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ABSTRACT

We propose an entropy-based sensor selection heuristic for localization. Given 1) a prior probability distribution of the target location, and 2) the locations and the sensing models of a set of candidate sensors for selection, the heuristic selects an informative sensor such that the fusion of the selected sensor observation with the prior target location distribution would yield on average the greatest or nearly the greatest reduction in the entropy of the target location distribution. The heuristic greedily selects one sensor in each step without retrieving any actual sensor observations. The heuristic is also computationally much simpler than the mutual-information-based approaches. The effectiveness of the heuristic is evaluated using localization simulations in which Gaussian sensing models are assumed for simplicity. The heuristic is more effective when the optimal candidate sensor is more informative.

Categories and Subject Descriptors

H.1.1 [**MODELS AND PRINCIPLES**]: Systems and Information Theory—*Information theory*; C.2.4 [**COMPUTER-COMMUNICATION NETWORKS**]: Distributed Systems—*Distributed applications*

General Terms

Algorithms, Management, Theory

Keywords

sensor selection, information-directed resource management, information fusion, target localization, target tracking, wireless sensor networks, mutual information, Shannon entropy

1. INTRODUCTION

The recent convergence of micro-electro-mechanical systems (MEMS) technology, wireless communication and networking technology, and low-cost low-power miniature digital hardware design technology has made the concept of wireless sensor networks viable and a new frontier of research [2, 1]. The limited on-board energy storage and the limited wireless channel capacity are the major constraints of wireless sensor networks. In order to save precious resources, a sensing task should not involve more sensors than necessary. From the information-theoretic point of view, sensors are tasked to observe the target in order to increase the information (or to reduce the uncertainty) about the target state. The information gain attributable to one sensor may be very different from that attributable to another when sensors have different observation perspectives and sensing uncertainties. Selective use of informative sensors reduces the number of sensors needed to obtain information about the target state and therefore prolongs the system lifetime. In the scenario of localization or tracking using wireless sensor networks, the belief state of the target location can be gradually improved by repeatedly selecting the most informative unused sensor until the required accuracy (or uncertainty) level of the target state is achieved.

There have been several investigations into information-theoretic approaches to sensor fusion and management. The idea of using information theory in sensor management was first proposed in [8]. Sensor selection based on expected information gain was introduced for decentralized sensing systems in [12]. The mutual information between the predicted sensor observation and the current target location distribution was proposed to evaluate the expected information gain about the target location attributable to a sensor in [11, 6]. On the other hand, without using information theory, Yao et. al. [16] found that the overall localization accuracy depends on not only the accuracy of individual sensors but also the sensor locations relative to the target location during the development of localization algorithms. We propose a novel entropy-based heuristic for sensor selection based on our experiences with target localization. It is computationally more efficient than mutual-information-based methods proposed in [11, 6].

We use the following notations throughout this paper:
1. S is the set of candidate sensors for selection, $i \in S$ is the sensor index;
2. $\mathbf{x}$ is the realization of the random vector that denotes the target location;
3. $\mathbf{x}^{\mathrm{t}}$ is the actual target location;
4. $\hat{\mathbf{x}}$ is the maximum likelihood estimate of the target location;
5. $\mathbf{x}_i$ is the deterministic location of sensor i;
6. z_i is the realization of the random variable that denotes the observation of sensor i about the target location;
7. z_i^{t} is the actual observation of sensor i about the target location;
8. z_i^{v} is the realization of the random variable that denotes the view of sensor i about the target location.

The rest of this paper is organized as follows. Section 2 describes the heuristic in detail. Section 3 evaluates the heuristic using simulations. Section 4 discusses the discrepancy between the heuristic and the mutual information based approaches. Section 5 outlines future work. Section 6 concludes the paper. Section 7 acknowledges the sponsors.

2. SENSOR SELECTION HEURISTIC

This Sect. formulates the sensor selection problem in localization, presents the details of the entropy-based sensor selection heuristic, and discusses the relation between the entropy difference proposed in this paper and mutual information used in previous work about sensor selection.

2.1 Sensor Selection Problem in Localization

There are several information measures. In this paper, we use Shannon entropy [14] to quantify the information gain (or uncertainty reduction) about the target location due to sensor observation. We adopt the greedy sensor selection strategy used in mutual-information-based approaches [11, 6]. The greedy strategy gradually reduces the uncertainty of the target location distribution by repeatedly selecting the currently unused sensor with maximal expected information gain. The observation of the selected sensor is incorporated into the target location distribution using sequential Bayesian filtering [3, 7]. The greedy sensor selection and the sequential information fusion continue until the uncertainty of the target location distribution is less than or equal to the required level. The core problem of the greedy sensor selection approach is how to efficiently evaluate the expected information gain attributable to each candidate sensor without actually retrieving sensor data.

The sensor selection problem is formulated as follows. Given
1. the prior target location distribution: $p(\mathbf{x})$,
2. the locations of candidate sensors for selection: $\mathbf{x}_i, i \in S$,
3. the sensing models of candidate sensors for selection: $p(z_i|\mathbf{x}), i \in S$,
the objective is to find the sensor $\hat{i}$ whose observation $z_{\hat{i}}$ minimizes the expected conditional entropy of the posterior target location distribution,

$$\hat{i} = \arg\min_{i \in S} H(\mathbf{x}|z_i) \ . \tag{1}$$

Equivalently, the observation of sensor $\hat{i}$ maximizes the expected target location entropy reduction,

$$\hat{i} = \arg\max_{i \in S} (H(\mathbf{x}) - H(\mathbf{x}|z_i)) \ . \tag{2}$$

$H(\mathbf{x}) - H(\mathbf{x}|z_i)$ is one expression of $I(\mathbf{x}; z_i)$, the mutual information between the target location $\mathbf{x}$ and the predicted sensor observation z_i,

$$I(\mathbf{x}; z_i) = \int p(\mathbf{x}, z_i) \log \frac{p(\mathbf{x}, z_i)}{p(\mathbf{x})p(z_i)} d\mathbf{x} dz_i \ , \tag{3}$$

where $p(\mathbf{x}, z_i) = p(z_i|\mathbf{x})p(\mathbf{x})$ and $p(z_i) = \int p(\mathbf{x}, z_i) d\mathbf{x}$. Thus, the observation of sensor $\hat{i}$ maximizes the mutual information $I(\mathbf{x}; z_i)$,

$$\hat{i} = \arg\max_{i \in S} I(\mathbf{x}; z_i) \ . \tag{4}$$

Sensor selection based on (4) is the maximal mutual information criterion proposed in [11, 6]. The target location $\mathbf{x}$ could be of up to three dimensions. The sensor observation z_i (e.g. the direction to a target in a three-dimensional space) could be of up to two dimensions. Therefore $I(\mathbf{x}; z_i)$ is a complex integral in the joint state space $(\mathbf{x}, z_i)$ of up to five dimensions. The complexity of computing $I(\mathbf{x}; z_i)$ could be more than that low-end sensor nodes are capable of. If the observation z_i is related to the target location $\mathbf{x}$ only through the sufficient statistics $z(\mathbf{x})$, then

$$I(\mathbf{x}; z_i) = I(z(\mathbf{x}); z_i) \ . \tag{5}$$

If $z(\mathbf{x})$ has fewer dimensions than $\mathbf{x}$, then $I(z(\mathbf{x}); z_i)$ is less complex to compute than $I(\mathbf{x}; z_i)$. In the above special scenario, $I(z(\mathbf{x}); z_i)$ has been proposed to replace $I(\mathbf{x}; z_i)$ to reduce the complexity of computing mutual information in [11]. In this paper, we propose an alternative entropy-based sensor selection heuristic. In general, the entropy-based sensor selection heuristic is computationally much simpler than the mutual information based approaches. However, the observation of the sensor selected by the heuristic would still yield on average the greatest or nearly the greatest entropy reduction of the target location distribution.

2.2 Entropy-based Sensor Selection Heuristic

During the development of wireless sensor networks for localization, we have observed that the localization uncertainty reduction attributable to a sensor is greatly effected by the difference of two quantities, namely, the entropy of the distribution of that sensor's view about the target location, and the entropy of that sensor's sensing model for the actual target location.

A sensor's view about the target location is the geometric projection of the target location onto that sensor's observation perspective. For example, a direction-of-arrival (DOA) sensor's view of the target location is the direction from the sensor to the target. The view of sensor i about the target location is denoted as z_i^{v},which is a function of the target location $\mathbf{x}$ and the sensor location $\mathbf{x}_i$,

$$z_i^{\mathrm{v}} = f(\mathbf{x}, \mathbf{x}_i) \ . \tag{6}$$

z_i^{v} usually has less dimensions than $\mathbf{x}$. The probability distribution of the view of sensor i about the target location, $p(z_i^{\mathrm{v}})$, is the projection of the target location distribution $p(\mathbf{x})$ onto the observation perspective of sensor i,

$$p(z_i^{\mathrm{v}}) dz_i^{\mathrm{v}} = \int_{z_i^{\mathrm{v}} \leq f(\mathbf{x}, \mathbf{x}_i) \leq z_i^{\mathrm{v}} + dz_i^{\mathrm{v}}} p(\mathbf{x}) d\mathbf{x} \ . \tag{7}$$

Alternatively, $p(z_i^{\mathrm{v}})$ can be regarded as the 'noise free' prediction of the sensor observation distribution $p(z_i)$ based on the target location distribution $p(\mathbf{x})$.

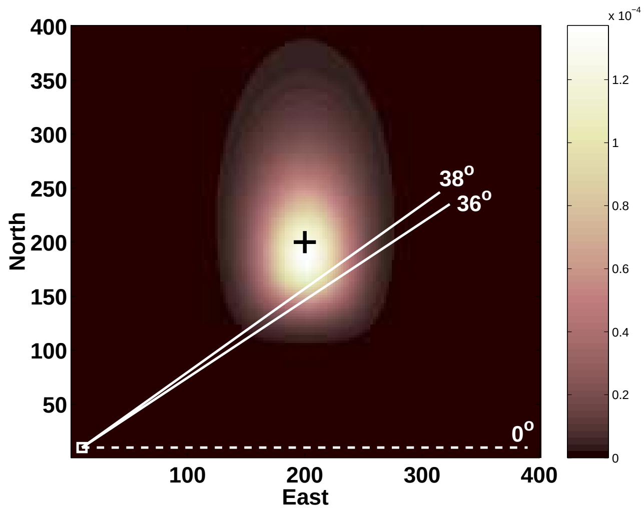

Figure 1: A DOA sensor's view about the target location. The state space of the target location is gridded in 1×1 cells. The image depicts the probability distribution of the target location. The actual target location is (200, 200), denoted by marker +. From the perspective of the DOA sensor denoted by the square, only the direction to the target is observable. The view of the DOA sensor about the target is in the interval $[36^o, 38^o]$ if and only if the target is inside the sector delimited by 36^o line and 38^o line.

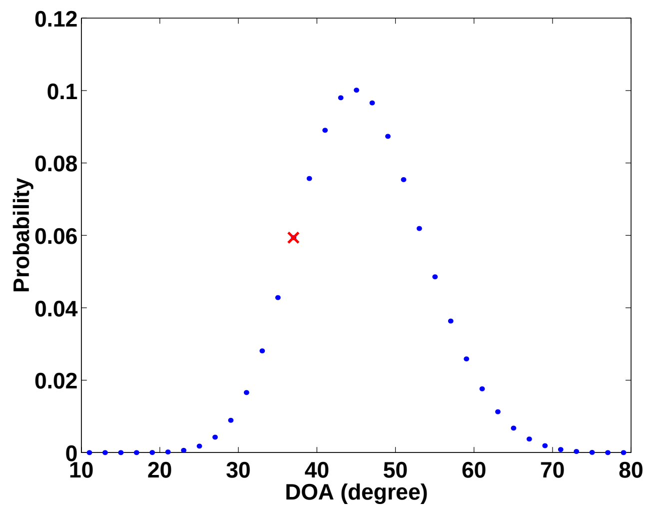

Figure 2: The discrete probability distribution of a DOA sensor's view. The state space of the DOA sensor view is gridded in 2^o intervals. The target location distribution and the DOA sensor location are illustrated in Fig. 1. Marker X denotes the probability of the DOA view interval $[36^o, 38^o]$, which is the summation of the probability of all target locations inside the sector delimited by 36^o line and 38^o line in Fig. 1. Please note that the sensor view distribution does not depends on the sensing uncertainty characteristics at all.

In practice, the state space of the target location and the sensor view can be discretized by griding for numerical analysis. The discrete representation of $p(z_i^{\mathrm{v}})$ can be computed as follows.
1. Let $\mathcal{X}$ be the grid set of the target location $\mathbf{x}$;
2. Let $\mathcal{Z}$ be the grid set of the sensor view z_i^{v};
3. For each grid point $z_i^{\mathrm{v}} \in \mathcal{Z}$, initialize $p(z_i^{\mathrm{v}})$ to zero;
4. For each grid point $\mathbf{x} \in \mathcal{X}$, determine the corresponding grid point $z_i^{\mathrm{v}} \in \mathcal{Z}$ using equation (6), and update its probability as $p(z_i^{\mathrm{v}}) = p(z_i^{\mathrm{v}}) + p(\mathbf{x})$;
5. Normalize $p(z_i^{\mathrm{v}})$ to make the total probability of the sensor view be 1.
The numerical computation of $p(z_i^{\mathrm{v}})$ for a DOA sensor is illustrated in Fig. 1 and Fig. 2.

The entropy of the probability distribution of the view of sensor i, H_i^{v}, is

$$H_i^{\mathrm{v}} = -\int p(z_i^{\mathrm{v}}) \log p(z_i^{\mathrm{v}}) dz_i^{\mathrm{v}} \ . \tag{8}$$

Given the discrete representation of $p(z_i^{\mathrm{v}})$ with a grid size of δz_i^{v}, H_i^{v} can be numerically computed as

$$H_i^{\mathrm{v}} = -\sum p(z_i^{\mathrm{v}}) \log p(z_i^{\mathrm{v}}) \delta z_i^{\mathrm{v}} \ . \tag{9}$$

The sensing model of sensor i for the actual target location $\mathbf{x}^{\mathrm{t}}$ is $p(z_i|\mathbf{x}^{\mathrm{t}})$, which describes the probability distribution of the observation of sensor i given that the target is at $\mathbf{x}^{\mathrm{t}}$. The sensing model incorporates observation uncertainty from all sources, including the noise corruption to the signal, the signal modeling error of the sensor estimation algorithm, and the inaccuracy of the sensor hardware. For a single-modal target location distribution $p(\mathbf{x})$, we can use the maximum likelihood estimate $\hat{\mathbf{x}}$ of the target location to approximate the actual target location $\mathbf{x}^{\mathrm{t}}$. Thus the entropy of the sensing model of sensor i for the actual target location $\mathbf{x}^{\mathrm{t}}$ is approximated as

$$H_i^{\mathrm{s}} = -\int p(z_i|\hat{\mathbf{x}}) \log p(z_i|\hat{\mathbf{x}}) dz_i \ . \tag{10}$$

For a multi-modal target location distribution $p(\mathbf{x})$ with M peaks $\hat{\mathbf{x}}^{(m)}$, where $m = 1, \ldots, M$, the entropy of the sensing model of sensor i for the actual target location $\mathbf{x}^{\mathrm{t}}$ can be approximated as a weighted average of the entropy of the sensing model for all modes,

$$H_i^{\mathrm{s}} = -\sum_{m=1}^{M} p(\hat{\mathbf{x}}^{(m)}) \int p(z_i|\hat{\mathbf{x}}^{(m)}) \log p(z_i|\hat{\mathbf{x}}^{(m)}) dz_i \ . \tag{11}$$

Given a target location distribution $p(\mathbf{x})$, the target location with maximum likelihood or local maximum likelihood can be found using standard search algorithms.

We have repeatedly observed that the incorporation of the observation of sensor i with larger entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ yields on average larger reduction in the uncertainty of the posterior target location distribution $p(\mathbf{x}|z_i)$. Therefore, given a prior target location distribution and the location and the sensing uncertainty model of a set of candidate sensors for selection, the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ can sort candidate sensors into nearly the same order as mutual information $I(\mathbf{x}; z_i)$ does. Specifically, the sensor with the maximal entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ also has the maximum or nearly the maximal mutual information $I(\mathbf{x}; z_i)$. Hence we propose to use the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$as an alternative to mutual information $I(\mathbf{x}; z_i)$ for selecting the most informative sensor. The entropy-based heuristic is to compute $H_i^{\text{v}} - H_i^{\text{s}}$ for every candidate sensor $i \in S$ and then to select sensor $\hat{i}$ such that

$$\hat{i} = \arg\max_{i \in S}(H_i^{\text{v}} - H_i^{\text{s}}) \ . \tag{12}$$

In Sect. 3, the validity of the heuristic is evaluated using simulations and the complexity of the heuristic is analyzed for two-dimensional localization. The entropy-based sensor selection heuristic works nearly as well as the mutual-information-based approaches. In addition, the heuristic is computationally much simpler than mutual information.

2.3 Relation of Entropy Difference and Mutual Information

A brief analysis of the relation between entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and mutual information $I(\mathbf{x}; z_i)$ helps to reveal fundamental properties of our sensor selection heuristic. Mutual information $I(\mathbf{x}; z_i)$ has another expression, namely, $H(z_i) - H(z_i|\mathbf{x})$. The entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ is closely related to $H(z_i) - H(z_i|\mathbf{x})$.

$H(z_i)$ is the entropy of the predicted sensor observation distribution $p(z_i)$,

$$H(z_i) = -\int p(z_i) \log p(z_i) dz_i \ . \tag{13}$$

The predicted sensor observation distribution $p(z_i)$ becomes the sensor's view distribution $p(z_i^{\text{v}})$ when the sensing model $p(z_i|\mathbf{x})$ is deterministic without uncertainty. The uncertainty in the sensing model $p(z_i|\mathbf{x})$ makes $H(z_i)$ larger than the sensor's view entropy H_i^{v} defined in (8). H_i^{v} closely approximates $H(z_i)$ when the entropy of the sensing model $p(z_i|\mathbf{x})$ is small relative to H_i^{v}.

$H(z_i|\mathbf{x})$ is actually the expected entropy of the sensing model $p(\mathbf{x})$ averaged for all possible target locations,

$$\begin{aligned} H(z_i|\mathbf{x}) &= -\int p(\mathbf{x}, z_i) \log p(z_i|\mathbf{x}) d\mathbf{x} dz_i \\ &= \int p(\mathbf{x}) \{-\int p(z_i|\mathbf{x}) \log p(z_i|\mathbf{x}) dz_i\} d\mathbf{x} \ . \end{aligned} \tag{14}$$

When $p(\mathbf{x})$ is a single-modal distribution, H_i^{s} is defined in (10), which is the entropy of the sensing model for the most likely target location estimate $\hat{\mathbf{x}}$. When $p(\mathbf{x})$ is a multi-modal distribution, H_i^{s} is defined in (11), which is the average entropy of the sensing model for all target locations with local maximal likelihood. When the entropy of the sensing model, $-\int p(z_i|\mathbf{x}) \log p(z_i|\mathbf{x}) dz_i$, changes gradually with $\mathbf{x}$, H_i^{s} can reasonably approximate $H(z_i|\mathbf{x})$.

The entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ reasonably approximates the mutual information $H(z_i) - H(z_i|\mathbf{x})$ when H_i^{s} is small relative to H_i^{v} and the entropy of the sensing model changes gradually with $\mathbf{x}$. However, selection of the most informative sensor does not require an exact evaluation of sensor information utility. Instead, an order of sensors in terms of information utility is needed. $H_i^{\text{v}} - H_i^{\text{s}}$ could sort sensors into approximately the same order as mutual information does. Therefore, a sensor with the maximal entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ also has the maximal or nearly the maximal mutual information. The correlation between the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and mutual information $I(\mathbf{x}; z_i)$ is analyzed using simulations in Sect. 3. Section 4 discusses the discrepancy between the heuristic and the mutual information based approaches.

3. HEURISTIC EVALUATION

This Sect. presents the evaluation of the entropy-based sensor selection heuristic using simulations. The computational complexity of the heuristic is also analyzed. The Gaussian noise model has been widely assumed for sensor observations in many localization and tracking algorithms, e.g. the Kalman filter [9]. Successes of these algorithms indicate that the Gaussian sensing model is a reasonable first-order-approximation of the reality. As a starting point, we assume Gaussian sensing models in the evaluative simulations for simplicity. The simple Gaussian sensing models assumed here are not accurate especially when sensors are very close to the target. To avoid the problem of over-simplified sensing models in the simulations, we only analyze sensors with some middle distance range to the target. The heuristic will be evaluated further under more realistic sensing models in the future. Four scenarios of sensor selection for localization have been studied. Three of them involve DOA sensors, range sensors, or time-difference-of-arrival (TDOA) sensors respectively. One of them involves all of the above sensors mixed together. In every sensor selection scenario, both the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and mutual information $I(\mathbf{x}; z_i)$ are evaluated and compared for all candidate sensors. In all sensor selection scenarios, the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ can sort all candidate sensors into nearly the same order as mutual information $I(\mathbf{x}; z_i)$ does. Therefore, the sensor with the maximal entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ selected by the heuristic always has the maximum or nearly the maximal mutual information $I(\mathbf{x}; z_i)$. The larger the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and mutual information $I(\mathbf{x}; z_i)$ are, the more consistent their sensor selection decisions are.

3.1 Selection of DOA Sensors

Consider now entropy-based sensor selection when all candidate sensors are DOA sensors, as depicted in Fig. 3. The prior probability distribution $p(\mathbf{x})$ of the target location $\mathbf{x}$ is non-zero in a limited area. We assume the unbiased Gaussian sensing models for DOA sensors in some middle distance range to the target. Specifically, given a target location such that $10 \leq \| \mathbf{x} - \mathbf{x}_i \| \leq 600$, the probability distribution of DOA observation z_i is assumed to be

$$p(z_i|\mathbf{x}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(z_i - z_i^{\text{v}})^2/(2\sigma^2)} \ , \tag{15}$$

where $z_i^{\text{v}} = f(\mathbf{x}, \mathbf{x}_i)$ is the direction from sensor i to the target location $\mathbf{x}$. For many DOA estimation algorithms like the approximate maximum likelihood (AML) algorithm [4], DOA estimation usually becomes much more uncertain when the candidate sensor is either very near or very far from the target. In this scenario, we exclude sensors that are either outside the study area or within a distance of 10 to the area of non-zero $p(\mathbf{x})$.

The entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and mutual information $I(\mathbf{x}; z_i)$ of DOA sensors are evaluated and compared in five cases. In each case, Gaussian sensing models of the same standard deviation σ are assumed for all 100 candidate sensors. However, the standard deviation σ varies with the case. As shown in fig. 4, mutual information $I(\mathbf{x}; z_i)$ vs the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ is plotted for all candidate sensors in all cases. Mutual information $I(\mathbf{x}; z_i)$ increases nearly monotonically with the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$. The larger the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and mutual information $I(\mathbf{x}; z_i)$ are, the more correlated they are. Therefore,

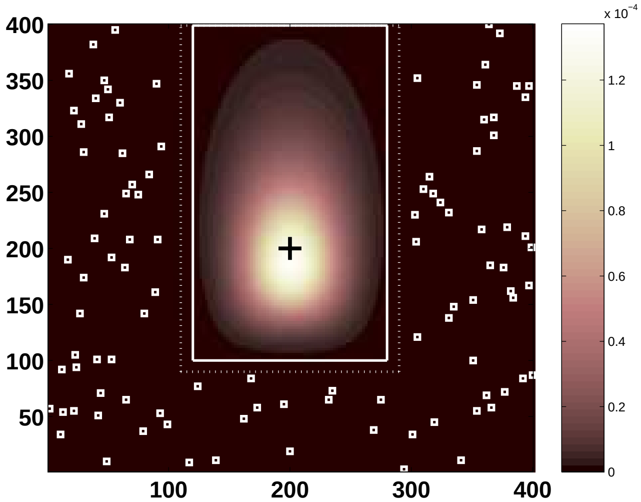

Figure 3: Scenario of sensor selection for localization using DOA sensors exclusively. The image depicts the prior probability distribution $p(\mathbf{x})$ of the target location $\mathbf{x}$. $p(\mathbf{x})$ is zero outside the solid rectangle. The actual target location is (200, 200), denoted by marker $+$. The squares denote candidate DOA sensors for selection. 100 DOA sensors are uniformly randomly placed outside the dotted rectangle. The gap between the solid rectangle and the dotted rectangle is 10.

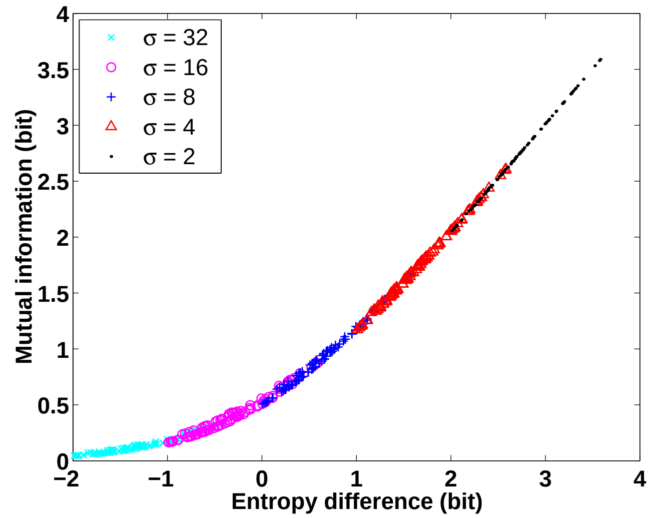

Figure 4: Mutual information $I(\mathbf{x}; z_i)$ vs entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ of DOA sensors. Each symbol denotes $(H_i^{\mathrm{v}} - H_i^{\mathrm{s}}, I(\mathbf{x}; z_i))$ pair evaluated for one candidate sensor. The prior target location distribution and the candidate sensor placements are shown in Fig. 3. Five cases with different standard deviation σ of Gaussian sensing models are studied. In each case, all candidate sensors are assumed to have the same σ value.

the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ sorts DOA sensors in nearly the same order as mutual information $I(\mathbf{x}; z_i)$ does, especially when the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ is large. The candidate DOA sensor selected by the proposed heuristic has the maximal entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$, and also has the maximal mutual information $I(\mathbf{x}; z_i)$.

3.2 Selection of Range Sensors and TDOA Sensors

This Subsect. evaluates the entropy-based sensor selection heuristic for range sensors and TDOA sensors respectively.

Fig. 5 shows the sensor selection scenario in which all candidate sensors can only measure the range to the target. The prior probability distribution $p(\mathbf{x})$ of the target location $\mathbf{x}$ is non-zero in a limited area. We assume the unbiased Gaussian sensing models $p(z_i|\mathbf{x})$ for range sensors used in [13]. When the actual range is small relative to the standard deviation σ of the Gaussian sensing model, $p(z_i|\mathbf{x})$ is significantly greater than zero even for negative values of range observation z_i. Because a range of negative value has no physical meaning, the above Gaussian sensing model is not valid for short ranges. To avoid the above difficulty of the Gaussian sensing model, we only consider candidate sensors in some middle distance range to the target. Specifically, in this range sensor selection scenario, we exclude sensors that are either outside the study area or within a distance of 32 to the area of non-zero $p(\mathbf{x})$.

Fig. 6 shows the sensor selection scenario in which only TDOA sensors are used. The prior probability distribution $p(\mathbf{x})$ of the target location $\mathbf{x}$ is non-zero in a limited area. As in [15], the signal arrival time difference observed by every TDOA sensor is relative to a common reference sensor. We also assume the unbiased Gaussian sensing models $p(z_i|\mathbf{x})$ for TDOA sensors. In order to be comparable with scenarios of DOA sensors and range sensors, we only consider TDOA sensors in middle range distance to the target. Specifically, we exclude TDOA sensors that are either outside the study area or within a distance of 10 to the area of non-zero $p(\mathbf{x})$.

Following the same approach to the heuristic evaluation for DOA sensors, the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ and mutual information $I(\mathbf{x}; z_i)$ of every candidate sensor are evaluated and compared for range sensor selection scenario in Fig. 5 and for TDOA sensor selection scenario in Fig. 6 respectively. Mutual information $I(\mathbf{x}; z_i)$ vs the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ is plotted in Fig. 7 for all range sensors and in Fig. 8 for all TDOA sensors. In both scenarios, mutual information $I(\mathbf{x}; z_i)$ increases nearly monotonically with the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$. The larger the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ and mutual information $I(\mathbf{x}; z_i)$ are, the more correlated they are. Using the proposed heuristic, both the selected range sensor and the selected TDOA sensor have the maximal entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$, and also have nearly the maximal mutual information $I(\mathbf{x}; z_i)$.

3.3 Selection of Mixed Sensors

In order to evaluate the entropy-based sensor selection heuristic across different sensing modalities, this Subsect. is devoted to the sensor selection scenario in which candidate sensors are a mixture of DOA sensors, range sensors and TDOA sensors.

Fig. 9 shows the sensor selection scenario for mixed candidate sensors. Each candidate sensor is randomly assigned one of three sensing modalities, namely, DOA, range, and TDOA. Gaussian sensing models are assumed for all candidate sensors with middle range distance to the target. Each

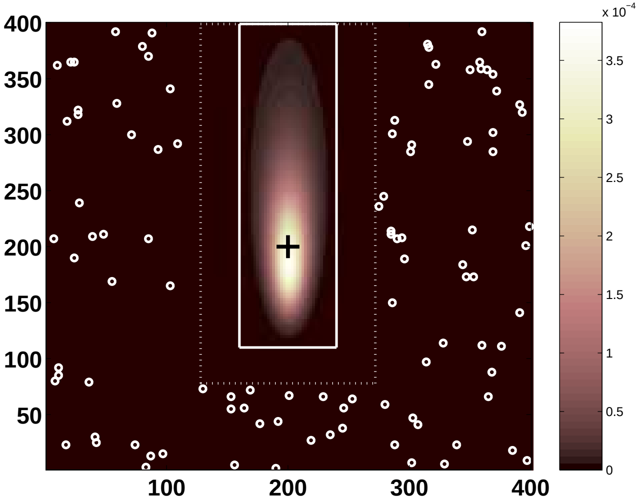

Figure 5: Scenario of sensor selection for localization using range sensors. The image depicts the prior probability distribution $p(\mathbf{x})$ of the target location $\mathbf{x}$. $p(\mathbf{x})$ is zero outside the solid rectangle. The actual target location is (200, 200), denoted by marker $+$. The circles denote candidate range sensors for selection. 100 range sensors are uniformly randomly placed outside the dotted rectangle. The gap between the solid rectangle and the dotted rectangle is 32.

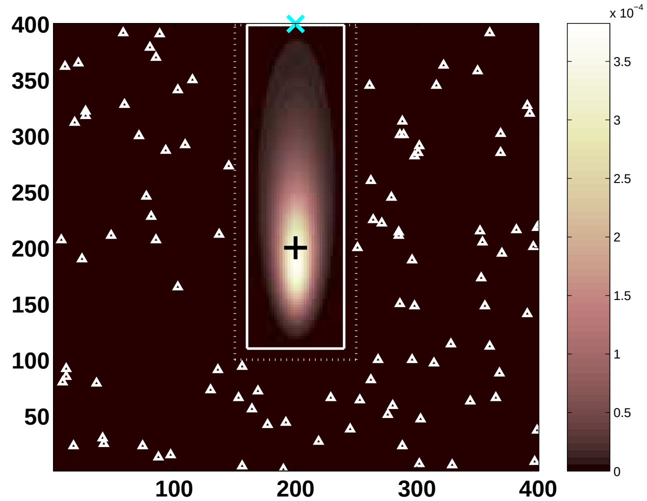

Figure 6: Scenario of sensor selection for localization using TDOA sensors. The image depicts the prior probability distribution $p(\mathbf{x})$ of the target location $\mathbf{x}$. $p(\mathbf{x})$ is zero outside the solid rectangle. The actual target location is (200, 200), denoted by marker $+$. The triangles denote candidate TDOA sensors for selection. Every TDOA observation is relative to a common reference sensor denoted by marker $\times$. 100 TDOA sensors are uniformly randomly placed outside the dotted rectangle. The gap between the solid rectangle and the dotted rectangle is 10.

candidate sensor is also randomly assigned one of five values of the standard deviation σ of the sensing model, namely, 2, 4, 8, 16, and 32. 100 candidate sensors are uniformly randomly placed in the vicinity of the prior target location estimation. In order to avoid the difficulties of Gaussian sensing models for DOA sensors and range sensors close to the target, we exclude sensors either outside the study area or within a distance of 32 to the non-zero area of the prior target location distribution $p(\mathbf{x})$.

The entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ and mutual information $I(\mathbf{x}; z_i)$ of every candidate sensor are evaluated and plotted in Fig. 10. The correlation between $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ and $I(\mathbf{x}; z_i)$ of mixed sensors is very similar to the correlation between $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ and $I(\mathbf{x}; z_i)$ of sensors with single modality. Across various sensing modalities, mutual information $I(\mathbf{x}; z_i)$ increases nearly monotonically with the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$. Therefore, across various sensing modalities, the candidate sensor with the maximal entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$, selected by the proposed heuristic, has the maximal mutual information $I(\mathbf{x}; z_i)$.

3.4 Computational Complexity

Computational complexity analysis is an important part of the evaluation of the heuristic. We will analyze the complexity of the heuristic and compare it to the complexity of the mutual-information-based approaches.

For two-dimensional localization, the target location $\mathbf{x}$ is two-dimensional. The sensor's view z_i^{v} of the target location $\mathbf{x}$ is one-dimensional. The sensor observation z_i is one-dimensional. We assume that all random variables are gridded for numerical computation. Specifically, the area with non-trivial $p(\mathbf{x})$ is gridded into $n \times n$. The interval with non-trivial $p(z_i)$ or $p(z_i^{\mathrm{v}})$ is also gridded into n. We assume there are K candidate sensors for selection. K is usually a small number.

The proposed heuristic evaluates the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ of all sensors and then selects the one with the maximal $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$. As shown in (7), $p(z_i^{\mathrm{v}})$ can be computed from $p(\mathbf{x})$ with cost $O(n^2)$. As shown in (8), H_i^{v} can be computed from $p(z_i^{\mathrm{v}})$ with cost $O(n)$. As shown in (10) and (11), H_i^{s} can be computed from $p(z_i|\mathbf{x})$ with cost $O(n)$. The cost to compute $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ for one candidate sensor is $O(n^2)$. Therefore, the total cost for the heuristic to select one out of K candidate sensors is $O(n^2)$.

The mutual-information-based approaches evaluate the mutual information $I(\mathbf{x}; z_i)$ of all sensors and then select the one with the maximal $I(\mathbf{x}; z_i)$. As shown in (3), $I(\mathbf{x}; z_i)$ can be directly computed from $p(\mathbf{x})$ and $p(z_i|\mathbf{x})$ with cost of $O(n^3)$. Therefore, the total cost to select one out of K candidate sensors is $O(n^3)$. As we mentioned early in Subsect. 2.1, the computational cost of mutual information $I(\mathbf{x}; z_i)$ could be reduced in some special scenarios. In general, however, the heuristic is computationally much simpler than the mutual-information-based approaches.

4. DISCREPANCY BETWEEN HEURISTIC AND MUTUAL INFORMATION

As shown in Sect. 3, when the mutual information $I(\mathbf{x}; z_i)$ is close to 0 bit, the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ might not sort candidate sensors into exactly the same order as the mutual information does. Such discrepancy is caused by the dispersion of the correlation between the entropy difference

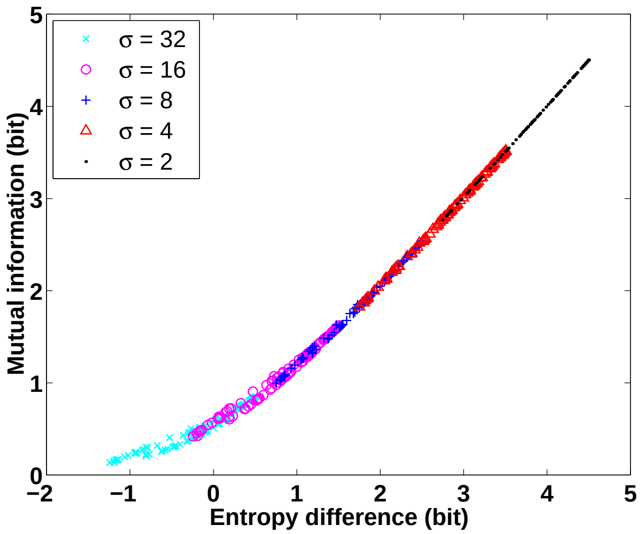

Figure 7: Mutual information $I(\mathbf{x}; z_i)$ vs entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ of range senors. Each symbol denotes $(H_i^{\text{v}} - H_i^{\text{s}}, I(\mathbf{x}; z_i))$ pair evaluated for one candidate sensor. The prior target location distribution and the candidate sensor placements are shown in Fig. 5. Five cases with different standard deviation σ of Gaussian sensing models are studied. In each case, all candidate sensors are assumed to have the same σ value.

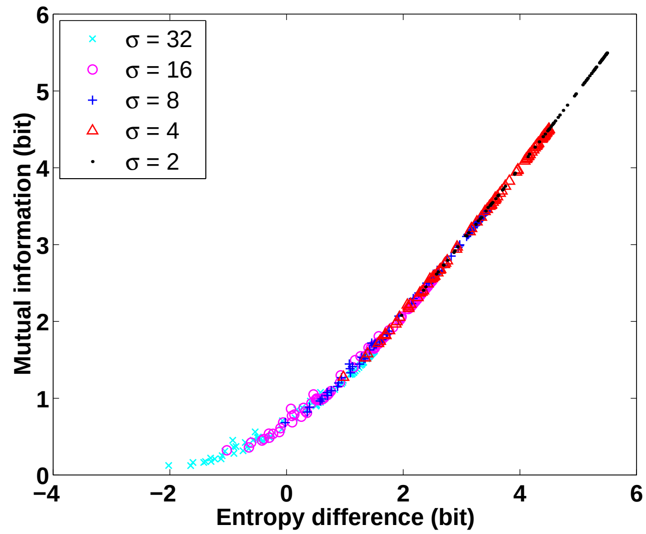

Figure 8: Mutual information $I(\mathbf{x}; z_i)$ vs entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ of TDOA senors. Each symbol denotes $(H_i^{\text{v}} - H_i^{\text{s}}, I(\mathbf{x}; z_i))$ pair evaluated for one candidate sensor. The prior target location distribution and the candidate sensor placements are shown in Fig. 6. Five cases with different standard deviation σ of Gaussian sensing models are studied. In each case, all candidate sensors are assumed to have the same σ value.

$H_i^{\text{v}} - H_i^{\text{s}}$ and the mutual information $I(\mathbf{x}; z_i)$ when the mutual information is small. In this Sect., we examine such correlation dispersion and evaluate its impact on the discrepancy of sensor selection decisions of the entropy-based heuristic and the mutual information based approaches.

4.1 Dispersion

In this Subsect., we describe the dispersion of the correlation between the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and the mutual information $I(\mathbf{x}; z_i)$ when the mutual information is small. We also examine possible sources for such correlation dispersion.

Close examination on the convex part of the mutual information vs. entropy difference curve in Fig. 7 and Fig. 8 reveals that the correlation between the mutual information $I(\mathbf{x}; z_i)$ and the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ is not strictly monotonic. Instead, there is obvious dispersion of the correlation. The convex part corresponds to the situation in which candidate sensors are not very informative because the mutual information between the target location distribution and the sensor observation is close to 0 bit. In another words, when candidate sensors are not very informative, the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ might not sort candidate sensors into the same order as the mutual information $I(\mathbf{x}; z_i)$ does. Given a set of candidate sensors whose observation could only reduce a little amount of uncertainty of the target location distribution, the sensor selected on the basis of the maximum entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ might not have the maximum mutual information $I(\mathbf{x}; z_i)$. Thus, there might be discrepancy between the sensor selection decision of the entropy-based heuristic and that of the mutual information based approaches if no candidate sensor is very informative.

There might be multiple causes of such correlation dispersion between the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and the mutual information $I(\mathbf{x}; z_i)$. As pointed out in Subsect. 2.3, the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ can be viewed as an approximation of the mutual information $I(\mathbf{x}; z_i)$. Thus, the order of sensors sorted by the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ is intrinsically an approximation of that by the mutual information $I(\mathbf{x}; z_i)$. In practice, the discretization of the state space of the target location random variable and the sensor view random variable might also introduce inaccuracy into the evaluation of H_i^{v}. Besides, as shown in (10) and (11), the maximum likelihood estimate of the target location is used to approximate the actual target location when evaluating the entropy of the sensing model for the actual target location.

4.2 Impact

In this Subsect., we examine the impact of the dispersion of the correlation between the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and the mutual information $I(\mathbf{x}; z_i)$ when the mutual information is small. The analysis shows that such correlation dispersion causes very little degradation to the quality of sensor selection decision of the entropy-based heuristic.

As shown by the convex part of the mutual information vs. entropy difference curve in Fig. 7 and Fig. 8, there is dispersion of the correlation between the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and the mutual information $I(\mathbf{x}; z_i)$ when candidate sensors are not very informative. We model such dispersion using a uniform distribution bounded by a parallelogram illustrated in Fig. 11. A candidate sensor could assume any position $(H_i^{\text{v}} - H_i^{\text{s}},\ I(\mathbf{x}; z_i))$ within the parallelogram with uniform probability. As shown in Fig. 11, the geometry of the parallelogram is defined by parameters a, b and c. a

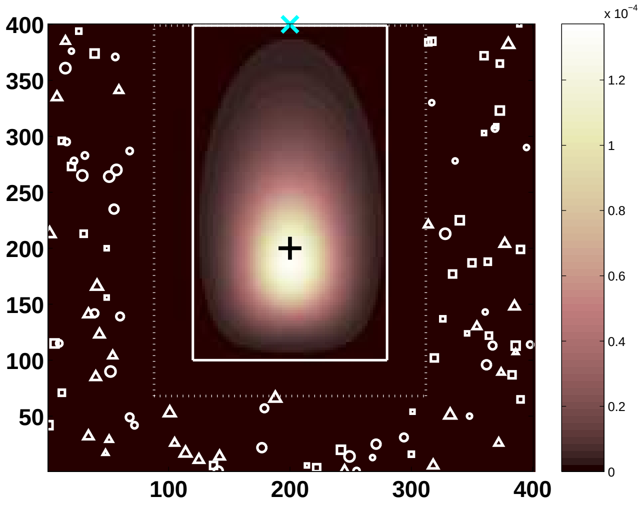

Figure 9: Scenario of sensor selection for localization using sensors with various modalities. The image depicts the prior probability distribution $p(\mathbf{x})$ of the target location x. $p(\mathbf{x})$ is zero outside the solid rectangle. The actual target location is (200, 200), denoted by marker +. The squares, the circles, and the triangles denote DOA sensors, range sensors and TDOA sensors respectively. Every TDOA observation is relative to a common reference sensor denoted by marker $\times$. Each sensor is randomly chosen to be a DOA sensor, a range sensor, or a TDOA sensor. Each sensor is also randomly assigned one of five values of the standard deviation σ of Gaussian sensing models, namely, 2, 4, 8, 16, and 32. The size of a symbol indicates the magnitude of σ. 100 sensors of various sensing modalities and σ values are uniformly randomly placed outside the dotted rectangle. The gap between the solid rectangle and the dotted rectangle is 32.

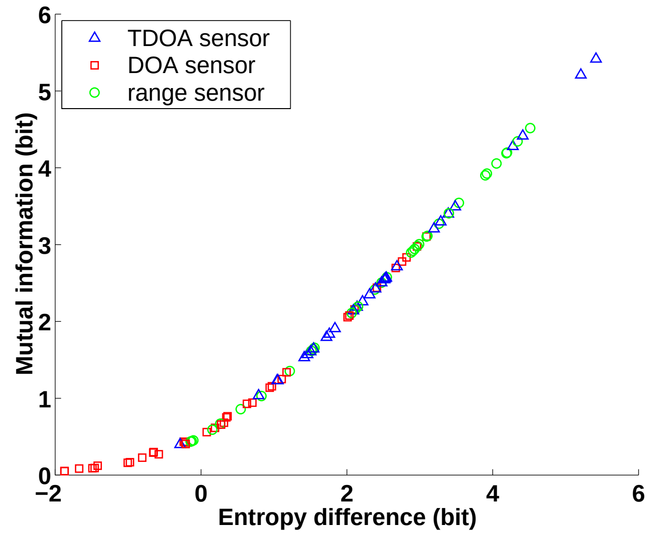

Figure 10: Mutual information $I(\mathbf{x}; z_i)$ vs entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ of mixed senors. Each symbol denotes $(H_i^{\mathrm{v}} - H_i^{\mathrm{s}}, I(\mathbf{x}; z_i))$ pair evaluated for one candidate sensor. The prior target location distribution and the candidate sensor placements are shown in Fig. 9.

is the variation scope of entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ among the set of candidate sensors. c indicates the variation scope of the mutual information $I(\mathbf{x}; z_i)$ among the set of candidate sensors. b describes the magnitude of dispersion of the correlation between the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ and the mutual information $I(\mathbf{x}; z_i)$. Although the bounded uniform distribution is not accurate, it captures the major features of the correlation dispersion revealed by simulations in Sect. 3. We choose this dispersion model for simplicity. As the first order approximation, the simple dispersion model does help to reveal some major characteristics of the impact of the correlation dispersion on the heuristics-based sensor selection.

A typical dispersion scenario is illustrated in Fig. 11. The mutual information $I(\mathbf{x}; z_i)$ of candidate sensors varies from 0 bit to 1 bit. Correspondingly, the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ of candidate sensors changes from -2 bit to 0 bit. For any value of the entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$, the disperse of the mutual information $I(\mathbf{x}; z_i)$ is 0.1 bit. Given the above scenario, we run $10,000$ simulations. In each simulation, 8 candidate sensors randomly assume their $(H_i^{\mathrm{v}} - H_i^{\mathrm{s}}, I(\mathbf{x}; z_i))$ pairs within the specified dispersion range. In each simulation, we identify both the sensor with the maximum entropy difference $H_i^{\mathrm{v}} - H_i^{\mathrm{s}}$ and the sensor with the maximum mutual information $I(\mathbf{x}; z_i)$. With 87.8% chance, the sensor selected by the entropy-based heuristic also has the maximum mutual information. Even when the heuristic fails to select the sensor of the maximum mutual information, the mutual information of the selected sensor is on average only about 0.026 bit less than the maximum mutual information. Overall, the mutual information of the sensor selected by the entropy-based heuristic is about $0.026 \times (1 - 87.8\%) = 0.0032$ bit less than the maximum mutual information. Therefore, most of the time, the correlation dispersion does not cause discrepancy of the sensor selection decisions between the entropy-based heuristic and the mutual information based approaches. Over all, the entropy-based heuristic introduces very little degradation to the quality of the sensor select decision even when candidate sensors are not very informative.

We have analyzed the impact of the correlation dispersion for different configurations of a, b, c, and the number of candidate sensors. In table 1 , $a = 2$ bit, $b = 0.1$ bit and $c = 1$ bit are fixed. We only change the number of candidate sensors. The chance for the heuristic to successfully select the sensor with the maximum mutual information decreases as the number of candidate sensors increases. When the heuristic fails to select the sensor with the maximum mutual information, the degradation of sensor selection decision based on the heuristic compared to that based on the mutual information does not change with the number of candidate sensors. Thus, the overall degradation of sensor selection decision based on the heuristic compared to that based on mutual information also increases as the number of candidate sensors increases.

In table 2 , $a = 2$ bit and $c = 1$ bit are fixed and the number of candidate sensors are fixed to be 8. We only change the dispersion width b. The chance for the heuristic to successfully select the sensor with the maximum mutual

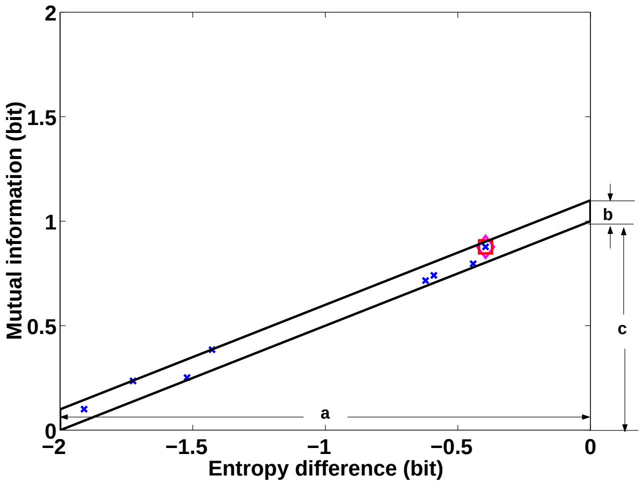

Figure 11: Discrepancy between the entropy-based sensor selection heuristic and the mutual information based approaches when candidate sensors are not very informative. The dispersion of the correlation between the entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and the mutual information $I(\mathbf{x}; z_i)$ is modeled by a uniform distribution bounded by a parallelogram. The geometry of the parallelogram is defined by parameters a, b and c. Candidate sensors are denoted by marker $\times$ whose coordinates are $(H_i^{\text{v}} - H_i^{\text{s}},\ I(\mathbf{x}; z_i))$. The entropy-based heuristic selects the rightmost sensor, which has the maximum entropy difference $H_i^{\text{v}} - H_i^{\text{s}}$ and is enclosed by a square marker. The mutual information based approaches selects the top sensor, which has the maximum mutual information $I(\mathbf{x}; z_i)$ and is enclosed by a diamond-shaped marker. The above two selected sensors might not be the same. In the scenario of this figure, $a = 2$ bits, $b = 0.1$ bit, $c = 1$ bit, and 8 candidate sensors are available for selection.

information decreases as the dispersion width b increases. When the heuristic fails to select the sensor with the maximum mutual information, the degradation of sensor selection decision based on the heuristic compared to that based on the mutual information increases as the dispersion width b increases. Thus, the overall degradation of sensor selection decision based on the heuristic compared to that based on mutual information also increases as the dispersion width b increases.

In table 3 , $a = 2$ bit and $b = 0.1$ bit are fixed and the number of candidate sensors are fixed to be 8. We only change the mutual information variation scope c. The chance for the heuristic to successfully select the sensor with the maximum mutual information increases as the mutual information variation scope c increases. When the heuristic

Table 1: Impact Change with Number of Sensors

Number of Candidate Sensors	4	8	16
Chance of Success (%)	93.6	87.8	78.2
Degradation per Failure (bit)	0.026	0.026	0.026
Overall Degradation (bit)	0.0016	0.0032	0.0058

Table 2: Impact Change with Dispersion Width

Dispersion Width b (bit)	0.05	0.1	0.2
Chance of Success (%)	93.6	87.8	78.1
Degradation per Failure (bit)	0.013	0.026	0.054
Overall Degradation (bit)	0.0008	0.0032	0.012

Table 3: Impact Change with Mutual Info. Scope

Mutual Info. Scope c (bit)	0.5	1	2
Chance of Success (%)	78.2	87.8	93.6
Degradation per Failure (bit)	0.027	0.026	0.025
Overall Degradation (bit)	0.0058	0.0032	0.0016

fails to select the sensor with the maximum mutual information, the degradation of sensor selection decision based on the heuristic compared to that based on the mutual information does not change much with the mutual information variation scope c. Thus, the overall degradation of sensor selection decision based on the heuristic compared to that based on mutual information decreases as the mutual information variation scope c increases.

In table 4 , $b = 0.1$ bit is fixed and the number of candidate sensors are fixed to be 8. We proportionally change the entropy difference variation scope a and the mutual information variation scope c so that $c/a = 1/2$ is fixed. The chance for the heuristic to successfully select the sensor with the maximum mutual information increases as the entropy difference variation scope a and the mutual information variation scope c proportionally increase. When the heuristic fails to select the sensor with the maximum mutual information, the degradation of sensor selection decision based on the heuristic compared to that based on the mutual information does not change. Thus, the overall degradation of sensor selection decision based on the heuristic compared to that based on mutual information decreases as the entropy difference variation scope a and the mutual information variation scope c proportionally increase.

5. FUTURE WORK

5.1 Prior Target Location Distribution

When the sensors is selected for tracking a temporally continuous source, the prior target location distribution at time $t+1$ can be obtained from the posterior target location distribution at time t by using the target dynamic model as described in [11]. However, when the sensor selection heuristic is applied to locate a temporally discontinuous source such as a bird call, it is not straightforward to obtain the prior target location distribution used in the sequential Bayesian fusion. One possible solution to the above problem could be as follows. First, all sensors buffer the signal once an event

Table 4: Impact Change with Entropy Diff. Scope c and Mutual Info. Scope a in Proportion

Entropy Diff. Scope a (bit)	1	2	4
Mutual Info. Scope c (bit)	0.5	1	2
Chance of Success (%)	78.2	87.8	93.6
Degradation per Failure (bit)	0.026	0.026	0.026
Overall Degradation (bit)	0.0058	0.0032	0.0016

such as a bird call is detected. Then, all triggered sensors elect a leader that received the strongest signal intensity using a protocol similar to that described in [10]. Finally, the leader can pick a few sensors to generate an initial prior target location distribution assuming a certain sensing model. With the initial prior target location distribution, we can apply the sensor selection heuristic to incrementally reduce the uncertainty of the target location distribution. We plan to implement and test the above mechanism in the future.

5.2 Discretization of State Space

There is a trade off of computational efficiency and numerical accuracy in the discretization of the state space of random variables such as the target location and the sensor view. The bigger the grid size is, the fewer grids are involved in the computation. However, a bigger grid size also introduces more inaccuracy into the evaluation of the entropy difference heuristic. In the future, we must study more details about the trade off in order to choose a proper grid size.

5.3 Sensing Uncertainty Model

We have assumed Gaussian sensing models in the simulations as the first step to evaluate the heuristic. Inaccuracy of sensing models diminishes the effectiveness of any sensor selection criterion. We plan to construct a more realistic sensing model for the AML-based DOA estimation. We have implemented AML algorithm for real-time DOA estimation on a wireless sensor network testbed [5]. We will first analyze the sensing uncertainty characteristic of the AML algorithm, and then experimentally validate and refine it using the testbed. We will also evaluate the effectiveness of the entropy-based sensor selection heuristic using realistic sensing models and implement the heuristic on the real-time wireless sensor network testbed for localization.

6. CONCLUSION

We have proposed an entropy-based sensor selection heuristic for localization. The effectiveness of the heuristic has been evaluated using simulations in which Gaussian sensing models are assumed for simplicity. Simulations have shown that the heuristic selects the sensor with nearly the maximal mutual information between the target location and the sensor observation. Given the prior target location distribution, the sensor locations, and the sensing models, on average, the sensor selected by the heuristic would yield nearly the greatest reduction in the entropy of the posterior target location distribution. The heuristic is more effective when the optimal candidate sensor is more informative. As mutual-information-based sensor selection approaches [11, 6] do, the heuristic greedily selects one sensor in each step without retrieving any actual sensor observations. In addition, in general, our heuristic is computationally much simpler than the mutual-information-based approaches.

7. ACKNOWLEDGMENTS

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