Title
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A DETERMINATION OF THE EFFECTIVE B AND ρ TRAJECTORIES FROM THE REACTION $\pi^+ p \to \omega\Delta^{++}$ *

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ABSTRACT

We present results of two high-statistics bubble chamber measurements of the reaction $\pi^+ p \to \omega \Delta^{++}$ at incident momenta of 2.67 and 7.1 GeV/c. The energy dependence of the natural parity exchange projection gives a Regge trajectory consistent with the accepted ρ trajectory. The unnatural parity exchange projection gives a B trajectory consistent with our current knowledge of the pion trajectory.

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We report the results of two high-statistics bubble chamber measurements of the differential cross section and $\omega°$ decay angular distributions for the reaction $\pi^+ p \rightarrow \omega° \Delta^{++}$ at the incident momenta of 2.67 and 7.1 GeV/c. The data are analyzed in terms of the quantum numbers exchanged in the t-channel. The ω decay distributions are used to project out the exchange contributions of specific spin-parity series and ω helicities. The high statistics allows us to use the energy dependence at several values of the momentum transfer to calculate the corresponding effective Regge trajectories.

The measurements are based on a 300K picture (2.67 GeV/c) exposure of the 25 inch hydrogen bubble chamber at the Bevatron and a 700K picture (7.1 GeV/c) exposure of the 82 inch bubble chamber at SLAC. The four-prong events were fit to the hypothesis $\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \pi°$. At 2.67 GeV/c events satisfying the 1-C hypothesis were additionally constrained to the 2-C hypothesis $\pi^+ p \rightarrow \pi^+ p \omega$ followed by $\omega \rightarrow \pi^+ \pi^- \pi°$. The criterion $\chi^2(2C) - \chi^2(1C) \leq 10$ was then used to define the ω sample. At 7.1 GeV/c the ω sample was defined by the mass interval 0.760 GeV $\leq M(\pi^+\pi^-\pi°) < 0.805$ GeV. At both momenta the mass interval 1.16 GeV $\leq M(\pi^+ p) \leq 1.28$ GeV was then used to select Δ^{++} events. Further details about the measurement, constraint, and normalization are described elsewhere [1,2], but a brief summary of the results is given in table 1.

For the total and differential cross sections a detailed background subtraction was made. For the decay angular distributions no correction for background was made. The selected $\omega° \Delta^{++}$ sample at 7.1 GeV/c contains about 8% background below the momentum transfer $|t| = 0.7$ $(\mathrm{GeV}/c)^2$ and about 16% for the higher $|t|$ values. The sample at 2.67 GeV/c contains about 20% background below $|t| = 0.8$ $(\mathrm{GeV}/c)^2$,

rising to about 40% for the higher $|t|$ values. The background is approximately evenly distributed between $\omega°$ non Δ^{++} and Δ^{++} non $\omega°$ events.

In order to isolate the contributions to the cross section of a given spin-parity series, we form the following combinations [3,4]:

(a) $\rho_{00}\, d\sigma/dt$ measures the ω helicity non-flip unnatural parity (B) exchange.

(b) $(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$ measures the ω helicity flip natural parity (ρ) exchange.

(c) $(\rho_{1,1} - \rho_{1,-1})\, d\sigma/dt$ measures the ω helicity flip unnatural parity (B) exchange.

(d) Letting $\rho^+ = \rho_{1,1} + \rho_{1,-1}$, the fraction of natural parity exchange, we form $\rho^+/(1-\rho^+)$, which measures the ratio of natural parity exchange to unnatural parity exchange; this ratio is independent of the normalization problems associated with $d\sigma/dt$.

In these expressions, ρ_{00} and $(\rho_{1,1} - \rho_{1,-1})$ may be evaluated in either the t-channel or s-channel helicity frame; $(\rho_{1,1} + \rho_{1,-1})$ and the ratio $\rho^+/(1-\rho^+)$ are frame independent. The above combinations are shown in fig. 1 for both momenta. A shoulder is observed in $\rho^t_{00}\, d\sigma/dt$ (fig. 1b) around $t = -0.3 (\mathrm{GeV}/c)^2$. Structure in this same t region is generally observed in this reaction [5-8], especially in ρ^t_{00}.

The combination $\rho^+/(1-\rho^+)$ (fig. 2a) shows that for all t values the fraction of natural parity exchange is substantially greater at 7.1 GeV/c than at 2.67 GeV/c. Discussion of some other features of figs. 1 and 2 is deferred to a later section.

We obtain effective trajectories from the relation

$$p^2_{lab}\, (d\sigma/dt)^c = G(t)\, \nu^{2\alpha_{eff}}; \quad \nu = (s-u)/2, \tag{1}$$

where $(d\sigma/dt)^c$ refers to one of the above combinations of density matrices and differential cross section. Assuming eq. (1) for both natural and unnatural parity exchange, it is readily shown that

$$\rho^+/(1-\rho^+) = H(t)\, \nu^{2(\alpha_N-\alpha_U)_{eff}} \qquad (2)$$

where α_N and α_U refer to natural and unnatural parity exchange, respectively. For a given t value, the log of the left-hand side of eq. 1 or 2 is plotted against the log of ν. The slope of a straight line fit to the data points yields $2\cdot\alpha_{eff}$ at that t value. In fig. 3 we show such plots at sample t values for $(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$, $\rho^+/(1-\rho^+)$ and $\rho^t_{00}\, d\sigma/dt$. The ν values are for our data together with published data at 3.7[5], 5.0[6], 5.5[7], and 11.7[8] GeV/c.

The results of a least-squares fit (to obtain $2\cdot\alpha_{eff}$) are shown for each plot[9]. It is clear that the high statistics data presented in this paper dominate these fits. It is interesting that (as previously indicated in fig. 2a) at all values of momentum transfer the fraction of natural parity exchange increases markedly with incident momentum. This effect has also been observed[12] in the reaction $\pi^-p\to\omega^\circ n$ and is expected if the higher-lying trajectories dominate at high energies.

Figure 4a shows the results for the effective trajectory from $(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$. The data are in remarkable agreement with the accepted ρ trajectory[13] (dashed line in fig. 4a) of 0.57 + 0.91t. A least-squares fit of the data points[14] to the expression $\alpha(t) = \alpha_0 + \alpha' t$ yields

$$\alpha_\rho = (0.56 \pm 0.10) + (0.78\pm 0.31)t;\ \delta\alpha_0\, \delta\alpha' = 0.029.$$

Figure 4b shows $(\alpha_N-\alpha_U)_{eff}$ from $\rho^+/(1-\rho^+)$. The slope is rather small; a least-squares fit yields

$$(\alpha_N-\alpha_U) = (0.52\pm0.11) + (0.18\pm0.32)t;\ \delta\alpha_0\delta\alpha' = 0.033.$$

To obtain α_U, we form $\alpha_U = \alpha_\rho - (\alpha_N-\alpha_U)$, where we take α_ρ to be a nominal trajectory of 0.57 + 0.91t. The subtraction is made point by point with the results shown in fig. 4c. This plot is taken as a measure of the B-meson trajectory. A least-squares fit yields

$$\alpha_B = (0.05\pm 0.11) + (0.73\pm 0.32)t.$$

This result has rather attractive features as a candidate for the B trajectory. It passes close to $\alpha = 0$ and $t = 0$, as would be expected from π-B exchange degeneracy. Further, for a B of spin-parity of 1^+, the trajectory passes quite close to the B mass in the unphysical t region. Estimates of the effective π trajectory from $pp\to n\Delta^{++}, \Delta^\circ\Delta^{++}$ have always indicated a relatively flat π trajectory with intercept near zero. The results of more recent determinations of the effective π trajectory using $\rho^s_{00}\, d\sigma/dt$ for the reaction $\pi^-p\to\rho^\circ n$ between 4.5 and 17.2 GeV/c[15], using compilations of data for the reactions $\pi^+p\to\rho^\circ\Delta^{++}$ and $K^+p\to K^{\circ *}\Delta^{++}$[16], and using the reaction $\gamma n\to\pi^-p$ with polarized photons[17] are in substantial agreement with our result.

The method of determining the above value of the B trajectory has the advantage that it does not depend on cross-section normalization. It has the disadvantage of incorporating a nominal value for the ρ trajectory. Figure 4d shows α_{eff} from $\rho^t_{00}\, d\sigma/dt$, which depends on cross-section normalization, but not the nominal ρ. The data points give the same general trend as for 4c. A least-squares fit[18] yields

$$\alpha_B = (0.0\pm 0.08) + (0.59\pm 0.23)t;\ \delta\alpha_0\delta\alpha' = 0.017.$$

The close agreement between this and the previous result is basically a consistency check on both the data and the assumption that the reaction is dominated by B + ρ exchange.

In obtaining the effective ρ and B trajectories we have necessarily ignored the possible effects of absorption on our reaction. If $(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$ could be completely explained by ρ-pole exchange, we would expect to see a dip in this distribution near $t = -0.6\ (\mathrm{GeV}/c)^2$ (fig. 1d), but no significant dip is evident. Figure 2a shows that the natural parity exchange contribution becomes very small in the region of the expected dip, making the observation of such a dip difficult in most experiments. The absence of such a dip could also be the result of absorptive corrections filling in the dip. This would be in accordance with an empirical rule which states that amplitudes involving no net helicity flip in the s-channel (n = 0) have strong absorptive corrections, while those involving one unit of helicity flip (n = 1) are only weakly affected by absorption. If we assume that the ρ chooses unit helicity flip at the nucleon vertex [19,20] then $(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$ should involve n = 0, 2, and therefore include strong absorptive effects. The status of n = 2 with respect to absorption is unclear. Chiu [20] has proposed that $\rho^s_{1,-1}$ should pass through zero when the ρ trajectory passes through zero if the n = 2 amplitude is not strongly affected by absorption. The quantity $\rho^s_{1,-1}$ is shown in fig. 2b. It does pass through zero near $t = -0.6\ (\mathrm{GeV}/c)^2$, but the crossing point moves to higher $|t|$ with increasing incident momentum.

In conclusion, we have determined the effective ρ and B trajectories from the reaction $\pi^+ p \to \omega\Delta^{++}$. The ρ trajectory is in good agreement with that determined from $\pi^- p \to \pi^0 n$. The B trajectory is consistent with what is known about the effective π trajectory. Although the data indicate that absorption effects may be present, the above agreement and the self-consistency of the B trajectory determination lead us to expect that the effective trajectory calculation may still be reliable.

References

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2. R. L. Ott (Ph. D. thesis), Lawrence Berkeley Laboratory Reports LBL-1547 (1972) and W. F. Buhl et al., LBL-2431.

3. The method of moments is used to calculate

$$\rho^{s,t}_{00} = \frac{1}{2}\ \langle 5\cos^2\theta_\omega - 1\rangle,\quad \rho^{s,t}_{1,-1} = -\left(\frac{5}{4}\right)\langle \sin^2\theta_\omega \cos 2\phi_\omega\rangle,$$

$$\mathrm{Re}\,\rho^{s,t}_{1,0} = -\left(\frac{5}{4\sqrt{2}}\right)\langle \sin 2\theta_\omega \cos\phi_\omega\rangle,$$

where θ_ω, ϕ_ω are the angles of the normal to the ω^0 decay plane $(\vec{q}_{\pi^-} \times \vec{q}_{\pi^+})$ in the t-channel helicity (Gottfried-Jackson) or s-channel helicity frame. Note that $\rho_{1,1} = 1/2(1-\rho_{00})$.

4. K. Gottfried and J. D. Jackson, Nuovo Cimento <u>33</u> (1964) 309; D. Grether, Powell-Birge Physics Memo 178 (Lawrence Berkeley Laboratory, unpublished). The combination $\rho^t_{00}\, d\sigma/dt$ is valid at all values of s, the rest to order 1/s.

5. G. S. Abrams et al., Phys. Rev. Lett. <u>25</u> (1970) 617.

6. C. L. Pols et al., Nucl. Phys. <u>B25</u> (1970) 109. The $d\sigma/dt$ of this work was used in conjunction with the more precise ρ_{ij} of Z. Carmel et al., WIS-72/28-P. Paper submitted to the XVI International Conference on High Energy Physics.

7. I. J. Bloodworth et al., Nucl. Phys. <u>B35</u> (1971) 79; erratum <u>B43</u> (1972) 641.

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9. It has been noted [10] that ρ-ω^0 interference can enhance the $\omega^0\Delta^{++}$ cross section at small momentum transfers, especially in the projection $\rho_{00}\ d\sigma/dt$, where the ρ and ω production amplitudes are thought to be most coherent. It is difficult to make a quantitative estimate of this effect without a priori knowledge of the production amplitudes, but a crude estimate indicates that the $\rho \to 3\pi$ decay could contribute as much as 30% of the events below $|t| = 0.2\ (\mathrm{GeV}/c)^2$ (in fig. 1b), depending on the relative ρ-ω production phase. If the coherence is only in the unnatural parity exchange (which decreases with increasing ν), the effect tends to raise the corresponding values of $\alpha(t)$ by about one standard error, and to raise the values of α_0 and α' by almost a standard error. If, as more recent measurements indicate [11], the coherence is also in the natural parity exchanges, the effect might be completely negligible. No correction has been made.

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11. D. S. Ayres et al., paper presented at the Berkeley Meeting of the American Physical Society, Division of Particles and Fields, August 1973.

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13. P. Bonamy et al., Phys. Letters 23 (1966) 501.

14. The fits are made to the solid data points in fig. 4. The lowest $|t|$ bin (open circle) is not used because it is below $|t_{min}|$ for the lowest energy data. The highest $|t|$ bin (open circle) is not used because of the increased background above $|t| \approx 0.7\ (\mathrm{GeV}/c)^2$.

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17. D. J. Sherden et al., Phys. Rev. Lett. 30 (1973) 1230.

18. Similar determinations of α_{eff} from $\rho^s_{00}\ d\sigma/dt$, $(\rho^t_{1,1} - \rho^t_{1,-1})\ d\sigma/dt$ and $(\rho^s_{1,1} - \rho^s_{1,-1})\ d\sigma/dt$ yield $\alpha = (-0.03 \pm 0.09) + (0.16 \pm 0.26)t$, $\alpha = (0.12 \pm 0.13) + (0.36 \pm 0.32)t$, and $\alpha = (0.10 \pm 0.12) + (0.71 \pm 0.29)t$.

19. The ρ couples to unit helicity flip at the nucleon vertex in the reactions $\pi^- p \to \pi^0 n$ and $\pi^+ p \to \Delta^{++}\pi^0$.

20. C. B. Chiu, "Workshop on Particle Physics at Intermediate Energies," California Institute of Technology Report CALT-68-300 (1971); "Evidence for Regge Poles and Hadron Collision Phenomena at High Energies," Ann. Rev. Nucl. Sci. 22, 255 (1972).

Table 1. Summary of experimental results

	2.67 GeV/c	7.1 GeV/c
Number of pictures	300 000	700 000
Events/μb	4.31	42.8
No. of scanned four-prongs	36 500	446 000
No. of $\pi^+ p\pi^+\pi^-\pi^0$ events	12 380	86 000
$\sigma(\pi^+ p \to \pi^+ p\pi^+\pi^-\pi^0)$mb	3.29 ± 0.17	2.16 ± 0.09
No. of $\omega\Delta^{++}$ events selected	3497[a]	2872[a]
$\sigma(\pi^+ p \to \omega\Delta^{++})$mb	0.96 ± 0.05	0.165 ± 0.014

[a]These are the samples used in studying the decay angular distributions of the ω^0. No background correction or any attempt to correct for the difference in selection procedures has been made.

Figure Captions

Fig. 1. Differential cross section projections as a function of t at 2.67 GeV/c (dotted error bars) and 7.1 GeV/c (solid error bars).

(a) $d\sigma/dt$

(b) $\rho^t_{00}\, d\sigma/dt$ for ρ^t_{00} evaluated in the t-channel helicity frame.

(c) $(\rho^t_{1,1} - \rho^t_{1,-1})\, d\sigma/dt$

(d) $(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$, which is frame independent

(e) $\rho^s_{00}\, d\sigma/dt$ for ρ^s_{00} evaluated in the s-channel helicity frame

(f) $(\rho^s_{1,1} - \rho^s_{1,-1})\, d\sigma/dt$.

Fig. 2. (a) $\rho^+/(1-\rho^+)$, where $\rho^+ = \rho_{1,1} + \rho_{1,-1}$ and is frame independent.

(b) $\rho^s_{1,-1}$.

Fig. 3. Samples of the effective trajectory calculations at t = - 0.175, -0.350, and -0.600 $(\mathrm{GeV}/c)^2$. The solid lines are from least-squares fits to the data points. The data from this experiment (2.67, 7.1 GeV/c) are shown as dots.

(a-c) $P^2_{lab}(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$ vs. ν, where $\nu = (s-u)/2$. In order of increasing ν, the data points are for incident momenta 2.67, 3.7, 5.0, 5.5, 7.1, and 11.7 GeV/c.

(d-f) $\rho^+/(1+\rho^+)$ vs. ν.

(g-i) $P^2_{lab}\, \rho^t_{00}\, d\sigma/dt$ vs. ν.

Fig. 4. Effective trajectories. The solid curves are linear fits to those data points with solid error bars.

(a) α_N^{eff} vs. t from $P^2_{lab}(\rho_{1,1} + \rho_{1,-1})\, d\sigma/dt$. The dashed curve is a nominal ρ-meson trajectory of $\alpha_\rho = 0.57 + 0.91t$. The fit gives $\alpha_\rho = (0.56 \pm 0.10) + (0.78 \pm 0.31)t$.

(b) $(\alpha_N - \alpha_U)^{eff}$ vs. t from $\rho^+(1-\rho^+)$, where α_N and α_U refer to trajectories corresponding to natural and unnatural parity exchange respectively. The fit gives $(\alpha_N - \alpha_U) = (0.52 \pm 0.11) + (0.18 \pm 0.32)t$.

(c) $\alpha_\rho - (\alpha_N - \alpha_U)^{eff}$ vs. t, where α_ρ is the nominal ρ trajectory of $0.57 + 0.91t$ and $(\alpha_N - \alpha_U)^{eff}$ is the same as in (b). The fit gives

$\alpha_B = (0.05 \pm 0.11) + (0.73 \pm 0.32)t$.

(d) α_U^{eff} vs. t from $P_{lab}^2 \rho_{00}^t \, d\sigma/dt$. The fit gives

$\alpha_B = (0.0 \pm 0.08) + (0.59 \pm 0.23)t$.

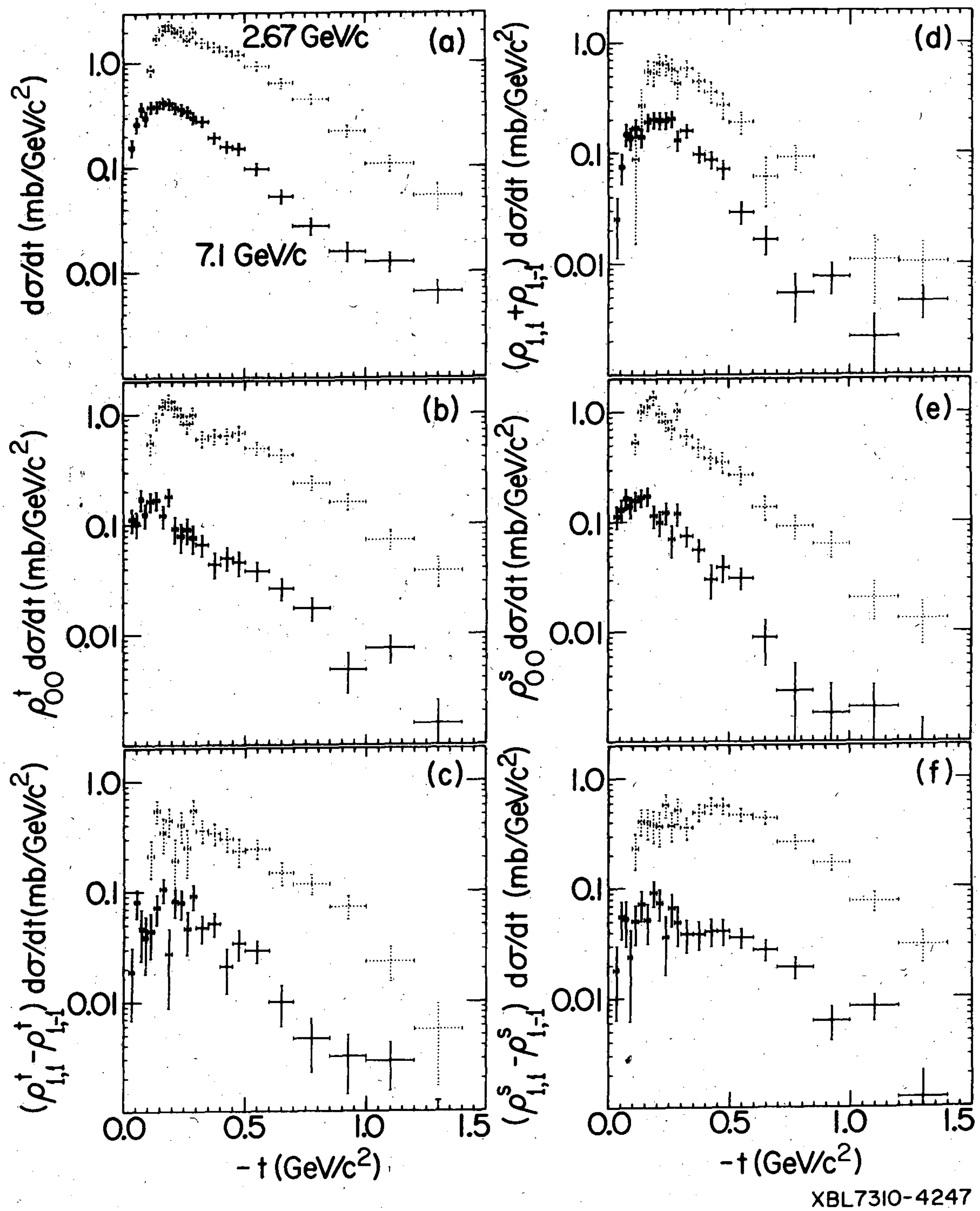
2.67 GeV/c
7.1 GeV/c
(a)
(b)
(c)
(d)
(e)
(f)
dσ/dt (mb/GeV/c²)
ρ^t_00 dσ/dt (mb/GeV/c²)
(ρ^t_1,1 - ρ^t_1,-1) dσ/dt(mb/GeV/c²)
(ρ_1,1 + ρ_1,-1) dσ/dt (mb/GeV/c²)
ρ^s_00 dσ/dt (mb/GeV/c²)
(ρ^s_1,1 - ρ^s_1,-1) dσ/dt (mb/GeV/c²)
1.0
0.1
0.01
0.0
0.5
1.0
1.5
-t (GeV/c²)
XBL7310-4247

Fig. 1

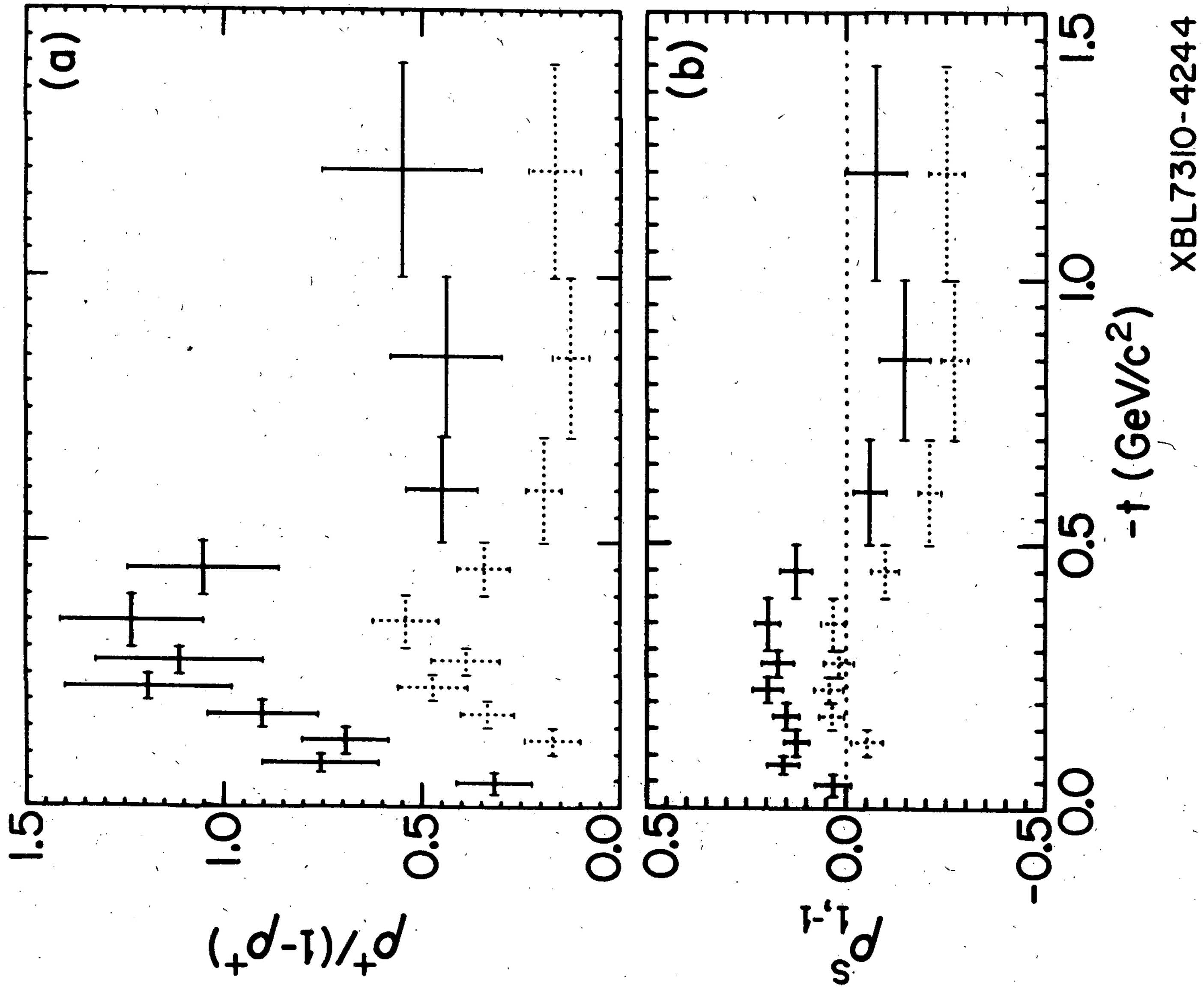

Fig. 2

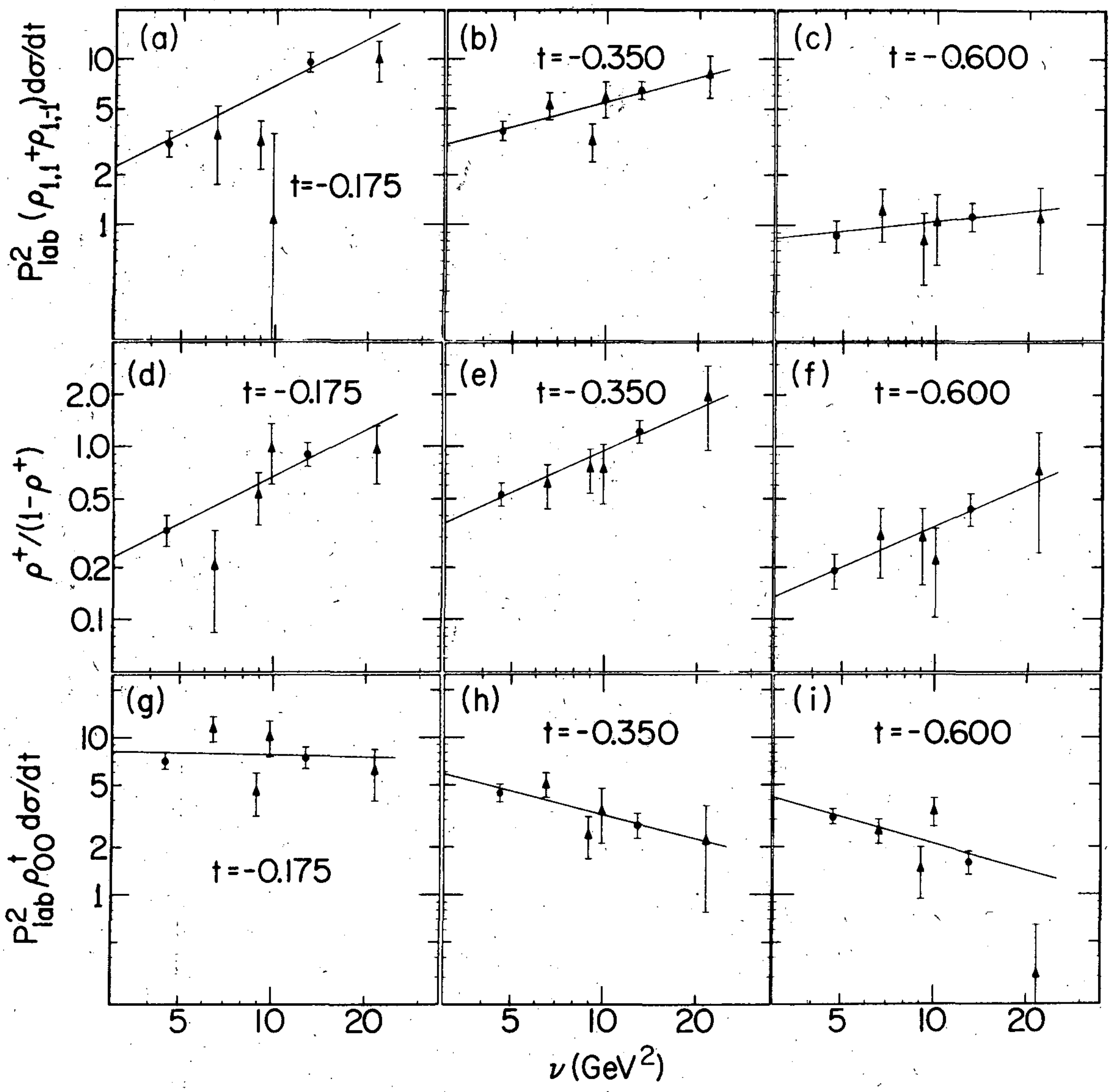

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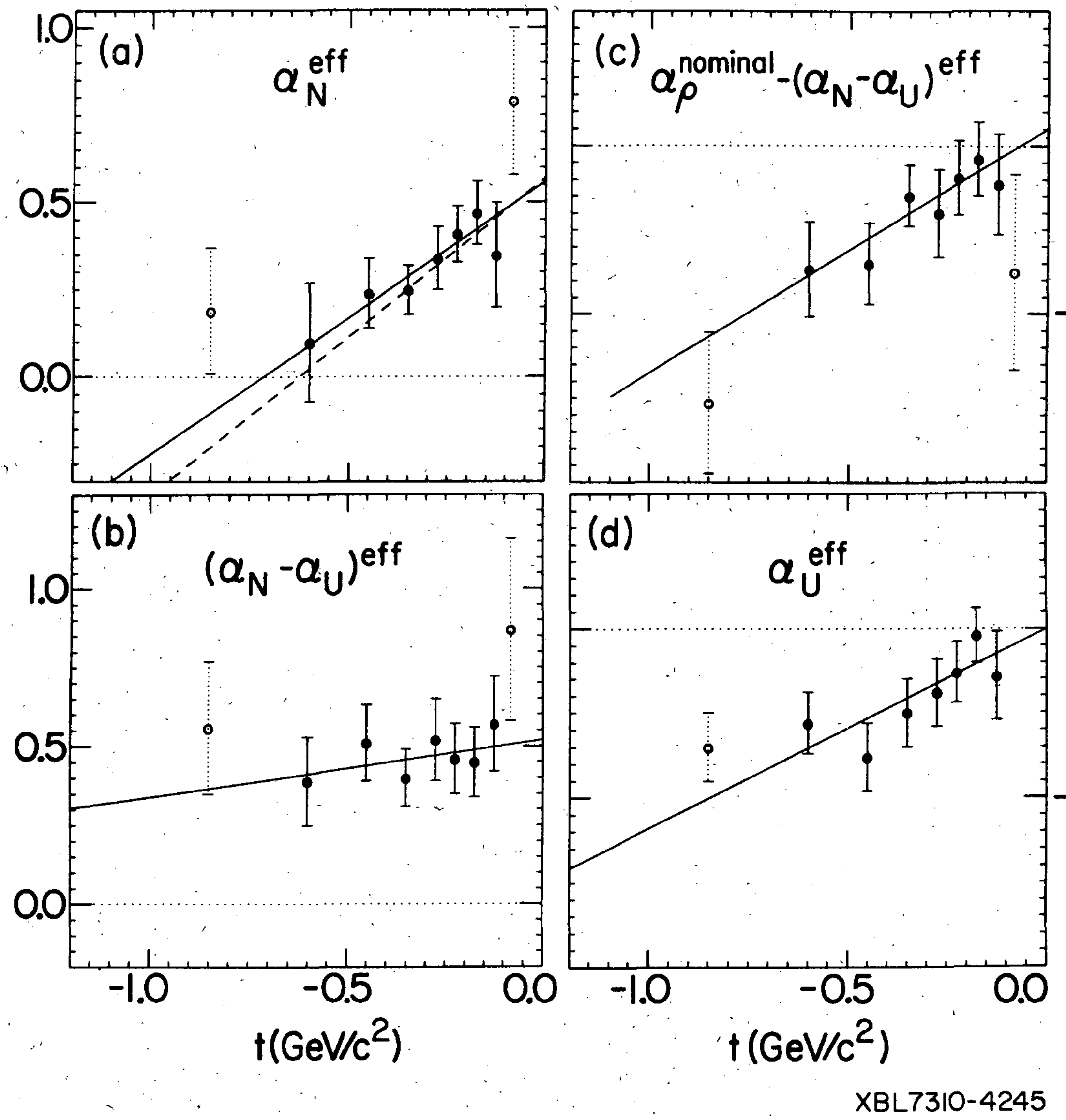
Fig. 3



(a)
α_N^{eff}
(b)
$(\alpha_N - \alpha_U)^{eff}$
(c)
$\alpha_\rho^{nominal} - (\alpha_N - \alpha_U)^{eff}$
(d)
α_U^{eff}
1.0
0.5
0.0
-0.5
-1.0
t(GeV/c²)
t (GeV/c²)
XBL7310-4245

Fig. 4

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