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## Abstract

This paper presents empirical tests of option price boundaries and of analytical and numerical models for pricing options. Results are based on a data set of about 38,000 option prices. Contracts studied include index options from the American, New York, and Chicago Board Exchanges, gold options from Comex, silver options from the Toronto Futures Exchange, debt options from the Amex and Chicago Board of Trade, and currency options from the Philadelphia stock exchange.

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## I. INTRODUCTION

The past several years have witnessed many innovations in contingent claims markets. Options on stock indices, government debt, foreign currencies, commodities, and futures contracts have appeared in North American markets in the past three years. A quick glance at The Wall Street Journal will reveal options trading on over a dozen indices, two metals, seven agricultural commodities, five foreign currencies, and half a dozen government debt instruments.

Aside from providing more data for academic study, the profusion of new contracts offers new opportunities for investors. Banks, insurance companies, institutional investors, and purchasers and suppliers of agricultural and industrial commodities can engage in hedging strategies previously unattainable. Individual investors, speculators, and futures traders also enjoy new choices with the emergence of these markets. Before plunging into these new instruments, the investor may pose several questions. Do the new markets provide the liquidity desired by investors who wish to trade frequently or trade in large volumes? Are the markets sufficienty broad and mature to provide rational prices at which to trade? Can the theory of finance provide option pricing models which are sufficiently accurate to be useful to the options investor? This paper will address some of the questions which the appearance of these new options gives rise to.

The paper is organized as follows. Section II describes the new contracts and the data base which has been created to study them. Section III presents empirical results on violations of pricing boundaries while Section IV describes and tests option pricing models for accuracy against actual prices. Results indicate that boundaries are infrequently violated and models are fairly accurate. Section V is a summary and conclusion.

## II. DESCRIPTION OF OPTION CONTRACTS AND DATA SET

Options on stock indices, precious metals, government debt instruments, and foreign currencies have appeared on North American options exchanges in the last three years. Table 1 is a summary of the characteristics of the contracts studied in this paper. Contracts included are from the American Stock Exchange (Amex), Chicago Board Options Exchange (CBOE), Chicago Board of Trade (CBT), Commodity Exchange (Comex), New York Stock Exchange (NYSE), Philadelphia Stock Exchange (PHLX), and Toronto Futures Exchange (TFE) while underlying assets include three stock market indices, five government debt instruments, two precious metals and five foreign currencies. Table 2 provides a summary of market activity for a randomly selected day.

Volume and open interest are particularly broad for the index, gold futures, and t-bond futures options while spot debt and silver options are less widely traded.

Table 3 summarizes the data set which has been collected for the empirical tests. Closing silver option prices were copied from The Globe & Mail, a Toronto newspaper. The Chicago Board of Trade supplied transactions prices for treasury bond options while transactions prices for foreign currency options are from the Ohio State University/Philadelphia Stock Exchange data base. All other option prices are closing prices as reported in The Wall Street Journal. Prices for the underlying asset and the rate of return on a treasury bill to match each option's time to expiration were also collected from these sources. In total, there are almost thirty-eight thousand option prices in the data set. Index, t-bond futures, and gold futures options are most heavily represented while there are relatively few prices for the less heavily traded spot debt and silver options. For each type of option listed in Table 3, the total number of option prices in the sample is reported along with the smaller number of option prices used to generate implied standard deviations which are used as the variance parameter for the option pricing models. The number of options used to calculate these variance estimates is smaller than the total number of options because low value or very short maturity options were excluded from these calculations.

## III. BOUNDARY CONDITIONS: THEORY & EMPIRICAL TESTS

This section presents and tests several boundary conditions on option prices. Frequent violations of these boundaries will suggest that the markets for the options are inefficient or, at the very least, are so thin that non-synchronous prices for options and the underlying asset lead to frequent boundary violations in the prices reported in the newspaper. Frequent violations also indicate that the prices may be too inaccurate to be of use in further empirical tests. As noted in Phillips & Smith (1980) and Bhattacharya (1983), the boundary violations do not necessarily represent opportunities for profitable arbitrage when closing prices are use in the tests.

The following symbols are employed in this section and throughout the paper:

C - price of an American call option

P - price of an American put option

S - price of underlying asset

X - exercise price of option

T - time to expiration of option

t - current time

r - risk free domestic interest rate

d - continuous dividend yield on underlying asset

PV(int,T) - present value of the interest accruing on underlying asset over option's life

F - price of underlying asset for a futures option

exp( ) - exponential function

log( ) - natural log function

Boundaries and early exercise conditions are derived in Merton (1973), Galai (1978), Ramaswamy & Sundaresan (1984), and Whaley (1984). The bondaries tested here are as follows.

i.) EARLY EXERCISE: C > S - X and P > X - S, also C > F - X and P > X - F for futures options.

ii.) EUROPEAN LOWER BOUNDARY

a. Underlying asset pays no dividend or a continuous dividend: C > S*exp(-dT) - X*exp(-rT)

b. Underlying asset is a bond paying fixed coupon: C > S - PV(int,T) - X*exp(-rT)

For a currency option, the foreign interest rate is the continuous "dividend" which the underlying asset pays. The above European lower boundaries will be valid for American calls if there is no likelihood of early exercise. Of course, if the underlying asset pays no dividends, early exercise will never occur and the boundaries hold. Additionally, a sufficient condition for no early exercise of an American call on an asset paying a continuous dividend is (Merton (1973)):

$$rTX > dTS \qquad (1)$$

The continuous dividend assumption is reasonable for a call option on a currency unless the option is deep in the money or has a very short time to expiration. For call options on coupon bonds, a sufficient condition for no early exercise is:

$$(1-\exp(-rT))X > PV(int, T) \qquad (2)$$

There is little likelihood of this condition being violated execpt for deep in the money or close to expiration options. While it is reasonable to test the prices of American calls on currencies and bonds against the European lower bound, this will not be the case for calls on futures[1] or for any American put.

Stock index options are difficult to test because the dividend yield is never known with certainty and because the index is not explicitly traded on any market.

iii.) AMERICAN PUTS AND CALLS ON FUTURES OPTIONS: $F*\exp(-rT) - X < C - P < F - X*\exp(-rT)$

This inequality may be broken into two parts for empirical testing:

a.) $P > C - F + X*\exp(-rT)$

b.) $P < C - F*\exp(-rT) + X$

Because these conditions are not as widely known as the others, proof of the inequalities follows.[2]

To prove part a.), set up a portfolio consisting of a written call, bought put, F-X*exp(-rT) in discount bonds, and a purchased "rollover" futures strategy. This strategy consists of buying exp{r} futures contracts the first day, exp{2r} futures contracts the second day, and so on so that exactly exp{rT} contracts are owned on the last day. The following table shows the initial cost of the portfolio, value if there is early exercise, and value at expiration:

| POSITION | COST | IF EXERCISED AT t<T | AT EXPIRATION: F(T)<X | F(T)>X |
|---|---|---|---|---|
| Write call | -C | -F(t)+X | 0 | -F(T)+X |
| Buy put | P | P(t) | X-F(T) | 0 |
| Buy futures rollover | 0 | (F(t)-F)*<br>exp{rt} | (F(T)-F)*<br>exp{rT} | (F(T)-F)<br>*exp{rT} |
| Lend F-X*<br>exp{-rT} | F-X*<br>exp{-rt} | Fexp{rt}-X*<br>exp{-r(T-t)) | Fexp{rT}<br>-X | Fexp{rT}<br>-X |
| Total | P+F<br>-X*<br>exp<br>{-rT}<br>-C | P(t)+F(t)*<br>(exp{rt}-<br>1)+X(1-<br>exp(-r(T-t))<br>>0 | F(T)*<br>(exp{rT}<br>-1)>0 | F(T)*<br>(exp{rT}<br>-1)>0 |

The portfolio always has positive value so the initial cost must be positive. Therefore, P>C-F+X*exp{-rT}. To prove part b.), set up a portfolio consisting of a bought call, written put, F*exp{-rT}-X borrowed, and a sold "rollover" futures strategy. This strategy consists of selling exp{-r(T-1)} futures contracts the first day, exp{-r(T-2)} futures contracts the second day, and so on so that exactly one contract is sold on the last day. The following table shows the initial cost of the portfolio, value if there is early exercise, and value at expiration:

| POSITION | COST | IF EXERCISED AT $t<T$ | AT EXPIRATION: $F(T)<X$ | $F(T)>X$ |
|---|---|---|---|---|
| Buy call | C | C(t) | 0 | F(T)-X |
| Write put | -P | -X+F(t) | -X+F(T) | 0 |
| Sell futures rollover | 0 | -(F(t)-F)* exp{r(T-t)} | -F(T)+F | -F(T)+F |
| Borrow F* exp(-rT) -X | -F* exp (-rT) +X | -F* exp{r(T-t)}+ X* *exp(-rt) | -F+ X* exp{rT} | -F+ X* exp{rt} |
| Total | C -P -F* exp(-rT) +X | C(t)+X* (exp{rt}- 1)+F(t)* (1-exp{-r* (T-t)})>0 | X* (exp {rT} -1) >0 | X* (exp {rT}- -1) >0 |

The portfolio always has positive value so the initial cost must be positive. Therefore, $P<C-F*\exp(-rT)+X$. Similar conditions hold for foreign currency options.

iv.) WEAK PUT-CALL PARITY FOR AN ASSET WHICH PAYS NO DIVIDEND: $P > C - S + X*\exp(-rT)$

This can be proved by starting with put call parity for European puts, p, and calls, c:

$$p=c-S+X*\exp(-rT) \qquad (28)$$

When the asset pays no dividend, there is no chance of early exercise and the European and American calls are equal in value so we may write:

$$p = C - S + X*\exp(-rT) \qquad (29)$$

However, the American put is at least as valuable as the European so we may write:

$$P > p = C - S + X*\exp(-rt) \qquad (5)$$

This is equivalent to iv.) above.

Table 4 is a summary of previous empirical research on violations of boundary conditions by American option prices. Galai (1978) and Bhattacharya (1983) find that CBOE call options on stocks violate boundaries only about 1% to 2% of the time. Beckers (1984), Bodurtha & Courtadon (1985b), and Shastri & Tandon (1984) find that prices for options on gold and foreign currencies violate boundaries anywhere from 0% to about 15% of the time. Halpern & Turnbull (1985) find violations of boundaries by Toronto Stock Exchange options among 5% or more of their transactions prices.

Tables 5, 6, 7, and 8 present results of boundary violation tests for the data set of index, metal, debt, and currency options collected. The foreign interest rates are imputed from prices of futures contracts on the currencies for each day.[3] The tables show boundary violation frequencies of 5% or less for most markets. Exceptions are early exercise of puts for some indices, currencies, and gold futures, and many boundaries for the less-heavily traded spot debt and silver options.

Although the frequency of boundary violations is interesting, the size of the violations is a more important indicator of market inefficiency or non-synchronous prices. The summary statistics for ex post violations describe the profits a trader would earn if he could initiate trades at the prices which violate the boundary. This is a somewhat unrealistic strategy because closing prices are used in the tests. Even if transactions prices were employed, it is likely that prices would correct themselves quickly and a trader would be unable to actually set up a position to exploit a violation he has observed. The ex post cash flows are, on average, so small and highly variable that they do not represent a profitable return to a riskless arbitrage strategy. The ex ante cash flows described in the tables summarize the profits from a strategy with which a trader observes a boundary violation but must wait until the next trading day to initiate transactions to attempt to exploit the violation. The profits from this strategy are, on average, negative or insignificant and highly variable. The ex ante strategy does not represent an opportunity for profitable riskless arbitrage.

These simple ex post and ex ante trading strategies using closing prices are not completely realistic or representative of all possible arbitrage trading strategies available to participants in these options markets. Nonetheless, the results indicate that no obvious large arbitrages are available in these markets. Furthermore, the low frequency of boundary violations suggest that the option prices in the sample are not seriously inaccurate.

## IV. OPTION PRICING MODELS: THEORY & EMPIRICAL TESTS

This section provides a brief description of the basic option valuation models used and tests their accuracy in predicting market prices for the options which have been collected for this study. Black & Scholes (1973) provide the analytical solution for a European call on a non-dividend paying stock and Merton (1973) adapts the formula for use when the underlying asset pays a continuous dividend yield, d. Assuming the stock index pays a continuous dividend and returning to expression (32) above, we can see that it is unlikely that such a call will be exercised early unless it is very deep in the money. However, we know that the dividend stream paid on a stock index is not smooth and continuous and must determine whether the "lumpiness" of the dividend stream will often induce early exercise and render the European formula inappropriate. Returning to expression (32), we may imagine an extreme case where the dividends for the entire quarter are paid in one lump. Assuming a dividend yield of 5% a year, this would represent a payment of 1.25% of the value of the stock index. Substituting into (1), the sufficient condition for no early exercise becomes:

$$rT > .0125\ S/X \qquad (6)$$

Assume an interest rate of 10%, rearrange, and we get:

$$T > .125\ S/X \qquad (7)$$

What this suggests is that early exercise due to an uneven dividend stream will be a possibility for deep in-the-money or short maturity calls on a stock index. However, we will employ the continuous level dividend stream assumption for simplicity's sake. Although a no early exercise assumption is appropriate for European calls on index options, no such assumption can be made for American calls on indices or futures options or any American put on any asset. These options may be exercised at any time without regard to the level or pattern of dividends paid on the underlying asset. Thus, we must employ a model which accounts for early exercise when valuing these contracts. In this paper, three basic types of models which incorporate the early exercise problem are employed.

The first model to be used is the binomial model of Cox, Ross, & Rubinstein (1979). Appendix A describes and illustrates the workings of this model. The basic idea is as follows. Assume the underlying asset can only jump up by a certain amount or jump down by a certain amount for a given unit of time. Knowing the size of the possible up or down jumps and given a riskless asset, we can set up a portfolio consisting of riskless assets and the underlying asset such that this portfolio exactly matches the return to an option spanning the unit of time. Given the portfolio weights for this mimicking portfolio and given the current price of the riskless asset and underlying asset, we can determine the current price of the option. Although the assumption that the underlying asset can only jump up or down by a certain amount over a given time period sounds unrealistic, we can assume that this time period is quite small and use the binomial method repeatedly to calculate a value which is quite accurate. In this paper, binomial models with up to 300 such "time-steps" over the life of an option are used. The added attraction of the binomial model is that it can be modified for early exercise conditions and payouts at each step.

The second model to be employed is the approximation created by Geske & Johnson (1984) to value American puts on stocks. Appendix B provides a description of the intuition behind the Geske-Johnson extrapolation method. The basic idea is as follows. Geske & Johnson (1984) recognize that an American option, W(T), is equivalent to an infinite series of compound options because the American option may be exercised at any instant prior to expiration. Although it would be computationally intractable to value such an infinite series of options, the authors recognize that the value of this series can be closely approximated by a weighted average of other option prices which can be calculated with relative ease. We can calculate the value of the corresponding European option, w(T), and the value of a "semi-American" option which can be exercised at only a few points in time. $w(T/2,T)$ is a "semi American" option which can be exercised only at times T/2 and T. We know that $W(T) > w(T/2,T) > w(T)$ because any additional opportunity to exercise carries positive value. Although it is hard to value $W(T)$ directly, Geske & Johnson (1984) provide a method to combine the values of the European options and the "semi-American" option to approximate the value of the American option. Additional accuracy can be obtained at additional computational cost by adding more "semi-American" options to the weighted average. The advantage of this method is that it accounts for early exercise, is computationally inexpensive, and can be made arbitrarily accurate by increasing the number of "semi-American" options in the approximation. Furthermore, it may be modified easily to account for a continuous dividend stream.

The third model to be used is a numerical procedure developed by Courtadon (1982) which allows for a stochastic short term rate of interest. Appendix C provides a more detailed description of the method. The idea is similar to that for the binomial model; we start at expiration and work backwards through time to get the value of the option. However, this method requires an estimate of a risk aversion parameter and specification of a particular stochastic process for the short interest rate. The solution algorithm employed is the explicit finite difference approach outlined by Brennan & Schwartz (1983). The advantage to the Courtadon method is that it assumes a stochastic short term interest rate and thus, is appropriate for valuing options on treasury bills.

We will use the Geske-Johnson two-point approximation, modified for continuous dividends, to value options on stock indices. Gold futures options are valued using a modified binomial model with 300 time steps. Silver calls are valued with the Black-Scholes formula while silver puts are valued with the Geske-Johnson method. Long term government bond options are valued using a 100 step binomial model modified for the receipt of bond interest payments. Options on treasury bills are valued with an explicit finite difference numerical procedure. Treasury bond futures options are valued with a binomial model with twenty time steps per month and modified to account for the effective rate of interest paid on the spot debt underlying the futures contract. Finally, options on foreign currencies are valued with the Geske-Johnson method modified for the receipt of interest which foreign currency deposits earn.

Table 9 presents a summary of previous empirical work on the accuracy of option pricing models. In a study of out-of-the-money Amsterdam gold options, Beckers (1984) finds that closing prices deviate from model prices by about 16%. Bodurtha & Courtadon (1985a) use numerical integration to value foreign currency options and find average deviations between model and transactions prices of about six to ten cents. Shastri & Tandon (1984) use the Geske-Johnson technique to value foreign currency options and find average deviations between model and transactions prices of about four to twenty-one cents. Using CBOE stock options, Whaley (1982) finds that prices generated from the Roll (1977) calls model deviate about 1% to 2% from actual prices while Blomeyer & Johnson (1984) find deviations between market and model prices which average thirty or forty cents. Dietrich-Campbell & Schwartz (1984) find that model generated prices for options on government bonds deviate from closing prices by about thirty to sixty cents. Studies of Chicago Mercantile Exchange S+P 500 and West German mark futures options by Shastri & Tandon (1985) and Whaley (1984) yield similar results.

Table 10 presents a summary of our empirical tests for index, precious metal, debt, and currency options. In all cases, the standard deviation used in the option pricing models was estimated with the implied standard deviations solved out of the previous day's option prices. for implied standard deviations. An efficient algorithm, the Newton-Raphson gradient method, is employed. Each asset ordinarily has several options trading each day and each of these does not necessarily produce an identical ISD. Thus, the ISD's must be aggregated to produce the best estimate of the true volatility of the underlying asset. Before aggregating, all ISD's which may be unreliable are discarded. An ISD is discarded if it was generated from an option which is:

i.) Low-priced, that is, less than $.50 for debt and silver options, $.10 for foreign currency options, $.05 for treasury bill options, and $1.00 for all other options. Because option prices move in increments of 1/8, 1/4, or 1/10, low priced options may not reflect underlying parameters accurately.

ii.) Close to expiring, that is, has less than four weeks until expiration.

iii.) Violating the immediate early exercise bound or is within ten cents of the bound for debt and silver options, within five cents for currency options, or within twenty five cents for other options.

The ISD's from the options remaining in the sample are combined into simple and weighted averages, the latter with weights proportional to the option's elasticity with respect to the volatility parameter. The idea behind this weighting scheme is to give the greatest weight to those options which are most sensitive to the volatility parameter. Regressions reported in Bailey (1985) are used to determine which WISD measure is the best predictor of volatility and this is used as the variance parameter in testing the models. Returning to Table 10, the deviations between market and model prices seem quite small on average. S+P 100 puts have an average market price of $3.79 while the model calculates an average price of $3.75. Calls on Comex gold futures have an average market price of $17.04 and an average model price of $16.35. Puts on 10.375% coupon long term U.S. Treasury bonds have an average market price of $3.63 and average model price of $3.58. Calls on German marks have an average market price of $1.10 and an average model price of $1.10. Although the deviations seem insignificant, we must determine what constitutes an acceptable level of model error: how small is "small enough"? There is no obvious answer to this question.

Tables 11 through 23 provide more detailed summary statistics on price deviations by grouping the options according to time to expiration and degree to which the option is or out-of-the-money. Most interesting are the statistics on the average absolute value of the deviation between model and actual prices. Although the average deviation may be small or zero, individual dollar differences may be quite large. For example, CBOE S+P 100 puts have an average deviation of only about four cents. However, the average absolute deviation is over thirty cents. The other options have similarly large average absolute deviations.

What this means is that there are many large positive and large negative deviations which happen to average to a few cents. Once again, it is difficult to judge whether the models are "accurate" or "inaccurate" just by examining these statistics.

Tables 24 through 36 present regressions which relate the pricing errors to different parameters in the option pricing models. The dependent variable is the difference between market and model prices while explanatory variables are the standard deviation estimate, degree to which the option is in or out-of the money, and time to expiration of the option. There is a strong negative relation between the price difference and the standard deviation and also between the price difference and the time to maturity. For some of the options, there is a significant relation between price difference and degree to which the option is in or out-of-the-money. However, the sign of the relationship is different for different options so it is not possible to generalize about this bias.

## V. SUMMARY AND CONCLUSIONS

Boundary conditions and option pricing models have been tested on a data set of 30,000 prices for options on stock indices, metals, debt, and currencies. Option prices adhere to the boundaries most of the time and the pricing models are reasonably accurate predictors of option prices. Lacking more information on how investors use option pricing models, it is difficult to judge whether the pricing models are sufficiently accurate to be useful. Finally, it is impoirtant to note that markets for some of the debt and currency options have a low level of volume and open interest. These markets may not offer sufficient liquidity for large investors.

FOOTNOTES:

1.) We would imagine that a call on an asset which will not pay any dividend will never be exercised early. However, this does not hold for calls on a futures option. The lower bound on a European futures option is:

$$c > \max(0, (F-X)*\exp(-rT)) \qquad (F1)$$

An American futures option which will not be exercised early will be subject to this bound. However, the American call is subject to the boundary for early exercise:

$$C > \max(0, F-X) \qquad (F2)$$

It is not always the case that the European bound will be larger than the early exercise bound. Thus, a hold-until-expiration strategy will not always dominate early exercise and we cannot assume that the American call on a futures option will never be exercised early.

2.) Thanks to Robert Whaley for this proof.

3.) If we can observe the futures price, F, the spot price, S, the riskless interest rate, r, and the time to maturity of the futures contract, T, we solve for the continuous dividend, d, using the cost of carry relationship:

$$F = S*\exp(T(r-d)) \qquad (F3)$$

The continuous dividend may be the dividend yield on a stock index, foreign interest rate paid on a currency, or effective interest rate paid on a bond. In some cases, such as treasury bond futures, we cannot directly observe the spot price of the asset. Nonetheless, we may solve for the effective rate of interest on bonds by using the prices of two futures contracts of differing maturities in a similar fashion. We may rewrite (F3) as:

$$S = F*\exp(-T(r-d)) \qquad (F4)$$

The relation will also hold for another futures contract on the same asset which has price F′ and time to maturity T′, where T′ does not equal T:

$$S = F'*\exp(-T'(r-d)) \qquad (F5)$$

Setting the right-hand sides of (F4) and (F5) equal, we may solve for d.

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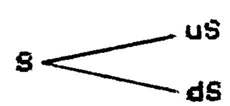
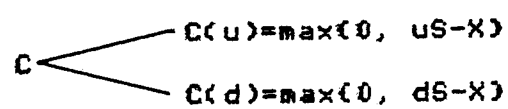
## APPENDIX A: AMERICAN BINOMAL OPTION VALUATION

Suppose that the price of an asset will either jump up at rate u or down at rate d over a period of time. We may set u and d as a function of the estimated standard deviation of the rate of price change of the asset (u=exp(SD*sqrt(T)) and d=exp(-SD*sqrt(T))). The initial cost of an asset with price S is S so we may draw a diagram representing the present and possible future values of a long position in the asset as follows:




Suppose that there also exists a riskless asset which yields one dollar with certainty at the end of the time period. Its present price will be 1/(1+R). Finally, suppose that there exists a call option on the asset with striking price X which will expire at the end of the time period. We do not know its present price, C, but know that at expiration it will be worth max(0,uS-X) or max(0,dS-X), depending on how the asset price moves. We may draw a diagram as follows:




To solve for C, we construct a portfolio consisting of a number of units of the asset, N(S), and a number of riskless assets, N(R), chosen to mimic the end-of-period return of the call option. Formally, N(S) and N(R) must satisfy two equations:

$$N(S)*uS + N(R)*(1) = C(u) \qquad (A1)$$

$$N(S)*dS + N(R)*(1) = C(d) \qquad (A2)$$

Using algebra, we can easily solve for N(S) and N(R). Because the portfolio {N(S),N(R)} exactly duplicates the payoffs on the call, it must sell for the same price as the call. The present cost of N(S) units of the asset is N(S)*S while the present cost of N(R) riskless assets is N(R)/(1+R). Thus, the price of the call must be N(S)*S + N(R)/(1+R).

A two period example illustrates how the binomial model can be expanded for greater accuracy and to account for possible early exercise:

```
                                             C(u,u)=max{0,uuS-X}
                    C(u)=max{C(u),uS-X}<
C=max{C,S-X}<                                C(u,d)=max{0,udS-X}
                    C(d)=max{C(d),dS-X}<
                                             C(d,d)=max{0,ddS-X}
```

Over the course of two time steps, the asset price may have two upticks, two downticks, or one of each. To value the call, we start at expiration and work back to solve for C(u) and C(d). If C(u) or C(d) are worth less than the cash flow from immediate exercise, the program sets them equal to this cash flow. Finally, we work back one more step to solve for C, the present price of the option, once again checking for early exercise.

The American binomial method may easily be modified to value puts by altering the terminal value of the option to correspond to a put, that is, $\max\{0, X-uS\}$ and $\max\{0, X-dS\}$. To value coupon debt, we recognize that an initial investment in a bond will yield a coupon, $c$, in addition to the uncertain future value of the bond:

$$B \begin{array}{l} \nearrow uB + c \\ \searrow dB + c \end{array}$$

Similarly, to value a foreign currency option, we recognize that an investment in foreign currency will yield interest at the foreign rate $r'$ so that the terminal value of such an investment will be either $u(1+r')S$ or $d(1+r')S$.

To value a futures option, we note that a futures contract requires a zero initial investment and pays off the difference between the initial futures price and the future uncertain price:

$$0 \begin{array}{l} \nearrow uF - F \\ \searrow dF - F \end{array}$$

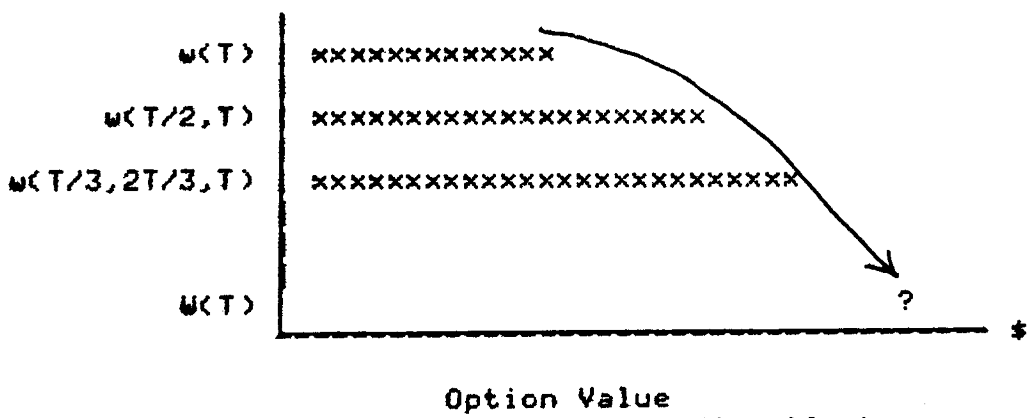
Alternately, we may assume that a certain relationship exists between spot and futures prices at all times and use the spot asset plus debt to create a hedge to duplicate the futures option. In this work, it is assumed that the futures contract equals the spot price times $\exp\{T*(r-d)\}$, where $T$ is the time until maturity of the contract, r the interest rate spanning this time, and d the cost (storage) or benefit (coupon or dividend payment) to holding the spot asset. For gold, we assume storage is costless so that d=0. For treasury bonds, we impute d out of two futures prices for each day. The advantage to this method is that the implied standard deviations produced this way are in terms of the spot asset, not futures contracts of varying maturities. Finally, for all options, we may set the size of the time period to be so small that there are hundreds of time steps over the life of the option and the true random behaviour of the underlying asset price is closely approximated.

## APPENDIX B: GESKE-JOHNSON VALUATION BY APPROXIMATION

We wish to value an American option, $W(T)$, which may be exercised at any instant prior to its expiration at time T. Begin by calculating the value of the corresponding European option, $w(T)$, which can be exercised only at time T. Continue by calculating the value of a "semi-American" option, $w(T/2,T)$, which can be exercised only at times T/2 and T, the value of $w(T/3,2T/3,T)$, and so on. Because the right to exercise conveys positive value we may write:

$$W(T) > w(T/3,2T/3,T) > w(T/2,T) > w(T) \qquad (B1)$$

We may graph the values as follows:




We wish to avoid calculating $W(T)$ directly because of the complexity of accounting for an infinite number of possible exercise points. However, the diagram shows that the values for the series of "semi-American" options approach the value of the American option $W(T)$. By taking an appropriately weighted combination of the European and "semi-American" options, we can extrapolate to estimate the value of the American option. The curved line represents this extrapolation process.

## APPENDIX C: COURTADON MODEL FOR VALUATION WITH STOCHASTIC INTEREST RATE

Amex treasury bill options allow the holder to buy or sell a fixed characteristic security, a U.S. Treasury bill with 13 weeks until maturity, at any time prior to option expiration. The bill price is sensitive to short term interest rates so we may not make the usual assumption of a constant interest rate spanning the life of the option. Courtadon (1982) assumes a flat term structure and a mean reverting process for the interest rate, r:

$$dr = a(m-r)dt + srdz \qquad (C1)$$

Parameters a and m are speed of adjustment and mean respectively while sr is the standard deviation of the process and dz is a Weiner process with zero mean and unit variance. The price, A(r,t), of an asset dependent on the interest rate must follow the valuation equation:

$$1/2\, s^2 r^2 A(r,r) + (a(m-r)-Lsr)A(r) - rA + A(t) = 0 \qquad (C2)$$

A(r), A(r,r), and A(t) are partial derivatives while L is the coefficient of risk aversion. Expiration, early exercise, and other rational boundary conditions are used with the above differential equation to value the asset.

An explicit finite difference scheme is used to calculate treasury bill and treasury bill option prices. The method is similar to the binomial in that it works backwards from the option's expiration to get its current value. However, the finite difference method does not rely on the binomial's riskless hedging technique but rather transforms the partial derivatives in the valuation equation to finite differences. We employ the transformed variable, s = 1/(1+40r), and use parameter estimates from Dietrich-Campbell & Schwartz (1984). Speed of adjustment, a, is estimated to equal .558 while risk aversion, L, is estimated to equal .26. The daily long interest rate implied in treasury bond futures prices is used as m, the value to which r tends to revert.

Table 1

OPTION CONTRACT SPECIFICATIONS

| Exchange | Underlying Asset | Price in newspapers |
|---|---|---|
| American Stock Exchange (AMEX) | 1. $100 times Amex Major Market stock index | per 1% of asset |
| | 2. $200000 face value of U.S. Treasury bills with 13 weeks to maturity | per $100 face value |
| Chicago Board Options Exchange (CBOE) | 1. $100 times S+P 100 Index | per 1% of asset |
| | 2. $100000 face value of U.S. Treasury 14%s Nov 06/11 bonds | per $100 face value |
| | 3. $100000 face value of U.S. Treasury 12%s Aug 08/13 bonds | per $100 face value |
| | 4. $100000 face value of U.S. Treasury 10.375%s Nov 07/12 bonds | per $100 face value |
| Chicago Board of Trade (CBT) | One $100000 face value CBT Treasury bond futures contract | per $100 face value |
| Commodity Exchange (COMEX) | One Comex gold futures contract (100 ounces) | per ounce |
| New York Stock Exchange (NYSE) | $100 times NYSE Index | per 1% of asset |
| Philadelphia Stock Exchange (PHLX) | 1. 50000 Canadian dollars<br>2. 6250000 Japanese yen<br>3. 62500 Swiss francs<br>4. 12500 U.K. pounds<br>5. 62500 West German marks | per 10000 units of foreign currency for yen, per 100 units for others. |
| Toronto Futures Exchange (TFE) | 100 ounces of silver | per ounce, U.S. funds |

Table 2

MARKET ACTIVITY: JANUARY 17TH, 1984

| | Trading Volume (# of contracts): | | Open Interest (# of contracts): | |
|---|---|---|---|---|
| | Puts | Calls | Puts | Calls |
| INDEX OPTIONS: | | | | |
| CBOE S+P 100 | 34567 | 41215 | 202976 | 249992 |
| Amex Major Market | 6241 | 6095 | 47433 | 68907 |
| NYSE | 2468 | 4010 | 50684 | 66244 |
| COMMODITY OPTIONS: | | | | |
| Comex Gold Future | 845 | 1562 | 8441 | 18639 |
| TFE Silver | 376 | 674 | 8471 | 2635 |
| DEBT OPTIONS: | | | | |
| CBOE U.S. Bonds | 191 | 339 | 4248 | 6619 |
| Amex U.S. T Bills & U.S. Notes | 90 | 70 | 592 | 900 |
| CBT T-Bond Future | 6121 | 5723 | 35450 | 64538 |
| CURRENCY OPTIONS: | | | | |
| PHLX C$,Y,fs,L,DM | 583 | 1442 | 16856 | 25434 |

Table 3

DATASET SUMMARY STATISTICS:

| Option | Data Begins | Data Ends | # of Calls Total | # of Calls Used for ISD's | # of Puts Total | # of Puts Used for ISD's |
|---|---|---|---|---|---|---|
| CBOE S+P 100 | 1JUL83 | 30APR84 | 2819 | 1403 | 2819 | 1237 |
| Amex Major Mkt | 1JUL83 | 30APR84 | 1929 | 1088 | 1844 | 906 |
| NYSE Index | 23SEP83 | 30APR84 | 1379 | 706 | 1223 | 438 |
| Comex Gold | 1JUL83 | 30APR84 | 4915 | 3487 | 4441 | 2588 |
| TFE Silver | 1JUL83 | 30APR84 | 1223 | 760 | 443 | 229 |
| CBOE 14% Nov 06 | 1JUL83 | 30APR84 | 342 | 209 | 219 | 137 |
| CBOE 12% Aug 08 | 9AUG83 | 30APR84 | 626 | 402 | 285 | 224 |
| CBOE 10.375% Nov 07 | 1JUL83 | 30APR84 | 452 | 274 | 255 | 156 |
| Amex T-Bill | 1JUL83 | 30APR84 | 183 | 99 | 198 | 81 |
| CBT T-Bond | 1JUL83 | 30APR84 | 3089 | 1292 | 2298 | 1390 |
| PHLX C$ | 1JUL83 | 30APR84 | 291 | 182 | 163 | 78 |
| PHLX Yen | 1JUL83 | 30APR84 | 1246 | 1082 | 554 | 351 |
| PHLX fs. | 1JUL83 | 30APR84 | 1223 | 964 | 595 | 336 |
| PHLX pound | 1JUL83 | 30APR84 | 839 | 752 | 438 | 385 |
| PHLX DM | 1JUL83 | 30APR84 | 1621 | 1218 | 797 | 431 |

Table 4

PREVIOUS RESULTS: VIOLATIONS OF BOUNDARIES BY AMERICAN OPTIONS

| Paper | Data | # of Options | Boundary | Ex Post Violations: Frequency | Ex Post Violations: Average $ Size |
|---|---|---|---|---|---|
| 1. Beckers (1984) | Closing prices for Amsterdam gold options | 581 | P>C-S+X* exp(-rT) | 81 | $ 1.94 per option |
| 2. Bhattacharya (1983) | Transactions prices for CBOE calls on stocks | 86137 | C>S-X | 120 | $12.57 per contract |
| | | 54735 | C>S-X* exp(-rT) -PV(DIV) | 4127 | $ 9.58 per contract |
| 3. Bodurtha & Courtadon (1985b) | Transactions prices for PHLX foreign currency options | 37824 | C>S-X | 359 | - |
| | | 14085 | P>X-S | 984 | - |
| | | 4511 | C<S+P-X* exp(-rT) | 3 | - |
| | | 3998 | P<X+C-S* exp(-r'T) | 22 | - |
| 4. Galai (1978) | Closing prices for CBOE calls on stocks | 16327 | C>S-X | 281 | $35.00 per contract |
| | | 16327 | C>S-X* exp(-rT) -PV(DIV) | 482 | $41.70 per contract |
| 5. Halpern & Turnbull (1985) | Transactions prices for Toronto Stock Exchange call options | 315202 | C>S-X | 12786 | $.348 per option |
| | | 315202 | C>S-X* exp(-rT) -PV(DIV) | 32989 | $.366 per option |

(table continued next page)

Table 4 (continued)

| Paper | Data | # of Options | Boundary | Ex Post Violations: Frequency | Average $ Size |
|---|---|---|---|---|---|
| 6. Shastri & Tandon (1984) | Closing prices for Philex foreign currency options | 3019 | C>Sexp(-r'T) -X* exp(-rT) | 81 | $59.25 per contract |
| | | 1726 | P>Xexp(-rT) -S* exp(-r'T) | 52 | $54.71 per contract |
| | | 3019 | C>S-X | 38 | $53.69 per contract |
| | | 1729 | P>X-S | 178 | $73.64 per contract |
| | | 1038 | P<C+X- S* exp(-r'T) | 17 | $114.88 per contract |

Notes:
1. "r'" represents foreign interest rate
2. Authors of studies 4. and 6. found that an ex ante or lagged test produced insignificant or highly variable "arbitrage profits" from boundary violations. Authors of studies 2. and 3. found that adding transactions costs also reduced "arbitrage profits" to insignificance.

Table 5

BOUNDARY VIOLATIONS: INDEX OPTIONS

| Index & Boundary | Observations: Ttl | Bound Violated | $ Per 1% of Underlying Asset: Ex Post Violations Mean | S.D. | Min | Max | Ex Ante Violations Mean | S.D. | Min | Max |
|---|---|---|---|---|---|---|---|---|---|---|
| CBOE S+P 100: | | | | | | | | | | |
| C>S-X | 2819 | 75 | .72 | 5.34 | 0.0 | 18.90 | .18 | 3.79 | - 4.34 | 18.17 |
| P>X-S | 2819 | 231 | .55 | 1.56 | 0.0 | 15.40 | -.27 | 1.42 | -12.64 | 3.46 |
| Amex Major Mkt: | | | | | | | | | | |
| C>S-X | 1929 | 33 | .44 | .62 | 0.0 | 2.23 | -.27 | .67 | - 1.51 | 2.05 |
| P>X-S | 1844 | 151 | .74 | 1.40 | 0.0 | 9.24 | -.01 | .99 | - 2.86 | 7.62 |
| NYSE Index: | | | | | | | | | | |
| C>S-X | 1379 | 43 | 2.33 | 3.03 | 0.0 | 8.64 | -1.28 | 2.05 | - 6.81 | .50 |
| P>X-S | 1223 | 110 | .85 | 2.18 | 0.0 | 10.16 | .04 | 1.38 | - 7.84 | 8.10 |

Table 6

BOUNDARY VIOLATIONS: COMMODITY OPTIONS

| Commodity& Boundary | Observations: Ttl | Bound Violated | $ Per Ounce: Ex Post Violations Mean | S.D. | Min | Max | Ex Ante Violations Mean | S.D. | Min | Max |
|---|---|---|---|---|---|---|---|---|---|---|
| Comex Gold Future: | | | | | | | | | | |
| C>F-X | 4915 | 85 | 1.25 | 4.57 | 0.0 | 36.3 | -.26 | 1.46 | -11.6 | 1.4 |
| P>X-F | 4441 | 602 | 1.51 | 4.47 | 0.0 | 86.4 | .15 | 3.65 | -29.1 | 20.9 |
| P>C-F+X* exp{-rT} | 4091 | 80 | 6.88 | 17.40 | 0.0 | 141.1 | 1.00 | 12.02 | -18.8 | 98.3 |
| P<C-Fexp{-rT} +X | 4091 | 28 | 11.18 | 4.60 | 0.0 | 21.9 | -7.30 | 8.17 | -21.7 | 0.0 |
| TFE Silver: | | | | | | | | | | |
| C>S-X | 1223 | 37 | .19 | .22 | .02 | 1.27 | -.03 | .20 | -.50 | .40 |
| C>S-Xexp{-rT} | 1223 | 82 | .16 | .19 | 0.0 | 1.47 | .02 | .31 | -1.02 | 1.47 |
| P>X-S | 443 | 123 | .76 | .68 | .02 | 3.07 | .60 | .82 | -2.73 | 3.07 |
| P>C-S+X* exp{-rT} | 261 | 77 | .11 | .14 | 0.0 | .82 | -.05 | .14 | -.29 | .43 |

Table 7

BOUNDARY VIOLATIONS: DEBT OPTIONS

| Issue & Boundary | Observations: Ttl | Bound Violated | $ Per $100 of Face Value: Ex Post Violations Mean | S.D. | Min | Max | Ex Ante Violations Mean | S.D. | Min | Max |
|---|---|---|---|---|---|---|---|---|---|---|
| CBOE 14% 11/06 | | | | | | | | | | |
| C>S(BID)-X | 342 | 8 | .53 | 1.04 | 0.0 | 3.09 | .37 | 1.13 | - .31 | 3.09 |
| P>X-S(ASK) | 219 | 9 | 1.39 | 3.60 | 0.0 | 11.00 | -.31 | .68 | -1.50 | .28 |
| C>S(BID)- PV(int,T)- Xexp{-rT} | 342 | 1 | 3.03 | 0.0 | 3.03 | 3.03 | -.50 | 0.0 | -.50 | -.50 |
| CBOE 12% 8/08 | | | | | | | | | | |
| C>S(BID)-X | 626 | 35 | 3.73 | 5.02 | 0.0 | 12.53 | .41 | 2.61 | -1.25 | 12.53 |
| P>X-S(ASK) | 285 | 8 | 1.63 | 1.90 | 0.0 | 5.53 | -.42 | .40 | - .97 | 0.0 |
| C>S(BID)- PV(int,T)- Xexp{-rT} | 626 | 20 | 5.89 | 4.87 | 0.0 | 11.72 | .77 | 3.22 | -1.73 | 11.69 |
| CBOE 10.375% 11/07: | | | | | | | | | | |
| C>S(BID)-X | 452 | 4 | .34 | .31 | 0.0 | .78 | -.72 | 1.16 | -2.44 | 0.0 |
| P>X-S(ASK) | 255 | 21 | 1.20 | 2.56 | 0.0 | 11.52 | -.17 | .39 | -1.50 | .16 |
| C>S(BID)- PV(int,T)- Xexp{-rT} | 452 | 3 | .26 | .29 | 0.0 | .57 | -.59 | .24 | -.86 | -.4 |
| Amex T-Bill: | | | | | | | | | | |
| C>S(BID)-X | 183 | 51 | .30 | .71 | 0.0 | 2.25 | .16 | .61 | -.20 | 2.25 |
| P>X-S(ASK) | 198 | 27 | .07 | .06 | 0.0 | .27 | .00 | .09 | -.23 | .10 |
| CBT T-Bond Fut: | | | | | | | | | | |
| C>F-X | 3089 | 31 | .03 | .02 | .01 | .09 | 0.0 | .04 | -.11 | .06 |
| P>F-X | 2298 | 119 | .02 | .02 | 0.0 | .06 | -.01 | .05 | -.45 | .06 |
| P>C-F+X* exp{-rT} | 1719 | 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| P<C-F* exp{-rT}+X | 1719 | 2 | .19 | .14 | .09 | .29 | -2.99 | 1.68 | -4.17 | -1.80 |

Table 8

BOUNDARY VIOLATIONS: FOREIGN CURRENCY OPTIONS

| | Observations: | | $ Per unit of foreign currency: | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Bound | Ex Post Violations | | | | Ex Ante Violations | | | |
| Issue & Boundary | Ttl | Violated | Mean | S.D. | Min | Max | Mean | S.D. | Min | Max |
| PHLX C$: | | | | | | | | | | |
| C>S-X | 291 | 1 | .03 | 0.0 | .03 | .03 | 0.0 | 0.0 | 0.0 | 0.0 |
| C<S+P-X* exp(-rT) | 79 | 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| P>X-S | 163 | 17 | .09 | .26 | 0.0 | .68 | -.03 | .11 | -.28 | .12 |
| P<X+C-S* exp(-r'T) | 79 | 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| PHLX Yen: | | | | | | | | | | |
| C>S-X | 1246 | 18 | .04 | .03 | .01 | .12 | 0.0 | .06 | -.12 | .12 |
| C<S+P-X* exp(-rT) | 414 | 2 | .22 | .12 | .13 | .31 | -.22 | .31 | -.44 | 0.0 |
| P>X-S | 544 | 18 | .07 | .17 | 0.0 | .74 | -.06 | .14 | -.39 | .05 |
| P<X+C-S* exp(-r'T) | 414 | 3 | .14 | .08 | .06 | .22 | -.05 | .05 | -.11 | -.01 |
| Phlx fs.: | | | | | | | | | | |
| C>S-X | 1223 | 1 | .03 | 0.0 | .03 | .03 | 0.0 | 0.0 | 0.0 | 0.0 |
| C<S+P-X* exp(-rT) | 390 | 10 | .09 | .18 | .01 | .58 | -.02 | .23 | -.21 | .58 |
| P>X-S | 595 | 74 | .05 | .10 | 0.0 | .88 | -.03 | .11 | -.26 | .14 |
| P<X+C-S* exp(-r'T) | 390 | 5 | .06 | .05 | .01 | .12 | -.07 | .09 | -.16 | .03 |

(Table 8 continued next page)

Table 8 (continued)

| Issue & Boundary | Ttl | Bound Violated | Ex Post Violations Mean | S.D. | Min | Max | Ex Ante Violations Mean | S.D. | Min | Max |
|---|---|---|---|---|---|---|---|---|---|---|
| PHLX pound: | | | | | | | | | | |
| C>S-X | 839 | 13 | .13 | .07 | .02 | .23 | -.04 | .20 | -.58 | .23 |
| C<S+P-X* exp(-rT) | 257 | 2 | .11 | .08 | .06 | .17 | -.71 | .66 | -1.18 | -.25 |
| P>X-S | 438 | 17 | .08 | .04 | .02 | .20 | -.04 | .16 | -.40 | .20 |
| P<X+C-S* exp(-r'T) | 257 | 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| PHLX DM: | | | | | | | | | | |
| C>S-X | 1621 | 12 | .03 | .02 | 0.0 | .05 | -.01 | .03 | -.07 | .05 |
| C<S+P-X* exp(-rT) | 638 | 5 | .07 | .06 | .01 | .13 | -.03 | .12 | -.20 | .13 |
| P>X-S | 797 | 92 | .06 | .20 | 0.0 | 1.90 | -.01 | .32 | -1.83 | 1.90 |
| P<X+C-S* exp(-r'T) | 638 | 7 | .04 | .04 | 0.0 | .10 | -.08 | .12 | -.33 | .02 |

Table 9

PREVIOUS RESULTS: DEVIATIONS OF MODEL PRICES FROM MARKET PRICES

| Paper | Data | # of Options | Model used and resulting average deviation (market - model) |
|---|---|---|---|
| 1. Beckers (1984) | Closing prices for Amsterdam gold calls (X>S only) | 658 | Black-Scholes; 16.68% |
| 2. Blomeyer & Johnson (1984) | Transactions prices for CBOE stock options | 10295 | Geske-Johnson four point extrapolation for American puts; $.386 or 23.9% |
| | | 18027 | Roll's model for calls with one remaining known dividend; $.268 or 5.95% |
| 3. Bodurtha & Courtadon (1985a) | Transactions prices for PHLX foreign currency options | 3326 calls | Parkinson numerical method; -7.07% |
| | | 1788 puts | Parkinson numerical method; -13.24% |
| 4. Dietrich-Campbell & Schwartz (1984) | Closing prices for CBOE and Amex options on U.S. Treasury securities | 2514 calls on bonds | 1 interest rate model; $.57 |
| | | 1279 puts on bonds | 1 interest rate model; $.55 |
| | | 2514 calls on bonds | 2 interest rate model; $.33 |
| | | 1279 puts on bonds | 2 interest rate model; $.30 |
| | | 395 calls on T-bills | 2 interest rate model; $.04 |
| | | 497 puts on T-bills | 2 interest rate model; $.03 |

(table continued next page)

Table 9 (continued)

| Paper | Data | # of options | Model used and resulting average deviation (market-model) |
|---|---|---|---|
| 5. Shastri & Tandon (1984) | Transactions prices for PHLX foreign currency options | 30655 calls | Geske-Johnson; absolute deviation of \$.048 to \$.21 |
| | | 8438 puts | Geske-Johnson; absolute deviation of \$.049 to \$.158 |
| 6. Shastri & Tandon (1985) | Transactions prices for Chicago Merc S+P 500 and DM futures options | 28524 calls and 19555 puts for S+P | Geske-Johnson; absolute deviations of \$92 to \$159 per contract |
| | | 22310 calls and 8214 puts for DM | Geske-Johnson; absolute deviations of \$24 to \$144 per contract |
| 7. Whaley (1982) | Closing prices for CBOE calls on stocks | 15582 | Roll's model for calls with one known remaining dividend; \$.0097 |
| 8. Whaley (1984) | Transactions prices for Chicago Merc S+P 500 futures options | 14886 calls | Geske-Johnson; \$-.08 for calls and \$.03 for puts |
| | | 13607 puts | |

Table 10

SUMMARY OF RESULTS ON DEVIATIONS OF MODEL PRICES FROM MARKET PRICES

| | Puts: | | Calls: | |
|---|---|---|---|---|
| | Average Market Price | Average Model Price | Average Market Price | Average Model Price |
| CBOE S+P 100 | $ 3.79 | $ 3.75 | $ 4.89 | $ 4.87 |
| Amex Major Market | 3.38 | 3.41 | 3.26 | 3.37 |
| NYSE Index | 2.25 | 2.27 | 2.51 | 2.60 |
| Comex Gold Futures | 27.79 | 27.87 | 17.04 | 16.35 |
| TFE Silver | 1.22 | 1.20 | 1.22 | 1.23 |
| CBOE 14% Nov 06 bond | 5.00 | 4.79 | 1.58 | 1.59 |
| CBOE 12% Aug 08 bond | 2.50 | 2.39 | 1.92 | 1.92 |
| CBOE 10.375% Nov 07 bond | 3.63 | 3.58 | 1.73 | 1.70 |
| Amex 13 week T-Bill | .51 | .65 | .38 | .49 |
| CBT T-Bond Futures | 1.78 | 1.79 | .99 | .98 |
| PHLX Canadian $ | .60 | .64 | .49 | .50 |
| PHLX Japanese Yen | .64 | .74 | 1.39 | 1.40 |
| PHLX Swiss franc | .75 | .83 | 1.02 | 1.02 |
| PHLX U.K. pound | 2.53 | 2.65 | 2.76 | 2.69 |
| PHLX West German DM | .71 | .76 | 1.10 | 1.10 |

Table 11

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: CBOE S+P 100 OPTIONS

MODELS: Geske-Johnson two point approximation with continuous dividend for American puts and calls.

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 2559 | .036 | .557 | .346 | .437 | 2724 | .022 | .991 | .351 | .927 |
| S>X | 1523 | .030 | .442 | .263 | .356 | 1144 | .154 | 1.41 | .508 | 1.32 |
| X>S | 1086 | .045 | .692 | .468 | .511 | 1580 | -.072 | .483 | .238 | .426 |
| t<.125 | 968 | .053 | .526 | .277 | .450 | 1100 | .043 | .349 | .224 | .221 |
| .125<t<.25 | 1092 | -.011 | .556 | .362 | .422 | 1170 | .018 | .582 | .351 | .464 |
| .25<t<.375 | 289 | .117 | .666 | .463 | .492 | 262 | .101 | 1.07 | .609 | .888 |
| .375<t<.5 | 209 | .101 | .509 | .422 | .300 | 189 | .090 | .618 | .474 | .406 |
| .5<t | 1 | .153 | .00 | .153 | .00 | 3 | .513 | 1.43 | .812 | 1.29 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

Table 12

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: AMEX MAJOR MKT OPTIONS

MODELS: Geske-Johnson two point approximation with continuous dividend for American puts and calls.

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 1642 | -.029 | .538 | .292 | .453 | 1838 | -.104 | .582 | .375 | .458 |
| S>X | 724 | -.099 | .605 | .371 | .478 | 682 | -.047 | .670 | .421 | .523 |
| X>S | 918 | -.045 | .479 | .230 | .423 | 1156 | -.137 | .521 | .347 | .412 |
| t<.125 | 544 | .026 | .548 | .223 | .501 | 642 | .029 | .481 | .254 | .409 |
| .125<t<.25 | 680 | -.073 | .560 | .322 | .464 | 764 | -.135 | .631 | .412 | .496 |
| .25<t<.375 | 208 | -.027 | .407 | .282 | .293 | 211 | -.150 | .570 | .455 | .373 |
| .375<t<.5 | 189 | -.022 | .553 | .388 | .394 | 192 | -.338 | .558 | .502 | .416 |
| .5<t | 21 | -.122 | .470 | .344 | .335 | 29 | -.343 | .818 | .631 | .615 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

Table 13

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: NYSE INDEX OPTIONS

MODELS: Geske-Johnson two point approximation with continuous dividend for American puts and calls.

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 1077 | -.019 | .690 | .319 | .612 | 1300 | -.094 | .320 | .235 | .237 |
| S>X | 628 | -.014 | .608 | .272 | .543 | 549 | -.009 | .358 | .267 | .239 |
| X>S | 449 | -.026 | .792 | .385 | .692 | 751 | -.156 | .273 | .212 | .232 |
| t<.125 | 446 | .044 | .761 | .285 | .707 | 518 | -.028 | .263 | .178 | .195 |
| .125<t<.25 | 472 | .005 | .667 | .342 | .595 | 594 | -.120 | .333 | .254 | .246 |
| .25<t<.375 | 140 | -.281 | .377 | .362 | .299 | 168 | -.220 | .383 | .341 | .281 |
| .375<t<.5 | 19 | -.205 | .224 | .261 | .151 | 20 | -.015 | .290 | .243 | .149 |
| .5<t | 0 | .0 | .0 | .0 | .0 | 0 | .0 | .0 | .0 | .0 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

Table 14

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: COMEX GOLD FUTURES OPTIONS

MODELS: Binomial model with 300 time steps for American puts and American calls.

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 3434 | - .08 | 2.68 | 1.21 | 2.39 | 3533 | .68 | 1.63 | 1.15 | 1.35 |
| F>X | 2433 | -.34 | 2.92 | 1.34 | 2.61 | 927 | .70 | 1.88 | 1.06 | 1.70 |
| X>F | 1001 | .55 | 1.85 | .88 | 1.72 | 2606 | .68 | 1.54 | 1.18 | 1.20 |
| t<.125 | 543 | .01 | 2.78 | 1.03 | 2.58 | 432 | .31 | 2.57 | 1.18 | 2.33 |
| .125<t<.25 | 859 | -.18 | 2.05 | .95 | 1.83 | 845 | .51 | 1.54 | .98 | 1.29 |
| .25<t<.375 | 892 | -.18 | 2.63 | 1.17 | 2.37 | 931 | .48 | 1.15 | .90 | .86 |
| .375<t<.5 | 445 | -.07 | 3.39 | 1.40 | 3.09 | 466 | .54 | 1.26 | 1.01 | .92 |
| .5<t | 695 | .09 | 2.82 | 1.59 | 2.83 | 859 | 1.34 | 1.56 | 1.67 | 1.22 |

The * denotes model price.

Options selling for less than $1 or violating the immediate early exercise boundaries are excluded.

Table 15

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: TFE SILVER OPTIONS

MODELS: Black-Scholes model for calls, Geske-Johnson two point approximation for puts

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 265 | .016 | .145 | .101 | .104 | 810 | -.008 | .186 | .106 | .152 |
| S>X | 59 | .048 | .170 | .109 | .139 | 447 | -.026 | .155 | .107 | .114 |
| X>S | 206 | .007 | .135 | .099 | .092 | 363 | .014 | .216 | .106 | .189 |
| t<.125 | 197 | .027 | .137 | .097 | .100 | 533 | -.005 | .175 | .097 | .146 |
| .125<t<.25 | 3 | -.034 | .048 | .049 | .023 | 7 | -.028 | .056 | .041 | .045 |
| .25<t<.375 | 13 | .000 | .139 | .105 | .086 | 28 | .007 | .185 | .123 | .135 |
| .375<t<.5 | 18 | .046 | .212 | .121 | .178 | 64 | -.030 | .173 | .089 | .151 |
| .5<t | 34 | -.052 | .137 | .115 | .090 | 178 | -.009 | .221 | .140 | .171 |

The * denotes model price.

Options selling for less than $.5 or violating the immediate early exercise boundaries are excluded.

Table 16

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: CBOE GOVERNMENT BOND OPTIONS

MODELS: Binomial model with 100 time steps for American puts and American calls.

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 548 | .115 | .654 | .437 | .508 | 923 | .007 | .496 | .301 | .394 |
| S>X | 251 | .137 | .656 | .435 | .509 | 380 | -.040 | .396 | .271 | .291 |
| X>S | 297 | .097 | .653 | .420 | .509 | 543 | .041 | .553 | .322 | .451 |
| t<.125 | 221 | .320 | .367 | .368 | .318 | 307 | .099 | .394 | .243 | .325 |
| .125<t<.25 | 165 | .136 | .695 | .415 | .573 | 302 | .025 | .580 | .312 | .489 |
| .25<t<.375 | 113 | -.123 | .669 | .444 | .513 | 215 | -.115 | .449 | .340 | .315 |
| .375<t<.5 | 17 | -.101 | 1.16 | .666 | .949 | 69 | -.009 | .525 | .342 | .396 |
| .5<t | 32 | -.438 | .933 | .707 | .43 | 30 | -.186 | .563 | .418 | .415 |

The * denotes model price.

Options selling for less than $.5 or violating the immediate early exercise boundaries are excluded.

Table 17

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES AMEX 13 WEEK T-BILL OPTIONS

MODEL: Courtadon stochastic interest rate model for American puts and calls.

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 129 | -.145 | .286 | .236 | .217 | 127 | -.114 | .265 | .192 | .214 |
| S>X | 67 | -.058 | .275 | .186 | .209 | 71 | -.141 | .293 | .232 | .226 |
| X>S | 62 | -.239 | .270 | .290 | .213 | 56 | -.081 | .222 | .142 | .188 |
| t<.125 | 13 | -.117 | .240 | .213 | .153 | 12 | -.095 | .283 | .145 | .258 |
| .125<t<.25 | 53 | -.108 | .271 | .195 | .216 | 53 | -.127 | .220 | .176 | .183 |
| .25<t<.375 | 34 | -.153 | .302 | .252 | .224 | 33 | -.036 | .273 | .185 | .202 |
| .375<t<.5 | 10 | -.252 | .196 | .270 | .167 | 18 | -.210 | .235 | .239 | .204 |
| .5<t | 19 | -.196 | .360 | .318 | .252 | 11 | -.148 | .413 | .272 | .337 |

The * denotes model price.

Options selling for less than $.05 or violating the immediate early exercise boundaries are excluded.

Table 18

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: CBT T-BOND FUTURES OPTIONS

MODELS: Binomial model with 20 time steps per month for American puts and calls.

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 2157 | -.003 | .116 | .089 | .075 | 2987 | 0.014 | .160 | .121 | .106 |
| F>X | 1232 | -.033 | .101 | .082 | .068 | 679 | -.205 | .150 | .212 | .141 |
| X>F | 925 | .036 | .123 | .098 | .082 | 2308 | 0.079 | .089 | .094 | .074 |
| t<.125 | 343 | -.090 | .111 | .109 | .092 | 331 | -.052 | .088 | .072 | .073 |
| .125<t<.25 | 539 | -.061 | .075 | .078 | .060 | 667 | -.040 | .128 | .092 | .098 |
| .25<t<.375 | 535 | .013 | .096 | .075 | .061 | 689 | .014 | .166 | .125 | .110 |
| .375<t<.5 | 375 | .030 | .090 | .074 | .058 | 581 | .031 | .162 | .131 | .100 |
| .5<t | 365 | .104 | .114 | .120 | .097 | 709 | .085 | .172 | .158 | .110 |

The * denotes model price.

Options selling for less than $.5 or violating the immediate early exercise boundaries are excluded.

Table 19

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: PHLX CANADIAN DOLLAR OPTIONS

MODELS: Geske-Johnson two point approximation for American puts and calls modified for receipt of interest on foreign deposits

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 96 | -.042 | .131 | .094 | .101 | 220 | -.01 | .116 | .081 | .083 |
| S>X | 48 | -.029 | .134 | .081 | .111 | 85 | -.016 | .110 | .084 | .073 |
| X>S | 48 | -.056 | .128 | .107 | .088 | 135 | -.006 | .120 | .080 | .089 |
| t<.125 | 5 | .033 | .096 | .059 | .080 | 8 | .019 | .071 | .064 | .026 |
| .125<t<.25 | 33 | -.049 | .087 | .073 | .067 | 46 | -.012 | .065 | .051 | .042 |
| .25<t<.375 | 31 | -.004 | .109 | .075 | .077 | 60 | .001 | .091 | .066 | .061 |
| .375<t<.5 | 15 | -.115 | .154 | .143 | .137 | 46 | -.033 | .131 | .088 | .101 |
| .5<t | 12 | -.064 | .214 | .152 | .158 | 60 | -.004 | .155 | .118 | .101 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

## Table 20

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: PHLX JAPANESE YEN OPTIONS

MODELS: Geske-Johnson two point approximation for American puts and calls modified for receipt of interest on foreign deposits

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 531 | -.099 | .092 | .110 | .078 | 1223 | -.012 | .128 | .090 | .092 |
| S>X | 309 | -.085 | .084 | .096 | .071 | 541 | -.037 | .139 | .099 | .104 |
| X>S | 222 | -.118 | .101 | .131 | .084 | 682 | .007 | .115 | .083 | .079 |
| t<.125 | 75 | -.054 | .071 | .067 | .059 | 140 | -.052 | .062 | .066 | .047 |
| .125<t<.25 | 195 | -.094 | .081 | .103 | .069 | 313 | -.039 | .074 | .067 | .051 |
| .25<t<.375 | 128 | -.105 | .091 | .116 | .075 | 231 | -.023 | .117 | .080 | .088 |
| .375<t<.5 | 71 | -.107 | .089 | .116 | .076 | 236 | .025 | .115 | .092 | .072 |
| .5<t | 62 | -.146 | .127 | .169 | .095 | 303 | .014 | .188 | .134 | .132 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

Table 21

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: PHLX SWISS FRANC OPTIONS

MODELS: Geske-Johnson two point approximation for American puts and calls modified for receipt of interest on foreign deposits

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 521 | -.080 | .130 | .112 | .103 | 1207 | .002 | .142 | .095 | .106 |
| S>X | 250 | -.065 | .091 | .087 | .071 | 244 | .011 | .165 | .112 | .112 |
| X>S | 271 | -.094 | .155 | .135 | .121 | 963 | -.001 | .135 | .088 | .103 |
| t<.125 | 83 | -.067 | .077 | .081 | .064 | 171 | -.029 | .085 | .059 | .068 |
| .125<t<.25 | 172 | -.077 | .084 | .092 | .067 | 335 | -.015 | .078 | .057 | .055 |
| .25<t<.375 | 113 | -.085 | .113 | .112 | .086 | 242 | .008 | .145 | .097 | .108 |
| .375<t<.5 | 82 | -.094 | .215 | .159 | .172 | 224 | .000 | .153 | .111 | .105 |
| .5<t | 71 | -.079 | .162 | .143 | .108 | 235 | .043 | .207 | .157 | .141 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

Table 22

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: PHLX U.K. POUND OPTIONS

MODELS: Geske-Johnson two point approximation for American puts and calls modified for receipt of interest on foreign deposits

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 400 | -.125 | .591 | .292 | .529 | 811 | .023 | .521 | .278 | .441 |
| S>X | 226 | -.079 | .583 | .265 | .525 | 256 | .043 | .531 | .347 | .403 |
| X>S | 174 | -.185 | .599 | .362 | .535 | 555 | .013 | .516 | .246 | .454 |
| t<.125 | 67 | -.055 | .241 | .185 | .163 | 103 | -.080 | .229 | .157 | .184 |
| .125<t<.25 | 125 | -.239 | .501 | .318 | .455 | 223 | -.093 | .578 | .237 | .535 |
| .25<t<.375 | 99 | -.054 | .238 | .195 | .145 | 170 | .006 | .289 | .216 | .191 |
| .375<t<.5 | 50 | -.151 | .736 | .378 | .647 | 137 | .001 | .667 | .317 | .586 |
| .5<t | 59 | -.061 | 1.10 | .446 | 1.00 | 178 | .261 | .538 | .430 | .415 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

Table 23

DEVIATIONS OF MODEL PRICES FROM MARKET PRICES: PHLX WEST GERMAN MARK OPTIONS

MODELS: Geske-Johnson two point approximation for American puts and calls modified for receipt of interest on foreign deposits

| | PUTS: | | | | | CALLS: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | P-P* | | /P-P*/ | | | C-C* | | /C-C*/ | |
| | N | Mean | SD | Mean | SD | N | Mean | SD | Mean | SD |
| All | 702 | -.052 | .101 | .084 | .076 | 1607 | -.003 | .143 | .186 | .113 |
| S>X | 395 | -.026 | .083 | .065 | .058 | 588 | -.014 | .149 | .102 | .110 |
| X>S | 307 | -.084 | .113 | .109 | .089 | 1019 | .003 | .138 | .077 | .115 |
| t<.125 | 115 | -.023 | .066 | .051 | .047 | 211 | -.018 | .062 | .048 | .043 |
| .125<t<.25 | 228 | -.064 | .078 | .079 | .062 | 389 | -.019 | .095 | .065 | .072 |
| .25<t<.375 | 164 | -.051 | .105 | .091 | .074 | 355 | -.005 | .158 | .087 | .132 |
| .375<t<.5 | 105 | -.051 | .112 | .089 | .084 | 342 | .014 | .141 | .091 | .109 |
| .5<t | 90 | -.058 | .153 | .121 | .109 | 310 | .010 | .201 | .134 | .149 |

The * denotes model price.

Options violating the immediate early exercise bound are excluded.

Table 24

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
CBOE S+P 100 OPTION

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | -.024 | -1.63 | 1.016 | 359.7 | .981 |
| P-P* = A0 + A1 WISD | .847 | 11.86 | -5.301 | -11.48 | .049 |
| P-P* = A0 + A1 (X-S) | .047 | 4.11 | .004 | 3.10 | .004 |
| P-P* = A0 + A1 t | .017 | .99 | .108 | 1.36 | .001 |
| C = A0 + A1 C* | .046 | 1.71 | .995 | 246.84 | .957 |
| C-C* = A0 + A1 WISD | .503 | 3.93 | -3.191 | -3.79 | .005 |
| C-C* = A0 + A1 (S-X) | .046 | 2.41 | .010 | 5.19 | .009 |
| C-C* = A0 + A1 t | .600 | 22.2 | -3.414 | -27.29 | .214 |

The * denotes model price.

Table 25

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
AMEX MAJOR MARKET OPTION

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | -.068 | -3.55 | 1.011 | 247.51 | .973 |
| P-P* = A0 + A1 WISD | .626 | 9.89 | -4.068 | -10.57 | .064 |
| P-P* = A0 + A1 (X-S) | -.024 | -1.85 | .008 | 3.56 | .008 |
| P-P* = A0 + A1 t | .010 | .45 | -.207 | -1.96 | .002 |
| C = A0 + A1 C* | -.038 | -1.89 | .980 | 219.70 | .963 |
| C-C* = A0 + A1 WISD | 1.193 | 21.05 | -8.123 | -23.41 | .229 |
| C-C* = A0 + A1 (S-X) | -.089 | -6.08 | .006 | 2.72 | .004 |
| C-C* = A0 + A1 t | .066 | 2.76 | -.901 | -8.46 | .037 |

The * denotes model price.

Table 26

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
NYSE INDEX OPTION

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | .002 | .09 | .990 | 106.91 | .914 |
| P-P* = A0 + A1 WISD | 1.550 | 11.54 | -10.49 | -11.82 | .114 |
| P-P* = A0 + A1 (X-S) | -.022 | -1.04 | -.004 | -.78 | .001 |
| P-P* = A0 + A1 t | .123 | 3.01 | -.910 | -4.05 | .015 |
| C = A0 + A1 C* | -.154 | -12.28 | 1.023 | 293.26 | .985 |
| C-C* = A0 + A1 WISD | .697 | 12.17 | -5.265 | -13.97 | .130 |
| C-C* = A0 + A1 (S-X) | -.074 | -8.12 | .011 | 7.15 | .037 |
| C-C* = A0 + A1 t | .003 | .17 | -.615 | -6.32 | .029 |

The * denotes model price.

Table 27

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
COMEX GOLD FUTURES OPTION

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | -.057 | - .88 | .999 | 588.7 | .990 |
| P-P* = A0 + A1 WISD | 2.302 | 6.09 | -13.37 | -6.35 | .011 |
| P-P* = A0 + A1 (X-F) | .029 | .55 | -.004 | -3.77 | .004 |
| P-P* = A0 + A1 t | -.196 | -2.12 | .363 | 1.44 | .001 |
| C = A0 + A1 C* | .724 | 18.47 | .997 | 583.8 | .989 |
| C-C* = A0 + A1 WISD | 2.224 | 9.82 | -8.59 | -6.84 | .013 |
| C-C* = A0 + A1 (F-X) | .484 | 14.41 | -.006 | -10.20 | .028 |
| C-C* = A0 + A1 t | .057 | 1.0 | 1.85 | 12.50 | .042 |

The * denotes model price.

## Table 28

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
TFE SILVER OPTION

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | .012 | .77 | 1.003 | 93.7 | .970 |
| P-P* = A0 + A1 WISD | .389 | 6.17 | -1.038 | -5.906 | .119 |
| P-P* = A0 + A1 (X-S) | .011 | 1.09 | .007 | .86 | .002 |
| P-P* = A0 + A1 t | .013 | 1.50 | -.029 | -1.29 | .006 |
| C = A0 + A1 C* | .095 | 6.33 | .915 | 82.4 | .893 |
| C-C* = A0 + A1 WISD | .638 | 15.15 | -1.845 | -15.5 | .224 |
| C-C* = A0 + A1 (S-X) | -.006 | -.97 | -.029 | -4.53 | .024 |
| C-C* = A0 + A1 t | -.009 | -1.34 | -.008 | -.59 | 0.000 |

The * denotes model price.

## Table 29

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
CBOE GOVERNMENT BOND OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | .242 | 5.73 | .962 | 101.7 | .949 |
| P-P* = A0 + A1 WISD | .738 | 7.50 | -5.19 | -6.57 | .073 |
| P-P* = A0 + A1 (X-S) | .128 | 4.41 | -.010 | -1.54 | .004 |
| P-P* = A0 + A1 t | .43 | 9.43 | -1.628 | -8.43 | .115 |
| C = A0 + A1 C* | .256 | 9.15 | .86 | 65.46 | .823 |
| C-C* = A0 + A1 WISD | .579 | 9.65 | -4.69 | -9.85 | .095 |
| C-C* = A0 + A1 (S-X) | -.010 | -.60 | -.023 | -3.90 | .016 |
| C-C* = A0 + A1 t | .150 | 4.92 | -.686 | -5.49 | .031 |

The * denotes model price.

Table 30

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
AMEX 13 WEEK T-BILL OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | .019 | .44 | .748 | 13.49 | .589 |
| P-P* = A0 + A1 WISD | .082 | 1.04 | -.781 | -3.02 | .067 |
| P-P* = A0 + A1 (X-S) | -.170 | -7.42 | -.096 | -5.78 | .208 |
| P-P* = A0 + A1 t | -.066 | -1.34 | -.281 | -1.83 | .026 |
| C = A0 + A1 C* | .024 | .73 | .716 | 13.47 | .592 |
| C-C* = A0 + A1 WISD | .231 | 3.56 | -1.137 | -5.63 | .202 |
| C-C* = A0 + A1 (S-X) | -.108 | -4.70 | -.093 | -2.68 | .054 |
| C-C* = A0 + A1 t | -.086 | -1.77 | -.101 | -.65 | .003 |

The * denotes model price.

Table 31

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
CBT T-BOND FUTURES OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | -.055 | -10.3 | 1.028 | 744.09 | .996 |
| P-P* = A0 + A1 WISD | 0.001 | 0.10 | -.039 | -0.25 | .000 |
| P-P* = A0 + A1 (X-F) | 0.001 | 0.41 | 0.010 | 13.28 | .075 |
| P-P* = A0 + A1 t | -0.012 | -28.4 | 0.368 | 31.68 | .317 |
| C = A0 + A1 C* | 0.115 | 40.0 | 0.907 | 542.30 | .989 |
| C-C* = A0 + A1 WISD | 0.014 | 1.42 | 0.004 | 0.04 | .000 |
| C-C* = A0 + A1 (F-X) | -.069 | -22.6 | -.024 | -41.79 | .369 |
| C-C* = A0 + A1 t | -.084 | 18.6 | 0.280 | 18.56 | .103 |

The * denotes model price.

Table 32

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
PHLX CANADIAN DOLLAR OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | .007 | .33 | .922 | 35.37 | .930 |
| P-P* = A0 + A1 WISD | .127 | 3.68 | -5.259 | -5.25 | .226 |
| P-P* = A0 + A1 (X-S) | -.042 | -3.09 | -.013 | -.72 | .005 |
| P-P* = A0 + A1 t | .001 | .03 | -.138 | -1.64 | .027 |
| C = A0 + A1 C* | .032 | 2.60 | .915 | 46.73 | .909 |
| C-C* = A0 + A1 WISD | .069 | 3.22 | -2.779 | -3.94 | .066 |
| C-C* = A0 + A1 (S-X) | -.016 | -1.95 | -.016 | -1.95 | .017 |
| C-C* = A0 + A1 t | -.004 | -.21 | -.015 | -.33 | .000 |

The * denotes model price.

## Table 33

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
PHLX JAPANESE YEN OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | -.064 | -10.34 | .952 | 144.32 | .975 |
| P-P* = A0 + A1 WISD | .212 | 7.92 | -3.02 | -11.73 | .206 |
| P-P* = A0 + A1 (X-S) | -.105 | -26.08 | -.019 | -5.92 | .062 |
| P-P* = A0 + A1 t | -.062 | -7.89 | -.132 | -5.48 | .053 |
| C = A0 + A1 C* | .005 | .75 | .987 | 261.23 | .982 |
| C-C* = A0 + A1 WISD | .205 | 7.37 | -2.13 | -7.88 | .048 |
| C-C* = A0 + A1 (S-X) | -.013 | -3.54 | -.011 | -4.64 | .017 |
| C-C* = A0 + A1 t | -.062 | -8.27 | .143 | 7.55 | .044 |

The * denotes model price.

Table 34

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
PHLX SWISS FRANC OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | -.090 | -10.42 | 1.012 | 128.61 | .970 |
| P-P* = A0 + A1 WISD | .445 | 8.71 | -5.261 | -10.34 | .171 |
| P-P* = A0 + A1 (X-S) | -.080 | -14.14 | .003 | .63 | .001 |
| P-P* = A0 + A1 t | -.073 | -6.66 | -.025 | -.79 | .002 |
| C = A0 + A1 C* | -.002 | -.26 | 1.003 | 204.14 | .972 |
| C-C* = A0 + A1 WISD | .605 | 16.48 | -6.031 | -16.51 | .184 |
| C-C* = A0 + A1 (S-X) | .002 | .45 | .000 | .16 | .000 |
| C-C* = A0 + A1 t | -.038 | -4.50 | .122 | 5.55 | .023 |

The * denotes model price.

## Table 35

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
PHLX U.K. POUND OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | .076 | 1.64 | .924 | 66.98 | .918 |
| P-P* = A0 + A1 WISD | 2.24 | 21.89 | -25.41 | -23.54 | .582 |
| P-P* = A0 + A1 (X-S) | -.132 | -4.39 | -.009 | -1.19 | .003 |
| P-P* = A0 + A1 t | -.121 | -2.10 | -.015 | -.09 | .000 |
| C = A0 + A1 C* | .027 | 1.03 | .998 | 134.48 | .957 |
| C-C* = A0 + A1 WISD | 1.491 | 14.44 | -15.88 | -14.40 | .204 |
| C-C* = A0 + A1 (S-X) | .013 | .67 | -.004 | -1.11 | .001 |
| C-C* = A0 + A1 t | -.203 | -5.60 | .674 | 7.14 | .059 |

The * denotes model price.

Table 36

REGRESSIONS OF MODEL DEVIATIONS ON MODEL PARAMETERS:
PHLX WEST GERMAN MARK OPTIONS

| Regression | A0 | t(A0) | A1 | t(A1) | R**2 (ADJ) |
|---|---|---|---|---|---|
| P = A0 + A1 P* | -.036 | -6.70 | .979 | 193.73 | .981 |
| P-P* = A0 + A1 WISD | .389 | 13.93 | -4.09 | -15.90 | .265 |
| P-P* = A0 + A1 (X-S) | -.054 | -14.39 | -.016 | -5.80 | .045 |
| P-P* = A0 + A1 t | -.045 | -5.93 | -.025 | -1.09 | .002 |
| C = A0 + A1 C* | .010 | 1.98 | .987 | 276.54 | .979 |
| C-C* = A0 + A1 WISD | .355 | 12.65 | -3.34 | -12.86 | .093 |
| C-C* = A0 + A1 (S-X) | -.010 | -2.79 | -.010 | -5.89 | .021 |
| C-C* = A0 + A1 t | -.027 | -3.70 | .073 | 3.70 | .008 |

The * denotes model price.