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Authors
Scalise, Nicole R
Daubert, Emily N
Ramani, Geetha B

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When One Size Does Not Fit All: A Latent Profile Analysis of Low-Income Preschoolers' Math Skills

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Nicole R. Scalise[1], Emily N. Daubert[2], and Geetha B. Ramani[3]

[1]University of California, Irvine

[2]University of Hawaiʻi at Mānoa

[3]University of Maryland, College Park

Author Note

Nicole R. Scalise ORCID https://orcid.org/0000-0003-1186-0375

Emily N. Daubert ORCID https://orcid.org/0000-0003-3261-7788

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Correspondence concerning this article should be addressed to Nicole R. Scalise, Department of Cognitive Sciences, 2201 Social & Behavioral Sciences Gateway Building, Irvine, CA, 92697. Email: nicole.scalise@uci.edu

Abstract

On average, preschoolers from lower-income households perform worse on symbolic numerical tasks than preschoolers from middle and upper-income households. Although many recent studies have developed and tested mathematics interventions for low-income preschoolers, the variability within this population has received less attention. The goal of the present study is to describe the variability in low-income children's math skills using a person-centered analysis. We conducted a latent profile analysis on six measures of preschoolers' (*N* = 115, mean age = 4.6 years) numerical abilities (non-symbolic magnitude comparison, verbal counting, object counting, cardinality, numeral identification, and symbolic magnitude comparison). The results show different patterns of strengths and weaknesses and revealed four profiles of numerical skills characterized by different patterns of strengths and weaknesses: (a) poor math abilities on all numerical measures (*n*=13); (b) strong math abilities on all numerical measures (*n*=41); (c) moderate abilities on all numerical measures (*n*=35); and (d) strong counting and numeral skills, poor magnitude skills (*n*=26). Children's age, working memory, and inhibitory control significantly predicted their profile membership. We found evidence of quantitative and qualitative differences between profiles, such that some profiles were higher performing across tasks than others but the overall patterns of performance varied across the different numerical skills assessed.

Keywords: cognitive development; numerical knowledge; low-income; preschool mathematics; latent profile analysis

When One Size Does Not Fit All: A Latent Profile Analysis of Low-Income Preschoolers' Math Skills

There is a consistent gap in the average mathematical performance of children from lower-income households compared to children from middle- and upper-income households (NCES, 2015; Reardon, 2011). This income-related gap is evident as early as preschool, and appears to widen as children progress through elementary and secondary school (Jordan et al., 2009; Starkey et al., 2004). It has spurred countless efforts to promote the early mathematics skills of low-income children (e.g., Baroody et al., 2009; Clements & Sarama, 2007; Dyson et al., 2013; Park et al., 2016; Ramani & Siegler, 2008; Starkey et al., 2004). The majority of these research efforts treat children from low-income households with a "one-size-fits-all" approach by implementing the same intervention regardless of children's initial mathematical skills. However, there is significant variability in children's performance on measures of early math skills within low-income samples (Authors, 2017; Geary & vanMarle, 2016; Wu et al., 2015).

A better understanding of the variability in children's early math skills is needed. Without that understanding, researchers may not know which of the many related early math skills to target and may spend valuable resources targeting specific skills that individual children have already mastered. The goal of the present study is to describe the variability in low-income children's numerical skills, including counting, recognizing numbers, and comparing numbers and sets of objects, which are consistent predictors of later mathematics achievement (Duncan et al., 2007; Watts et al., 2014). We use a person-centered analysis to accomplish this goal, and in turn, provide relevant insight for the design of future intervention studies. Characterizing the variability in children's numerical skills can inform best practices for early math research, by identifying whether children from low-income households need a variable amount of support across all early numerical skills or whether there are specific skills that most low-income children need support on.

We first review the early numerical skills that lay the foundation for mathematical achievement, followed by a summary of the differences in performance between children from lower-income households and higher-income households on specific numerical skills. We next describe research that has used person-centered latent profile analysis techniques to characterize variability across children's mathematical skills and summarize the gaps in previous research that the present study aims to fill.

Early Mathematics Foundations

Unpacking the variability in low-income children's early numerical skills is particularly important because these skills lay the foundation for later mathematics achievement. Preschool children's number knowledge, for example counting, recognizing numbers, and comparing numbers and sets of objects, predicts their mathematics achievement in later elementary school and at age 15 (Duncan et al., 2007; Watts et al., 2014). Moreover, preschoolers' initial numerical skills predict their rates of growth in math across early elementary school (Aunola et al., 2004). However, children's number knowledge consists of several unique but interrelated skills that develop during early childhood.

From infancy, humans have the ability to discriminate between sets of different numerical magnitudes referred to as Approximate Number Sense (ANS; Izard et al., 2009; Cordes & Brannon, 2008; Xu & Spelke, 2000). Children and adults' ability to discriminate between sets with smaller ratios, often termed non-symbolic magnitude understanding, is predictive of performance on broader math achievement measures (Schneider et al., 2016). However, preschoolers' knowledge of symbolic numbers referenced by verbal labels (i.e., number words) and written symbols (i.e., numerals) is a more consistent predictor of later mathematics success (e.g., Kolkman et al., 2013; Purpura et al., 2013; vanMarle et al., 2014).

Verbal counting knowledge is often the first type of symbolic numerical knowledge that children master (for a review, see Authors, 2019a). Children develop knowledge of the verbal counting sequence as early as 2 and 3 years old (Gelman & Gallistel, 1978), and counting proficiency in preschool predicts later mathematics achievement (Nguyen et al., 2016), however, they are not immediately proficient at connecting number words to the correct numerosities (Wynn, 1992). Three and four-year old children typically know the verbal count sequence for numbers one through ten (Fuson, 1988; Siegler & Robinson, 1982), but many continue to make errors in applying those same counting words to sets of objects. However, even children who can successfully count sets of objects up to ten are not always accurate at linking number words to their corresponding set sizes (e.g., Le Corre & Carey, 2007).

The ability to link numerical symbols, like number words, to their corresponding quantities is termed cardinality. Cardinality skills in preschool predict children's more advanced number skills in kindergarten and later math but not reading achievement, signaling that it may be a domain-specific gateway skill (Geary et al., 2017; Moore et al., 2016; Nguyen et al., 2016). On average, children master cardinality around the age of 3 - 4 years old, meaning they can reliably give someone the correct number of objects associated with a number word, like three spoons or eight rubber ducks (Sarnecka, 2015).

Young children also develop knowledge of numerals early. By age four, approximately one-quarter of children can correctly label numerals 1 – 9 (Ginsburg & Baroody, 2003). Numeral knowledge is theorized to link children's informal mathematical knowledge, such as counting and discrimination of sets of objects, and formal mathematical knowledge, such as basic arithmetic (Pupura et al., 2013). However, children must learn to connect verbal number words, numerals, and quantities over time. Cross-sectional studies have demonstrated that children first understand the cardinal values of number words, then begin to recognize written numerals, and ultimately are able to match written numerals to their cardinal values (Knudsen et al., 2015). Longitudinal research has shown that preschool children's ability to identify written numerals is a significant predictor of their later formal mathematics ability (Chard et al., 2005; Clarke & Shinn, 2004; Purpura et al., 2013).

As children begin to understand the quantities associated with individual symbolic numbers, they also begin to develop an understanding of symbolic magnitude, or the relations between the quantities represented by different numbers. A child who understands the symbolic magnitude of the numbers 1 - 4 would be able to say that 1 cookie is less than 4 cookies, and to approximate the relative positions of those numbers on a bounded number line (Siegler, 2016). Understanding of non-symbolic and symbolic magnitudes is theorized to underlie numerical development across the lifespan, grounding the higher order manipulations of more advanced math problems by helping students learn arithmetic, select appropriate strategies when solving math problems, and identify more plausible answers (Mussolin et al., 2016; Siegler, 2016). However, meta-analytic estimates have shown that the overall effect size of the relation between symbolic magnitude comparison tasks and mathematics achievement is significantly larger than the estimated effect of non-symbolic magnitude comparison tasks and mathematics achievement (Schneider et al., 2016). Indeed, preschoolers' symbolic magnitude understanding predicts their concurrent and later arithmetic skills (Kolkman et al., 2013; Authors, under review; Siegler & Ramani, 2009; Toll & Van Luit, 2014), and fully mediates the relation between kindergartners' non-symbolic magnitude understanding and math achievement (Xenidou-Dervou et al., 2013).

Children's performance on counting, numeral knowledge, cardinality, non-symbolic and symbolic magnitude tasks are positively correlated (e.g., *r*s: 0.16 - 0.56; vanMarle et al., 2014). These skills all develop in early childhood and inform one another. Although each type of number knowledge has been linked to later mathematics achievement, it is important to consider the patterns of children's performance across these different skills as there is significant variability among specific numerical abilities (Dowker, 2008; Gray & Reeve, 2016). Understanding and promoting children's mathematical development requires taking a comprehensive view of numerical skills; it is unlikely that focusing on any single numerical skill could lead to general mathematics proficiency. Moreover, although children are often considered "good" or "bad" at mathematics, these general ratings of mathematics achievement mask children's strengths and weaknesses on specific numerical skills (Gray & Reeve, 2016).

Income-Related Gaps in Early Number Skills

On average, children from low-income households perform worse on mathematical tasks than children from middle-income households. Early gaps in foundational numerical skills may widen over time into larger gaps in mathematical achievement. For example, among a nationally representative sample of U.S. eighth graders, only 18% percent of students from low-income households performed met the criteria for proficiency in math, compared to 48% of students from middle- and upper-income households (U.S. Department of Education, 2019).

Preschoolers from middle-income households show higher performance on average on tasks of verbal counting and cardinality (Jordan et al., 2006), counting objects (Starkey et al., 2004), numeral recognition (Jordan et al., 2006), ordinality (Starkey et al., 2004), symbolic magnitude comparison (Authors, 2017; Starkey et al., 2004), and number line estimation (Siegler & Ramani, 2008). Furthermore, the numerical skills (e.g., verbal and object counting, numeral identification, symbolic magnitude comparison, addition) of children from low-income households typically lag behind those of children from mid- and upper-income households by up to 8 months (Ramani & Siegler, 2011; Starkey et al., 2004). Interestingly, there is less evidence of income-related gaps on children's performance on non-symbolic tasks, including non-symbolic arithmetic problems (Gilmore et al., 2010), non-symbolic estimation tasks (Mejias & Schiltz, 2013), and non-symbolic magnitude comparison tasks with small quantities (Sarnecka et al., 2018; Authors, 2017).

The gap in average performance on mathematical tasks is generally attributed to children from low-income households having fewer high-quality experiences with math at home and in classrooms (Clements & Sarama, 2007; Dyson et al., 2013; Siegler, 2009). Indeed, after relatively brief experiences with math games children from low-income households show significant improvements in their numerical skills, so much so that their post-intervention performance is statistically equivalent to children from middle-income households (e.g., Authors, 2017; Siegler & Ramani, 2008). This suggests that the income-related differences in performance are quantitative in nature, such that all children share similar developmental trajectories of their mathematical skills but children from lower-income households are a few months older on average when they reach proficiency than children from higher-income households (Wu et al., 2015; Yang et al., 2005).

However, children from low-income households may be more different than they are alike. Studies that focus on low-income preschoolers' numerical skills show significant variability in their performance on measures of counting, cardinality, and numeral identification (e.g., one standard deviation is equivalent to 55 to 97 percent of the mean score; Geary & vanMarle, 2016). The same is true for studies of middle-income preschoolers: standard deviations are large relative to the group-level average performance. Although the aforementioned studies that show group-level average performance gaps for children from low-income households compared to middle-income households provide motivation to develop interventions for children from low-income households (e.g., Authors, 2017; Jordan et al., 2006; Siegler & Ramani, 2008; Starkey et al., 2004), the focus on between-group comparisons in these studies masks the variability within each group.

Characterizing Children's Skills

To best support children from low-income households' early mathematical development, it is critical to understand the patterns in their numerical understanding. Several studies have used Latent Profile Analysis (LPA), a statistical method used to identify common profiles of participants based on their patterns of responses to different tasks, in order to characterize children's early mathematics skills. Wu and colleagues (2015) described patterns in low-income preschool children's performance across several early mathematical skills (verbal counting, object counting, measurement, numbers and shapes, pattern recognition, and subitizing). They identified three profiles reflecting quantitative differences in children's performance that they labeled a high-achieving class, a typical-achieving class, and a low-achieving class. Children in the high-achieving class outperformed children in the other classes on all measures, children in the typical-achieving class had lower performance than children in the high-achieving class and higher performance than children in the low-achieving class (on all measures except verbal counting), and children in the low-achieving class had the lowest performance of all three classes. In addition to the early mathematical skills used to identify children's profile membership, children in the high-achieving class also had significantly higher scores on a general mathematics achievement test than children in the typically-achieving or low-achieving classes. The results of this study imply that children's early mathematical skills differ by performance level but reflect similar patterns of performance across skill types. However, the authors did not investigate individual differences in children's numerical skills specifically, focusing instead on broader mathematics skills across subdomains. This leaves an open question as to whether low-income children's numerical skills would show a similar pattern of results.

More recently, Gray and Reeve (2016) found evidence for five distinct profiles of numerical skills, drawing from a sample of middle-income Australian preschool children. They assessed children's math abilities with tasks of verbal counting, object counting, cardinality, knowledge of Arabic numerals, ordinal relations, and nonverbal arithmetic. The profiles indicate both quantitative (i.e., performance level) and qualitative (i.e., patterns of skills) differences in children's numerical skills: excellent abilities across all mathematical tasks, good arithmetic ability, good abilities across all mathematical tasks except poor counting, average abilities across all mathematical tasks, and poor ability across all mathematical tasks. Furthermore, children's profile membership was significantly related to their domain-specific cognitive abilities, such as spontaneous focus on numbers in the environment. The results of this study suggest that although children in the excellent abilities, average abilities, and poor abilities profiles showed similar levels of proficiency across math tasks, a notable portion of children have heterogenous performance across skills. These patterns of heterogeneity implied to the study authors that mathematical instruction should target the development of individual skills rather than assume some skills serve as prerequisites for others, as their findings showed that some children had strong performance on more complex mathematical tasks with poor performance on tasks considered to be foundational (e.g., the good math abilities, poor count sequence profile; Gray & Reeve, 2016). Moreover, this finding suggests that there may also be significant qualitative differences across the specific numerical skills of preschool children from low-income households.

In addition to characterizing differences in children's mathematical skills, latent profile analyses have also been used to illustrate relations between domain-general executive functioning skills and domain-specific mathematical skills. Across the lifespan, executive functioning skills including working memory, inhibitory control (e.g., the ability to override natural responses), and attention shifting (e.g., the ability to shift between tasks) relate to math performance (Bull & Lee, 2014; Diamond, 2013). From a theoretical perspective, executive functioning may allow children to inhibit incorrect responses, maintain and manipulate relevant information, and shift their attention to the relative features of mathematical tasks, all of which support higher mathematical performance (Geary & Hoard, 2005). Indeed, two additional LPAs found that children's domain-general executive functioning skills relate to their mathematical skills profiles. Chew and colleagues (2016) categorized 5- to 7-year old children into four profiles based on children's accuracy and speed on non-symbolic and symbolic magnitude tasks. Profile membership was predicted by child age and working memory ability, or children's ability to store and update information. Similarly, Hannula and colleagues (2017) compared math skill profiles between premature and full-term 5-year olds, and found that working memory was related to math profiles in full-term children. Among samples of young children, executive functioning skills including working memory, inhibitory control (e.g., the ability to override natural responses), and attention shifting (e.g., the ability to shift between tasks) relate to math performance (Bull & Lee, 2014). Taken together, these results suggest that domain-general cognitive abilities such as working memory and inhibitory control can help to explain differences in low-income children's numerical ability profiles.

The Present Study

The present study aimed to identify distinct profiles of low-income preschoolers' numerical skills. While previous research has categorized low-income preschoolers' early mathematics skills broadly and middle-income children's numerical skills specifically (e.g., Gray & Reeve, 2016; Wu et al., 2015), this is the first study to our knowledge to characterize patterns of low-income preschoolers' numerical skills. We considered a range of interrelated numerical skills previously shown to predict future math achievement. Furthermore, in the study we examined whether children's numerical skills profile membership related to their age and domain-general executive functioning abilities. Identifying different profiles of low-income preschoolers' numerical skills provides important information on children's relative performance on specific skills, as well as implications for how to support children's early mathematic success. We focus primarily on numerical skills that previous research has shown children from low-income and middle-income households differ in their performance. The results of this study have important implications for early intervention research. Although there is a growing literature on early mathematics interventions for low-income children, not all children from low-income households may need interventions. Furthermore, not all existing interventions may be optimally designed to help children from low-income households, whose performance may vary significantly across numerical skills.

Thus, our study had three distinct aims. Our first aim was to estimate two to five profile models to characterize children's numerical skills and determine the best-fitting model. This aim is based upon previous studies demonstrating children's skills are most likely to fall between a similar range of profiles (e.g., Wu et al., 2015; Gray & Reeve, 2016; Hannula et al., 2017). Our second aim was to determine whether the resulting profiles revealed quantitative or qualitative differences in children's skills. Quantitative differences - such as three profiles with children's performance as low, medium, and high across tasks - would imply that different groups of children need varying amounts of additional instruction in all early numerical skill areas (Wu et al., 2015; Yang et al., 2005). Qualitative differences - such as different patterns of accuracy across different numerical measures - would imply that there were some types of skills training that all children could benefit from, and other types of skill training that would benefit only specific subgroups of children (Wu et al., 2015; Yang et al., 2005). Our third aim was to determine whether demographic and domain-general executive functioning skills predicted children's numerical skills profile membership. Specifically, we expected that children's age, working memory, and inhibitory control skills would predict their profile membership, such that children who were older and had higher executive functioning skills would be more likely to belong to higher performing profiles. Relations between children's age and executive functioning skills and their profile membership would replicate previous studies with middle-income children (e.g., Chew et al., 2016; Hannula et al., 2017), and would further suggest that future research consider instructional supports for both children's mathematical and executive functioning skills.

Method

Participants

Participants were 115 3 – 5-year-old preschoolers (*M* = 4.6 years, *SD* = 7 months; 51% female; 46% Hispanic/Latino; 37% African American/Black, 10% Multiracial, 6% Asian, 1% White). We conducted a posthoc Monte Carlo simulation with 100 replications and confirmed there was sufficient power (> 80%) with 115 participants to detect the estimated latent means and variances of each skill from the best-fitting latent profile model (Nylund-Gibson & Choi, 2018; Muthén & Muthén, 1998-2017). A subset of the participants (*n* = 82) were part of a larger math intervention study (data collected 2015 - 2016; Authors, 2019b). The remaining participants (n = 33) were recruited to participate in the present study only (data collected in 2018). Children were recruited from four Head Start centers in the mid-Atlantic region of the United States. Head Start is a federally funded early childhood education program for families living at or below the federal poverty line. One additional child was recruited for the larger math intervention study but repeatedly declined to participate.

Procedure

Parental consent forms were sent home with all children attending the participating Head Start centers. Only children who received parental consent to participate and who provided verbal assent to an experimenter were allowed to participate in the study. Participating children completed one 15- to 20-min session individually with an experimenter in a quiet area of their school or classroom. This session served as the pretest assessment for the subset of children participating in the larger intervention study (Authors, 2019b). During this session, children completed six numerical knowledge and two executive functioning measures presented in the following order to maintain participant engagement: verbal counting, numeral identification, inhibitory control, symbolic magnitude comparison, object counting, cardinality, working memory, and non-symbolic magnitude comparison. Children were thanked for their participation with a sticker. This study was approved by the Institutional Review Board at the [University Name] (Refs. 783158 and 869486).

Measures

Numerical knowledge. Six measures of numerical knowledge were administered.

Non-symbolic magnitude. Children were asked to compare sets of dots using the Panamath program (Libertus et al., 2013). Children saw two sets of colored dots on a laptop computer, and were asked to press a button to indicate which side had more dots. Children were shown 32 pairs, displayed for 2.3 seconds each, with set sizes ranging from four to 15 dots. Each quantity was counterbalanced for the side of presentation and controlled for dot area and density. The numerical comparisons included ratios of 2.0 (e.g., 4 vs. 8, 25% of trials); 1.5 (e.g., 4 vs. 6, 25% of trials); 1.3 to 1.4 (e.g., 8 vs. 11, 25% of trials); and 1.1 to 1.2 (e.g., 8 vs. 9 objects, 25% of trials). The dependent measure was the percentage of correct comparisons.

Verbal counting. Children were asked to count aloud starting with one. They were stopped by the experimenter after they made an error or successfully counted to 25 (adapted from Ramani & Siegler, 2008). The dependent measure was the highest number reached without errors divided by the highest possible score (i.e., 25).

Object counting. Children were asked to count a set of stars on a piece of paper (adapted from Ramani et al., 2015). There were three trials total, with one set each of four, five, and nine stars. The dependent measure was the percentage of trials in which the child correctly counted the number of stars.

Cardinality. Children were given 10 plastic tokens and asked to give the experimenter a specific number of tokens. There were three trials total with requests for three, five, and seven tokens, respectively. The dependent measure was the percentage of trials in which the child correctly provided the number of tokens.

Numeral identification. Children were shown the numerals from 1 to 10 presented in random order and asked to verbally label the numeral (Ramani & Siegler, 2008). The dependent measure was the percentage of numerals correctly labeled.

Symbolic magnitude. Children were asked to compare pairs of numerals ranging from 1 to 9 (Ramani & Siegler, 2008). Following two practice trials with experimenter feedback, children completed 18 test trials. On each trial, the experimenter showed and read aloud each pair of numbers and asked the child to indicate which number was larger. Each number was counterbalanced for side of presentation (i.e., 3 vs. 8, 8 vs. 3). The ratio between pairs ranged from 1.1 (e.g., 8 vs. 9) to 9.0 (e.g., 1 vs. 9). The dependent measure was the percentage of correct comparisons.

Executive functioning. Two measures of executive functioning were administered.

Working memory (WM). Children were shown a piece of paper with a 3 x 3 matrix of lily pads in a pond. Children watched the experimenter tap a sequence of lily pads, then were asked to repeat the sequence in the same order (adapted from Morales et al., 2013). Children completed two practice trials of sequence length 2, followed by two trials at each sequence length from 2 to 6. The experimenter ended the task if a child could not complete either trial at a given sequence length, or if the child completed the trials at sequence length 6. Children were given one point for each lily pad correctly tapped (accuracy) and one point for each tap that occured in the correct order within a sequence (order). The dependent measure was the sum of accuracy and order scores across all trials, for a maximum possible score of 80 points.

Inhibitory control. Children were asked to touch their head when the experimenter said “feet” and touch their feet when the experimenter said “head” (adapted from Ponitz et al., 2008).

After three trials with experimenter feedback, children completed 16 test trials. The dependent measure was the percentage of correctly completed trials.

Analytic Approach

We conducted a latent profile analysis (LPA) on the measures of numerical knowledge using Mplus's mixture model analysis (Vermunt & Magidson, 2013). We estimated LPA models ranging from two to five profiles and evaluated each to determine the best-fitting model. Our evaluation criteria included statistical tests, parsimony, and theoretical interpretability (Collins & Lanza, 2009). We used the Vuong Lo Mendell Rubin likelihood ratio test (VLMR) to test the likelihood of the k-profile solution compared against the likelihood of the k - 1 profile solution. A VLMR test with $p < .05$ suggests that the k-profile model has a higher likelihood than the k - 1 profile model, given the observed data. We also compared the values of the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), where lower values indicate better fitting models. As our final statistical criterion, we assessed the entropy value of each profile model. Entropy values characterize the probability of accurate profile membership, with values closer to 1 indicating a high probability of participants belonging to their assigned profile and a low probability of belonging to the other profiles. Entropy values above 0.8 are considered acceptable (Clark & Muthen, 2009). In keeping with our criterion of parsimony, we prioritized models suggesting acceptance statistical fit with the fewest possible latent profiles. Finally, we assessed the theoretical interpretability of the resulting profile model to ensure that it aligned with previous empirical and theoretical findings.

Next, we conducted a 3-step LPA to examine the association between child age, working memory, and inhibitory control skills and profile membership. Age and executive functioning skills were treated as covariates predicting profile membership. The 3-step method corrects for the probability of classification errors that arise from assigning profile memberships. If uncorrected, the classification error probability can lead to attenuated estimates of the relations between hypothesized covariates and profile memberships (Bakk et al., 2013). If a relation between a covariate and profile membership resulted in $p < .05$ in the 3-step LPA, the covariate was considered a significant predictor of profile membership.

Results

Similar to previous research, performance across all numerical knowledge and executive functioning measures showed significant variability (Table 1). Children's age (in months) was significantly and positively correlated with all measures of executive functioning and numerical knowledge. With the exception of cardinality and inhibitory control, all bivariate two-tailed Pearson correlations between measures were statistically significant and positive. The significant correlation coefficients correspond to medium to large effect sizes (Cohen's *d*s = 0.43 – 2.02; Cohen, 1969).

Table 1

Descriptive statistics and correlations between numerical knowledge measures

	Mean (*SD*)	1	2	3	4	5	6	7	8	9
1. Age (mos)	54.6 (7.0)	1								
2. Working memory	29.2 (14.8)	.52***	1							
3. Inhibitory control	42.3 (37.6)	.45***	.32**	1						
4. Non-symbolic magnitude	63.4 (13.6)	.37***	.30**	.30**	.30**	.21*	.23*	.38***	.56***	1

5. Verbal count	62.3 (28.6)	.48***	.31**	.26**	1					
6. Object count	85.5 (27.3)	.36***	.38***	.25**	.48***	1				
7. Cardinality	60.9 (39.3)	.34***	.39***	.15	.62***	.46***	1			
8. Numeral ID	71.5 (33.5)	.41***	.37***	.26**	.57***	.40***	.71***	1		
9. Symbolic magnitude	68.6 (19.5)	.38***	.33***	.32***	.40***	.33***	.45***	.44***	1	

Note. Children's age is reported as months old (mos) at the time of the assessment. All other measures are reported as percent accurate (out of 100), except working memory, which is reported as the sum of accuracy and order scores across all trials for a maximum possible score of 80 points. *p<.05, **p<.01, ***p<.001

We compared LPA models ranging from two to five profiles and found that the best fitting model was the four-profile solution (Table 2). The four-profile solution was significantly more likely than the three-profile solution (VLMR $p < .01$), but was not significantly less likely than the five-profile solution (VLMR $p > .05$). The four-profile solution also had the lowest BIC, second-lowest AIC, and an entropy of 0.92, greater than the benchmark of 0.80. Finally, the patterns of performance on numerical knowledge measures from the four-profile solution correspond well with previous theoretical and empirical findings. Like Wu et al. (2015), we show evidence of quantitative differences in low-income preschoolers' mathematical skills, with profiles of low, medium, and high performance across numerical skills. In addition, we see evidence of one profile of children who are highly proficient at verbal counting, counting objects, cardinality, and labeling numerals, but are less accurate at magnitude comparison tasks.

Table 2

Model fit comparison of profile solutions

Model solution	VLMR (*p*)	AIC	BIC	Entropy
2 profiles	.0005	-133.830	-81.677	0.942
3 profiles	.0068	-224.196	-152.828	0.953
4 profiles	**.0093**	**-262.891**	**-172.308**	**0.919**
5 profiles	.5255	-279.086	-169.289	0.898

Specifically, children were classified as having: (a) poor math abilities on all numerical measures (*n*=13); (b) moderate math abilities on all numerical measures (*n*=35); (c) strong counting and numeral skills, poor magnitude skills (*n*=26); and (d) strong math abilities on all numerical measures (*n*=41, Table 3). The average latent profile probabilities of being assigned a specific profile in the four-profile model were high (0.89 - 0.99), indicating effective classification.

Table 3

Means and standard deviations of numerical skills for individual profiles

	Non-symbolic magnitude	Verbal count	Object count	Cardinality	Number identification	Symbolic magnitude
Poor math (*n* = 13)	56.6 (11.0)	24.6 (20.6)	20.5 (16.9)	7.7 (14.6)	34.6 (31.0)	50.9 (7.1)
Moderate math (*n* = 35)	59.8 (12.7)	49.5 (15.3)	92.4 (14.2)	23.8 (19.1)	44.9 (30.8)	60.0 (14.9)
Strong counting and numeral skills, poor magnitude (*n* = 26)	54.5 (9.5)	70.8 (25.6)	89.4 (18.3)	91.0 (15.1)	86.9 (17.4)	56.6 (12.9)
Strong math (*n* = 41)	74.1 (9.9)	79.7 (25.2)	97.6 (8.8)	90.2 (17.1)	96.1 (8.3)	89.2 (8.9)

Note. Means with standard deviations in parentheses.

To determine whether profiles of children had significantly different performance on the numerical tasks, we conducted a one-way analysis of variance (ANOVA) for each numerical knowledge measure, followed by Bonferroni-corrected post hoc comparisons between each profile pair. The ANOVAs for the numerical knowledge measures were all statistically significant ($F(3, 111) = 21.9 - 164.9$, ps $< .001$), indicating that the profiles differed from each other in mean non-symbolic magnitude accuracy, verbal counting, object counting, cardinality, numeral identification, and symbolic magnitude accuracy. Specifically, children in the strong abilities profile had significantly higher accuracy on all numerical tasks than children in the poor abilities profile, significantly higher accuracy on all tasks except object counting than children in the moderate abilities profile, and significantly higher accuracy on non-symbolic and symbolic magnitude tasks than children in the strong counting and numeral skills but poor magnitude skills profile. Children in the strong counting and numeral skills but poor magnitude skills profile had significantly higher accuracy on all tasks except the non-symbolic and symbolic magnitude tasks compared to children in the poor math abilities profile, and significantly higher accuracy on the verbal counting, cardinality, and numeral identification tasks than children in the moderate abilities profile. Finally, children in the moderate abilities profile had significantly higher accuracy than children in the poor abilities profile on the verbal counting, object counting, and cardinality tasks. See Tables 4 and 5 for the ANOVA and pairwise comparison results. All of the significant pairwise comparisons between profiles correspond to large to very large effect sizes (absolute value of Cohen's ds = 0.91 – 7.02; Cohen, 1969). The largest effect size differences between profiles were on the object count and cardinality measures, particularly the poor math abilities profile compared to the other three profiles. The smaller (but significant) effect size differences were on the non-symbolic comparison and verbal count measures.

Table 4

Analysis of variance (ANOVA) comparing numerical skills accuracy across profiles

	Poor math	Moderate math	Strong counting and numeral skills, poor magnitude	Strong math		
	M (SD)				*F*-value	*p*-value
Non-symbolic magnitude	56.6 (11.0)	59.8 (12.7)	54.5 (9.5)	74.1 (9.9)	21.93	<.001
Verbal count	24.6 (20.6)	49.5 (15.3)	70.8 (25.6)	79.7 (25.2)	25.97	<.001
Object count	20.5 (16.9)	92.4 (14.2)	89.7 (18.3)	97.6 (8.8)	107.20	<.001
Cardinality	7.7 (14.6)	23.8 (19.0)	91.0 (15.1)	90.2 (17.1)	164.88	<.001
Number ID	34.6 (31.0)	44.9 (30.8)	86.9 (17.4)	96.1 (8.3)	50.26	<.001
Symbolic magnitude	50.9 (7.1)	60.0 (14.9)	56.6 (12.9)	89.2 (8.9)	66.62	<.001

Note. Degrees of freedom for all *F*-tests were 3, 111.

Table 5

Bonferroni-corrected comparisons of numerical skills between pairwise sets of profiles

	Reference Group: Mean difference (Cohen's *d*)		
	Poor math	Moderate math	Strong count and numeral, poor magnitude
Non-symbolic magnitude	Moderate: -3.3 (-0.27) Strong count and numeral, poor magnitude: 2.0 (0.21) Strong: -17.5* (-1.75)	Strong count and numeral, poor magnitude: 5.3 (0.47) Strong: -14.3* (-1.28)	Strong: -19.6* (-2.03)
Verbal count	Moderate: -24.9* (-1.51) Strong count and numeral, poor magnitude: -46.2* (-1.96) Strong: -55.1* (-2.32)	Strong count and numeral, poor magnitude: -21.3* (-1.07) Strong: -30.2* (-1.44)	Strong: -8.9 (-0.36)
Object count	Moderate: -71.9* (-4.91) Strong count and numeral, poor magnitude: -69.2* (-3.98) Strong: -77.0* (-7.02)	Strong count and numeral, poor magnitude: 2.6 (0.17) Strong: -5.2 (-0.45)	Strong: -7.8 (-0.60)
Cardinality	Moderate: -16.1* (-0.91) Strong count and numeral, poor magnitude: -83.3* (-5.73) Strong: -82.6* (-5.09)	Strong count and numeral, poor magnitude: -67.2* (-3.91) Strong: -66.4* (-3.74)	Strong: 0.1 (0.05)

Number ID	Moderate: -10.2 (-0.34) Strong count and numeral, poor magnitude: -52.3* (-2.37) Strong: -61.5* (-3.78)	Strong count and numeral, poor magnitude: -51.2* (-1.65) Strong: -42.1* (-2.39)	Strong: -9.1 (-0.74)
Symbolic magnitude	Moderate: -9.1 (-0.70) Strong count and numeral, poor magnitude: -5.8 (-0.52) Strong: -38.3* (-4.59)	Strong count and numeral, poor magnitude: 3.4 (0.24) Strong: -29.2* (-2.46)	Strong: -32.5* (-3.12)

Note. * indicates the Bonferroni corrected mean difference is significant at the .05 level.

A 3-step LPA including age, working memory, and inhibitory control revealed all of the hypothesized covariates were significantly related to profile membership (Table 6).

Table 6

Three-step latent profile model with covariates predicting numerical skills profile membership

			Numerical skills profiles *M (SE)*			
	Wald	*p*	Poor math	Moderate math	Strong count and numeral skills, poor magnitude	Strong math
Age (mos)	35.37	<.001	48.8 (1.3)	53.1 (1.1)	53.2 (1.4)	58.7 (1.1)
WM	24.04	<.001	14.8 (3.7)	26.7 (2.3)	30.7 (2.9)	35.3 (2.3)
Inhibitory control	9.84	.02	25.5 (9.3)	38.1 (5.6)	32.3 (7.3)	57.1 (6.6)

Pairwise Comparisons of Covariates Between Numerical Skills Profiles

Children with strong math abilities were older than children with strong counting and numeral skills but poor magnitude abilities ($\chi^2(1)=8.93$, $p<.01$, Cohen's $d = 0.78$), moderate abilities ($\chi^2(1)=12.97$, $p<.001$, Cohen's $d = 0.84$), and poor abilities ($\chi^2(1)=33.90$, $p<.001$, Cohen's $d = 1.62$). Children with poor magnitude abilities were also younger than children with strong counting and numeral skills but poor magnitude ($\chi^2(1)=5.31$, $p<.05$, Cohen's $d = 0.85$) and children with moderate abilities ($\chi^2(1)=6.08$, $p<.05$, Cohen's $d = 0.70$). Children with strong abilities had higher WM than children with moderate ($\chi^2(1)=7.07$, $p<.01$, Cohen's $d = 0.30$) or poor abilities ($\chi^2(1)=22.07$, $p<.001$, Cohen's $d = 0.25$), and children with strong counting and numeral skills but poor magnitude and moderate abilities had higher WM than children with poor abilities ($\chi^2(1)=11.51$, $p<.01$, Cohen's $d = 1.23$ and $\chi^2(1)=7.43$, $p<.01$, Cohen's $d = 0.89$, respectively). Finally, children with strong abilities had greater inhibitory control than children with strong counting and numeral skills but poor magnitude ($\chi^2(1)=5.69$, $p<.05$, Cohen's $d = 0.57$), children with moderate abilities ($\chi^2(1)=4.79$, $p<.05$, Cohen's $d = 0.49$), and children with poor abilities ($\chi^2(1)=7.68$, $p<.01$, Cohen's $d = 0.79$). There were no other significant pairwise differences between profile membership and covariates.

Discussion

Recently, more research has been focused on the early mathematics skills of preschoolers from low-income households, however, less attention has been paid to the *variability* within this population. The present study is the first to our knowledge to characterize the variability of the numerical skills of preschoolers from low-income households. To address this issue, we conducted a latent profile analysis on low-income preschoolers' numerical skills. We focus on skills that show performance gaps between low-income preschoolers and middle-income preschoolers (Authors, 2017; Jordan et al., 2006; Siegler & Ramani, 2008; Starkey et al., 2004). We identified the best-fitting model as a four-profile solution, with children categorized as having poor math abilities, moderate math abilities, strong math abilities, or strong counting and numeral skills but poor magnitude skills. We also found that children's age, WM, and inhibitory control predicted their profile membership.

The findings from our study both align and extend previous research in several important ways. First, prior research on low-income children's mathematical skills found quantitative differences in profiles, with children categorized as high, moderate, or low performers (Wu et al., 2015). Like Wu and colleagues (2015), we found support for children who were poor, moderate, and high performers. Second, we extend these findings by focusing on numerical skills. The study by Wu and colleagues focused broadly on children's mathematics performance and included measures of their shape knowledge and pattern recognition skills. Although shape and patterning knowledge fall within the domain of early childhood mathematics, they have less theoretical overlap than measures of numerical knowledge, which may explain the broader categorization of children into three profiles compared to our finding four profiles.

Our finding is further aligned with previous work focusing specifically on children's numerical skills with a middle-income population, which found five profiles including high, moderate, and low performers (Gray & Reeve, 2016). One profile of children in our study demonstrated a qualitatively different pattern of skills than the high, moderate, and low performing groupings. Approximately one quarter of children (23%) in our sample were considered to have "strong counting and numerical skills, poor magnitude skills" due to their high performance on verbal counting, counting objects, cardinality, and labeling numerals, but lower accuracy on magnitude comparison tasks. This pattern of findings parallels the “excellent math” profile from Gray and Reeve (2016), who created profile labels based on middle-income children’s performance on tasks of verbal counting, object counting, cardinality, labeling numerals, ordinal relations, and simple arithmetic, and also related children’s profile membership to their accuracy and reaction times on magnitude comparison tasks. Like our strong counting and numeral skills but poor magnitude skills profile, children in the “excellent math” profile had high performance on tasks of verbal counting, object counting, cardinality, and labeling numerals, but were less efficient at magnitude comparison tasks than children in the “average math” and “poor math” profiles.

Relations Between Numerical and Executive Functioning Skills

Like previous studies, we found a significant relation between children’s executive functioning and their numerical abilities (Chew et al., 2016; Hannula et al., 2017; c.f., Gray & Reeve, 2016). Executive functioning may directly affect children’s ability to inhibit incorrect responses, keep relevant information in mind, and shift their attention to the relative features of numerical tasks (Geary & Hoard, 2005). In our study, WM differentiated between more profiles of numerical skills than inhibitory control. Among children from lower-income households, inhibitory control skills are thought to play an important role in explaining mathematical performance, particularly on measures that require inhibiting behaviors like selecting one of two response options, such as the magnitude comparison tasks (Fuhs & McNeil, 2013). However, theoretical and empirical research suggest that working memory may be the executive functioning skill most directly related to early mathematical performance (for a review, see Bull & Lee, 2014). Moreover, longitudinal research on children in early elementary school similarly found that working memory and numerical skills (e.g., counting, non-symbolic and symbolic addition and comparison) at the beginning of first grade predicts mathematics achievement at the end of second grade, providing further evidence for the importance of both domain-general EF and domain-specific numerical skills as early predictors of later math achievement (Xenidou-Dervou et al., 2018).

Implications for Future Research and Mathematical Learning

Our findings provide several important implications for researchers and practitioners. First, only a small proportion (11%) of our sample of low-income preschoolers had very low performance across the numerical skills measures. It is possible that this profile of children is at risk for future low mathematics achievement. Moreover, a closer investigation of children in this profile could help to identify children who may have a domain-specific mathematical deficit, or dyscalculia. An estimated 3 to 8 percent of school-aged children have dyscalculia (Geary, 2017). Future longitudinal research tracking the performance of children in a low-performing subgroup could help to determine whether or not individual children met criteria for persistent low-performance or dyscalculia. Notably, the low-performing children in the present study also had significantly lower working memory abilities, possibly signalling a more general cognitive deficit for some children. Previous research has demonstrated that measures that combine executive functioning and numerical skills, such as the number-surface area incongruent trials of non-symbolic magnitude comparison tasks, serve as stronger predictors of the math performance than measures of EF or numerical skills alone (Wilkey et al., 2020). Specifically, the significant association between number-specific EF skills and math achievement held among children categorized as dyscalculic, low-performing, and typically-performing in mathematics.

In contrast, the majority of children across the profiles were proficient at object counting, while many children were also successful at verbal counting, labeling numerals, and providing the correct cardinal value when asked. However, we found evidence of quantitative and qualitative differences between profiles, such that the patterns of performance varied across the different numerical skills assessed. In particular, most of the children in our sample would benefit from additional support on their magnitude understanding. Magnitude understanding is theorized to support numerical understanding across the lifespan (Siegler, 2016). Furthermore, symbolic magnitude understanding selectively predicts children's later arithmetic skills (Authors, under review; Kolkman et al., 2013; Siegler & Ramani, 2009; Toll & Van Luit, 2014), and serves as a domain-specific skill that is as important to later math achievement as early phonological awareness is to later reading (Vanbinst & De Smedt, 2016; Vanbinst et al., 2020).

In response to theoretical and empirical findings, there are a number of existing interventions that could be adapted to match the patterns of low-income children's needs. For early magnitude understanding, researchers have found that playing linear numbered board games and number comparison card games can lead to improvements in preschoolers' symbolic magnitude skills (Authors, 2017; Ramani & Siegler, 2008; Whyte & Bull, 2008). Our findings suggest that some subgroups of children would also benefit from additional practice on verbal counting, numeral identification, and cardinality, all skills that are incorporated in number games involving dice, spinners, or numerical cards. Interventionists should consider using pretest numerical skills measures as a screening tool to direct minor adaptations to experimental protocols. For example, a child whose pretest scores suggested they were in the moderate ability profile may benefit from more explicit practice with cardinality during game play (e.g., "How many dots are on your card?"), whereas a child with strong counting and numeral skills but poor magnitude skills could focus on symbolic number comparisons (e.g., "What is another number that is less than 3?"). These types of minor adaptations to existing intervention protocols would allow for a more direct targeting of low-income preschoolers' specific needs. However, the success of more targeted intervention designs needs to be tested experimentally in future research. In addition, future experimental studies could expand the assessment measures used to better capture the overlap between multiple numerical and executive functioning skills, such as tasks that combine non-symbolic and symbolic representations of numbers (e.g., sets-to-numerals mapping task, Purpura et al., 2013) or number-specific executive functioning (e.g., incongruent non-symbolic comparison trials, Wilkey et al., 2020).

Finally, although our sample size was sufficient for conducting a latent profile analysis, future research should seek to replicate the present results with a larger sample size given the number of measures included. Moreover, including participants from a range of socioeconomic backgrounds rather than a targeted sample of children from low-income households (such as the present study) or middle-income households (such as Gray and Reeve, 2016) would provide more evidence as to whether the results are consistent across SES groups.

Conclusion

In sum, the present study underscores the need to consider variability within low-income preschoolers' numerical skills to inform intervention designs. Children from low-income households are not a monolith, with both quantitative and qualitative differences in their numerical performance that have important implications for parents, educators, and researchers. With this understanding, researchers can spend valuable time and resources targeting the specific skills that children need to master. As we move as a field towards a broader consideration of individual differences in children's mathematical abilities (Dowker et al., 2019), it is critical to remember that one-size approaches do not fit the skill profiles of all children.

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