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UNDER UNIAND BIAXIAL STRESS

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**MICROMECHANICAL STRESS CONCENTRATIONS  
IN TWO-PHASE BRITTLE MATRIX CERAMIC COMPOSITES**

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TWO-PHASE BRITTLE MATRIX CERAMIC COMPOSITES

D. P. H. Hasselman and R. M. Fulrath

December 8, 1966

MICROMECHANICAL STRESS CONCENTRATIONS IN TWO-PHASE BRITTLE  
MATRIX CERAMIC COMPOSITES

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December 8, 1966

ABSTRACT

A quantitative investigation was conducted on the effect of micro-mechanical stress concentrations on the strength of two-phase brittle matrix ceramic systems. The materials consisted of a continuous brittle matrix containing dispersions with elastic properties different than those of the matrix. A soda borosilicate glass was used as the matrix and the dispersions consisted of 60 micron diameter spherical alumina particles and 60 micron diameter spherical pores. Stress concentrations were varied by measuring the strength of the composite under uniaxial and biaxial tensile stress conditions.

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The experimental results showed that micromechanical stress concentrations strongly affect the macroscopic strength of the composite. Under biaxial tensile stress additions of either alumina microspheres or spherical porosity to the glass matrix resulted in a decrease in strength equal to the maximum calculated stress concentration factor. Under uniaxial tensile stress conditions, however, the reduction in strength for the glass-alumina system was negligible. The glass-porosity system gave a reduction in uniaxial strength which was not equal to the maximum calculated stress concentration factor.

Experimental results suggest that differences in strength of brittle multi-component systems under uniaxial and biaxial stress states can in part be attributed to microstructural features.

On the basis of the experimental work a hypothesis is developed relating the relative size of the region in the glass matrix over which stress concentrations act to the size of the Griffith flaws responsible for failure. This hypothesis is extended to the effect of porosity on the strength of polycrystalline brittle ceramic materials.

## I. INTRODUCTION

Many brittle ceramic materials of technological importance are composed of two or more phases which differ in physical and chemical properties. Under mechanical loading differences in elastic properties of individual components of a ceramic body can lead to stress inhomogeneities generally referred to as "stress concentrations". Numerous theoretical solutions have appeared in the literature for stress concentrations associated with elastic inhomogeneities of various shapes contained in an infinite matrix.<sup>1-4</sup> Although these theoretical solutions appear to be useful for the prediction of mechanical behavior of engineering structures, their applicability to the prediction of the mechanical properties of ceramic systems appears limited.

The purpose of the present study was to investigate the effect of stress concentrations on strength of composite materials, in order to determine whether engineering stress concentration theories can be applied to these types of materials. The composite system selected consisted of a continuous glass matrix containing (a) spherical alumina particles and (b) spherical pores, as dispersed phases. These two dispersants will result in distinctly different values and distributions of the stress concentrations. Use of a glass matrix assures one of a homogeneous uniform, pore-free matrix material.

In order to investigate the effect of different stress concentrations for each type of dispersant, specimens were subjected to uniaxial as well as biaxial tensile loading.

## II. EXPERIMENTAL

### A. Materials

The glass for the composite systems investigated consisted of a sodium borosilicate glass of the same composition (14% Na<sub>2</sub>O, 16% B<sub>2</sub>O<sub>3</sub>, and 70% SiO<sub>2</sub> by weight) as the D-glass used in previous investigations.<sup>5,6,7,8</sup> This glass composition has a coefficient of thermal expansion nearly equal to that of alumina, which eliminates internal stresses as a variable.

The spherical alumina particles were obtained by spheroidization of alumina powder in a dc plasma jet. Alumina particle size was about 60μ. This particle size was selected such that over the whole range of volume fraction alumina investigated, strength would not be controlled by glass "mean-free-path" considerations.<sup>8</sup> Interfacial bonding is obtained between the D-glass and the alumina particles as shown previously.<sup>8</sup>

Spherical porosity was simulated by incorporating spherical nickel particles in the glass matrix. Nickel particle size was approximately 60μ. Due to the much larger coefficient of thermal expansion of the nickel as compared to the glass and the nonbonding characteristics of the glass-nickel interface, on cooling the composite from the hot-pressing temperature a spherical pore is produced. By comparing the total shrinkage of the nickel with the calculated elastic strains in the glass induced in the strength tests,<sup>9</sup> it is easily verified that the nickel cannot contribute to the load-bearing ability of the glass for volume concentrations of nickel investigated.



### B. Specimen Preparation

All composites were prepared by the vacuum hot-pressing technique described previously.<sup>6,7,8</sup> Glass-alumina biaxial specimens consisted of 1 in. diam disks (0.075 in. thick) sliced with a high-precision diamond saw from hot-pressed slugs 1 in. in diam by approximately 1 in. high. Uniaxial strength specimens (0.075 in. thick by 1/4 in. wide) were sliced from disks 2 in. diam by 1/4 in. thick.

For purposes of comparison, it was attempted to manufacture the glass-nickel specimens in a manner identical to the one followed for the glass-alumina specimens. However, considerable difficulty was encountered attempting to slice 1 in. diam biaxial specimens, presumably because of the ductile nature of the metal phase. Instead, a sufficient quantity of 1 in. diam disks (0.075 in. thick) were hot-pressed individually, followed by a light grinding (1000-grit silicon carbide) to remove carbon and graphite adhering to the surface. This also insured a uniform surface treatment. Uniaxial glass-nickel strength specimens (approximately 1/8 in. wide) were cut from 1 in. diam hot-pressed disks.

### C. Strength Measurements

Strength was determined by bend tests carried out at room temperature. Uniaxial strength was determined by four-point loading using a 3/4 in. overall span. Biaxial tensile bend strength was obtained by means of a ring test similar to the one employed by Binns<sup>10</sup> which, in effect, simulates two equal mutually perpendicular tensile stresses. The 1 in. diam strength specimens were placed on a stiff rubber O-ring

and loaded by placing a 3/16 in. diam ball-bearing in the center of the specimen. The disk was loaded to failure in an Instron machine. A rubber O-ring was selected as specimen support in order to duplicate the loading rates used in the uniaxial bend test. Time to failure was approximately 30 sec. The value of strength obtained by the ring test was calculated by means of an expression given by Timoshenko et al.<sup>11</sup> For simplicity this value of strength will hereafter be referred to as biaxial strength. Although the strength of glass is governed by numerous variables<sup>12</sup> (i.e., humidity, rate of loading, temperature, etc.) the value of glass strength obtained will be characteristic for the experimental conditions selected.

### III. EXPERIMENTAL RESULTS

Figures 1, 2, and 3 illustrate the experimental values for the uniaxial and biaxial strength for the glass-alumina and glass-porosity systems. Average strength values with statistical data are also listed in Tables I and II. Although different surface preparation techniques were employed for each composite system, the values of glass strength are sufficiently close together to conclude that no major differences exist between the surface conditions. The values of the relative slope which can be drawn through the glass-porosity data at low porosities (see Figs. 2 and 3) are approximately minus two and minus unity for the uniaxial and biaxial stress conditions, respectively.

Figure 4 illustrates a fracture surface of a glass-nickel specimen, clearly showing the nonbonding characteristics between the glass and the nickel. The interaction between spherical pores and propagating

crack front as indicated by the "steps" in the glass surface appears to be similar to the observation of Passmore et al.<sup>14</sup> of the fracture surfaces of polycrystalline alumina.

The theoretical stress concentrations were calculated on the basis of the solutions of Goodier.<sup>1</sup> Since glass fracture generally is nucleated in the specimen surface<sup>12</sup> and because of the high stress gradients in the strength test employed, it was felt that Goodier's solutions for the stress concentrations around a circular inclusion in a flat plate (two dimensions) better describe the stress conditions in the surface than the solutions for the stress concentrations around a spherical inclusion in a three-dimensional matrix. Figure 5 illustrates the polar coordinate system employed to describe the stress. Using numerical values 331,<sup>5</sup> 1635,<sup>13</sup> and 0 kilobars for the shear moduli for the glass, alumina, and porosity, respectively; and for Poisson's ratio for the glass and alumina the values 0.197<sup>5</sup> and 0.257,<sup>13</sup> respectively, the results for the stress concentrations in the glass matrix under conditions of uniaxial loading obtained were:

Glass-alumina system: (Uniaxial)

$$\sigma_r = 2T \left[ 0.107 \frac{a^2}{r^2} + \left( -0.267 \frac{a^4}{r^4} + 0.356 \frac{a^2}{r^2} \right) \cos 2\theta \right] + T \cos \theta \quad (1)$$

$$\sigma_\theta = 2T \left[ -0.107 \frac{a^2}{r^2} + 0.267 \frac{a^4}{r^4} \cos 2\theta \right] + T \sin \theta \quad (2)$$

where:

T = the stress applied to the composite

$\sigma_r$ ,  $\sigma_\theta$ , r, a, and  $\theta$  are defined in Figure 4

Glass-porosity system: (Uniaxial)

$$\sigma_r = 2T \left[ -\frac{a^2}{4r^2} + \left( \frac{3}{4} \frac{a^4}{r^4} - \frac{a^2}{r^2} \right) \cos 2\theta \right] + T \cos \theta \quad (3)$$

$$\sigma_\theta = 2T \left[ \frac{a^2}{4r^2} - \frac{3}{4} \frac{a^4}{r^4} \cos 2\theta \right] + T \sin \theta \quad (4)$$

Under conditions of biaxial loading, the stress concentrations became:

Glass-alumina system: (Biaxial)

$$\sigma_r = 4T \left[ 0.107 \frac{a^2}{r^2} \right] + T \quad (5)$$

$$\sigma_\theta = 4T \left[ -0.107 \frac{a^2}{r^2} \right] + T \quad (6)$$

Glass-porosity system: (Biaxial)

$$\sigma_r = 4T \left[ -\frac{a^2}{4r^2} \right] + T \quad (7)$$

$$\sigma_\theta = 4T \left[ \frac{a^2}{4r^2} \right] + T \quad (8)$$

Under conditions of a tensile load (T positive), tensile stresses greater than the applied stress occur for Eqs. (1), (4), (5), and (8). The other equations result in stresses smaller than the applied tensile stress. As a consequence, if stress concentrations are to affect tensile strength, Eqs. (1) through (8) suggest that in the glass-alumina

system failure will be due to the radial component of the stress concentration. (Eqs. (1) and (5)) whereas for the glass-porosity system the tangential component (Eqs. (4) and (8)) will lead to failure. Equations (1) through (8) also show that maximum stress occurs at the interface ( $r = a$ ) and that under conditions of biaxial loading the stress concentrations are independent of the angular position.

For the glass-alumina system the values of the maximum radial stresses are 1.39 T and 1.43 T for uniaxial and biaxial loading, respectively, whereas the maximum tangential stresses for the glass-porosity system are 3 T and 2 T under uniaxial and biaxial stress, respectively. The values of stress concentration divided by the applied stress are often referred to as stress concentration factors.

#### IV. DISCUSSION AND CONCLUSIONS

The most interesting results of the present investigation is that the experimental data of the strength for the glass-porosity system show an apparent precipitous decrease in strength even at a small volume fraction porosity, followed by a rather small further decrease in strength at further increases in porosity. This appears to be in disagreement with observations on polycrystalline ceramics,<sup>14,15,16,17,18,19</sup> which generally are thought to show a smooth decrease in strength from the zero-porosity strength upon introduction of the pore phase. In the present investigation extrapolation to zero porosity of the glass-porosity strength data results in zero-porosity values of strength much smaller than the observed values of strength of the glass itself. The zero-porosity extrapolated value of biaxial strength of the glass

(6600 psi) is approximately one-half the observed value of the glass strength, in agreement with a calculated stress concentration of twice the applied stress (Eq. (8)). The relative slope of the straight line in Fig. 3, with a value of approximately minus unity, suggests that under conditions of biaxial stress the average stress level within the glass is inversely proportional to the glass volume fraction.

The uniaxial strength data for the glass-porosity system is less distinct. Although a precipitous decline in strength on addition of porosity is observed, the extrapolated value of zero-porosity strength does not correspond to the glass-strength divided by the calculated stress concentration factor equal to 3 (see Eq. 4). The relative slope of the strength-porosity curve under uniaxial stress ( $\nu = 2$ ) as compared to the relative slope under biaxial load ( $\nu = 1$ ) is thought to be a reflection of the fact that under uniaxial load the stress distribution around a pore is considerably more complex than under a biaxial load, as may be ascertained by comparing Eqs. (3), (4), (7), and (8).

Of interest to note is the upward curvature of both the uniaxial and biaxial strength data for the glass-porosity system as shown in Figs. 2 and 3. In a previous investigation<sup>8</sup> it was suggested that at the interparticle spacings employed neglecting stress concentration effects (other variables held constant) that the strength of a composite system should follow the relation:

$$S = S_0 (1 - \phi)^{-1/2} \quad (9)$$

where  $S$  and  $S_0$  represent the strength of the composite and matrix, respectively, and  $\phi$  is the volume fraction dispersant (porosity  $P$  for

the present system).

The experimental results for biaxial strength of the glass-porosity system suggests that Eq. (9) should reflect the effect of the stress concentration, as well as the decrease in amount of material carrying the load. Dividing the right-hand side of Eq. (9) by the maximum stress concentration factor (K) and introducing the factor (1-P) to correct for the reduction of load bearing volume results in:

$$S = \frac{S_0}{K} (1-P)^{1/2} \quad (10)$$

which indeed describes the observed strength values for biaxial stress (K = 2) to within 300 psi of the observed average values. Equation (10) however, does not appear to be applicable to describe the results obtained for the uniaxial strength of the glass-porosity system.

As the relative effect of porosity on the biaxial strength of the glass appears to be different than the relative effect of the porosity on uniaxial strength as shown in Figs. 2 and 3, it is suggested that a similar effect may be observed for polycrystalline ceramics. At present the writers are not aware of experimental data which would substantiate this hypothesis.

The experimental results for strength for the glass-alumina system can be interpreted in a manner similar to the results for the glass-porosity system, with the difference that now the alumina dispersed phase is under stress and contributes to the load-bearing ability. The biaxial strength results (Fig. 1) show a zero-volume-fraction intercept at a value of strength which is close to the value of glass strength

divided by the calculated stress-concentration factor for this system (1.43). At least qualitatively similar observations were made by Binns.<sup>10</sup> In fact, the biaxial strength value can be described quite accurately, using Eq. (9) corrected for the effect of stress concentration, by

$$S = \frac{S_0}{K} (1 - \phi)^{-1/2} \quad (11)$$

The uniaxial strength results of the glass-alumina system in contrast to the glass-porosity system do not show a precipitous decline in strength on addition of the alumina phase.

It should be noted here that the values of stress concentration factors as described by Eqs. (1) through (8) should be valid for very small concentrations of dispersed phase only. The agreement between theory (Eqs. 10 and 11) and experiment suggests that, at least under conditions of biaxial stress, the calculated values of stress concentrations (Eqs. (1) to (8)) can be applied to relatively high volume concentrations of a dispersed phase.

For the four different systems studied (i.e., porous matrix under uniaxial and biaxial loading and a dispersed phase system under uniaxial and biaxial loading) it is apparent that the micromechanical stresses must be considered in discussing fracture behavior of brittle materials. See  $t$  values as given in Tables I and II. A reasonable explanation for the observed differences in the effect of micromechanical stress concentrations on the uniaxial and biaxial tensile strength of the composite systems investigated, can be based on the hypothesis that fracture in brittle materials generally originates at structural



defects, i.e., "Griffith flaws". The existence of these flaws in brittle materials is well substantiated.<sup>12,20</sup> For this investigation the surface treatment of the glass matrix would produce Griffith flaws of the order of 40 to 50 microns as found previously.<sup>8</sup>

The original expression as derived by Griffith for the macroscopic strength (S) of an infinitely large, uniformly stressed flat plate, with an elliptical flaw of length (d) can be expressed by

$$S \sim \left( \frac{\gamma E}{d} \right)^{1/2} \quad (12)$$

where  $\gamma$  is the surface energy and E is Young's modulus of elasticity of the material.

The material systems presently investigated contained spherical dispersions with a diameter of approximately 60 microns. Examination of Eqs. (1) through (8) will reveal that the concentration of stress in excess of the internally applied stress extends over distances considerably smaller than the size of the dispersions. In fact, the maximum value of stress concentration under uniaxial stress conditions occurs at two points only ( $\theta = 0$  and  $\theta = 180^\circ$ , at  $r = a$ ). As a consequence a Griffith flaw located near one of the dispersed alumina particles or pores is subjected to a highly non-uniform stress field, such that only a small segment of the flaw is subjected to the higher level of stress due to the stress concentration effect. Under these conditions it is to be expected that, at least qualitatively, the Griffith criterion (12) no longer is valid. For failure to occur, a stress level will be required higher than calculated on the basis of

the maximum stress concentration factor and lower than the macroscopic strength of the matrix phase.

Examination of Eqs. (1) to (4) shows that for the glass-alumina system the region of the matrix under a high stress concentration is considerably smaller than the region under a high stress concentration in the glass-porosity system. This at least qualitatively explains the precipitous decline in the uniaxial strength of the glass porosity system but not for the glass-alumina system. Examination of Eqs. (5) to (8) shows that under biaxial stress conditions the high tensile stress concentrations act over a considerably larger volume of the matrix as compared to the uniaxial stress conditions. And indeed, as suggested by the observations, the stress concentrations under biaxial conditions act over a sufficiently large volume compared to the size of the flaws that failure can occur at the calculated level of stress.

At this stage the development of a fracture theory of strength based on Griffith flaws located in non-uniform stress fields would be desirable. However, the development of such a theory appears mathematically prohibitive, no doubt made even more complex by interactions between the stress field around the dispersed phase and the stress distributions around the Griffith flaw.

The hypothesis that the effect of micromechanical stress concentrations on the strength of a brittle material depend on the size of the Griffith flaw relative to the region over which the stress concentration acts is vital to the development of any fracture theory. Based on this hypothesis, the effect of porosity on strength can be

viewed from two extremes. The first case is where the pore size and resultant micromechanical stress fields are large relative to the size of the Griffith flaw. The reverse forms the basis of the opposite case.

In case I the pore size is substantially larger than the flaw size such that a flaw lies entirely in material stressed to the maximum value of stress concentration. Engineering structures with drilled holes, grooves, etc., in otherwise pore free materials, fall in this region. Here the stress concentration approach can be applied successfully, the structure failing when the maximum stress concentration exceeds the strength of the nonporous material. In this region the effect of porosity on tensile strength will exhibit an instantaneous decrease in strength upon introduction of the first pore in the body. The decrease in strength will correspond to the maximum stress concentration factor. The results obtained in this investigation for the biaxial tensile strength of a glass containing spherical pores appear to be representative of the effect of porosity on tensile strength for case I.

For case III the pore is considerably smaller than the Griffith flaw. The flaw will be completely unaffected by the stress concentrations near the pores. Strength should exhibit a monotonic decrease with increasing porosity, without the precipitous decrease in strength characteristic of case I. The effect of porosity on high strength polycrystalline industrial ceramics should fall in this region, as well as many high strength ceramics investigated in the laboratory, such as the high-density alumina investigated by Passmore et al.<sup>14</sup> who found

a flaw size of the order of the grain size and a pore size an order of magnitude smaller than the grain size. It is in this region where the "cross-sectional area" approach by Knudsen<sup>17</sup> and Brown et al.<sup>21</sup> for the effect of porosity on strength appear to be most applicable. As these theories are not restricted to pore size, they should be applicable to engineering structures as well as brittle ceramics containing small pores. However, care should be taken in predicting the effect of porosity on strength based on these theories in regions where stress concentrations are the governing factor.

In intermediate case II the flaw size is of the order of the pore size such that only a segment of the flaw is subjected to the stress concentration. The effect of porosity on strength in this region will exhibit a precipitous decrease in strength upon introduction of the first pore but not to a value corresponding to the calculated maximum stress concentration. The present results for the uniaxial tensile strength of the glass containing spherical pores appear to fall in this region.

The results for the hypothesis that micromechanical stress concentrations affect the porosity-stress relationship of brittle materials is summarized in Fig. 6. The relative slopes shown in the figure for the three cases were arbitrarily chosen.

#### V. SUMMARY

An investigation was carried out to determine the effect of micromechanical stress concentrations on the macroscopic strength of multi-component brittle ceramic materials. Composite systems investigated

consisted of a continuous glass matrix containing (a) spherical particles of alumina and (b) spherical pores. Experimental data obtained show that for the systems investigated stress concentrations have a detrimental effect of macroscopic tensile strength. Under biaxial tensile stress conditions the decrease in strength corresponds to the maximum calculated stress concentration factor. Under uniaxial tensile stress, however, the decrease in strength is less than expected from stress concentration factors.

A discussion is presented which explains these observations in terms of the size of the region of the glass matrix over which the stress concentrations act, compared to the size of the Griffith flaws, responsible for failure.

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Table I. Crossbending strength and statistical data for a sodium borosilicate glass containing dispersions of 60 $\mu$  diameter spherical alumina particles

Volume content Al <sub>2</sub> O <sub>3</sub> (%)	Uniaxial strength			Biaxial strength			Values of t† for difference between uniaxial and biaxial strength	Significant difference‡ between uniaxial and biaxial strength?
	Average (psi)	No. of samples	Standard deviation (% of average)	Average (psi)	No. of samples	Standard deviation (% of average)		
0	14,700*	38	12.7	13,100	18	7.2	3.4	Yes
2.75	--	--	--	9,650	10	8.9	--	--
5.5	14,300	32	4.7	9,610	9	6.0	11.2	Yes
10.9	13,200*	25	10.2	9,600	9	9.5	7.3	Yes
21.6	13,200*	34	5.6	9,900	9	5.0	12.7	Yes
32.0	14,100*	35	5.9	11,000	10	5.9	10.7	Yes
42.3	16,000*	32	5.3	13,400	10	2.5	9.3	Yes
47.5	16,900*	31	5.4	13,000	9	5.4	12.2	Yes

\* Data obtained from reference 8.

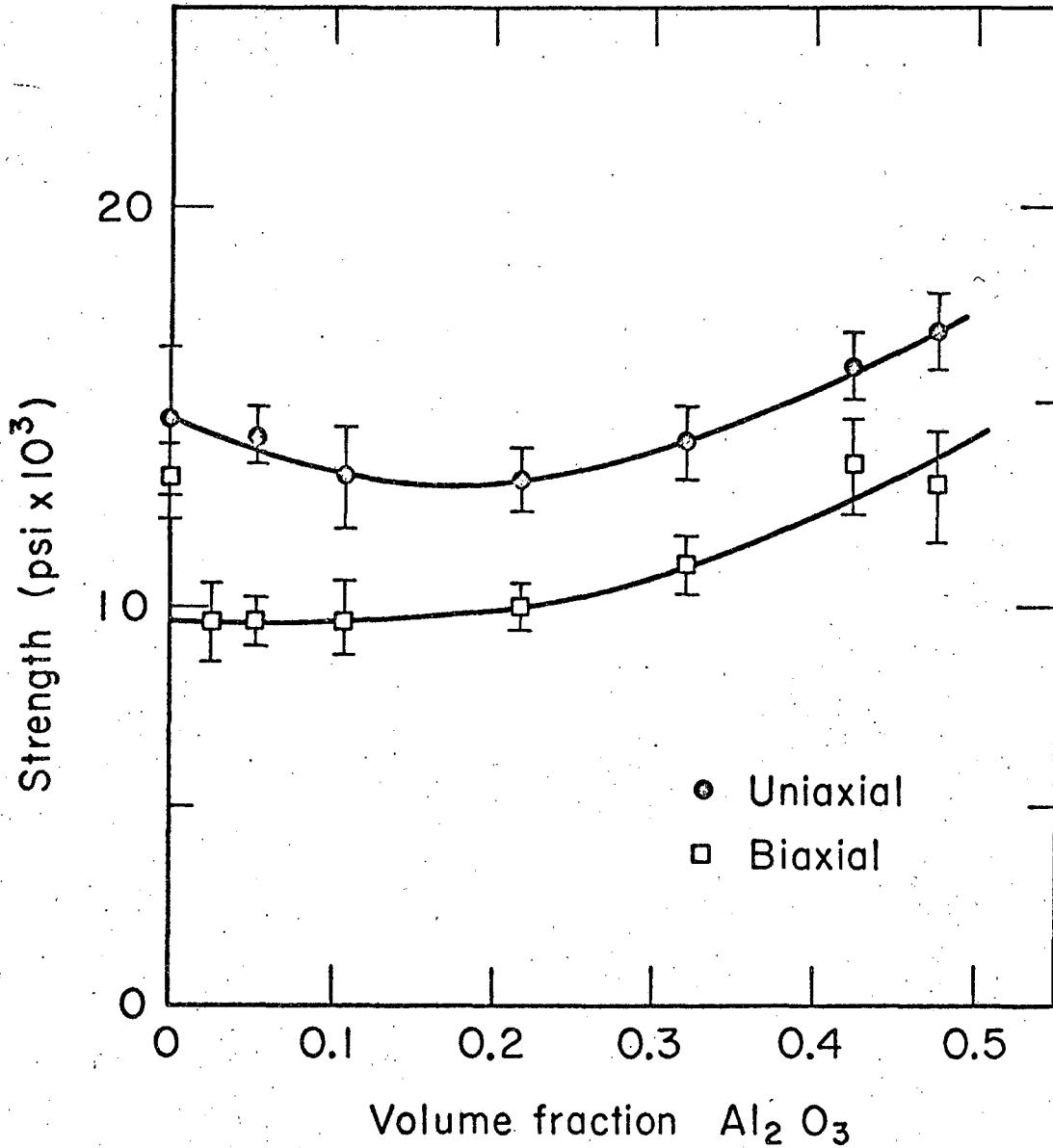
† See W. J. Dixon and F. J. Massey, Jr., Introduction to Statistical Analysis, Chapter 9, Second Edition. McGraw-Hill Book Co., New York, 1957. 488 pp.

‡ At the 95% confidence level.

Table II. Crossbending strength and statistical data for a sodium borosilicate glass containing 60 $\mu$  diameter spherical pores

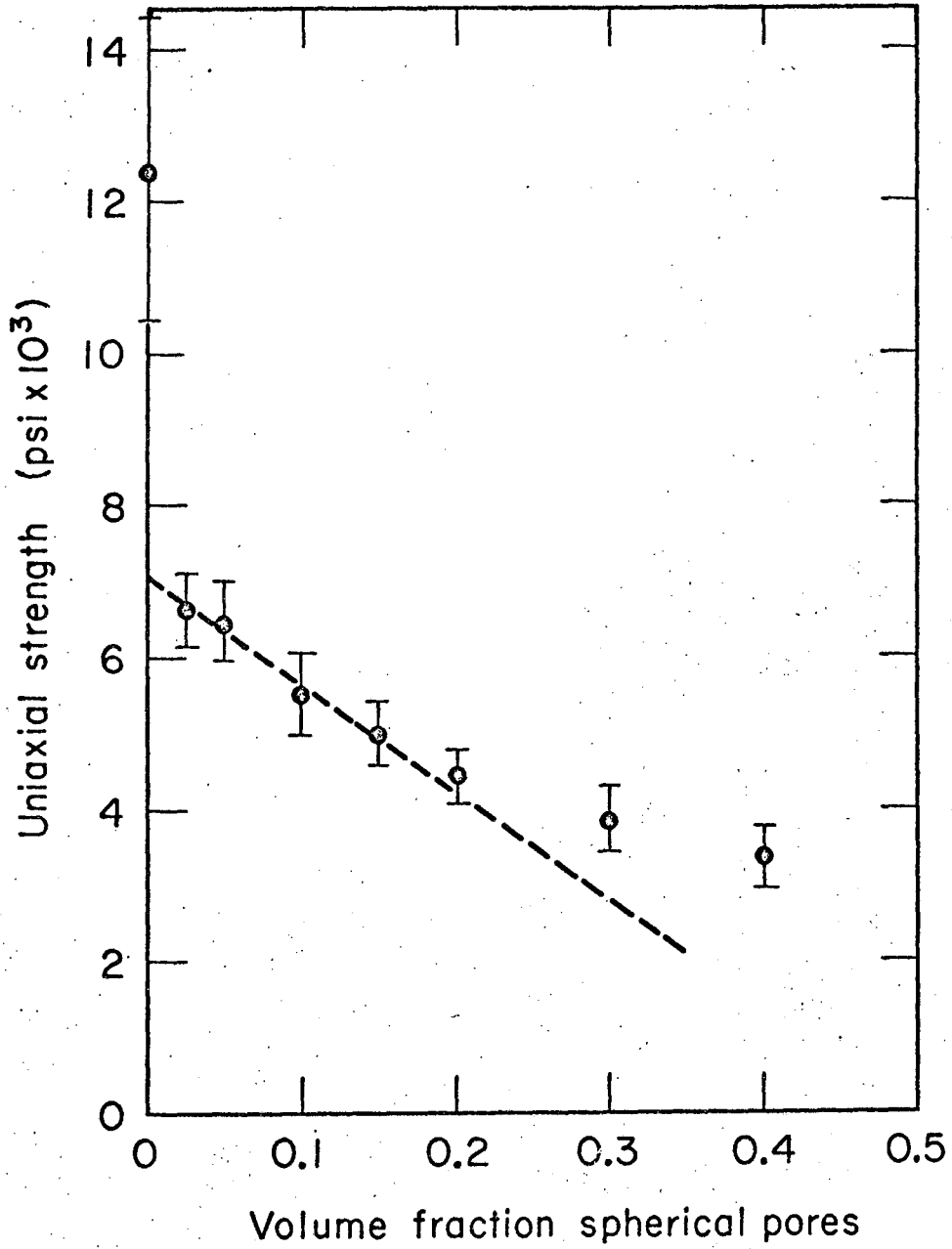
Porosity (%)	Uniaxial strength			Biaxial strength			Values of t <sup>+</sup> for difference between uniaxial and biaxial strength	Significant difference <sup>+</sup> between uniaxial and biaxial strengths?
	Average (psi)	No. of samples	Standard deviation (% of average)	Average (psi)	No. of samples	Standard deviation (% of average)		
0	12,400	15	16.1	12,500	12	5.5	0.16	No
2.5	6,610	15	7.5	6,300	9	11.0	1.22	No
5	6,450	18	8.2	6,360	8	6.0	0.41	No
10	5,500	10	10.1	5,980	9	6.9	2.00	No
15	4,970	12	8.9	5,400	9	3.7	2.12	No
20	4,410	14	8.2	5,280	9	8.0	5.12	Yes
30	3,870	16	11.4	5,170	9	5.2	8.02	Yes
40	3,340	10	12.5	5,180	9	7.9	9.01	Yes

+ See Table I.

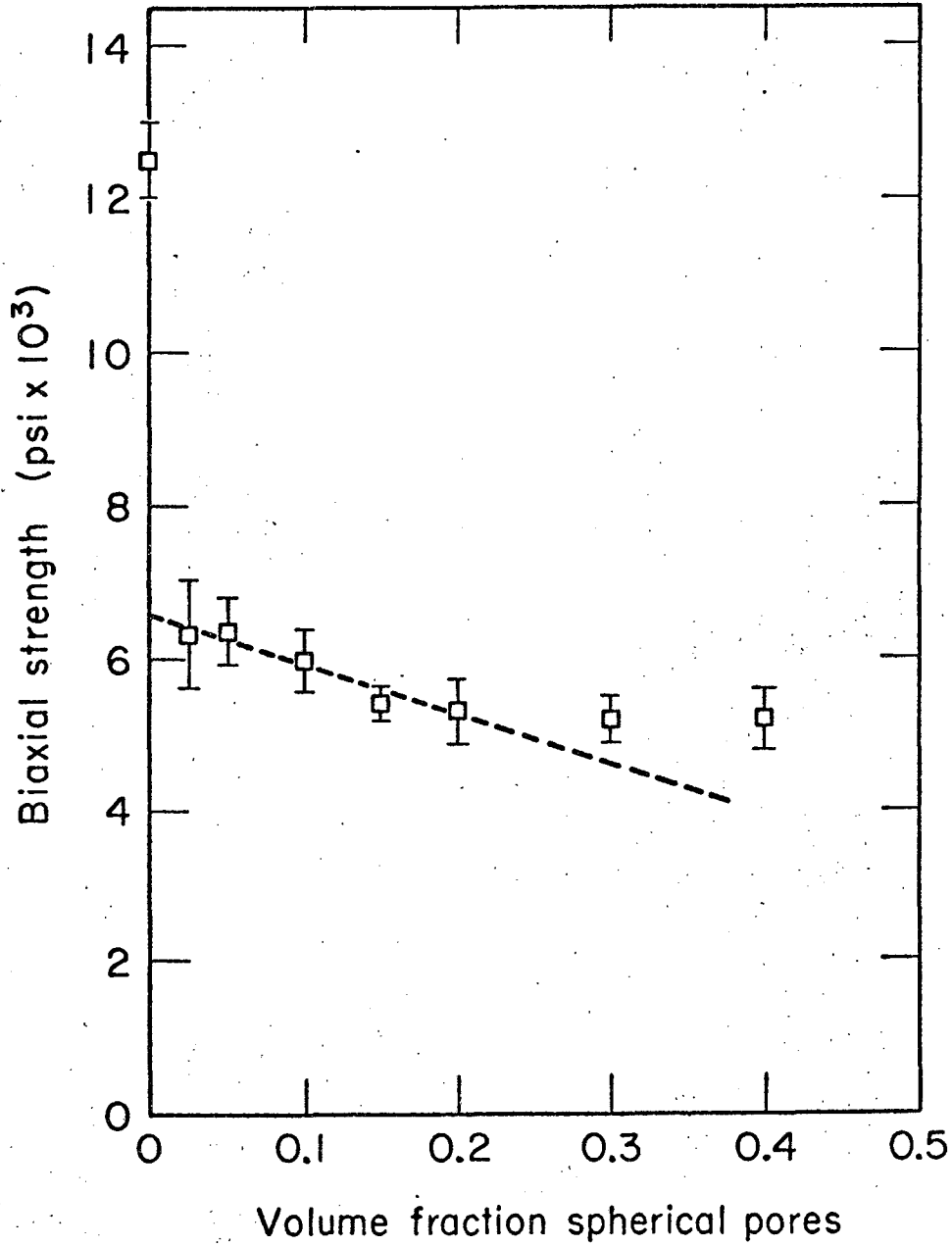


MUB-7856

Fig. 1 Uniaxial and biaxial strength of a soda borosilicate glass containing 60 $\mu$  diam alumina spheres

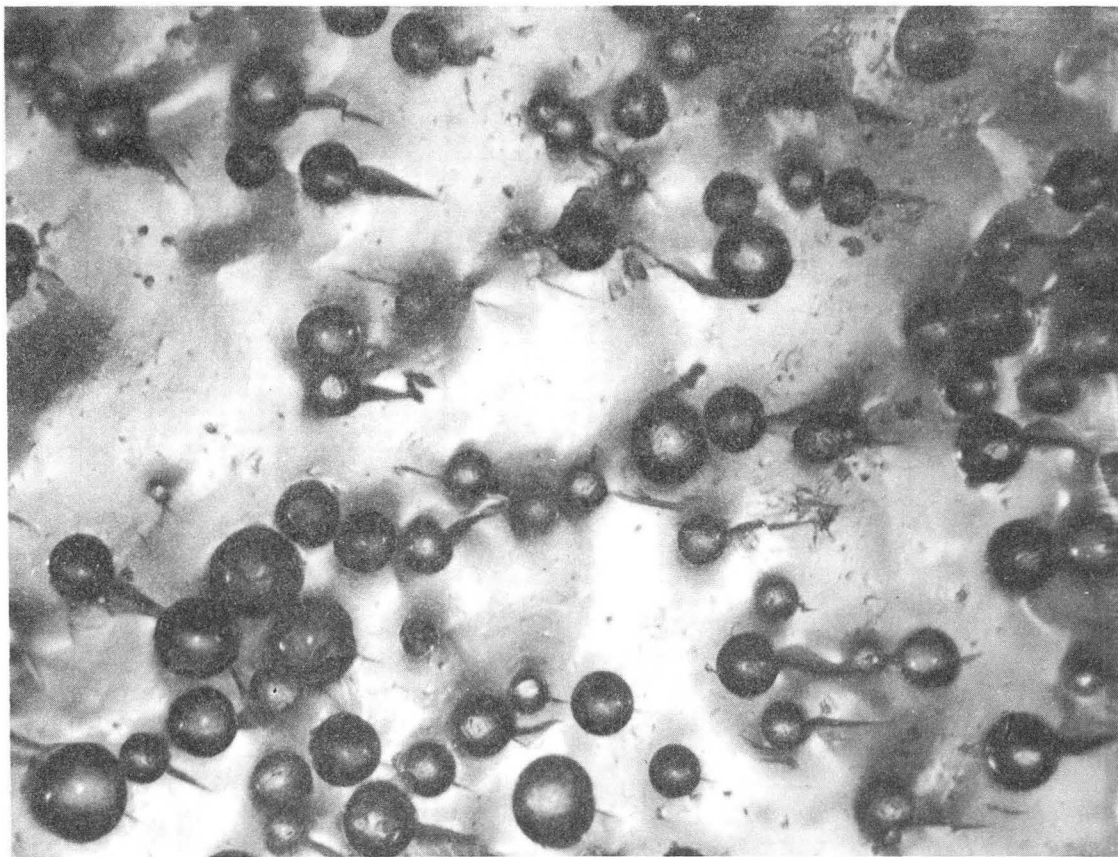


MUB-7857  
Fig. 2 Uniaxial strength of a soda borosilicate glass containing 60 $\mu$  diam spherical pores



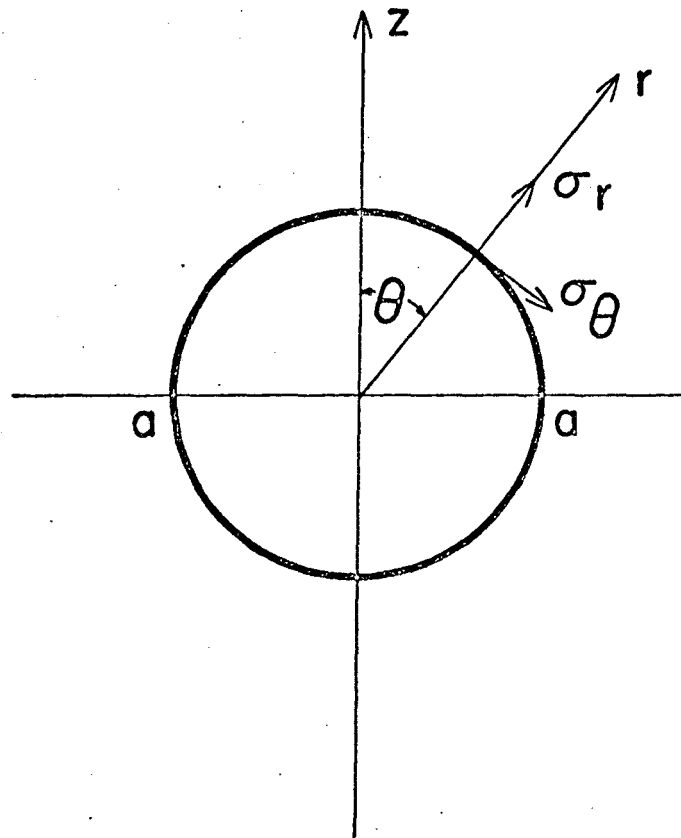
MUB-7855

Fig. 3 Biaxial strength of a soda borosilicate glass containing 60 $\mu$  diam. spherical pores



ZN-5269

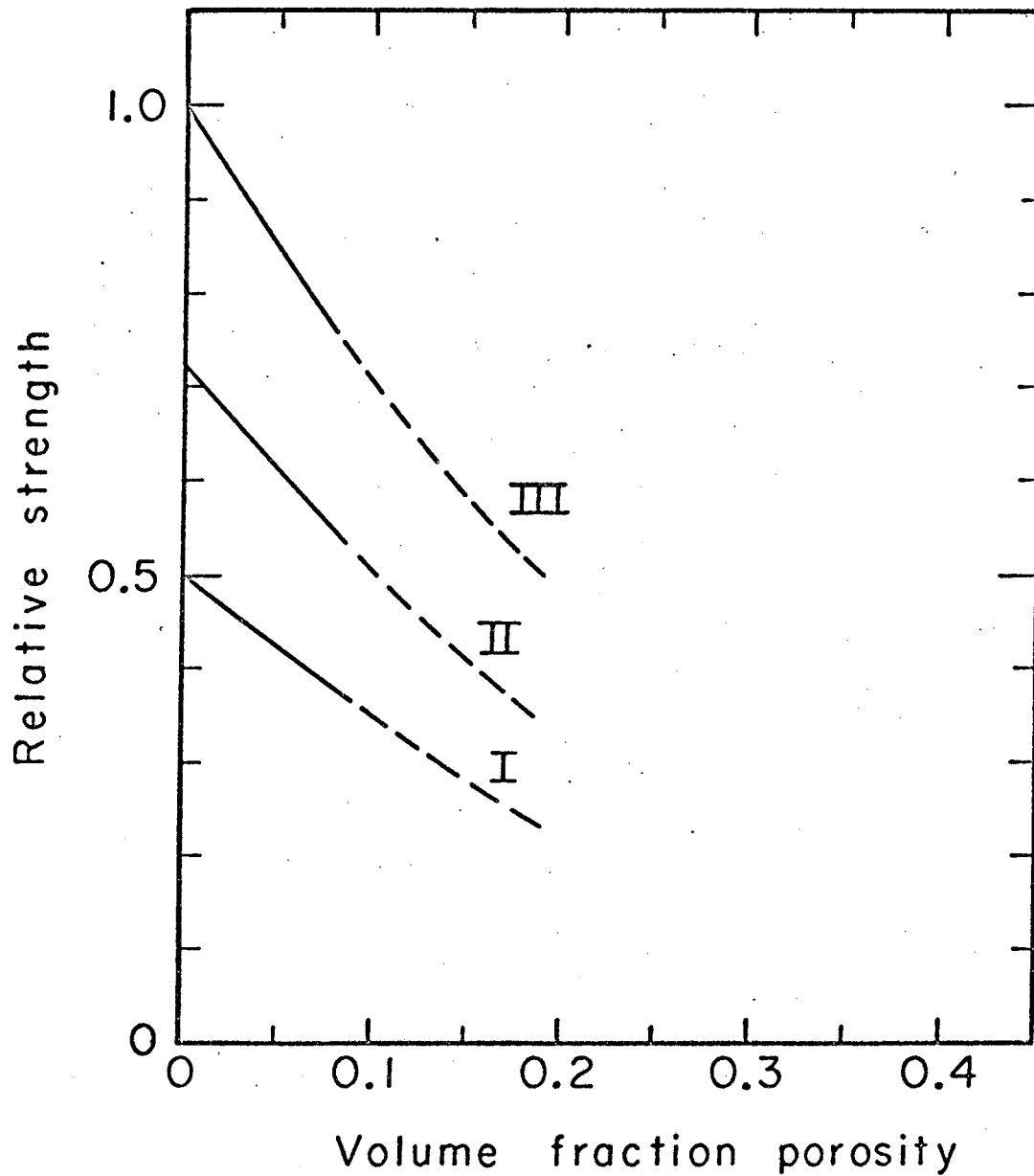
Fig. 4 Fracture surface of a soda borosilicate glass containing 15 vol %, 60 $\mu$  diam nickel spheres



MUB-7874

Fig. 5 Polar-coordinate system for the description of micromechanical stress concentrations





MUB-8103

Fig. 6 Proposed effect of micromechanical stress concentrations on the strength of a porous brittle material where: Case I. Flaw size  $\ll$  pore size; Case III. Flaw size  $\gg$  pore size; and Case II. Flaw size  $\approx$  pore size.

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