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**Backward-Looking Contracts, Credibility and
Inflation Convergence**

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## Abstract

In this paper, we build a model that incorporates a backward-looking component to the exclusively forward-looking staggered prices model of Calvo (1983). The objective of this formulation is to include the effect of the history of high inflation on the price formation, reflected in the existence of widespread backward-looking indexation (-*de facto or de jure*-). Thus, the model is able to isolate the effects of history in price setting from the genuinely forward-looking lack of credibility arising from the intertemporal inconsistency between fiscal and monetary policy. One remarkable feature of the model is that it remains analytically tractable despite its enhanced dynamics. When used to simulate, the model replicates inflation persistence and real appreciation. Immediate inflation convergence is not achievable even if the economy's fundamentals would say so.
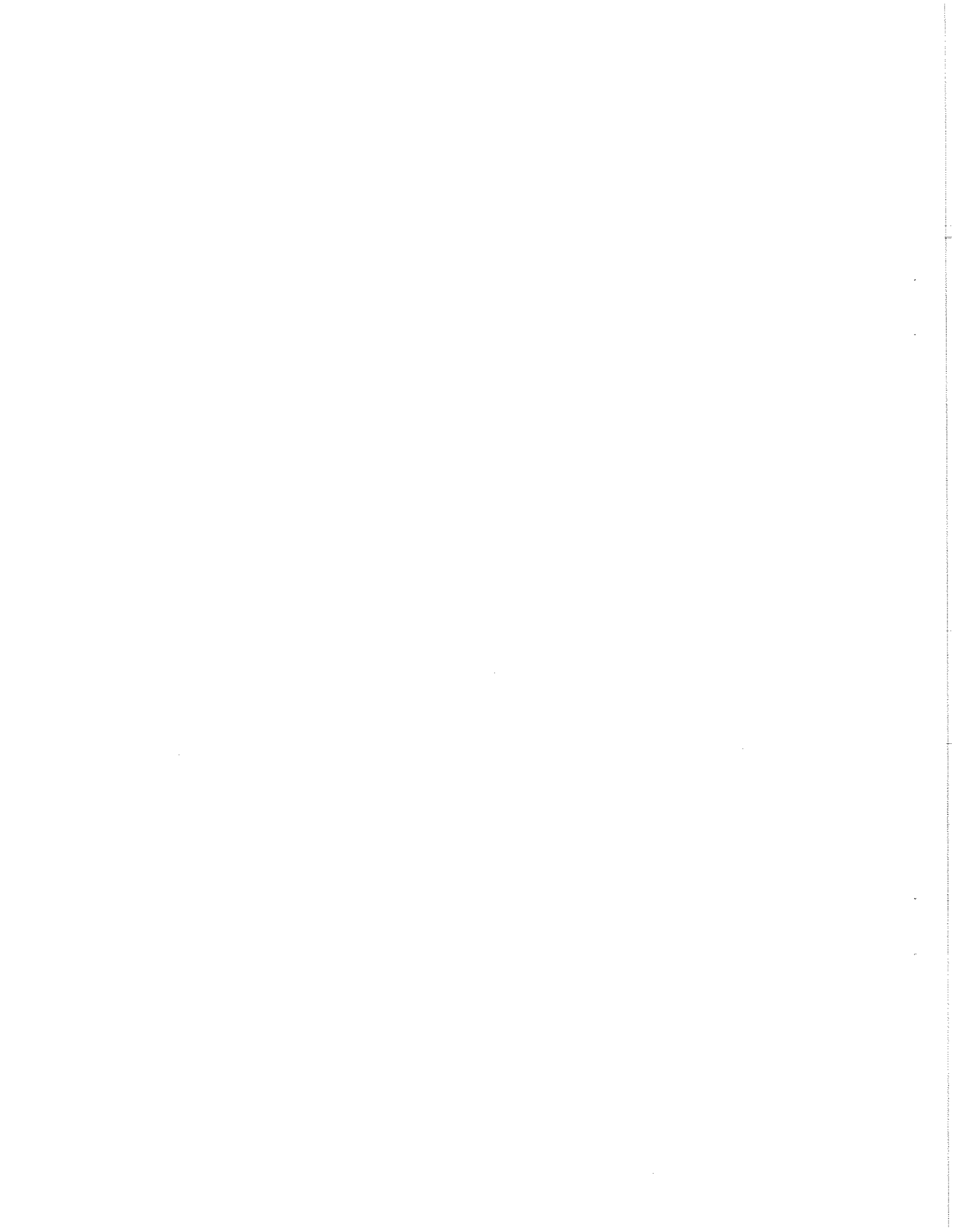
## 1. Introduction

Most of the efforts to disinflate in the last 20 years have found that the inflation rate has converged only slowly to the growth rate of the nominal anchor. One explanation for the lack of inflation convergence has been the role played by staggered price settings. However, the most popular overlapping contracts (Phelps (1978), Taylor (1980), Calvo (1983)) share a remarkable feature: while price levels are sticky, the inflation rate is relatively flexible and rapid disinflation is achieved without major output cost. One can show that inflation persistence is found in discrete time Taylor contracts, but only for one period, while no inflation persistence is found in the continuous time Calvo model.[1]

Miller and Sutherland (1993), trying to explain the slow inflation adjustment in the European Monetary System, stress that there are two assumptions consistent with low or no inflation persistence in staggered contracts. First, anti-inflationary monetary or exchange rate policy are fully credible. Second, rational expectations hold: the inflation process is assumed to be common knowledge. They argue that both assumptions might not be appropriate after a major change of regime.

Calvo and Vegh (1993) set up a model which shows that the lack of inflation convergence to the growth rate of the nominal anchor can be attributed to price setters' lack of belief in the permanent duration of the new policy. As they put it, "…[I]n any well-structured model of public belief's, history should weigh heavily during the first stages of the stabilization program. This of course means that there will be an element of exogeneity in the public's expectations." Based on this assertion, they are able to justify the assumption of an exogenous lack of credibility which is embedded in new prices, generating a forward-looking inflation stickiness.

Nonetheless, credibility is a very vague concept for which we have a relatively poor understanding. One could think of an exclusively forward-looking 'lack of credibility", in the spirit of the unpleasant monetarist arithmetic of Sargent and Wallace (1981), which arises from a fundamental inconsistency between the growth rate of the

nominal anchor and the sustainable inflation rate resulting from the authorities' preferences. But, one could also think that the history of high inflation is reflected in the widespread use of backward-looking indexation mechanisms. Indeed, one should agree that together with the forward-looking inflation stickiness, it is undeniable that backward-looking stickiness (as a result of indexation schemes *-de facto or de jure-*) has played a major role in inflation persistence.

The purpose of this paper is to incorporate explicitly the effect of history in price formation trying to isolate it from the lack of credibility attributed to inconsistent monetary and fiscal policies. With that objective, we build a staggered-prices model in the spirit of Calvo (1983). In the original model, home goods price setters change prices after receiving a price-change signal. New prices, set at time *t*, are exclusively forward-looking and consider expected future prices set by other firms and a forecast of excess demand. In this model, we modify the exclusively forward-looking price setting mechanism by one in which a fraction of the price setters at *t* is under backward-looking contracts and simply update prices by the inflation rate between the last time they changed prices and *t*. The other fraction is still forward-looking price setters that consider expected future prices of home and foreign goods, and expected state of the market (excess demand). One obvious result of the above formulation is that inflation shows significant persistence, even in cases in which the nominal anchor's growth rate has been permanently reduced to zero and it is fully believed by forward-looking price setters.

This is not the first attempt to introduce indexation in staggered prices. Ambler and Cardia (1992) introduce contracts that are continuously indexed by a fraction of past inflation. In their formulation, the resulting prices are adjusted for current rather than past inflation, missing a very important result of indexation: inflation inertia. Fuhrer and Moore (1995) present a model in which wage setters care about relative real wages over the time of the contract. Their model is capable of generating inflation persistence, without

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[1]The immediate inflation convergence in the Calvo model is specially striking because it is not dependent on the continuous time assumption.

imposing ex-ante real rigidities.[2] The closest formulation to ours is presented by Obstfeld (1995). In his formulation, past inflation is embodied in new prices in proportion to the degree of indexation.[3]

In section 2, we present the staggered-prices model. One of the noteworthy findings is that despite its enhanced features, it remains tractable. In section 3, we show the effect of several disinflation experiments. Not surprisingly, we obtain inflation persistence, even in the cases in which the fundamental source of inflation has disappeared and the forward-looking price setters fully believe it.

## 2. A model of Staggered Prices with Partial Backward-Looking Behavior

In this model, there is a very large number of firms which sets prices only when a price signal is received.[4] The probability (density) of receiving the signal $h$ periods from today is independent from the last signal received and it is given by the geometric distribution:

$$\delta \exp(-\delta h), \qquad \delta > 0 \qquad\qquad (1)$$

which is stochastically independent across firms. The expected length of the contract at the time the price changes is $(1/\delta)$.[5] The aggregate (log of the) domestic price level follows:

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$$p_t = \delta \int_{-\infty}^{t} x_v \exp(-\delta(t-v))dv \qquad (2)$$

where prices posted at time $v$, $x_v$, are weighted by the probability that they continue in effect at $t$, $\exp(-\delta(t-v))$. The (log of) newly posted prices are a weighted average of a backward-looking and a forward-looking component:

$$x_t = \omega \left\{ p_t + \frac{1}{\delta} \psi_t \right\} + (1-\omega) \left\{ \delta \int_{t}^{\infty} [\beta s_v + (1-\beta)p_v + B y_v] \exp(-\delta(v-t))dv \right\}$$

$$(3)$$

where $\psi_t = \delta \int_{-\infty}^{t} \pi_v \exp(-\delta(t-v))dv$, $\qquad (4)$

$\omega$ is the economy's degree of backward indexation-*de facto or de jure*-, $s$ (the spot exchange rate) is also the domestic price of foreign output[6], $p$ is the domestic GDP deflator, $\beta$ indicates the weight of domestically produced goods on the CPI, $B$ reflects responses of newly posted prices to future excess demand conditions, $y_t$, with:

$$y_t = \alpha(s_t - p_t) \qquad (5)$$

where $\alpha > 0$, which implies that the world aggregate demand for domestic output is positively related to the relative price of foreign and domestic goods.[7]

The backward-looking component of equation 3 reflects indexation mechanisms-*de facto or de jure*- inherited from a history of high inflation. It can be obtained by assuming that backward-looking price setters at $t$ update prices by the inflation rate

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[6]We have assumed that the law of one price holds and that foreign GDP deflator is normalized to zero. Therefore, the domestic price of foreign goods equals the nominal exchange rate, $s$.

[7]Without loss of generality, the equilibrium real exchange and the long run level of output are normalized to zero.

between $t$ and the previous time prices were revised, $v$. That is, an individual price setter which last revised its price at $v$ and receives a price signal at $t$ will set according to:

$$x_{t,v} = x_v + \int_v^t \pi_u du \qquad (6)$$

which weighted by the probability that a price setter at $v$ receives a new price signal at $t$, results in the backward-looking component of the right hand side of equation 3[8].
Right hand side differentiation of (4) yields:

$$\dot{\psi}_t = \delta(\pi_t - \psi_t) \qquad (7)$$

One could find an alternative interpretation for the backward-looking component of equation 3. Price setters at $t$ index prices to the current aggregate price level plus a constant forecast of future inflation $\psi$, which is a weighted average of past inflation, adjusted by the expected length of the contract $(1/\delta)$. In that case, (7) shows the learning process in which expected inflation is updated by a fraction of the difference between today's inflation and expected inflation. Both formulations, backward indexation and backward-looking expectations, are observationally equivalent.[9]

The second component of the left hand side of equation 3 reflects the usual forward-looking behavior of staggered-prices models in which expected future average price levels (the geometric mean of outstanding prices set at $v$) and excess demands, which under the perfect foresight assumption are equal to their actual values, are discounted by the probability that the contracts survive until then.

Making use of (5), (3) can be expressed as:

$$x_t = \omega\left\{p_t + \frac{1}{\delta}\psi_t\right\} + (1-\omega)\left\{\delta\int_t^\infty [p_v + \varphi e_v]\exp(-\delta(v-t))dv\right\} \qquad (8)$$

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[8]See appendix A for a derivation.
[9] Under the backward-looking expectations alternative, one does not need to force the speed of adjustment parameter of (7) to be equal to $\delta$.

where $\varphi = (\beta + B\alpha)/(1-\beta)$, and $e(=(1-\beta)(s-p)))$ is the (log of the) real exchange rate.

Also, at points in time in which continuity follows, GDP inflation is obtained by differentiating (2):

$$\pi = \dot{p} = \delta(x-p) \tag{9}$$

The change in newly posted prices is obtained by differentiating (8):

$$\dot{x} = 2\omega(\pi-\psi) + \delta(x-p) - (1-\omega)\delta\varphi e \tag{10}$$

The differentiation of the inflation rate (9) yields:

$$\dot{\pi} = \varpi_1(\pi-\psi) - \varpi_2 e \tag{11}$$

where $\varpi_1 = 2\omega\delta$ and $\varpi_2 = (1-\omega)\delta^2\varphi$. Thus, inflation reduction is coming from the usual excess demand mechanism of the Calvo staggered-prices formulation (second term of the right hand side of equation 11). The logic for this *higher order inverse Phillips Curve* is that the higher the real exchange rate at $t$, the higher excess demand at $t$, and the higher prices at $t$. However, since forward-looking price setters at $t'>t$ do not consider excess demand at $t$, the greater the excess demand at $t$, the greater inflation reduction for $t'>t$.[10] Inflation reduction is also a result of a learning process (first term of the right hand side of equation 11). The lower $\pi$ with respect to $\psi$, the greater the reduction of $\psi$ (see equation 7) and the lower the premium over the price level in the first term of the right hand side of (8). A lower level in newly posted prices $x_t$ with respect to the price level $p_t$ will be translated in lower inflation through equation 9.

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[10]This explanation is due to Calvo and Vegh (1994).

## 3. Effect of Disinflationary Policy under Alternative Exchange Rate Regimes

### 3.1 Exchange Rate-Based Disinflation

Exchange rate pegging to a low inflation currency (or to a basket of currencies) has been widely used to reduce inflation. It has been usually assumed that tradable prices will be anchored by the exchange rate and non-tradable inflation will converge to international levels.

One can use the model of the previous section to see the effects of exchange rate-based stabilization in a context in which backward-looking contracts are relevant. The system of differential equations consists of (7), (11) and the definition of the real exchange rate:[11]

$$
\begin{bmatrix} \dot{\pi} \\ \dot{\psi} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 2\omega\delta & -2\omega\delta & -(1-\omega)\delta^2\varphi \\ \delta & -\delta & 0 \\ -(1-\beta) & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi - \bar{\varepsilon} \\ \psi - \bar{\varepsilon} \\ e - \bar{e} \end{bmatrix}
\tag{12}
$$

where $\varepsilon(=\dot{s})$ is the crawling rate. Having derived the system, we need to examine the conditions for its stability. There are two predetermined variables ($\psi$ and the real exchange rate) and one jump variable (the inflation rate). The existence of a unique path which converges to the steady state will require two convergent (non-positive) roots, which will always be the case (see appendix B)[12].

Before using the model to see the effects of disinflation, it is interesting to stress some characteristics of the model. In steady state, independently of the degree of indexation, the economy is running constant inflation and devaluation rates equal to $\upsilon$. Excess aggregate demand is zero and newly posted prices are:

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[11] We are making use of the fact that $\bar{\psi} = \bar{\pi}$ and both equal the long run devaluation rate $\bar{\varepsilon}$.

[12] For most plausible parameter values, the system has two complex conjugates with negative real parts.

$$x_t = p_t + \frac{1}{\delta}\upsilon \qquad\qquad (13)$$

If the economy were fully indexed ($\omega = 1$), inflation reduction would not be possible at all. We can clearly see this looking at equations 7, 8 and 9. Equation 8 shows that, in a fully indexed economy, $x_t = p_t + (1/\delta)\psi_t$, which implies that $\psi_t = \pi_t$ and through (7) that the inflation rate is constant.

On the other hand, if price setting were exclusively forward-looking ($\omega = 0$), the model would have a remarkable result: a permanent reduction in the rate of devaluation, which is perfectly credible, will result in an immediate jump in inflation to the new devaluation rate. For example, if the permanent devaluation rate is zero, we can see that $x_t = p_t$ and therefore, the inflation rate is zero. This result, originally obtained by Calvo and Vegh (1994) in an exclusively forward-looking model, is not dependent on the assumption of continuity in the contracts technology. In fact, one could believe that since contracts signed at each point in time have zero weight, current contracts could follow directly to their new equilibrium. Nonetheless, a discrete time version of the Calvo model would exhibit the same lack of inertia.[13]

It is hard to imagine cases of solely forward-looking economy, especially after a history of high inflation. With backward-looking indexation included, $x_t$ has a predetermined component (in proportion to the degree of indexation) and a component which is free to jump. Since $x_t$ determines the inflation rate through (9) and $p_t$, is predetermined, the inflation rate has a sticky component.

This can be seen in figure 1, which shows the effects of a permanent fixation of the exchange rate, which is fully believed by forward-looking price setters.[14] The results are hardly surprising, the inflation rate jumps but not fully to the new long run inflation rate (zero). Under a fixed currency, this implies a real appreciation and through the negative effects on aggregate demand, a real output contraction. In the experiment set, the

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[13]Obstfeld (1995) presents a discrete time relative of the Calvo model and shows that there is no inflation inertia if a credible program is implemented.

[14]The parameter values used in the simulations are: $\omega = 0.6$, $\delta = 0.5$, $\alpha = 0.3$, $\beta = 0.5$, $B = 0.2$,

quarterly inflation rate jumps from a pre-stabilization level of 20% to 13% and after 4.4 quarters reaches a zero level. However, during that period, an important real appreciation is built and the only possibility to restore competitiveness is through a prolonged deflation. Figure 2 shows that the country's competitiveness is fully restored more than 15 quarters after the implementation of the initial program.

In order to avoid an appreciated currency from the start, a possibility is to engineer an initial real devaluation. Figure 3 shows the effects of fixing the currency after an initial step devaluation of 15%. It can be seen that even if the country gains competitiveness in the short run, this is at a cost of higher inflation and eventually domestic currency appreciates (figure 4),[15] albeit less than in the case of no initial devaluation.[16]

## 3.2 Money-Based Disinflation

An alternative to exchange rate pegging is to let the exchange rate to float and to use the money supply as the nominal anchor. In order to derive the system, we assume that demand for nominal domestic money-balances, $m$, is:

$$m_t = \beta s_t + (1-\beta)p_t + y_t \tag{14}$$

Differentiating (14) with respect to time, making use of equation 5, and fixing the money supply growth rate, $\dot{m}$, equal to $\mu$, we obtain:

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$\psi(0) = 0.2$. Thus, contracts are expected to last for 2 quarters, and 60% of prices are under backward-looking indexation contracts. It should be obvious that a credible peg requires a reform in monetary and fiscal institutions which make the long run inflation rate consistent with the peg.

[15]One empirical regularity found in exchange-rate-based stabilizations is that there is a consumption boom at the beginning, specially in non-tradables and a real output contraction after several periods. Studying several hypothesis available, Vegh and Rebelo (1995) conclude that the real effects of exchange rate based stabilization remain puzzling. It is interesting to notice that this model with an initial stepwise devaluation is consistent with most of the real effects of exchange-rate-based stabilization.

[16]The initial stepwise devaluation is ad-hoc and does not necessarily respond to any optimizing behavior. For a more completely specified model in which the optimal initial devaluation is obtained endogenously see Ghezzi (1996).

$$\mu - \pi_t = (\alpha + \beta)(\varepsilon_t - \pi_t)$$ (15)

Thus, under a monetary rule the system of equations becomes:

$$\begin{bmatrix} \dot{\pi} \\ \dot{\psi} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 2\omega\delta & -2\omega\delta & -(1-\omega)\delta^2\varphi \\ \delta & -\delta & 0 \\ -(1-\beta)/(\alpha+\beta) & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi - \bar{\mu} \\ \psi - \bar{\mu} \\ e - \bar{e} \end{bmatrix}$$ (16)

that has the same stability properties as system $(12)$[17] Figures 5 and 6 show the path for the inflation rate and the real exchange rate. Under the parameter assumptions, the real exchange rate will appreciate more than in the case of a pegged exchange rate rule.[18] In particular, a zero monetary growth rule will be consistent with a negative depreciation of the nominal exchange rate ($\varepsilon < 0$) which will reinforce the real appreciation resulting from positive inflation rates.[19]

If instead of a fixed money supply growth rule, the authority would follow an accommodative monetary policy:

$$\dot{m}_t = v + \chi\pi_t$$ (17)

where $v$ is a fixed trend and $\chi$ measures the degree of accommodation to current

inflation, the long run inflation bias would be $\dfrac{v}{1-\chi}$ and the system of equations would

change to:

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[17]Since nominal money demand does not depend on the interest rate (equation 14) there will be no jump in money demand once disinflation takes place. Thus, the nominal exchange rate, and hence the real exchange rate, will be predetermined provided that there is not an initial discrete jump in the money supply.

[18]It should be clear that under pegged exchange rates, money supply would be endogenous and determined by equation (14).

[19] This result depends on the fact that $\alpha + \beta < 1$. If $\alpha + \beta > 1$, the opposite result would hold. Real appreciation would be sharper under a pegged exchange rate.

$$\begin{bmatrix} \dot{\pi} \\ \dot{\psi} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 2\omega\delta & -2\omega\delta & -(1-\omega)\delta^2\varphi \\ \delta & -\delta & 0 \\ -(\chi-1)(1-\beta)/(\alpha+\beta) & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi - \overline{\mu} \\ \psi - \overline{\mu} \\ e - \overline{e} \end{bmatrix} \qquad (18)$$

The effect of a disinflation when the monetary authority sets the long run bias $v$ to zero but still accommodates inflation partially, can be seen in figures 7 and 8.[20] As expected, inflation will be more persistent under an accommodative rule than under a zero growth rule. For example, while with an accommodative rule inflation reaches zero after 5.3 quarters, it will be zero after 4 quarters under a fixed money supply rule.[21]

However, the greater flexibility in monetary policy allows a nominal devaluation which more than compensates for the larger inflation. Therefore, the real appreciation will be lower under accommodative monetary policy than under both a pegged exchange rate and a fixed monetary rule.

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[20] $\chi = 0.5$ in the simulations.

[21] Of course, long run inflation will be determined by the long run inflation bias consistent with authorities preferences ($v$ in (17)). A more accommodative policy will increase the persistence while leaving the unconditional mean constant.

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**Appendix A**

In this appendix we will prove that the backward-looking component of (3) can be derived from a setting in which backward-looking price contracts that last were changed at $v$ and receive a price signal at $t$ are updated by the inflation rate in the period in which prices are constant:

$$x_{t,v}^B = x_{v,v}^B + \int_v^t \pi_u du \qquad \text{(A. 1)}$$

If we assume that the process starts very early in the past, that initial conditions do not matter, we have that $x_{t,v}^B = x_t^B$, $\forall t$. The probability that those prices were set at $v$ and survived by $(t-v)$ periods is determined by:

$$\delta \exp(-\delta(t-v)) \qquad \text{(A. 2)}$$

Aggregating each price by the probability it was set $(t-v)$ periods ago, we obtain:

$$\delta \int_{-\infty}^t x_t^B \exp(-\delta(t-v)) dv = \delta \int_{-\infty}^t (x_v + \int_v^t \pi_u du) \exp(-\delta(t-v)) dv$$

$$= \omega \left( p_t + \delta \int_{-\infty}^t \int_v^t \pi_u du \exp(-\delta(t-v)) dv \right) \qquad \text{(A. 3)}$$

Thus, we need to prove that:

$$\delta \int_{-\infty}^t \int_v^t \pi_u du \exp(-\delta(t-v)) dv = \int_{-\infty}^t \pi_v \exp(-\delta(t-v)) dv \qquad \text{(A. 4)}$$

If we denote by $\overline{\chi_t}$ the left hand side term of equation A.4, and we differentiate:

$$\dot{\overline{\chi_t}} = \pi_t - \delta \overline{\chi_t}, \qquad \text{(A. 5)}$$

whose solution for an initial condition $\overline{\chi}_s$ is:

$$\overline{\chi}_t = \overline{\chi}_s \exp(-\delta(t-s)) + \int_s^t \pi_v \exp(-\delta(t-v))dv \qquad \text{(A. 6)}$$

If the initial condition is far away in the past, we will obtain:

$$\lim_{s \to -\infty} \overline{\chi}_t = \int_{-\infty}^t \pi_v \exp(-\delta(t-v))dv$$

which is exactly equal to the right hand side of A.4. QED.

**Appendix B**

In this Appendix, we will show that there is a unique path which converges to the steady state. We know that we have two predetermined variables ($\psi$ and $e$) and one jump variable ($\pi$), and we need to show that the system has two roots with negative real parts.[22]

The eigenvalues of the system

$$\begin{bmatrix} \dot{\pi} \\ \dot{\psi} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 2\omega\delta & -2\omega\delta & -(1-\omega)\delta^2\varphi \\ \delta & -\delta & 0 \\ -(1-\beta) & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi - \overline{\pi} \\ \psi - \overline{\psi} \\ e - \overline{e} \end{bmatrix} \qquad \text{(B. 1)}$$

have the following property:

$$\phi_1\phi_2\phi_3 = \det A = (1-\omega)(1-\beta)\delta^3\varphi \qquad \text{(B. 2)}$$

$$\phi_1\phi_2 + \phi_2\phi_3 + \phi_1\phi_3 = -(1-\omega)(1-\beta)\delta^2\varphi \qquad \text{(B. 3)}$$

From equation (B.2), we know that either the three eigenvalues are positive or one is positive and the other two are negative. However, equation (B.3) shows that at least one root is negative. Thus, we exactly obtain two negative roots. The system will uniquely converge to the steady state. QED

In general, the solution will be:

for $0 \leq t \leq T$

$\pi_t = A_1 \exp(\phi_1 t) + A_2 \exp(\phi_2 t) + A_3 \exp(\phi_3 t)$

$\psi_t = A_1 \upsilon_1 \exp(\phi_1 t) + A_2 c_1 \exp(\phi_2 t) + A_3 d_1 \exp(\phi_3 t)$

$\ln e_t = \ln \overline{e}_1 + A_1 \upsilon_2 \exp(\phi_1 t) + A_2 c_2 \exp(\phi_2 t) + A_3 d_2 \exp(\phi_3 t)$

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[22] We will proceed only with the pegged exchange rate case. The proof is almost the same under the monetary rule alternatives.

where $\phi_1$ is the positive eigenvalue and $\phi_2$ and $\phi_3$ are the negative eigenvalues. The solution is obtained for the initial conditions for $\ln e$ and $\psi$ and the transversality condition that implies that $A_1.=0$.

*Complex roots.*

In this case, the system has one positive eigenvalue and the other two are complex conjugates with negative real part. We will name them:

$$\phi_1 \text{ and } h \pm vi$$

respectively. The solution to the system implies:

$$\pi_t = A_1 \exp(\phi_1 t) + A_2 \exp((h + vi)t) + A_3 \exp((h - vi)t)$$
$$\psi_t = A_1 \upsilon_1 \exp(\phi_1 t) + A_2(a + bi)\exp((h + vi)t) + A_3(a - bi)\exp((h - vi)t)$$
$$e_t = \bar{e}_1 + A_1 \upsilon_2 \exp(\phi_1 t) + A_2(c + di)\exp((h + vi)t) + A_3(c - di)\exp((h - vi)t)$$

We also know that:
$$\exp(vi) = \cos(v) + i\sin(v)$$

Working with the first equation we obtain:

$$\pi_t = A_1 \exp(\phi_1 t) + \exp(ht)[(A_2 + A_3)\cos(vt) + (A_2 - A_3)i\sin(vt)]$$

If we make $B_2 = A_2 + A_3$ and $B_3 = (A_2 - A_3)i$

$$\pi_t = A_1 \exp(\phi_1 t) + \exp(ht)[B_2 \cos(vt) + B_3 \sin(vt)]$$
$$\psi = A_1 \upsilon_1 \exp(\phi_1 t) + \exp(ht)[(aB_2 + bB_3)\cos(vt) + (aB_3 - bB_2)\sin(vt)]$$
$$e_t = A_1 \upsilon_2 \exp(\phi_1 t) + \exp(ht)[(cB_2 + dB_3)\cos(vt) + (cB_3 - dB_2)\sin(vt)]$$

In the long run, transversality conditions imply $A_1 = 0$ and the initial conditions on $\psi$ and $e$ determine the solution for $B_2$ and $B_3$.

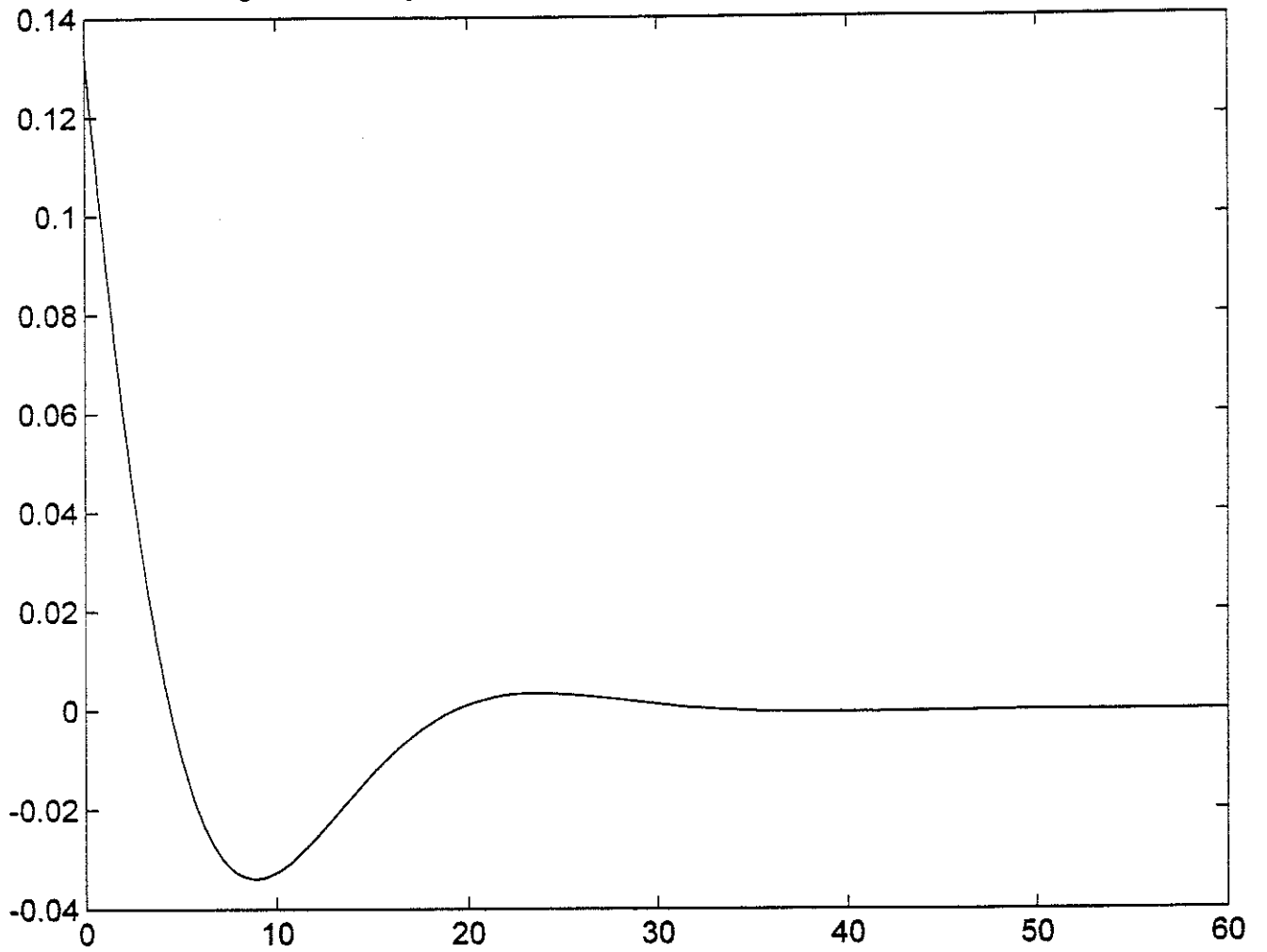Fig.1. Exchange Rate-Based Disinflation: Effect on GDP Inflation

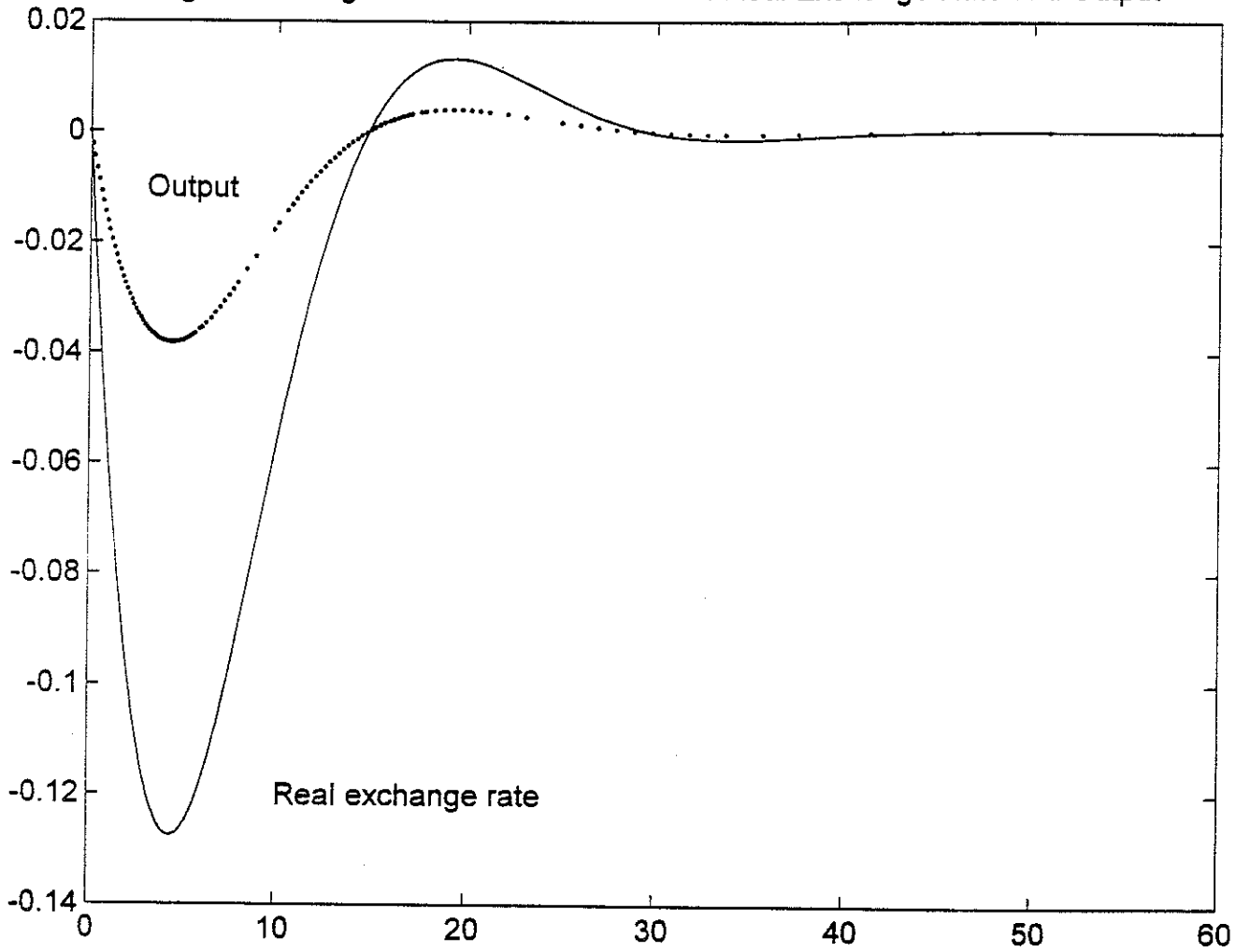Fig.2. Exchange Rate-Based Disinflation: Real Exchange Rate and Output

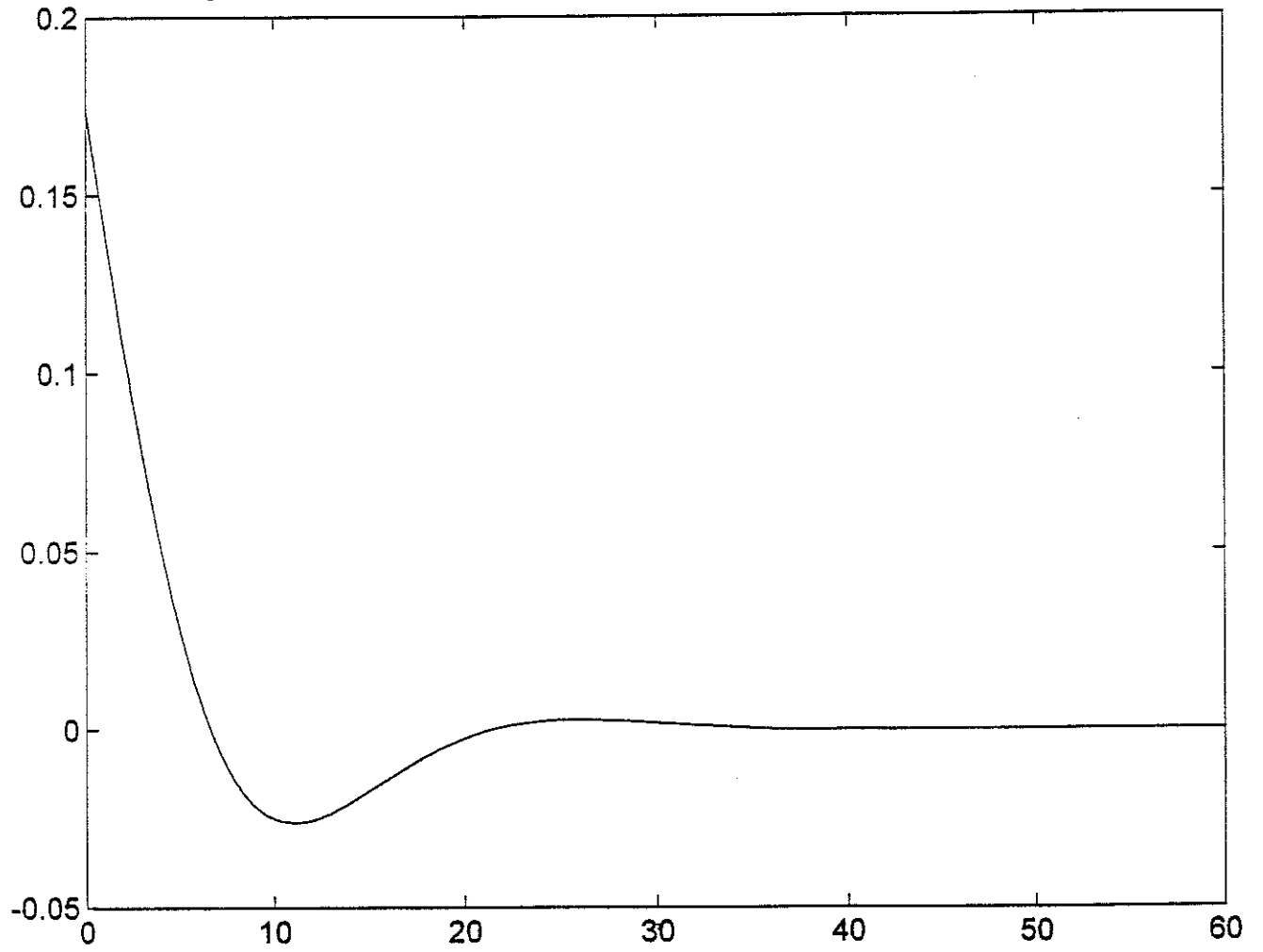Fig.3. Disinflation and 15% Initial Devaluation: Effect on GDP Inflation

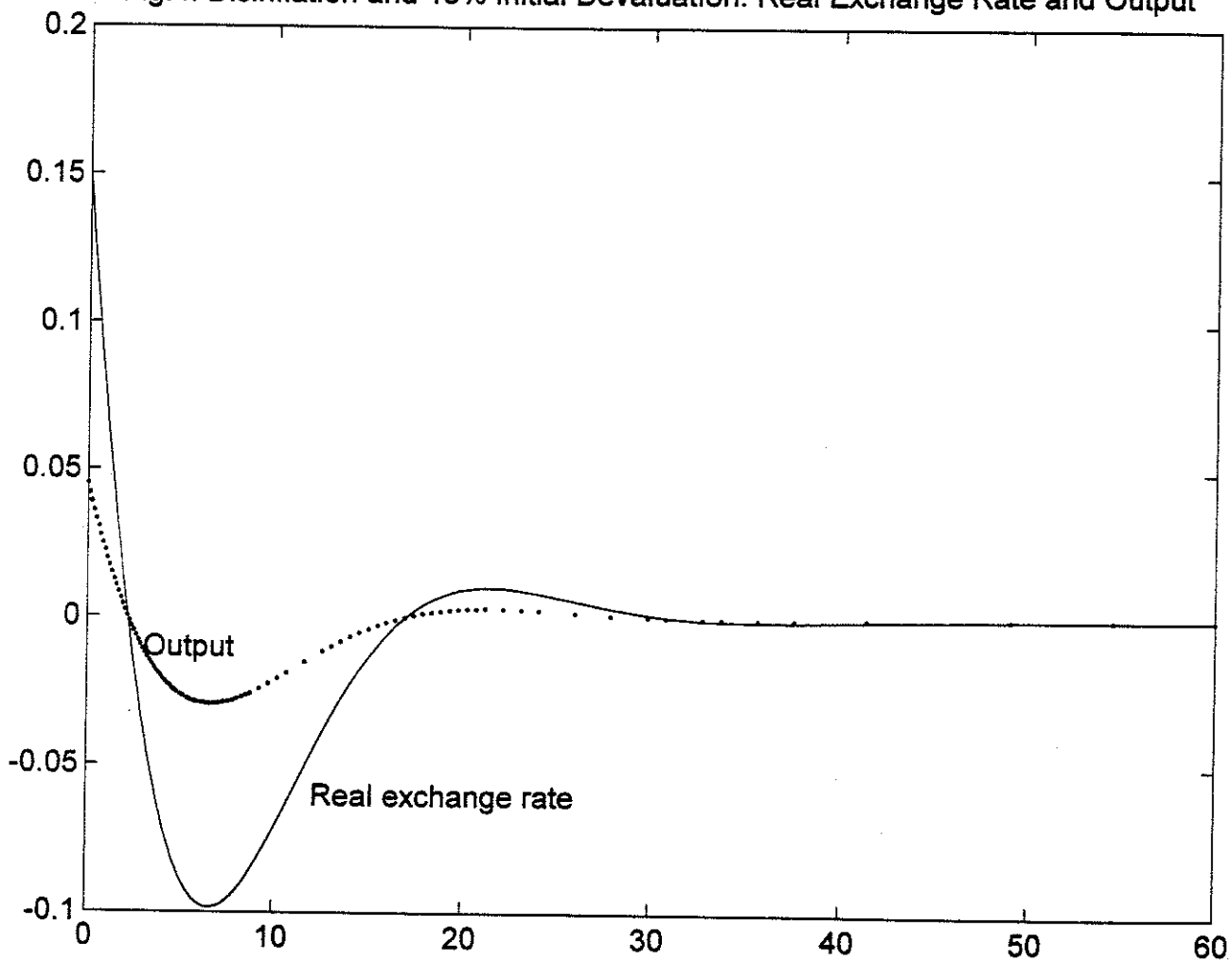Fig.4. Disinflation and 15% Initial Devaluation: Real Exchange Rate and Output

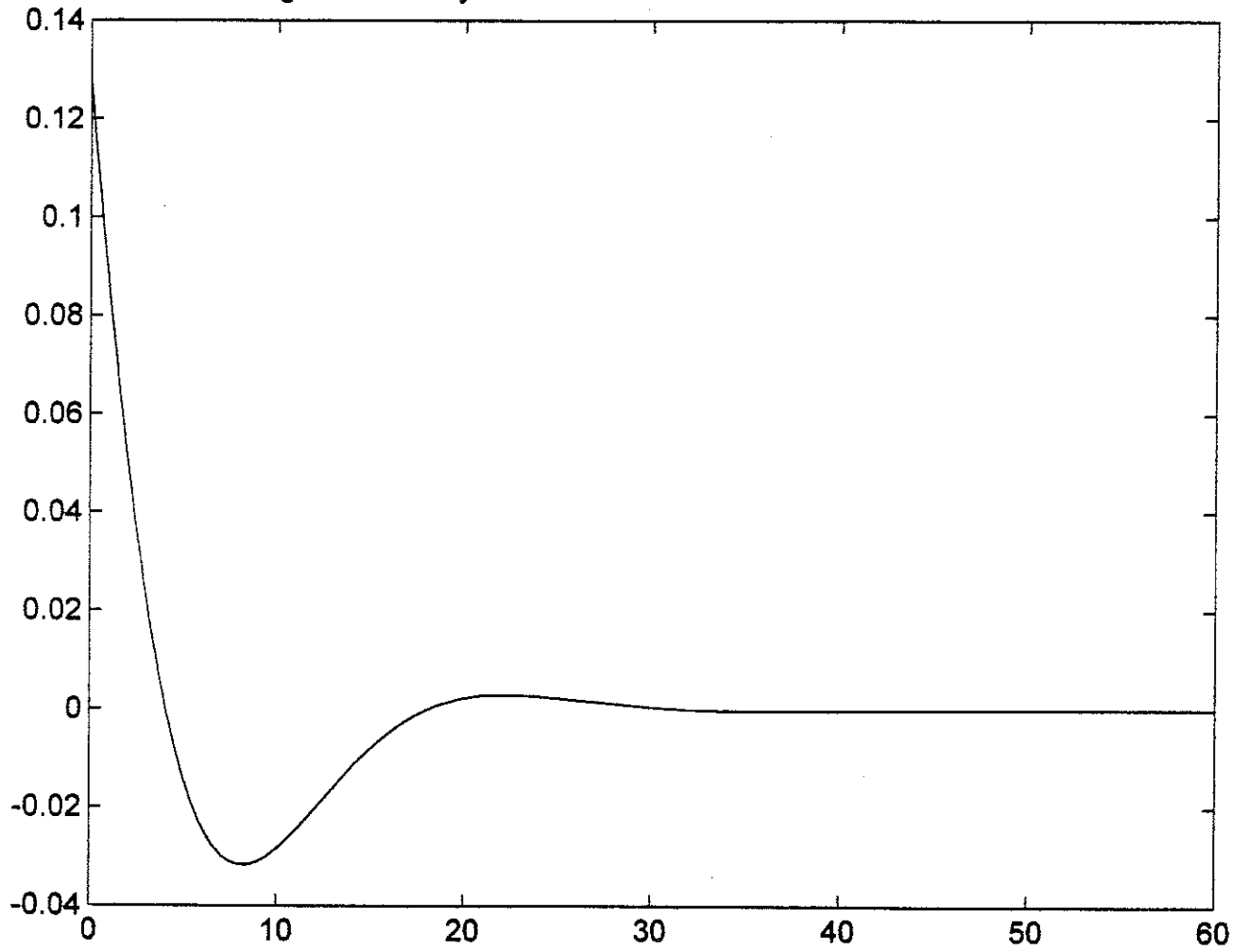Figure 5. Money-Based Disinflation: Effect on GDP Inflation

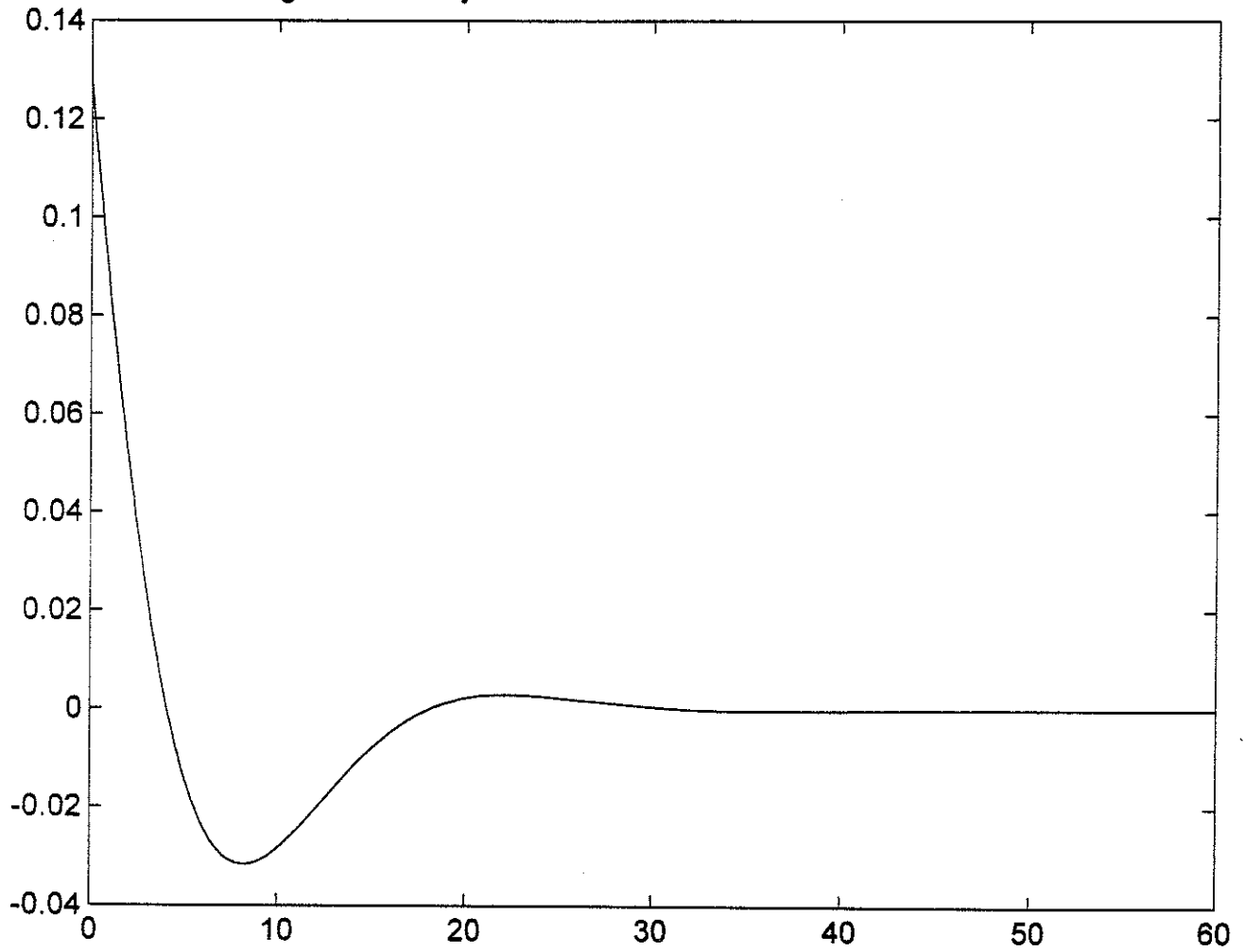Figure 5. Money-Based Disinflation: Effect on GDP Inflation

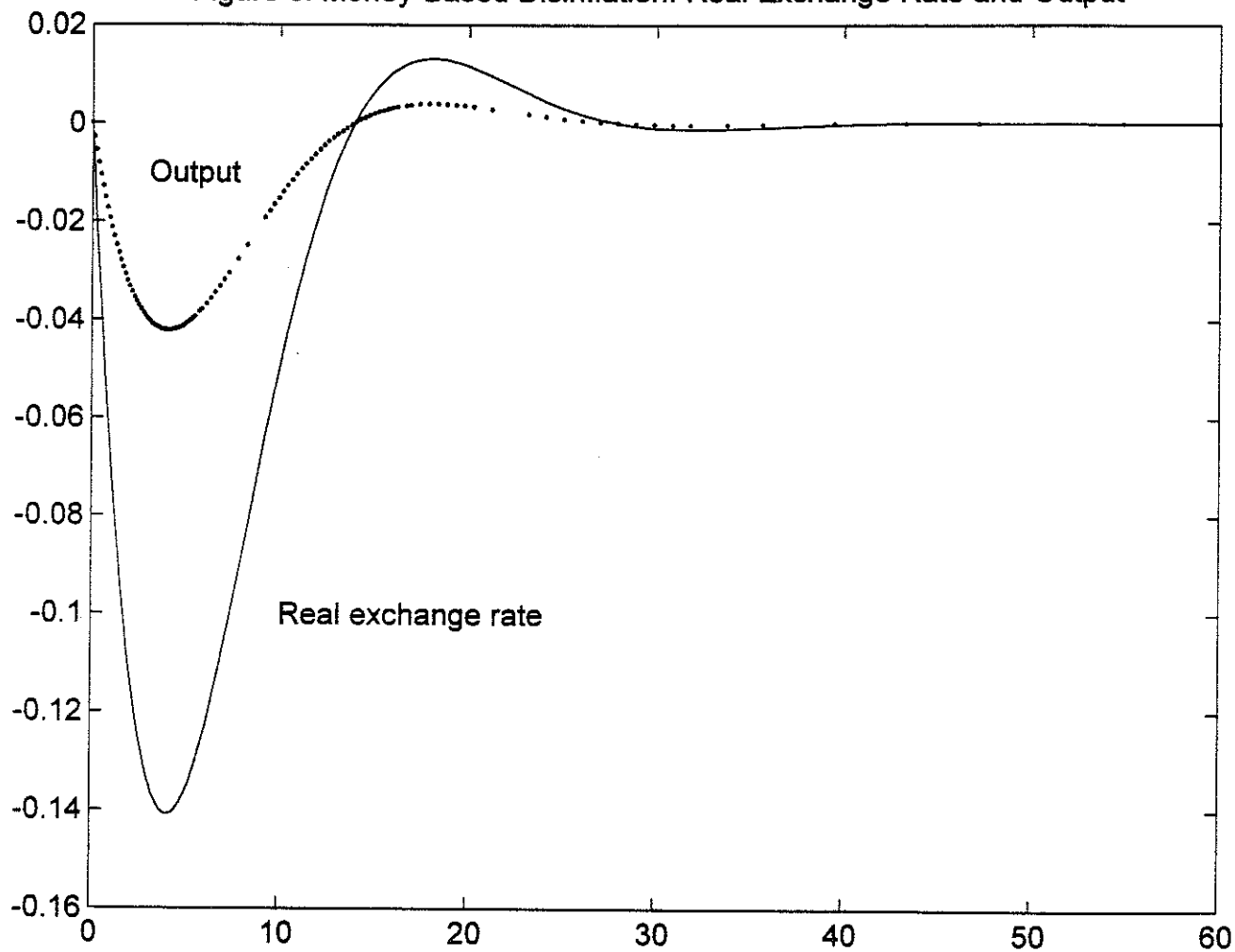Figure 6. Money-Based Disinflation: Real Exchange Rate and Output

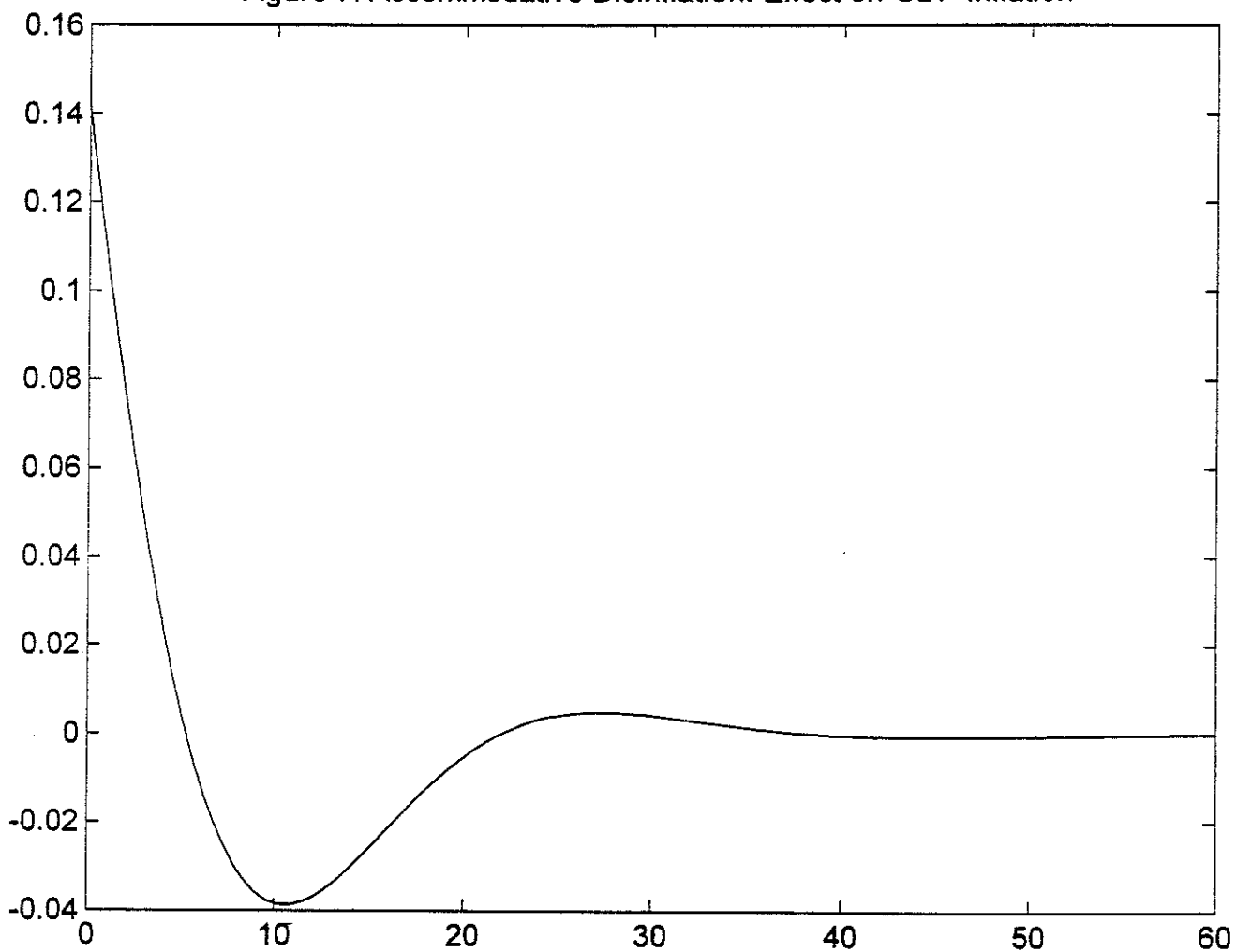Figure 7. Accommodative Disinflation: Effect on GDP Inflation
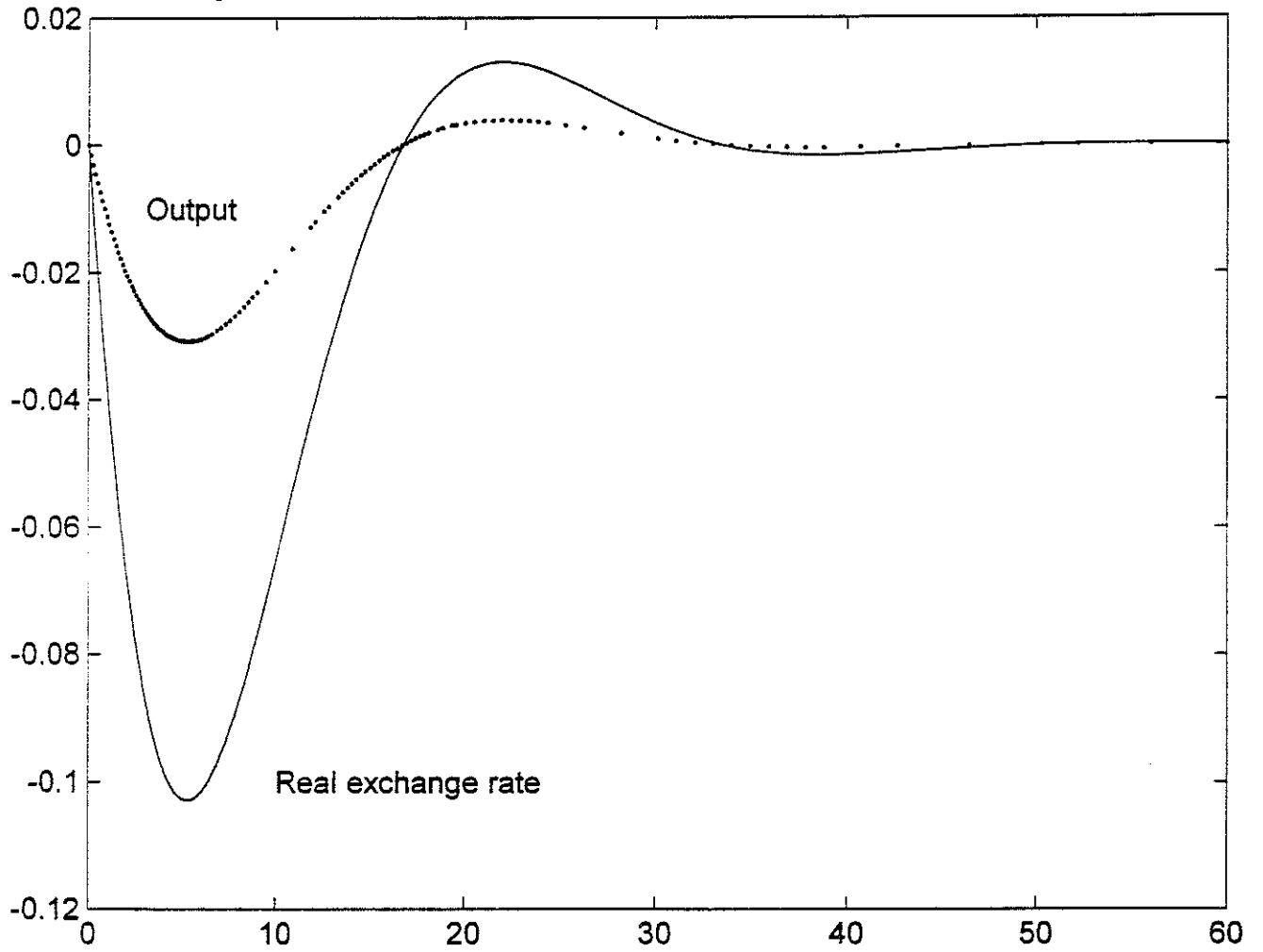
Figure 8. Accomodative Disinflation: Real Exchange Rate and Output

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