ISOSPIN AND HYPERCHARGE

The extension to include isospin and hypercharge is trivial once we realize, following ABFST, that a definite isospin and hypercharge in the t channel (see Fig. 4) is carried through the multiperipheral chain because of the conservation laws. If we work with an amplitude of definite isospin and hypercharge in the t channel, then their inclusion introduces merely a couple of parameters I_t, Y_t in our Eq. (II.7). One has to know $A^{R,I_t,Y_t}_{\mu\mu'}(s,t,t')$, of course, and that is accomplished by the use of the hypercharge-isospin crossing matrix β; so that

$$A_{\mu\mu'}^{R,I_t,Y_t}(s,t,t') = \sum_{I_s,Y_s} \beta_{I_s,Y_s}^{I_t,Y_t} A_{\mu\mu'}^{R,I_s,Y_s}(s,t,t'), \quad \text{(III.1)}$$

where $A_{\mu\mu'}^{R,I_s,Y_s}(s,t,t')$ is given by (II.11).

We give a detailed description of $\sigma_{e\ell,I_s,Y_s}^{\mu\mu'}(s)$ in the next section. Here we want to explain the procedure used to evaluate β.

As we are working in a problem invariant under the product $SU(2)_I \otimes U(1)_Y$, our crossing matrices are the direct product of the crossing matrices for isospin and hypercharge.

We are going to study $\pi\pi$ scattering. Therefore in (II.7) the intermediate states can be $\pi\pi$, $K\bar{K}$, or $\eta\eta$ if we are looking for the output vacuum pole $(I_t = 0 = Y_t;\ G_t = +1)$. For the ρ pole (now $I_t = 1$, $Y_t = 0$; $G_t = +1$) there are only two channels, $\pi\pi$ and $K\bar{K}$. There are two ways of forming an eigenfunction of zero hypercharge out of two kaons. One is symmetric, the other antisymmetric. However, we are looking for G parity +1 in the t channel, and therefore we have to choose the antisymmetric combination. Another way of seeing this is to recall that the toal wave function of the system of two kaons must be symmetric, and we have, for the vacuum pole, that $K\bar{K}$ must be in an even orbital state; the isospin function is odd, therefore its hypercharge wave function must be odd. The same result holds for the rho. Here the orbital state is now odd, but the total isospin eigenfunction $(I = 1)$ is even, so again the hypercharge wave function must be antisymmetric.

This problem does not arise with the $\pi\pi$ and $\eta\eta$ systems because they are already eigenstates of zero total hypercharge. There

remains to specify that we use the phase convention of Refs. 8-10 for the antiparticle states. Namely, if we represent a particle state $|C\rangle$ by the hypercharge-isospin state $|I_C,Y_C,I_{zC}\rangle$, then the antiparticle state $|\bar{C}\rangle$ is given by

$$|\bar{C}\rangle = (-1)^{I_{zC}+Y_C/2}\,|I_C, -Y_C, -I_{zC}\rangle\,. \quad \text{(III.2)}$$

Once Eq. (II.7) has been projected onto the three allowed intermediate states, we require $A_{\mu\mu'}^{R,I_s,Y_s}(s,t,t')$ for the following six processes:

$$\begin{array}{llll} \text{(a)} & \pi\pi \to \pi\pi & \text{(b)} & K\bar{K} \to \pi\pi \\ \text{(c)} & \eta\eta \to \pi\pi & \text{(d)} & \eta\eta \to \eta\eta \\ \text{(e)} & K\bar{K} \to \eta\eta & \text{(f)} & K\bar{K} \to K\bar{K}\,. \end{array} \quad \text{(III.3)}$$

The matrix $\beta_{I_s,Y_s}^{I_t,Y_t}$ needed for Eq. (III.1) is given in the Appendix.

IV. LOW ENERGY MESON-MESON AMPLITUDES

As we are mainly interested on the effect that the inclusion of K exchange has on the kernel, we are going to make the assumption that the input, low energy meson-meson scattering amplitude is primarily given by the on-shell transition via a few sharp resonances. This way of parametrizing the low energy kernel has proven to be meaningful in previous works.[2,3,11]

Contracting the Breit-Wigner forms of the resonances to Dirac deltas we get for $\sigma_{e\ell}^{\mu\mu'}(s)$ of Eq. (II.11) [12]

$$\sigma_{e\ell}^{\mu\mu'}(s) = \sum_i \frac{4\pi^2 k_i}{q^2}\,(2J_i + 1)(x_i^{\mu\mu'})^2\, m_i\,\Gamma_i\,\delta(s - m_i^2)\,, \quad \text{(IV.1)}$$

where J_i, m_i, Γ_i and $x_i^{\mu\mu'}$ are, respectively, the spin, mass, total width, and elasticity of resonance i in the system $\mu\mu'$, and q and s are the c.m. momentum and total energy squared, respectively; k_i is a factor to take into account the distinguishability or not of particles μ and μ'.

This is the general form of the input elastic cross sections we have used. The assumed properties of the resonances are given in Table I.

If we parametrize the low energy region by the total cross sections--i.e., allowing the $K\bar{K}$ resonances to decay in $\pi\pi$--and assume the same resonance dominance as before, we get for the cross section an equation almost identical to (IV.1), the only difference being that the elasticities $x_i^{\mu\mu'}$ are raised to the first power instead of to the second.

These models for the low-energy cross sections were fed into our equations and the output Regge poles were determined by searching for zeroes of the Fredholm determinant of the homogeneous system (II.7). We used standard numerical methods based on Gaussian quadratures to compute this quantity. The results are presented in the next section.

V. RESULTS AND DISCUSSION

As we have stated in the preceding section, we investigated our model using two different approximations for the low energy amplitudes. They consisted in using the elastic and total cross sections in (II.11), respectively.

Furthermore, we repeated the computations several times, varying the elasticities shown in Table I, even including as resonance $\pi_N(1016)$ and doubling the $\rho_N(1670)$ elasticities,[13] but the results showed almost negligible dependence on the channels other than $\pi\pi$, so

the output poles suffered little if any change at all in position. The results are shown in Table II.

Our results agree with those of Ref. 2, in which only π exchange and low-energy total cross sections were used. Although our parametrization was slightly different, the agreement shows again how small is the dependence of the output pole positions on slight variations of the input amplitudes. This effect has already been noted by D. M. Tow.[2]

One result that is not totally unexpected is that the effect of including K and η exchange, in this model, can be taken into account by simply replacing the elastic cross sections by the total ones in the input amplitudes (II.11).

Nonetheless, there remains the main result that K exchange has very little effect on the strength of the kernel in spite of the fact that the use of unmodified propagators for K's and η's presumably overestimates their effect. Let us remark that Dale R. Snider[14] has reached results similar to ours.

In conclusion, alternative mechanisms for increasing the strength of the kernel must be sought. At present, it seems that only two ways remain to be explored. One is the inclusion of the exchange of other types of mesons such as the ρ and ω mesons, although here the pole approximation seems not to be a good one. Another way would be a systematic investigation of off-shell continuations of the kernel. Such a progam is currently under way and will be detailed elsewhere.

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APPENDIX

We quote here the crossing matrices for the six processes (III.3) computed according to the rules outlined in Sec. III.

We want to emphasize that within this formalism there is no basic difference between the systems $K\bar{K}$ and KK apart from their total hypercharge, which plays the same role here as the third component of isospin plays within an isomultiplet.

The crossing matrices, in self-explanatory notation, are the following:

(a) t ch: $\pi\bar{\pi} \to \pi\bar{\pi}$

s ch: $\pi\pi \to \pi\pi$

I_t	Y_t	I_s: 0 Y_s: 0	I_s: 1 Y_s: 0	I_s: 2 Y_s: 0
0	0	1/3	1	5/3
1	0	1/3	1/2	-5/6
2	0	1/3	-1/2	1/6

(b) t ch: $K\bar{K} \to \pi\bar{\pi}$

s ch: $K\pi \to K\pi$

I_t	Y_t	I_s: 1/2 Y_s: +1	I_s: 3/2 Y_s: +1	I_s: 1/2 Y_s: -1	I_s: 3/2 Y_s: -1
0	0	$1/\sqrt{3}$	$2/\sqrt{3}$	$1/\sqrt{3}$	$2/\sqrt{3}$
1	0	$\sqrt{2}/3$	$-\sqrt{2}/3$	$\sqrt{2}/3$	$-\sqrt{2}/3$

(c) t ch: $\eta\bar{\eta} \to \pi\bar{\pi}$

s ch: $\eta\pi \to \eta\pi$

I_t	Y_t	I_s:	1
		Y_s:	0
0	0		$-\sqrt{3}$

(d) t ch: $\eta\bar{\eta} \to \eta\bar{\eta}$

s ch: $\eta\eta \to \eta\eta$

I_t	Y_t	I_s:	0
		Y_s:	0
0	0		1

(e) t ch: $K\bar{K} \to \eta\bar{\eta}$

s ch: $K\eta \to K\eta$

I_t	Y_t	I_s:	1/2	1/2
		Y_s	+1	-1
0	0		-1	-1

(f) t ch: $K\bar{K} \to K\bar{K}$

s ch: $KK \to KK$

I_t	Y_t	G_t \ I_s / Y_s / G_s	0 / 0_A / +1	1 / 0_A / +1	0 / 0_S / -1	1 / 0_S / -1	0 / +2	1 / +2	0 / -2	1 / -2
0	0_A	+1	1/4	3/4	1/4	3/4	1/4	3/4	1/4	3/4
1	0_A	+1	1/4	-1/4	1/4	-1/4	1/4	-1/4	1/4	-1/4
0	0_S	-1	1/4	3/4	1/4	3/4	-1/4	-3/4	-1/4	-3/4
1	0_S	-1	1/4	-1/4	1/4	-1/4	-1/4	1/4	-1/4	1/4
0	+2		1/4	3/4	-1/4	-3/4	0	0	0	0
1	+2		1/4	-1/4	-1/4	1/4	0	0	0	0
0	-2		1/4	3/4	-1/4	-3/4	0	0	0	0
1	-2		1/4	-1/4	-1/4	1/4	0	0	0	0

In the last matrix the subscripts A and S mean antisymmetric and symmetric respectively. We have specified the G parity of the state where so corresponds.

Table I: Estimated properties of low energy resonances.

Resonance	Mass (GeV)	Full Width (GeV)	Elasticities $\Gamma_{\mu\mu'}/\Gamma_{total}$					
			μ: π	K	η	π	π	K
			μ': π	K	η	η	K	η
ρ	0.765	0.125	1.0	--	--	--	--	--
ϵ	0.765	0.450	1.0	--	--	--	--	--
K^*	0.892	0.050	--	--	--	--	1.0	--
ϕ	1.019	0.004	--	0.8	--	--	--	--
f	1.260	0.150	1.0	--	--	--	--	--
A_2	1.300	0.020	--	0.02	--	0.16	--	--
K_N	1.412	0.096	--	--	--	--	0.5	0.02
f'	1.514	0.073	0.1	0.8	0.1	--	--	--
ρ_N	1.670	0.170	0.45	0.03	--	--	--	--

Table II: The computed $t = 0$ intercepts of the leading output Regge trajectories for $\pi\pi$ elastic scattering. A and B refer to the parametrization of the low-energy amplitudes by $\sigma_{e\ell}$ and σ_{tot}, respectively.

Type of particles exchanged	Output trajectory: Pomeron A	Pomeron B	Rho A	Rho B
π exchange only	0.28	0.30	0.12	0.15
π, K and η exchange	0.30	0.33	0.16	0.18

FOOTNOTES AND REFERENCES

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† Fellow of the National Research Council of Argentina.

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FIGURE CAPTIONS

Fig. 1. A diagram representing Eq. (II.1) for the 2-to-n amplitude, showing momentum assignment. Each blob is an off-shell 2-to-2 amplitude.

Fig. 2. One term of the amplitude squared of Eq. (II.2), showing the labeling of exchanged particles. For example, μ_2 refers to the mass of the particle exchanged at the second link. It can take the values μ_π, μ_K, μ_η.

Fig. 3. A diagram not contained in our formalism.

Fig. 4. Diagrammatic representation of Eq. (II.7). The labels μ, μ', and μ'' refer to the different types of particles being considered. The encircled letters give our channel convention.

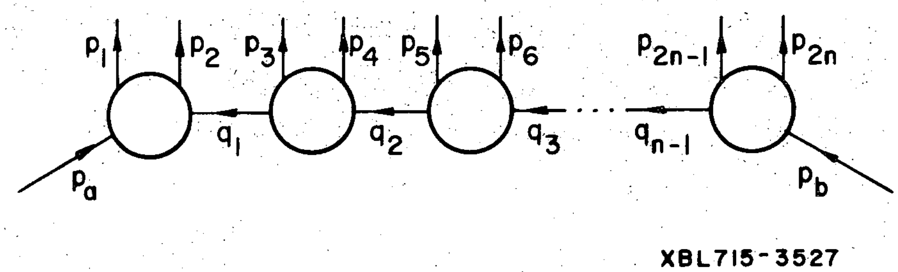

Fig. 1

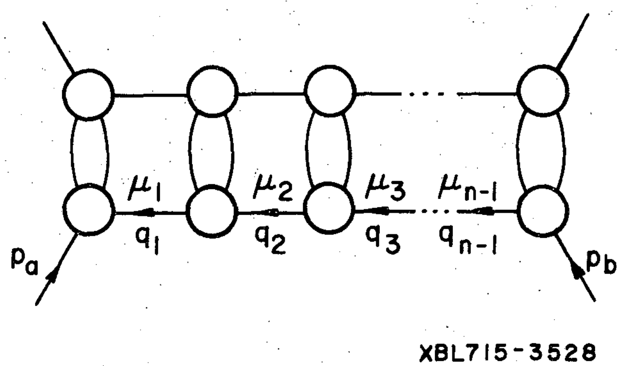

Fig. 2

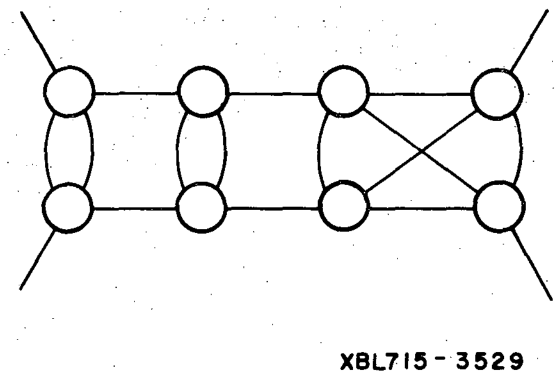

Fig. 3

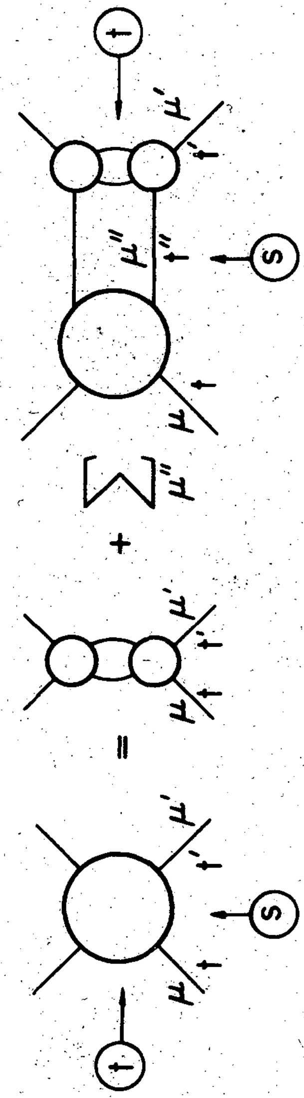

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Fig. 4

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