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A STATE SPACE MODEL OF THE ECONOMIC FUNDAMENTALS

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Key words: Dynamic programming, state space, nonstationary, unit root

Abstract

This paper presents a state space model of the economic fundamentals. In theory the economic fundamentals--tastes, technology, stochastic shocks, and initial wealth--determine the allocation of real resources and the values of financial assets. We show that in a recursive competitive equilibrium the minimal dimensional dynamic programming state vector is a sufficient statistic for the economic fundamentals. The dynamic programming state vector drives the allocation of real resources and the values of financial assets. We test this representation using the state space time series techniques recently introduced by Aoki. Financial and real capital do not have the same state space representation.

JEL Classification: 023, 131, 211

Introduction

In October of 1987 the Dow Jones industrial average fell more than 30% in a week and recorded the largest single day decline in the twentieth century. The US stock market collapse wiped out nearly a trillion dollars of financial wealth. Stock markets around the world shuddered in sympathy with the US market, declining anywhere from 15 to 50% over the same interval.

In theoretical economic models the value of financial assets reflects the value of the economic fundamentals. And most economists believe that, at least in the long run, financial asset values depend on the economic fundamentals. Yet almost a year after the massive October '87 meltdown no one has identified a change in the fundamentals that triggered the stock market crash. Nor did the crash send a reliable signal of a slowdown in future real economic activity. Recent GNP and corporate profit growth in the US exceeded the pre-crash estimates. In short, the stock market runup and subsequent crash in 1987 seems to have been an isolated incident independent of real economic activity.

This paper takes a more systematic look at the theoretical and empirical relationships between the values of financial and real assets. Stochastic general equilibrium models give precisely specified descriptions of economies where the economic fundamentals--tastes, technology, and stochastic shocks--determine the allocation of real resources and the value of

financial assets. But the testable implications of the famous and elegant Arrow-Debreu general equilibrium representation are few. Prescott and Merha (1980) had the keen insight to recognize that dynamic programming provides an extremely useful representation for testing general equilibrium theories with time-series data. Dynamic programming represents the equilibrium as a set of functions while Arrow-Debreu represents the equilibrium as a set of outcomes. A recursive dynamic programming state transition equation completely characterizes the essential elements of the economic system. The state vector is a minimal dimensional vector that summarizes all past decisions and current information. The state vector is a sufficient statistic for the economic fundamentals. Real allocation decisions are functions of the state vector. And the values of financial assets are functions of the state vector. Section 1 shows the theoretical relationship between the state vector (the economic fundamentals) and the value of real and financial assets.

Section 2 presents the results of tests of the theoretical restrictions implied by dynamic programming representation using the state-space times-series techniques developed by Aoki (1987). Aoki models observables as linear functions of the unobservable state vector. In theory, the same state vector should explain both physical and financial capital. In fact, bivariate and univariate modelling of the series give very different representations rejecting the hypothesis that the same

state vector describes both series. The values of financial and real capital do not appear to be driven by the same forces even in the very long run.

Systematic examination of the data for the post-WWII period leads to essentially the same conclusion as casual empiricism from the '87 stock market crash; financial asset values are not tightly linked to the economic fundamentals.

Section 1: A Simple General Equilibrium Model

In theoretical economic models households save to transfer consumption from the present into the future. In equilibrium a higher saving ratio implies more capital investment which increases future output and potential consumption. Adding financial markets puts another loop in the sequence, but it does not change the basic story. Households increase saving to accumulate financial assets which they plan to sell in the future for consumption. The increased demand for financial assets bids up their price. The portion of output not consumed gets invested in physical capital which increases future real output and potential consumption. Since financial assets are a claim on future real output an increase in the expected stream of future output is consistent with higher financial asset prices. Any economic model where financial asset values reflect economic fundamentals is a particular specification of this basic process.

This section presents a representative individual general equilibrium model to illustrate the restrictions imposed by the dynamic programming representation. We also present an example with a closed-form solution.

The Model

Household Preferences

The representative household is a stand-in for all households. The utility of the (infinitely lived) household depends on the

expected value of the time-separable discounted utility function,

$$1.1.1 \qquad \sum_{\tau=0}^{\infty} \beta^{\tau} E_t U(c_{t+\tau}, 1-z_{t+\tau}).$$

Instantaneous utility is strictly concave in consumption, c, and leisure, 1-z. β, the household time discount factor, is between zero and one.

Technology

The stochastic production function is a concave function of the factor inputs,

$$1.1.2 \qquad y_t = f(k_t, I_t, z_t, e_t)$$

$$k_{t+1} = I_t + (1-\delta)k_t$$

Capital, k_t, is predetermined. Current investment, I_t, adds to next period's productive capital but uses up some of current output, ie, there is a cost to adjusting capital. Labor, z_t, is a current choice variable. The exogenous productivity shock, e_t, is a strictly positive random variable that follows a first-order Markov process. δ is the depreciation rate.

1.2 The Central Planning Problem

The direct mathematical solution to the problem of efficiently allocating resources is the so-called central planning solution. An omnipotent planner selects a contingent plan for capital and labor (a real resource allocation plan) that maximizes the household utility function subject to the resource constraint

that,

1.2.1 $c_t + I_t = y_t$

consumption plus capital accumulation not exceed production. The economic fundamentals determine the solution to the central planning problem. A commodity's contribution to utility, its shadow price, measures its value. The central planning solution maximizes welfare and the allocation of resources is Pareto optimal.

Necessary Conditions

At a maximum capital must satisfy the Euler equation,

1.2.2 $1 - f_{It} = E_t[D_{t+1}\{f_{kt+1} + (1-\delta)(1-f_{It+1})\}]$,

where, $D_{t+1} \equiv \beta U_{ct+1}/U_{ct}$.

The Euler equation states that the expected discounted value of an additional unit of capital (the payoff in terms of increased output plus the consumption value of the unit of capital next period) equals the cost in terms of lost current consumption. The discount factor is the marginal intertemporal rate of substitution for consumption weighted by the household time discount factor.

And at a maximum labor must satisfy the condition,

1.2.3 $U_{1-zt}/U_{ct} = f_{zt}$,

that the marginal product of labor equals the ratio of the marginal utilty of leisure to the marginal utility of consumption--the shadow real wage.

The Dynamic Programming Solution

There are many ways to solve concave maximization problems. The dynamic programming solution is an extremely useful representation comparing the properties of a theoretical model with time-series data generated by the actual economy.

The dynamic programming solution for an infinite-horizon concave problem consists of three time-invariant recursive functions, eg, see Sargent (1987) Chapter 1. A state transition equation,

1.2.4 $$S_{t+1} = g(S_t, u_t, e_{t+1}),$$

summarizes the system; here S denotes the dynamic programming state vector and u the decision or control vector. The state vector is the **minimal dimensional** representation of the system. In general the state vector is neither unique nor observable, but the state vector has a unique minimum dimension. The state vector summarizes all past decisions and current information. Additional variables or functions of additional variables add no information that would change decisions. The state vector is a sufficient statistic for the economic fundamentals.

A decision function,

1.2.5 $$u_t = h(S_t),$$

gives the optimal decisions, u, as a function of the current state, here u is the vector of real allocations k and z.

The decision function maximizes the value of the objective function,

$$1.2.6 \qquad P(S_t) = \max_{u_{t+\tau}} \sum_{\tau=0}^{\infty} \beta^{\tau} E_t U(S_{t+\tau}, u_{t+\tau}); \; c_t = c(S_t, u_t),$$

subject to the transition equation 1.2.4. Recursively substituting the decision function and the transition equation into 1.2.6 gives,

$$1.2.6' \qquad P(S_t) = U(h(S_t), S_t) + \beta E_t[P(g(S_t, h(S_t), e_{t+1})],$$

$$= \max_{u_t} [U(u_t, S_t) + \beta E_t P(S_{t+1})],$$

the recursive form of the objective function.

The economic fundamentals are tastes, technology, the random shocks, and society's accumulated wealth. The dynamic programming state vector is a sufficient statistic for the fundamentals in the sense that all decisions can be written as functions of only the state vector.

1.3 A Decentralized Market Equilibrium

To determine the relationship between the economic fundamentals and financial asset values we need to examine a market economy. Decentralized decision making and free exchange in markets characterize a market economy. Firms produce commodities and demand labor and capital. Households demand commodities and supply labor and savings. Labor, commodities, and equities (financial assets) trade in competitive spot markets. Agents treat market prices as exogenous in their decision rules and form

rational expectations about future economic outcomes. It is well-known that a competitive equilibrium supports the Pareto optimal allocation when the constraint set is convex, eg, see Varian Chapter 5.

Households

The representative household wants to maximize the utility function 1.1.1 subject to its budget constraint. The budget constraint limits household consumption plus asset accumulation to,

1.3.1 $$c_t + (n_{t+1}-n_t)V_t = w_t z_t + n_t d_t$$

labor income plus dividend income.[1] Here V_t denotes the current (spot market) price of the firm's equity and d_t the dividend. n_t the "number of shares"[2] owned by the household at the beginning of the period and n_{t+1} is the number of shares owned by the household at the end of the period. The Modigliani-Miller theorem holds in this environment so V would represent the market value of the firm (equity plus debt) in a model with a richer set of financial contracts. w_t is the spot market wage. The spot market prices are relative to the price of consumption which we normalize at one. The household chooses contingent plans for

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[1] The additional constraint that $\beta^\tau V_{t+\tau}$ goes to zero as τ goes to infinity is required to rule out unbounded borrowing (short sales).

[2] We assume there is one share of infinitely divisible stock outstanding in the firm. So, $0 \leq n \leq 1$, is the fraction of the firm owned by the household, and V is the equity value of the firm.

labor and asset accumulation.

At a maximum the household chooses to accumulate (or sell) shares of stock until,

1.3.2 $V_t = E_t[D_{t+1}\{V_{t+1}+d_{t+1}\}]$,

where, $D_{t+1} \equiv \beta U_{ct+1}/U_{ct}$,

the expected discounted value of the stock equals the current cost. This is the deservedly famous consumption-capital asset pricing equation. The household supplies labor until,

1.3.3 $U_{1-zt}/U_{ct} = w_t$,

the shadow real wage equals the spot market wage.

Firms

The owners of the firm instruct the firm manager to choose contingent plans for capital and labor that maximize the expected value of the stream of discounted dividends,

1.3.4 $$W_t = \max_{k_{t+1+\tau}, z_{t+\tau}} \sum_{\tau=0}^{\infty} E_t D_{t+\tau} d_{t+\tau} = \max_{k_{t+1}, z_t}\{d_{t+1} + E_t[D_{t+1}W_{t+1}]\}.$$

The firm returns net earnings to the shareholders in dividends,

1.3.5 $d_t = y_t - w_t z_t - I_t \equiv f(k_t, I_t, z_t, e_t) - w_t z_t - I_t$.

Substituting the definition of dividends into the household budget constraint, equation 1.3.1, and aggregating over households so $n_{t+1}=n_t=n$ (ie, households own all the outstanding shares of stock) gives the central planning resource constraint, equation 1.2.1. Real resource decisions constrain household

consumption. An individual household can rearrange its intertemporal consumption path by trading financial assets but society cannot. Firms are households' agents. Maximizing W maximizes the current equity value of the firm and the dividend, or the owners' wealth.

At a maximum the firm invests until,

1.3.6 $$1 - f_{It} = E_t[D_{t+1}\{f_{kt+1} + (1-\delta)(1-f_{It+1})\}],$$

the expected discounted value of an additional unit of capital equals the cost of a unit of capital in terms of lost sales. And it hires labor until,

1.3.7 $$f_{zt} = w_t,$$

the marginal product of capital equals the real wage.

It is easy to verify that the market equilbrium is Pareto optimal. The firm's necessary condition for capital accumulation is the Euler equation in the central planning problem. And in equilibrium the marginal product of labor equals the spot market wage, equation 1.3.7, which equals the household's shadow wage, equation 1.3.3.

Now that we have specifed the equilibrium conditions we can write the value of equity as a function of the dynamic programming state vector--the economic fundamentals. Since the competitive equilibrium allocation of resources equals allocation in the central planning problem we can write the firm's decision rules

as,

$$1.3.10 \qquad \begin{bmatrix} k_{t+1} \\ z_t \end{bmatrix} = \begin{bmatrix} k(S_t) \\ z(S_t) \end{bmatrix} = u_t = h(S_t).$$

functions of the dynamic programming state vector. The transition equation 1.2.4 gives the evolution of the state variables.

Recursive substitution of the decision rules, the transition equation, and the definition of variables, into the firm's objective function 1.3.4 gives,

$$1.3.11 \qquad W(S_t) = d(S_t) + E_t[D(S_{t+1})W(S_{t+1})],$$

a recursive form of the objective function that only depends on the dynamic programming state vector. And since,

$$1.3.12 \qquad W(S_t) - d(S_t) \equiv V_t$$
$$= E_t[D_{t+1}(V_{t+1}+d_{t+1})] = E_t[D(S_{t+1})W(S_{t+1})],$$

the value of financial assets depends on the dynamic programming state vector. Furthermore, if an element of the state affects the real allocation it also affects the equity value.

The asset valuation equation 1.3.12 is similar to Ross's (1976) popular Arbitrage Pricing Theory representation. Ross focuses on a partial equilibrium model of asset prices. Unobservable "factors" determine the asset prices. In a general equilibrium the state vector determines asset values and the real allocation. In principle this provides a testable restriction. The factor

models used to implement Ross's Arbitrage Pricing Theory, should also explain real allocations, and vice-versa. Section 2 tests these restrictions using Aoki's state-space modelling techniques.

1.4 An Example

This example illustrates the linkage between the state vector and the real allocation and financial values. The example is based on the examples in Brock (1982) and Long and Plosser (1983). Let the instantaneous utility function in 1.1.1 be,

1.4.1 $$U(c_t, 1-z_t) = \ln(c_t) + U(1-z_t),$$

a logarithm function of consumption plus a concave function of leisure. And define technology by a homogeneous power function in the factor inputs times a random productivity shock,

1.4.2 $$y_t = f(k_t, z_t, e_t) = k_t^{a} z_t^{1-a} e_t.$$

And, assume capital has a one-period life (δ=1) so the resource constraint becomes,

1.4.3 $$c_t + I_t = y_t = c_t + k_{t+1}.$$

The solution to this example is well known. Let y_t be the state variable. y summarizes all past decisions, k, and current information, e. Conjecture that the capital accumulation rule is a linear function of the state,

1.4.4 $$k_{t+1} = a\beta y_t$$

$$z_t = z,$$

and that labor is constant. Then the resource constraint, 1.4.3, defines consumption,

1.4.5 $c_t = \{1-a\beta\}y_t,$

as a linear function of the state.

The state transition equation is a log-linear function,

1.4.6 $y_t = g(y_{t-1},u_t,e_t) = k_t^a z^{1-a} e_t,$ or

$\ln(y_t) = a\ln(y_{t-1}) + \ln(e_t) + \text{constant}.$

Notice the transition equation can be nonstationary, eg, e_t could be a log-normally distributed random walk with drift, as long as the discounted programming problem is bounded.

To verify that the conjectured solution gives the Pareto optimal allocation substitute the decision rules into the necessary conditions for the central planning problem giving,

1.4.7 $1/c_t = \beta E_t[ay_{t+1}/k_{t+1}c_{t+1}],$

$k_{t+1}/\{1 - a\beta\}y_t = \beta E_t[ay_{t+1}/\{1 - a\beta\}y_{t+1}] = a\beta/\{1-a\beta\},$

or, $k_{t+1} = a\beta y_t,$

and,

1.4.8 $\{1-a\beta\}y_t U_{1-zt} = (1-a)y_t,$ or

$U_{1-zt} = \{1-a\beta\},$

confirming the conjecture.

To express the equity value as an explicit function of the state, note that,

1.4.9 $d_t = y_t - w_t z - k_{t+1} = ay_t - a\beta y_t,$

dividends are a linear function of the state variable and that the discount factor equals,

1.4.10 $D_{t+\tau} \equiv \beta U_{ct+\tau}/U_{ct} = \beta y_t/y_{t+\tau}$.

Now substituting 1.4.9 and 1.4.10 into the firm's objective function, equation 1.3.4, gives,

$$1.4.7\quad W(y_t) - d(y_t) = V(y_t) = \max_{k_{t+1+\tau}, z_{t+\tau}} \sum_{\tau=1}^{\infty} E_t D_{t+\tau} d_{t+\tau}$$

$$= \beta y_t \sum_{\tau=1}^{\infty} E_t[(1/y_{t+\tau})a(1-\beta)y_{t+\tau}] = a\beta y_t.$$

the equity value of the firm as a linear function of the state.

In this example a single observable state variable, y, summarizes the economic fundamentals. A single variable is sufficient to represent accumulated wealth, k_t, and the current shock, e_t. The real allocations, c_t and k_{t+1}, and the equity value, V_t, are the same linear functions of the state.[3] Of course the particular solution depends on the parameterization. But the example illustrates the general proposition that the state vector--the proxy for the economic fundamentals--drives real allocation decisions and financial asset values.

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[3] The production function is homogeneous of degree one and there are no cost to adjusting capital so Tobin's q equals one.

Section 2: Empirical Evidence

In principle the dynamic programming representation of the recursive general equilibrium imposes testable restrictions. The dynamic programming state vector, S, is a sufficient statistic for the economic fundamentals. The dynamic programming state vector drives the allocation of real resources and the value of financial asset. In principle one could test the restriction that the economic fundamentals drive financial asset value by estimating an equation of the form,

2.1.1 $y_t = C(S_t)$,

where y is a vector containing real variables, such as the capital stock, and financial asset values, such as the equity value of the firm.

In practice one must make some additional assumptions to confront the data. The functional forms are unknown and the state vector is unobservable. We use Aoki's (1987,1988) state space modelling procedures to estimate a linearized version of the 2.1.1 and to identify and a linear transition equation for the unobservable states.

2.2 Estimation Procedure

Aoki uses a state space "innovation" model of the form:

2.2.1a $y_t = Cs_t + r_t$

2.2.1b $s_{t+1} = As_t + Br_t$

where y_t is a k x 1 vector of data observed at time t, s_t is a

n x 1 (minimal dimensional) vector of unobserved state components, and r_t is the weakly stationary innovation of the orthogonal projection of y_t onto its past values, i.e. $r_t = y_t - y_{t|t-1}$, where $y_{t|t-1}$ denotes the linear projection of y_t onto the space spanned by past observations.

Given the dimension of the state vector, n, the form in 2.2.1 imposes enough structure to estimate the parameter matrices and recover the unobserved "states" from the data. The "states" in the estimation model, s, must lie in the space spanned by the past observations. So s_{t+1} is a linear projection of the true state onto the past observations, $S_{t+1|t}$. The residual innovation, r, contains the systems error, e, and any projection error.[1]

A, B, and C are matrices whose elements are to be estimated; the transition matrix, A, is n x n, B is n x k, and C is k x n in dimension. The dimension of the state vector (n) is in general not known a priori, and is chosen on the basis of information contained in the autocovariances of the data and on the goodness of fit of the final model.

When the data contains unit root, or near unit root, components

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[1] Suppose the true system were linear,
$y_t = CS_t$
$S_{t+1} = AS_t + e_{t+1}$,
then the best the econometrican can recover from the data is a system like 2.2.1 where $S_t = s_t + e_t$.

they will tend to overwhelm any state components whose dynamics are less long lasting, this fact leads Aoki (1988) to recommend a two step procedure in estimation. In the first step a model like 2.2.1 is fit for the trend components, the eigenvalues of the transition matrix estimated for these components should have large magnitude (usually close to one). Since whatever nontrend, or cyclical, components present in the data are ignored in this first step the residuals will typically be autocorrelated. A second innovation model is then fit to the residuals to capture these components; the residuals in the second step should look like white noise and the eigenvalues of the transition matrix should be much smaller than the eigenvalues from the first step. If we let s_{1t} denote the n_1 trend components, and s_{2t} the n_2 cyclical components ($n_1 + n_2 = n$), then this procedure results in a trend-cycle decomposition which can be written as:

$$2.2.2a \quad y_t = [C_1 \; C_2] \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + r_t$$

$$2.2.2b \quad \begin{bmatrix} s_{1t+1} \\ s_{2t+1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1C_2 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} r_t$$

The structure of this system is block recursive, the cyclical components are assumed to affect, but not be affected by, the trend components. This method allows for variables which share common trend components, as in Engle and Granger's (1987) definition of co-integration. The structure is not restrictive

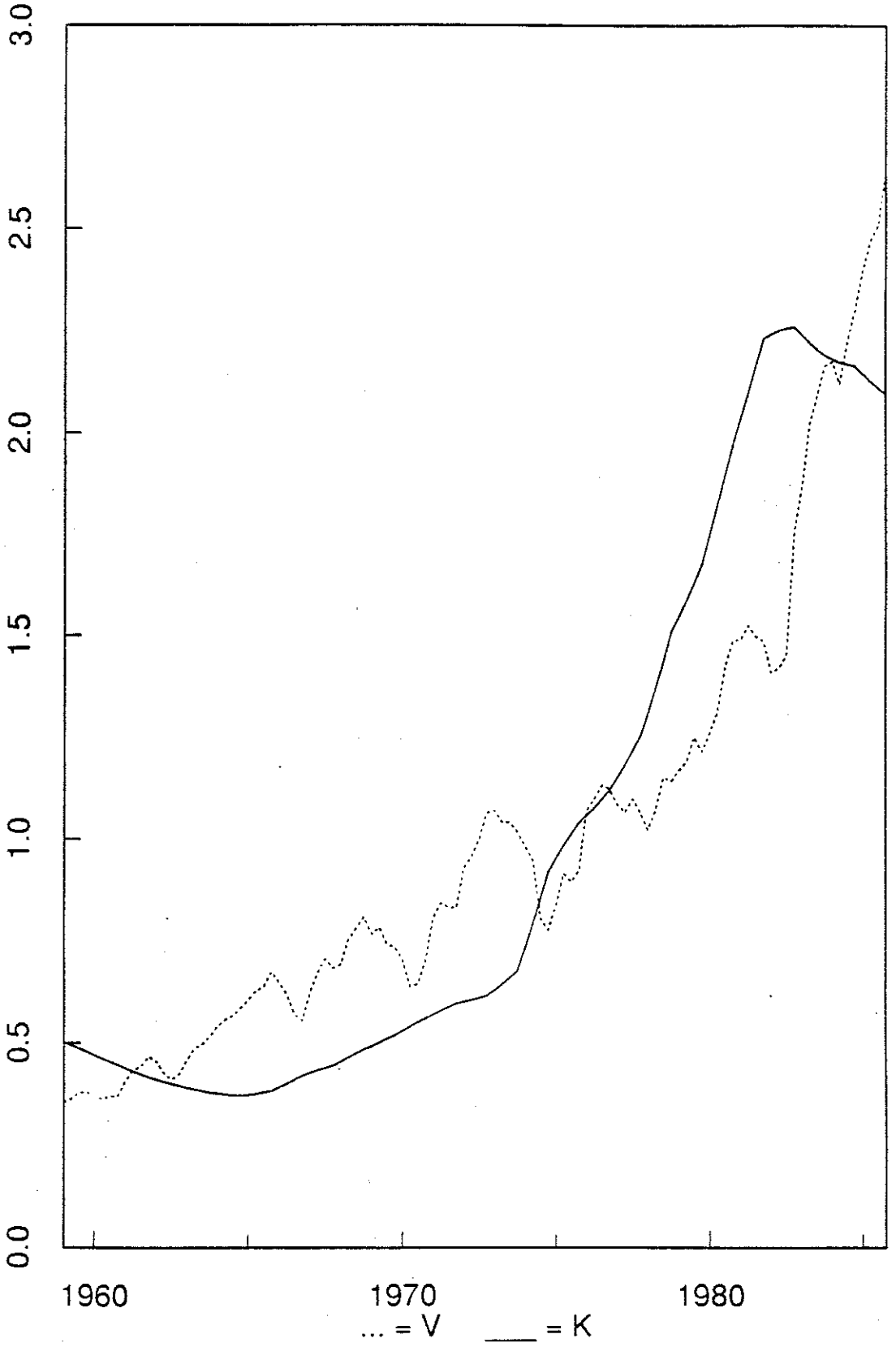
since the state variables may always be redefined so that they have a recursive form.

Estimation

The data appendix gives a detailed description of the data. V_t, the market valuation of the firm, is the real value of equity plus debt of all nonfinancial firms. V represents the financial valuation. We chose the capital stock, K_t, to represent the real allocation decisions. Capital is a choice variable that allows society to transfer consumption between periods. It seems likely that the same economic factors affect real capital and financial values even if the model we used in section 1 doesn't hold. K is the net real capital stock (using an annual depreciation rate of 10%) in all manufacturing as measured by DRI. The observations run from the third quarter of 1958 through the fourth quarter of 1985. Figure 1 shows the series scaled by their sample means. Both series display a strong upward drift over the sample and the renowned volatility of the stock market shows up in the V series.

The null hypothesis is that the same economic factors--the fundamentals--explain both series. To test the hypothesis we fit univariate models to each series. Under the null the univariate models should have the same state transition equation since the same states drive both series. Then we fit a bivariate model that restricts the state generating process to be the same for the two series. Under the null the bivariate model is simply the vector

Fig.1 Capital and Market Valuation



of the univariate models coupled with a transition equation.

In a univariate fit of the V_t series we found strong evidence of a single trend component in the first step of estimation (the transition matrix for this component, which in this case is scaler, was .94). It was only necessary to fit one further cyclical component in order to match the series well, the final model is:

$$V_t = [623.8 \; 58.7] \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + r_t$$

$$\begin{bmatrix} s_{1t+1} \\ s_{2t+1} \end{bmatrix} = \begin{bmatrix} .94 & .09 \\ 0 & .81 \end{bmatrix} \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + \begin{bmatrix} .002 \\ .007 \end{bmatrix} r_t$$

The residuals from this model have good general characteristics, the first two autocorrelations are .028 and .046 respectively; and only one out of the first ten autocorrelations (the ninth) is significant.

The dynamics of a univariate fit of K_t were somewhat more complex, after some experimentation the following model was chosen[2]:

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[2]There is another representation with three state components which also fits well, in the first step two trend components were fit (the eigenvalues of the estimated transition matrix are both .976 in magnitude), and a further component was fit in the second step (with an eigenvalue of .912) - thus this representation fits three very long lasting state components. We chose to deal with the model shown in the text because its representation with two state components seems more parsimonious, and because it seems,

$$K_t = [205.3 \;\; 8.5] \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + r_t$$

$$\begin{bmatrix} s_{1t+1} \\ s_{2t+1} \end{bmatrix} = \begin{bmatrix} .98 & .04 \\ 0 & .94 \end{bmatrix} \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + \begin{bmatrix} .005 \\ .106 \end{bmatrix} r_t$$

This representation has essentially two unit root, or near unit root, components, with a recursive structure so that the second component is not affected by the first. Again, the residuals have good characteristics, the first two autocorrelations are .035 and .06 respectively, and none of the first ten autocorrelations is significant.

The transition equations for the univariate representations have the same general form, but, the second state component of in the two models seems quantitatively different. After five years about 29% of an innovation in the second component of the capital stock series will remain, while only 1.5% of an innovation in the second component of the market valuation series will still be present. (Figure 1 shows these characteristics. The V series is very volatile reflecting the infamous random walk stock market component. The K series is much smoother although it also contains nonstationary components.)

If the two series are indeed run by the same state components

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if anything, to be less favorable to our conclusions.

then a bivariate model of the same form as the univariate models (two state components with a recursive structure) should do roughly as well as either univariate fit. The estimated bivariate model is:

$$\begin{bmatrix} K_t \\ V_t \end{bmatrix} = \begin{bmatrix} 199.6 & -30.6 \\ 609.6 & 54.6 \end{bmatrix} \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + r_t$$

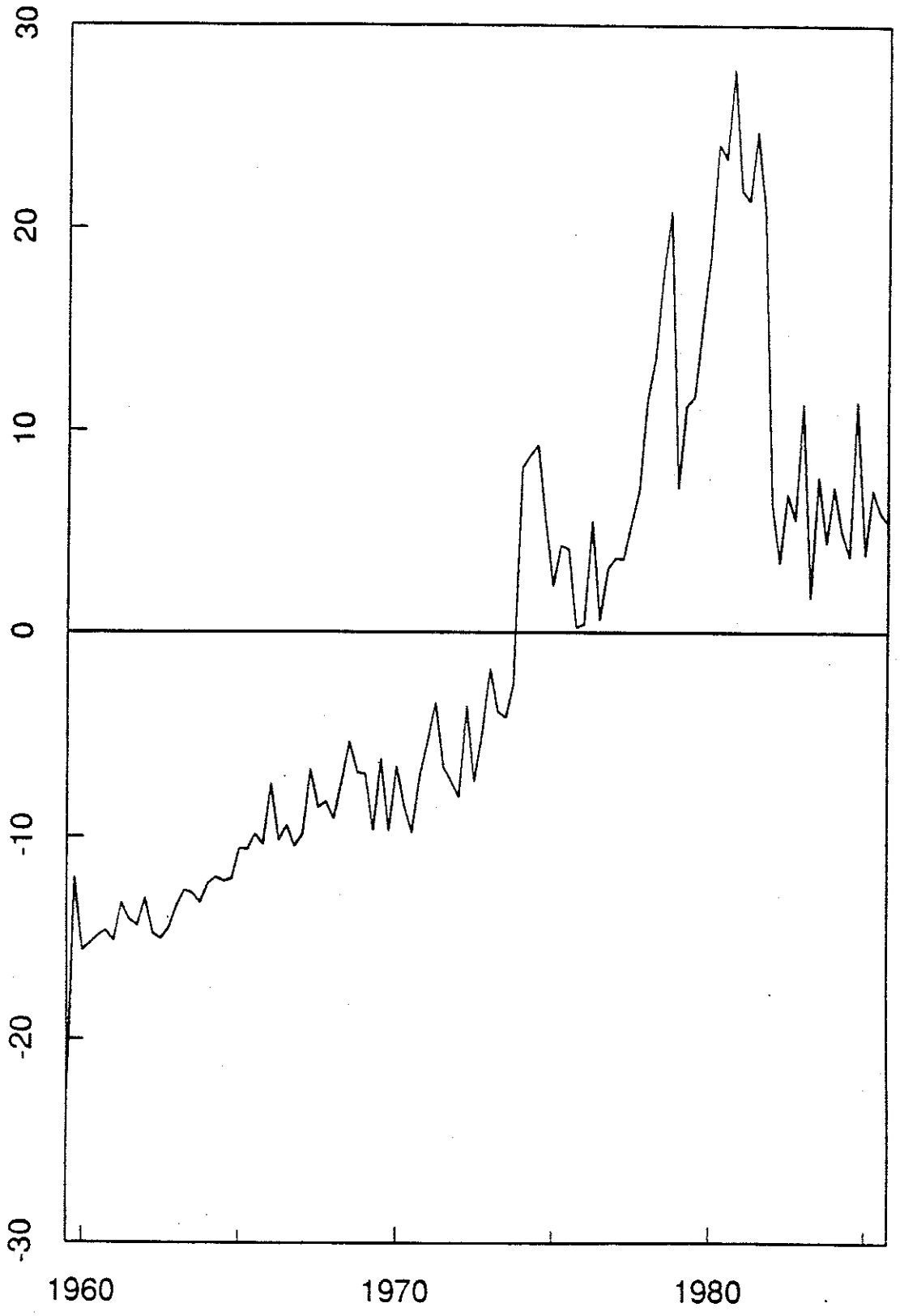
$$\begin{bmatrix} s_{1t+1} \\ s_{2t+1} \end{bmatrix} = \begin{bmatrix} .97 & -.05 \\ 0 & .88 \end{bmatrix} \begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} + \begin{bmatrix} .004 & .004 \\ -.011 & .015 \end{bmatrix} r_t$$

Restricting the explanation of the financial valuation series and the capital stock series to the same state vector (the economic fundamentals) wreaks havoc.

The capital stock appears to have a low frequency component which smooths the series that is not present in the financial series. Forcing the series to share a common state vector creates major problems. The first autocorrelation of the residuals for the capital stock is .92; in addition, the residuals have a noticeable upward trend (see Fig.2). The first autocorrelation of the residuals for the market valuation is smaller (.34) but significant. The constrained model produces a series too smooth to fit the observed market valuation series and a series that does not grow fast enough to fit the capital series.

The data do not support the hypothesis that the same state vector drives financial and physical assets. The series seem to share a common stochastic trend but capital requires a second trend or

Fig. 2 Residuals for capital stock from bivariate model



very low frequency component to explain the data. Since the second component for the capital stock series is a near unit root component, the two series may drift apart for long periods of time, perhaps permanently.[3]

Section 3: Summary

This paper examines the theoretical and empirical relationships between the economic fundamentals and financial and physical capital. We use the dynamic programming representation of a recursive competitive equilibrium to define the economic fundamentals. The dynamic programming transition equation is a minimal dimensional representation of the system. In Section 1 we show that the dynamic programming state vector drives both real allocation decisions and the values of financial assets. The dynamic programming state vector is a sufficient statistic for the economic fundamentals.

In Section 2 we test the restrictions implied by the theory using Aoki's state space modelling techniques. The data are not kind to the restrictions. The value of financial and real capital do not appear to be driven by the same forces even in the very long run.

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[3] This evidence is consistent with the results from co-integration tests and Stock-Watson tests that indicate the series may drift apart for very long periods, see Craine (1988).

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Data Appendix:

Definitions

NV = MVD + MVE

MVD = INT/YA, the market value of debt

MVE = DIV/YSP, the market value of equity

This follows Abel and Blanchard's construction of the financial value of the firm, see their appendix. The data come from DRI's data bank with the DRI mnemonic in parenthesis.

INT is net interest payments by nonfinancial business corporations (INTBUSCORPNF)

YA is the yield on Moody's A corporate bonds (RMMBCANS)

DIV is dividends paid by nonfinancial business corporations (NFCDIV)

YSP is the quarterly average of the monthly yield on the S&P 500.

NK is nonresidential manufacturing capital (KGFIXNRM) interpolated to follow the quarterly pattern of investment in plant and equipment (IP&EM)

More Definitions

V = NV/PUNEW, financial value of the firm in consumption units

K = NK/GDIF, real value of capital

The remaining data series come from CITIBASE. All capital letters indicate the CITIBASE mnemonic.

PUNEW is the consumer price index for all urban consumers

GDIF is the implicit price deflator for gross private domestic investment.