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Line Defects and Interfaces from Holography

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ABSTRACT OF THE DISSERTATION

Line Defects and Interfaces from Holography

by

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In this dissertation, we discuss half-BPS solutions of gauged supergravity that are holographic realizations of conformal line defects and interfaces. These solutions are constructed by taking a suitable ansatz for the geometry, consisting of a warped product of an AdS spacetime and a sphere, if necessary, over a line, and solving the supersymmetry variations. Quantities such as one-point functions in the presence of the defect and the entanglement entropy are calculated holographically.

In chapter 1, we review the AdS/CFT correspondence and holography. In chapter 2, we relate two different formulations of AdS_6 solutions in type IIB supergravity. In chapter 3, we construct solutions in six-dimensional $F(4)$ gauged supergravity that are dual to line defects. In chapter 4, we construct solutions in four-dimensional $N = 2$ gauged supergravity that are dual to line defects, obtained by a double-analytic continuation of BPS black hole solutions. In chapter 5, we construct Janus solutions in three-dimensional $\mathcal{N} = 8$ gauged supergravity that are dual to interface CFTs.

The dissertation of Kevin Chen is approved.

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Per Kraus

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University of California, Los Angeles

2022

To my son . . .

who has much to teach me

in the coming years.

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CONTRIBUTION OF AUTHORS

Chapter 2 is based on [1] in collaboration with Michael Gutperle. Chapter 3 is based on [2] in collaboration with Michael Gutperle. Chapter 4 is based on [3] in collaboration with Michael Gutperle and Matteo Vicino. Chapter 5 is based on [4] in collaboration with Michael Gutperle.

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PUBLICATIONS

K. Chen, M. Gutperle, and C. Hultgreen-Mena, "Janus and RG-flow interfaces in three-dimensional gauged supergravity," arXiv:2111.01839 [hep-th]

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CHAPTER 1

Introduction

1.1 The AdS/CFT correspondence

The AdS/CFT correspondence [5] is an important tool for understanding quantum gravity and field theories. It states that, in certain cases, a theory of gravity formulated on an anti-de Sitter (AdS) background in $d+1$ dimensions is equivalent, or *dual*, to a conformal field theory (CFT) that lives on its d-dimensional boundary. This equivalence is a realization of the holographic principle of quantum gravity [6, 7], and part of the work leading up to this discovery had been on studying black hole thermodynamics using field theoretic methods [8]. In the end, both directions of the duality are important—observables of the strongly-coupled field theory can be calculated in the gravitational theory [9, 10]. It is with this particular application that this dissertation will mostly be concerned. For reviews and lecture notes on the AdS/CFT correspondence, see [11–14].

To motivate discussion, let us briefly summarize the example given in [5]: consider N coincident D3 branes in type IIB string theory. In the low-energy limit, where we send the string length scale $\ell_s \to 0$ but keep N and the string coupling g_s fixed, we only have massless string states—namely those of four-dimensional $\mathcal{N}=4$ U(N) supersymmetric Yang-Mills on the D-branes, in addition to the bulk free supergravity. On the other hand, we can take the same low-energy limit in the D3 brane solution of supergravity,

$$\begin{aligned} \mathrm{d}s^2 &= f^{-1/2}\big(-\mathrm{d}t^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2\big) + f^{1/2}\big(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_5^2\big) \ , \\ f &= 1 + \frac{R^4}{r^4} \ , \qquad R^4 \equiv 4\pi g_s \alpha'^2 N \ . \end{aligned} \tag{1.1}$$

The excitations coming from the near-horizon region around $r = 0$ are red-shifted to low energies, as viewed by an observer at infinity, and they decouple from the bulk supergravity. In the limit $r \ll R$, the geometry becomes that of $\mathrm{AdS}_5 \times S^5$,

$$\mathrm{d}s^2 \approx \left[\frac{r^2}{R^2} \left(-\,\mathrm{d}t^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2 \right) + \frac{R^2}{r^2}\,\mathrm{d}r^2 \right] + R^2\,\mathrm{d}\Omega_5^2 \ . \tag{1.2}$$

This suggests that four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is the same as type IIB string theory on $\mathrm{AdS}_5 \times S^5$, and numerous checks support this correspondence. For one, the global symmetries of both theories agree: the $\mathrm{SO}(4,2)$ isometry group of AdS_5 is the same as the conformal group in four dimensions, and the $\mathrm{SO}(6)$ group of S^5 rotations is identified with the $\mathrm{SU}(4)$ R-symmetry of the field theory.

The classical supergravity description is valid when N is large[1] and the AdS_5 radius R is much larger than the string length,

$$1 \ll \frac{R^4}{\ell_s^4} \sim g_s N \ . \tag{1.3}$$

But since $g_s N \sim g_{\mathrm{YM}}^2 N$, this regime corresponds to large 't Hooft coupling in the field theory. So the AdS/CFT correspondence is a strong-weak duality—a strongly-coupled field theory is dual to a weakly-coupled gravitational theory, and vice versa. This allows us to study the dual CFT using classical supergravity in a regime where traditional field theoretic calculations may be intractable.

In this dissertation, we study line defects and interfaces in the CFT. These objects are realized holographically in the gravitational theory as excitations above the AdS vacuum. However, the equations of motion for fluctuations around the vacuum can be difficult to analyze. In many cases, the ten- or eleven-dimensional supergravity can be consistently truncated to a lower-dimensional gauged supergravity, where the infinite tower of Kaluza-Klein modes of a compact submanifold is truncated to just a finite subset and the equations

[1] We can also interpret this as the AdS_5 radius R being much larger than the Planck length, since $R^4/\ell_p^4 \sim N$. This suppresses loop contributions in the gravitational theory.

of motion close on the remaining modes. Solutions to the equations of motion in the lower-dimensional gauged supergravity are often easier to find as there are fewer fields to consider, and they can be uplifted to solutions in the higher-dimensional supergravity. For instance, type IIB supergravity on $\mathrm{AdS}_5 \times S^5$ can be consistently truncated to five-dimensional SO(6) gauged supergravity [15–17]. Even in cases where a consistent truncation has not yet been established, it is still fruitful to study solutions of the lower-dimensional gauged supergravity in order to study general properties of these excitations.

1.2 Holography

In this section, we outline how CFT observables can be computed from the gravitational theory, according to [9, 10]. For concreteness, consider (Euclidean) AdS_{d+1} in Poincaré coordinates,

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(\mathrm{d}z^2 + \mathrm{d}x_1^2 + \cdots + \mathrm{d}x_d^2\right) . \tag{1.4}$$

The d-dimensional dual CFT is located at the $z \to 0$ boundary. Note that the CFT vacuum state is dual to this pure AdS solution; excited states are dual to asymptotically AdS solutions of the supergravity, where the metric approaches an AdS metric at the boundary. The asymptotic behavior of the bulk fields near the boundary determine correlation functions of the CFT. To illustrate this, consider a free massive scalar in the bulk,

$$\int_{\mathrm{bulk}} \mathrm{d}^{d+1}x \, \sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2\right) . \tag{1.5}$$

Solving the equations of motion on the AdS background and ignoring the backreaction, we obtain two linearly independent solutions whose leading-order expansions in z are

$$\phi(x,z) = \phi_0(x)z^{\lambda_-} + \phi_1(x)z^{\lambda_+} + \cdots , \tag{1.6}$$

where

$$\lambda_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2} , \tag{1.7}$$

are the roots of the equation $\lambda(\lambda - d) = m^2$. Note that unitarity requires $m^2 \geq -d^2/4$, so $\lambda_\pm$ are real [18, 19]. The boundary value $\phi_0(x)$ couples to an operator $\mathcal{O}(x)$ of the CFT via a coupling term $\int_{\text{bdy}} \phi_0 \mathcal{O}$ and we identify λ_+ as the scaling dimension of this dual operator.[2]

The statement of the AdS/CFT correspondence is the identification of the bulk AdS partition function as a generating function for correlation functions in the boundary CFT:

$$\left\langle \exp \int_{\text{bdy}} \phi_0 \mathcal{O} \right\rangle = \exp(-S_{\text{on-shell}}[\phi_0]) \;, \tag{1.8}$$

where $S_{\text{on-shell}}[\phi_0]$ is the classical on-shell supergravity action. Then, CFT correlation functions can be calculated by functional derivatives,

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle = \frac{\delta}{\delta\phi_0(x_1)} \cdots \frac{\delta}{\delta\phi_0(x_n)} S_{\text{on-shell}}[\phi_0] \bigg|_{\phi_0 = 0} \;. \tag{1.9}$$

A similar procedure applies for other types of fields, such as vectors and tensors. This identification is made more precise below.

Correlation functions in the CFT have UV divergences that require renormalization to make them well-defined. Likewise, the gravitational on-shell action is divergent due to the infinite region of integration at the boundary. To control the divergences of the gravitational theory, we use *holographic renormalization* [21, 22]: we expand, as a power series in z near the boundary, the bulk metric,

$$\begin{aligned} \mathrm{d}s^2 &= \frac{1}{z^2}\left(\mathrm{d}z^2 + g_{ij}(x,z)\,\mathrm{d}x^i\,\mathrm{d}x^j\right) \;, \\ g_{ij}(x,z) &= g_{(0)ij}(x) + z g_{(1)ij}(x) + \cdots + z^d g_{(d)ij}(x) + z^d \log z\, \tilde{g}_{(d)ij}(x) + \cdots \;, \end{aligned} \tag{1.10}$$

and all bulk fields of the theory. For a general bulk field $\mathcal{F}(x,z)$, this expansion is

$$\mathcal{F}(x,z) = z^m \Big(f_{(0)}(x) + z f_{(1)}(x) + \cdots + z^n f_{(n)}(x) + z^n \log z\, \tilde{f}_{(n)}(x) + \cdots \Big) \;, \tag{1.11}$$

(for the scalar field, the powers m and $m+n$ correspond to λ_- and λ_+ above). The boundary value $f_{(0)}$ that multiplies the leading order term corresponds to the source for a CFT operator

[2] If $-d^2/4 < m^2 < 1 - d^2/4$, there exists a second possible quantization where instead we take λ_- to be the scaling dimension and $\phi_1(x)$ to be the source that couples to the dual operator [20].

(e.g. $g_{(0)ij}$ sources the stress tensor [23]). If we plug these expansions into the equations of motion and group the terms order by order in z, the coefficients $f_{(1)}, \ldots, f_{(n-1)}, \tilde{f}_{(n)}$ can be solved for entirely in terms of the $f_{(0)}$ sources. The coefficient $f_{(n)}$ is undetermined by the equations of motion, and will be associated with the expectation value of the corresponding CFT operator in the presence of the sources. The coefficient $\tilde{f}_{(n)}$ that multiplies a $\log z$ is generically needed in order to satisfy the equations of motion, and will be related to the conformal anomaly.

With these expansions, we can regulate the on-shell action by restricting the integral to $z \geq \varepsilon$ for some small constant $\varepsilon > 0$. Upon integrating z, we obtain a regularized action on the boundary with a finite number of divergences as $\varepsilon \to 0$,

$$S_{\text{reg}} = \int_{z=\varepsilon} \mathrm{d}^d x \, \sqrt{g_{(0)}} \big(\varepsilon^{-k} a_{(0)} + \varepsilon^{-k+1} a_{(1)} + \cdots + \log \varepsilon \, a_{(k)} + \mathcal{O}\big(\varepsilon^0\big) \big) \ , \tag{1.12}$$

where $a_{(0)}, \ldots, a_{(k)}$ are local functions of $f_{(0)}$ and do not depend on $f_{(n)}$. The coefficient $a_{(k)}$ that multiplies a $\log \varepsilon$ is associated with the conformal anomaly [24]. These divergences can be canceled out by local counterterms S_{ct} which are expressed in terms the boundary fields $\mathcal{F}(x, \varepsilon)$ and induced metric $h_{ij}(x) = g_{ij}(x, \varepsilon)/\varepsilon^2$ on the boundary. In other words, we invert the series in (1.11) to obtain $f_{(0)} = f_{(0)}(\mathcal{F}(x, \varepsilon), \varepsilon)$, which we substitute into the $a_{(0)}, \ldots, a_{(k)}$ coefficients. The renormalized action,

$$S_{\text{ren}} = S_{\text{reg}} + S_{\text{ct}} \ , \tag{1.13}$$

is then finite as we take $\varepsilon \to 0$ and can be used to calculate well-defined CFT correlation functions in the presence of sources, in the manner described by (1.9). For instance, if $\mathcal{O}(x)$ is the CFT operator dual to the bulk field $\mathcal{F}(x, z)$, the one-point function of $\mathcal{O}(x)$ in the presence of sources is

$$\langle \mathcal{O} \rangle_{\text{sources}} = \lim_{\varepsilon \to 0} \frac{1}{\sqrt{g_{(0)}}} \frac{\delta S_{\text{ren}}}{\delta f_{(0)}} = \lim_{\varepsilon \to 0} \left(\varepsilon^{m-d} \frac{1}{\sqrt{h}} \frac{\delta S_{\text{ren}}}{\delta \mathcal{F}(x, \varepsilon)} \right) \ . \tag{1.14}$$

Explicit evaluation of this limit yields

$$\langle \mathcal{O} \rangle_{\text{sources}} \sim f_{(n)} + C(f_{(0)}) \ , \tag{1.15}$$

where C is a local function of the sources, and so leads to contact terms in higher order functions.

Another quantity that we can calculate using the AdS/CFT correspondence is the entanglement entropy. Let A be a $(d-1)$-dimensional static submanifold of the boundary CFT, and let B denote its compliment. The entanglement entropy S_{EE} of the region A can be calculated holographically using the Ryu-Takayanagi prescription [25, 26],

$$S_{\mathrm{EE}} = \frac{\mathrm{Area}(\Gamma_A)}{4G_N^{(d+1)}} \ , \tag{1.16}$$

where Γ_A is the $(d-1)$-dimensional minimal surface in the bulk AdS_{d+1} whose boundary coincides with that of A. Intuitively, since the entanglement entropy is defined in the CFT by tracing out B and making that region inaccessible to an observer in A, we can think of this from the bulk point of view as hiding the region B behind an event horizon. Then (1.16) is a measure of entropy on the horizon, analogous to the Bekenstein-Hawking entropy of a black hole [27]. A covariant version of this prescription was subsequently developed in [28], and allows for time-dependence of the entanglement entropy.

1.3 Conformal defects

In addition to local operators, QFTs also contain important extended objects. The most famous are Wilson and 't Hooft lines in four-dimensional gauge theories, which play a role in characterizing the phases of the theory [29, 30]. Related to these are Gukov-Witten surface operators [31, 32], which generate one-form global symmetries that the Wilson and 't Hooft lines are charged under [33, 34]. A special class of extended objects in CFTs are conformal defects, which preserve a subgroup of the conformal symmetry. For example, boundaries and line defects have been studied extensively in two-dimensional CFTs [35–40]. In general, a p-dimensional conformal defect in a d-dimensional CFT preserves a $\mathrm{SO}(p,2)\times\mathrm{SO}(d-p)$ subgroup of the $\mathrm{SO}(d,2)$ conformal group, which corresponds to the residual conformal transformations of the defect and the rotations in the transverse directions [41].

In a SCFT, an extended object that additionally preserves a superconformal subgroup is called a superconformal defect.

In addition to the usual bulk CFT operators, whose operator product expansion (OPE) take the schematic form,

$$\mathcal{O}_1(x)\mathcal{O}_2(y) \sim \sum_k C_{12k}|x-y|^{\Delta_k-\Delta_1-\Delta_2}\mathcal{O}_k(y) \ , \tag{1.17}$$

there can also be local operators $\widehat{\mathcal{O}}$ living on the defect, which can fuse amongst themselves according to their own OPE. Also, when a bulk operator is brought towards the defect, it excites defect operators according to a bulk-to-defect OPE,

$$\mathcal{O}(x) \sim \sum_{\widehat{\mathcal{O}}} B_{\mathcal{O}\widehat{\mathcal{O}}}|x_\perp|^{\widehat{\Delta}-\Delta}\widehat{\mathcal{O}}(x_\parallel) \ , \tag{1.18}$$

where we have assumed for simplicity a flat defect and separated x into the $x_\perp$ perpendicular and $x_\parallel$ parallel directions. In particular, bulk operators acquire a non-vanishing one-point function in the presence of the defect, due to the OPE with the defect identity operator,

$$\langle\mathcal{O}(x)\rangle = A_{\mathcal{O}}|x_\perp|^{-\Delta} \ . \tag{1.19}$$

Conformal defects can be realized holographically in the gravitational theory by probe branes. For example, the fundamental Wilson line in four-dimensional $\mathcal{N}=4$ $\mathrm{U}(N)$ supersymmetric Yang-Mills corresponds to an insertion of a fundamental string in $\mathrm{AdS}_5\times S^5$ that ends at the boundary along the curve of the Wilson line [42,43]. The generalization to higher representations, which involves insertions of D3 branes in $\mathrm{AdS}_5\times S^5$ carrying fundamental string charge, was subsequently worked out in [44–48]. When the number of probe branes becomes large, the backreaction can not be neglected and a fully backreacted supergravity solution replaces the probe description. For the case of the Wilson lines mentioned above, this solution was found in [49]. This is done by taking a geometry whose isometries match the preserved $\mathrm{SO}(p,2)\times\mathrm{SO}(d-p)$ subgroup, which is most easily done by taking a warped product of AdS_{p+1} and S^{d-p-1} over a line,

$$\mathrm{d}s^2 = f_1(r)\,\mathrm{d}s^2_{\mathrm{AdS}_{p+1}} + f_2(r)\,\mathrm{d}\Omega^2_{d-p-1} + f_3(r)\,\mathrm{d}r^2 \ . \tag{1.20}$$

In this dissertation, we use this ansatz geometry to consider holographic realizations of two types of superconformal defects in gauged supergravity: line defects ($p = 1$) and interfaces ($p = d - 1$). Interfaces, in this context, are conformal defects with the additional requirement that the defect has no degrees of freedom itself—local operators on the interface only involve fields of the bulk CFT. For example, the Janus solution in [50] is a deformation of type IIB supergravity on $\mathrm{AdS}_5 \times S^5$ which divides the asymptotic AdS boundary into two halves, and the bulk dilaton approaches a different constant value in each half. Since the asymptotic value of the dilaton is associated with a source for the Lagrangian density of four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills,[3] the corresponding dual CFT contains a planar interface, across which the gauge coupling is discontinuous [51]. Solutions with a spatially-varying axion are obtained by taking $\mathrm{SL}(2, \mathbb{R})$ transformations [52]. These Janus solutions break all supersymmetries, but Janus solutions that preserve some supersymmetry were subsequently discovered [52–55], which correspond to supersymmetric interfaces [56,57]. For a partial list of other examples of supergravity solutions corresponding to interfaces, see [58–67].

[3]More precisely, it is dual to the fourth descendent $Q^4\mathcal{O}$ of the $\Delta = 2$ chiral primary $\mathcal{O}^{IJ} = \mathrm{Tr}\big(\Phi^{\{I}\Phi^{J\}}\big)$.

CHAPTER 2

Relating AdS_6 solutions in type IIB supergravity

Five-dimensional superconformal field theories take an interesting place among superconformal field theories. They realize a unique superconformal algebra $F(4)$, they are strongly coupled in the UV, and many exhibit unusual properties such as enhanced exceptional flavor symmetries [68–70]. However, until recently, very few supergravity solutions in ten or eleven dimensions dual to five-dimensional SCFTs were known. The first solutions [71–73] were constructed in massive type IIA supergravity. Special examples of type IIB solutions were constructed from the massive IIA solutions using (non-Abelian) T-duality in [74–76]. In [77], type IIB supergravity solutions were constructed from first principles. The solutions take the form of a fibration of AdS_6 over a four-dimensional base manifold and pure spinor geometry was used to determine the conditions for sixteen unbroken supersymmetries. It was found that the manifold M_4 is a S^2 fibration over a two-dimensional space and the problem was reduced to solving two partial differential equations on this two-dimensional space. In [78], a different approach utilizing Killing spinors on an $AdS_6 \times S_2$ fibration over a two-dimensional Riemann surface Σ_2 was used to reduce the BPS equations of the bosonic background. It was shown that local solutions can be expressed in terms of two holomorphic functions on the Riemann surface Σ_2. Later, regular global solutions were constructed [79, 80] and shown to be related to the conformal fixed-points of field theories derived from taking a conformal limit of (p, q) 5-brane webs [81, 82]. Various aspects of these solutions have been studied recently, see e.g. [83–95].

The goal of this chapter is to relate the form of the local IIB solutions found in [77] to the ones found in [78] and determine the exact map between the two. In addition, we analyze the regularity conditions and the map for global regular solutions. The structure of this chapter is as follows. In sections 2.1 and 2.2, we briefly review the two supergravity solutions of [77] and [78], respectively. In section 2.3, we determine the exact map between these two solutions, and illustrate the relation with some explicit examples. In section 2.4, we look at how the global regular solution of [80] is mapped into framework of [77]. We conclude with a discussion in section 2.5.

2.1 Review of AFPRT solutions

Here we outline the solution in [77] by Apruzzi, Fazzi, Passias, Rosa, and Tomasiello (AFPRT). The spacetime takes the form of $\mathrm{AdS}_6 \times S^2 \times \Sigma_2$ and the supergravity fields depend on the two-dimensional space Σ_2 through four quantities (x, α, A, ϕ), two of which are actually independent and can be used to parameterize Σ_2. Following [77] we take (x, α) to be independent and $A = A(x, \alpha)$ and $\phi = \phi(x, \alpha)$ to be dependent functions. Here e^A is a warping function and e^ϕ is the dilaton. These quantities satisfy two partial differential equations,

$$
\begin{aligned}
&\mathrm{d}\left[\frac{e^{4A-\phi}}{x}\cot\alpha\,\mathrm{d}(e^{2A}\cos\alpha) + \frac{1}{3x}e^{2A}\sqrt{1-x^2}\,\mathrm{d}(e^{4A-\phi}\sqrt{1-x^2}\sin\alpha)\right] = 0\ , \\
&3\sin(2\alpha)\,\mathrm{d}A\wedge\mathrm{d}\phi = \mathrm{d}\alpha\wedge\left[6\,\mathrm{d}A + \sin^2\alpha\big(-\mathrm{d}(x^2) - 2(x^2+5)\,\mathrm{d}A + (1+2x^2)\,\mathrm{d}\phi\big)\right]\ . \qquad (2.1)
\end{aligned}
$$

The metric in the string frame is

$$
\mathrm{d}s^2_{\mathrm{S}} = \frac{\cos\alpha}{\sin^2\alpha}\frac{\mathrm{d}q^2}{q} + \frac{1}{9}q(1-x^2)\frac{\sin^2\alpha}{\cos\alpha}\left[\frac{1}{x^2}\left(\frac{\mathrm{d}p}{p} + 3\cot^2\alpha\frac{\mathrm{d}q}{q}\right)^2 + \mathrm{d}s^2_{S^2}\right] + e^{2A}\,\mathrm{d}s^2_{\mathrm{AdS}_6}\ , \qquad (2.2)
$$

where p, q are quantities defined by

$$
\begin{aligned}
q &= e^{2A}\cos\alpha\ , \\
p &= e^{4A-\phi}\sin\alpha\sqrt{1-x^2}\ . \qquad (2.3)
\end{aligned}
$$

The one-form field strength F_1 is

$$
F_1 = s_1 s_2\frac{e^{-\phi}}{6x\cos\alpha}\left[\frac{12\,\mathrm{d}A}{\sin\alpha} + 4e^{-A}(x^2-1)\,\mathrm{d}(e^A\sin\alpha) + e^{2\phi}\sin\alpha\,\mathrm{d}\big(e^{-2\phi}(1+2x^2)\big)\right]\ , \qquad (2.4)
$$

and the three-form NS-NS and R-R field strengths, H_3 and F_3, are

$$H_3 = s_1 \frac{1}{9x} e^{2A} \sqrt{1-x^2} \sin\alpha \left[-\frac{6\,\mathrm{d}A}{\sin\alpha} + 2e^{-A}(1+x^2)\,\mathrm{d}(e^A \sin\alpha) + \sin\alpha\,\mathrm{d}(\phi + x^2) \right] \wedge \mathrm{vol}_{S^2} \ ,$$

$$F_3 = s_2 \frac{e^{2A-\phi}}{54} \sqrt{1-x^2} \frac{\sin^2\alpha}{\cos\alpha} \left[\frac{36\,\mathrm{d}A}{\sin\alpha} + 4e^{-A}(x^2-7)\,\mathrm{d}(e^A \sin\alpha) + e^{2\phi} \sin\alpha\,\mathrm{d}\big(e^{-2\phi}(1+2x^2)\big) \right] \wedge \mathrm{vol}_{S^2} \ , \qquad (2.5)$$

where s_1 and s_2 are $\pm$ signs and vol_{S^2} denotes the volume form of S^2 with unit radius. The self-dual five-form field strength F_5 vanishes in this background. These field strengths satisfy the Bianchi identities,

$$\begin{aligned} 0 &= \mathrm{d}F_1 \ , \\ 0 &= \mathrm{d}F_3 - H_3 \wedge F_1 \ , \\ 0 &= \mathrm{d}H_3 \ . \end{aligned} \qquad (2.6)$$

The signs s_1, s_2 depend on the specific supergravity solution, which we discuss later in this chapter.

2.2 Review of DGKU solutions

Here we outline the solution in [78] by D'Hoker, Gutperle, Karch, and Uhlemann (DGKU). The spacetime takes the form of $\mathrm{AdS}_6 \times S^2 \times \Sigma_2$, where Σ_2 is a Riemann surface parametrized by complex coordinates $w, \bar{w}$. The supergravity fields depend only on Σ_2 through two holomorphic functions $\mathcal{A}_\pm(w)$. For completeness we present the following quantities which are useful to express the supergravity fields in a concise form. We use the notation $\partial \equiv \partial_w$ and

$\bar{\partial} \equiv \partial_{\bar{w}}$.

$$\begin{aligned} \kappa_{\pm} &= \partial \mathcal{A}_{\pm} \ , \\ \kappa^2 &= -|\kappa_+|^2 + |\kappa_-|^2 \ , \\ \partial \mathcal{B} &= \mathcal{A}_+ \partial \mathcal{A}_- - \mathcal{A}_- \partial \mathcal{A}_+ \ , \\ \mathcal{G} &= |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}} \ , \\ \mathcal{D} = \left(\frac{1+R}{1-R}\right)^2 &= 1 + \frac{2|\partial \mathcal{G}|^2}{3\kappa^2 \mathcal{G}} \ . \end{aligned} \tag{2.7}$$

The metric in the Einstein frame is

$$\mathrm{d}s_{\mathrm{E}}^2 = f_6^2 \, \mathrm{d}s_{\mathrm{AdS}_6}^2 + f_2^2 \, \mathrm{d}s_{S^2}^2 + 4\rho^2 |\, \mathrm{d}w \,|^2 \ , \tag{2.8}$$

where the metric factors are

$$\begin{aligned} f_6^2 &= \frac{\kappa^2}{\rho^2} \sqrt{\mathcal{D}} \ , \\ f_2^2 &= \frac{\kappa^2}{9\rho^2} \frac{1}{\sqrt{\mathcal{D}}} \ , \\ (\rho^2)^2 &= \frac{\kappa^4}{6\mathcal{G}} \sqrt{\mathcal{D}} \ . \end{aligned} \tag{2.9}$$

Note that to make contact with the parameterization in section 2.1, the metric should be transformed into the string frame,

$$\mathrm{d}s_{\mathrm{S}}^2 = e^{\phi/2} \, \mathrm{d}s_{\mathrm{E}}^2 \ . \tag{2.10}$$

Here the dilaton is normalized in the standard fashion to $\tau = \chi + i e^{-\phi}$. The solution [78] utilizes an $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$ parametrization of the complex scalar field in terms of B, which is related to the axion-dilaton field via

$$B = \frac{1 + i\tau}{1 - i\tau} \ , \tag{2.11}$$

and is given by [91] in terms of the defined quantities as

$$B = \frac{\mathcal{S} + \mathcal{T}/\sqrt{\mathcal{D}}}{\bar{\mathcal{S}} - \bar{\mathcal{T}}/\sqrt{\mathcal{D}}} \ , \tag{2.12}$$

where for notational convenience we introduced the quantities,

$$\mathcal{S} = -\mathcal{A}_+ + \bar{\mathcal{A}}_- \ ,$$
$$\mathcal{T} = \frac{\kappa_+ \bar{\partial}\mathcal{G} + \bar{\kappa}_- \partial\mathcal{G}}{\kappa^2} \ . \tag{2.13}$$

This gives expressions for the axion χ and the dilaton e^ϕ,

$$e^\phi = -\frac{(\mathcal{S}+\bar{\mathcal{S}})^2 - (\mathcal{T}-\bar{\mathcal{T}})^2/\mathcal{D}}{2(\mathcal{S}\bar{\mathcal{T}}+\bar{\mathcal{S}}\mathcal{T})/\sqrt{\mathcal{D}}} \ ,$$
$$\chi = -i\,\frac{(\mathcal{S}^2-\bar{\mathcal{S}}^2) - (\mathcal{T}^2-\bar{\mathcal{T}}^2)/\mathcal{D}}{(\mathcal{S}+\bar{\mathcal{S}})^2 - (\mathcal{T}-\bar{\mathcal{T}})^2/\mathcal{D}} \ . \tag{2.14}$$

If we also define

$$\mathcal{U}_\pm = (\kappa_+ \pm \kappa_-)\bar{\partial}\mathcal{G} \ , \tag{2.15}$$

then noting the relations,

$$\mathcal{U}_- + \bar{\mathcal{U}}_- = \kappa^2(\mathcal{S}+\bar{\mathcal{S}}) \ , \qquad \mathcal{U}_- - \bar{\mathcal{U}}_- = \kappa^2(\mathcal{T}-\bar{\mathcal{T}}) \ ,$$
$$\mathcal{U}_+ + \bar{\mathcal{U}}_+ = \kappa^2(\mathcal{T}+\bar{\mathcal{T}}) \ , \qquad \mathcal{U}_+ - \bar{\mathcal{U}}_+ = \kappa^2(\mathcal{S}-\bar{\mathcal{S}}) \ , \tag{2.16}$$

we have yet another expression for the axion and dilaton,

$$e^\phi = -\frac{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2/\mathcal{D}}{(\mathrm{Re}\,\mathcal{U}_-\,\mathrm{Re}\,\mathcal{U}_+ + \mathrm{Im}\,\mathcal{U}_-\,\mathrm{Im}\,\mathcal{U}_+)/\sqrt{\mathcal{D}}} = \frac{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2/\mathcal{D}}{|\partial\mathcal{G}|^2\kappa^2/\sqrt{\mathcal{D}}} \ , \tag{2.17}$$
$$\chi = \frac{\mathrm{Re}\,\mathcal{U}_-\,\mathrm{Im}\,\mathcal{U}_+ - \mathrm{Im}\,\mathcal{U}_-\,\mathrm{Re}\,\mathcal{U}_+/\mathcal{D}}{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2/\mathcal{D}} \ . \tag{2.18}$$

The one-form field strength F_1 is given in terms of the axion χ by

$$F_1 = \mathrm{d}\chi \ . \tag{2.19}$$

The complex two-form potential $\mathcal{C}_2$ is given by

$$\mathcal{C}_2 = \frac{2i}{9}\left[\frac{\mathcal{T}}{\mathcal{D}} - 3(\mathcal{A}_+ + \bar{\mathcal{A}}_-)\right]\mathrm{vol}_{S^2} \ . \tag{2.20}$$

This can be written in terms of the real two-form potentials C_2 and B_2,

$$\mathcal{C}_2 = B_2 + iC_2 \ , \tag{2.21}$$

where now

$$
\begin{aligned}
B_2 &= -\frac{1}{9i}\left[\frac{\mathcal{T}-\bar{\mathcal{T}}}{\mathcal{D}} - 3(\mathcal{A}_+ + \bar{\mathcal{A}}_- - \bar{\mathcal{A}}_+ - \mathcal{A}_-)\right]\mathrm{vol}_{S^2} \ , \\
C_2 &= \quad \frac{1}{9}\left[\frac{\mathcal{T}+\bar{\mathcal{T}}}{\mathcal{D}} - 3(\mathcal{A}_+ + \bar{\mathcal{A}}_- + \bar{\mathcal{A}}_+ + \mathcal{A}_-)\right]\mathrm{vol}_{S^2} \ .
\end{aligned} \tag{2.22}
$$

This gives the R-R and NS-NS three-form field strengths F_3 and H_3,

$$
\begin{aligned}
F_3 &= \mathrm{d}C_2 - H_3\chi \ , \\
H_3 &= \mathrm{d}B_2 \ .
\end{aligned} \tag{2.23}
$$

The self-dual five-form field strength F_5 vanishes. These field strengths satisfy the same Bianchi identities (2.6) given previously.

2.3 Mapping local solutions

Given these two different approaches to finding half-BPS solutions with AdS_6 factors in type IIB supergravity, our goal is to determine how they are related. We note that the difference in the parameterization of the solution lies in the two-dimensional Riemann surface Σ_2. The DGKU solution uses a uniformized form with complex coordinates $w, \bar{w}$ whereas the AFPRT solution uses coordinates which are adapted to the pure spinor construction leading to the PDEs (2.1).

In order to relate the DGKU solution with the AFPRT solution the goal is to identify the four quantities (x, α, A, ϕ) of AFPRT in terms of the coordinates $w, \bar{w}$ given the holomorphic data $\mathcal{A}_\pm(w)$ of DGKU. We use the fact that the four quantities $(f_2^2, f_6^2, \chi, \phi)$ are scalars with respect to Σ_2 and hence are independent of coordinate choices. Consequently they can be used to obtain a map between the two parameterizations. We show in the following that the coordinates x and α and the independent functions A and ϕ can be expressed in terms of the holomorphic functions $\mathcal{A}_\pm(w)$ and that they satisfy the PDEs (2.1).

2.3.1 Positivity

We start by slightly adapting the discussion of positivity in [78]. On the Riemann surface Σ_2, we consider solutions where the supergravity fields remain finite and the metric components are strictly positive,

$$0 < f_2^2, f_6^2, \rho^2 \ . \tag{2.24}$$

Then the definitions (2.7) imply that κ^2, $\mathcal{G}$, and $\sqrt{\mathcal{D}}$ are non-zero, finite, and have the same sign. From the definition of $\mathcal{D}$, we necessarily have $\mathcal{D} \geq 1$. In taking the square-root we have a sign ambiguity, so without loss of generality we can take $\sqrt{\mathcal{D}} \geq 1$. This gives us the equivalent constraints,

$$0 < \kappa^2, \mathcal{G} < \infty \text{ on } \Sigma_2 \ . \tag{2.25}$$

2.3.2 Matching metric factors

We can start by identifying the metrical factors of $\mathrm{d}s^2_{S^2}$ and $\mathrm{d}s^2_{\mathrm{AdS}_6}$ in the two string frame metrics (2.2) and (2.8),

$$\begin{aligned} f_2^2 e^{\phi/2} &= \frac{1}{9} e^{2A} (1 - x^2) \sin^2\alpha \ , \\ f_6^2 e^{\phi/2} &= e^{2A} \ . \end{aligned} \tag{2.26}$$

Then using the definitions in (2.9) we have

$$\mathcal{D} = \frac{1}{(1 - x^2)\sin^2\alpha} \ . \tag{2.27}$$

The dilaton e^{ϕ} is given explicitly in (2.17), and so is e^A through (2.26). Then including (2.27) above, we can express three quantities of AFPRT in terms of $w, \bar{w}$:

$$(1 - x^2)\sin^2\alpha \ , \qquad e^A \ , \qquad e^{\phi} \ .$$

Only one more quantity needs to be matched. If we equate the remaining portions of the two metrics, which correspond to $\mathrm{d}s_\Sigma^2$, we can simplify to get

$$\cot^2\alpha\left(\frac{\mathrm{d}q}{q}\right)^2+\frac{1}{9\mathcal{D}x^2}\left(\frac{\mathrm{d}p}{p}+3\cot^2\alpha\frac{\mathrm{d}q}{q}\right)^2=\frac{2\kappa^2}{3\mathcal{G}}\,\mathrm{d}w\,\mathrm{d}\bar{w}\ . \tag{2.28}$$

We may also make the replacement $\cot^2\alpha=\mathcal{D}(1-x^2)-1$. This equation turns out to be not very helpful because it contains derivatives of α in $\mathrm{d}q$. We can write the left-hand side in terms of α, its first-order derivatives $\partial_w\alpha$ and $\partial_{\bar{w}}\alpha$, and other quantities we already know how to write in terms of $w,\bar{w}$. Matching the differentials $\mathrm{d}w\,\mathrm{d}w$, $\mathrm{d}w\,\mathrm{d}\bar{w}$, and $\mathrm{d}\bar{w}\,\mathrm{d}\bar{w}$ on both sides will give first-order non-linear PDEs for $\alpha(w,\bar{w})$. We will not attempt this approach as it turns out there is a more direct way match the last remaining quantity.

2.3.3 Matching one-forms

The last remaining quantity can be matched using the one-form field strength. In AFPRT, F_1 is given in (2.4). In DGKU, we have an expression for the axion χ in (2.18). We can then simplify the equation $F_1=\mathrm{d}\chi$ to

$$\begin{aligned}4(3\mathcal{D}-1)(1-x^2)\,\mathrm{d}A-2(1+2x^2)\,\mathrm{d}\phi+2(1-x^2)\,\mathrm{d}\ln\mathcal{D}&\\=6s_1s_2xe^{\phi}\,\mathrm{d}\chi\,(\mathcal{D}(1-x^2)-1)^{1/2}\ .&\end{aligned} \tag{2.29}$$

It is important to note that the $\mathrm{d}\alpha$ dependence drops out. Now apart from x^2, every quantity appearing in this equation (i.e. A, ϕ, $\mathcal{D}$, and χ) can be written in terms of $w,\bar{w}$. By making the replacement $\mathrm{d}\to\partial_w$ and squaring of both sides of the equation, we obtain a complex-valued quadratic equation for x^2. This quadratic equation is very complicated for general $\mathcal{A}_\pm$ functions, but will always have a real root with the surprisingly simple form,

$$1-x^2=\frac{(\mathcal{S}+\bar{\mathcal{S}})^2-(\mathcal{T}-\bar{\mathcal{T}})^2/\mathcal{D}}{(\mathcal{S}+\bar{\mathcal{S}})^2-(\mathcal{T}-\bar{\mathcal{T}})^2}\ . \tag{2.30}$$

This was arrived at by firstly taking explicit examples for $\mathcal{A}_\pm$ where the one-form equation (2.29) was simple enough to be solved and guessing a general solution, and then verifying this solution algebraically for general $\mathcal{A}_\pm$ using Mathematica. So far no insightful simplifications have been found; due to the simplicity of the solution despite the complexity of the quadratic equation, it is very possible that this conclusion can be obtained from simpler considerations. Another useful form is given by

$$1 - x^2 = \frac{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2/\mathcal{D}}{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2} \ . \tag{2.31}$$

2.3.4 Explicit examples

Before continuing, let us verify this mapping for two established solutions which were discussed as examples in [77].

Example 1 – The first example of a type IIB solution in [77] is given by

$$e^A = \frac{c_1}{\cos^{1/6}\alpha} \ , \qquad e^\phi = \frac{c_2}{\sin\alpha\cos^{2/3}\alpha} \ , \qquad x = 0 \ , \tag{2.32}$$

for $0 < \alpha < \pi/2$. As the only independent variable is α, this solution is slightly degenerate. The holomorphic data which reproduces this solution is given in [78], and is slightly changed here for convenience,

$$\mathcal{A}_\pm = -\frac{a}{2}w^2 \pm ibw \ , \qquad \mathcal{B}(w=0) = \frac{ab}{6} \ . \tag{2.33}$$

Below we give some relevant derived quantities. In this example, we use coordinates $w = (X + iY)/2$.

$$\begin{aligned}
\kappa^2 &= 2abY \ , \\
\mathcal{G} &= \frac{1}{3}ab(1 - Y^3) \ , \\
\mathcal{D} &= \frac{1}{1 - Y^3} \ , \\
\mathcal{U}_- &= 2ab^2Y^2 \ , \\
e^\phi &= \frac{2b}{aY}\frac{1}{\sqrt{1 - Y^3}} \ ,
\end{aligned}$$

$$e^A = \frac{\sqrt{2b}}{Y^{1/4}} \ ,$$
$$\chi = \frac{aX}{2b} \ . \tag{2.34}$$

For positivity, we take $0 < Y < 1$. The one-form equation (2.29) simplifies to

$$\frac{9x^2 Y(4 - 20x^2 + 25x^2 Y^3)}{(1 - Y^3)^2} = 0 \ , \tag{2.35}$$

and has a solution $x^2 = 0$. This is consistent with our solution for x^2 in (2.31) as $\mathcal{U}_-$ has zero imaginary part. Then the equation for $\mathcal{D}$ in (2.27) implies the coordinate change,

$$Y = \cos^{2/3}\alpha \ . \tag{2.36}$$

Plugging this into our expressions for e^ϕ and e^A in (2.34) above gives exactly the desired solution (2.32), with the identifications,

$$c_1 = \sqrt{2b} \ , \qquad c_2 = 2b/a \ . \tag{2.37}$$

If we write out the metric starting from (2.8) with $w = (X + i\cos^{2/3}\alpha)/2$, we can match the metric in [75, Eq. (A.1)] if we identify

$$\frac{1}{4}\hat{W}^2\hat{L}^2 = \frac{2b}{9}\frac{1}{\cos^{1/3}\alpha} \ , \qquad \hat{\theta} = \alpha \ , \qquad \hat{\phi}_3 = \frac{2b}{3}X \ , \tag{2.38}$$

and the axion and dilaton if we identify

$$a = \frac{27}{16}\hat{L}^4\hat{m}^{1/3} \ , \qquad b = \frac{9}{8}\hat{L}^2\hat{m}^{-1/3} \ , \tag{2.39}$$

where quantities with ˆ are those of [75]. Then the three-form field strengths F_3 and H_3 also match. In the literature, this solution is obtained by a Hopf T-duality on the $\mathrm{AdS}_6 \times S^4$ Brandhuber-Oz solution to massive IIA supergravity [71].

Example 2 – A second solution given in [77] is

$$e^A = \frac{c_1}{\cos^{1/6}\alpha} \ , \qquad e^\phi = \frac{x c_2}{\sin^3\alpha\cos^{1/3}\alpha} \ , \tag{2.40}$$

for $0 < \alpha < \pi/2$ and $0 < x < 1$. The holomorphic data which reproduce this solution is

$$\mathcal{A}_\pm = -\frac{a}{3}w^3 \mp ibw + i18a \ , \qquad \mathcal{B}(w=0) = 0 \ . \tag{2.41}$$

Let us now try to rederive this AFPRT solution with this as our starting point. Below we give some relevant derived quantities. In this example, we use coordinates $w = X + 3iY$.

$$\begin{aligned}
\kappa^2 &= -24abXY \ , \\
\mathcal{G} &= -144abX(1-Y^3) \ , \\
\mathcal{D} &= \frac{X^2Y + (1-Y^3)^2}{X^2Y(1-Y^3)} \ , \\
\mathcal{U}_- &= 144ab^2(XY^2 + i(1-Y^3)) \ , \\
e^\phi &= \frac{b}{6a}\frac{1}{\sqrt{Y(1-Y^3)(X^2Y+(1-Y^3)^2)}} \ , \\
e^A &= \frac{\sqrt{12b}}{Y^{1/4}} \ , \\
\chi &= \frac{a}{b}(-X^2 + 15Y^2 - 6Y^5) \ .
\end{aligned} \tag{2.42}$$

For positivity, we take $0 < Y < 1$ and $X < 0$. The one-form equation (2.29) has two solutions for x^2: one complex-valued, which does not admit a simple expression, and one which coincides with our solution (2.31),

$$x^2 = \frac{(1-Y^3)^2}{X^2Y + (1-Y^3)^2} \ . \tag{2.43a}$$

From the equation for $\mathcal{D}$ in (2.27), we also have

$$\sin^2\alpha = 1 - Y^3 \ . \tag{2.43b}$$

Together, these imply the coordinate change,

$$\begin{aligned}
X &= -\frac{\sin^2\alpha}{\cos^{1/3}\alpha}\frac{\sqrt{1-x^2}}{x} \ , \\
Y &= \cos^{2/3}\alpha \ .
\end{aligned} \tag{2.44}$$

This gives us the desired e^{ϕ} and e^{A} with the identifications,

$$c_1 = \sqrt{12b} \ , \qquad c_2 = b/6a \ . \tag{2.45}$$

We can also match the metric, dilaton, and field strengths in [74, Eq. (11)] if we identify

$$\frac{1}{4}\hat{W}^2\hat{L}^2 = \frac{4b}{3}\frac{1}{\cos^{1/3}\alpha} \ , \qquad \hat{\theta} = \alpha \ , \qquad \hat{r} = \frac{4b}{3}\frac{\sin^2\alpha}{\cos^{1/3}\alpha}\frac{\sqrt{1-x^2}}{x} \ ,$$
$$a = \frac{3}{512}\hat{L}^6 \ , \qquad b = \frac{3}{16}\hat{L}^2\hat{m}^{-1/3} \ , \tag{2.46}$$

where quantities with ˆ are those of [74]. Then the three-form field strengths F_3 and H_3 also match. In the literature, this solution is obtained by a non-Abelian T-duality on the Brandhuber-Oz solution.

2.3.5 Summary of the relation

We have shown in this section the four quantities (x, α, A, ϕ) of AFPRT can be expressed in terms of the holomorphic functions of DGKU in the following way:

$$\begin{aligned} e^{\phi} &= \frac{(\mathrm{Re}\mathcal{U}_-)^2 + (\mathrm{Im}\mathcal{U}_-)^2/\mathcal{D}}{|\partial\mathcal{G}|^2\kappa^2/\sqrt{\mathcal{D}}} \ , \\ e^{4A} &= \frac{(\mathrm{Re}\mathcal{U}_-)^2\mathcal{D} + (\mathrm{Im}\mathcal{U}_-)^2}{|\partial\mathcal{G}|^2\kappa^2/6\mathcal{G}} \ , \\ 1 - x^2 &= \frac{(\mathrm{Re}\mathcal{U}_-)^2 + (\mathrm{Im}\mathcal{U}_-)^2/\mathcal{D}}{(\mathrm{Re}\mathcal{U}_-)^2 + (\mathrm{Im}\mathcal{U}_-)^2} \ , \\ \sin^2\alpha &= \frac{(\mathrm{Re}\mathcal{U}_-)^2 + (\mathrm{Im}\mathcal{U}_-)^2}{(\mathrm{Re}\mathcal{U}_-)^2\mathcal{D} + (\mathrm{Im}\mathcal{U}_-)^2} \ . \end{aligned} \tag{2.47}$$

We have verified that the map holds for two previously known solutions related to T-duals of type IIA solutions. For general local solutions the algebra becomes very extensive. The following steps in verifying the map of DGKU to AFPRT have been performed algebraically for general $\mathcal{A}_\pm$ using Mathematica:

1. *Match the remaining parts of the metric corresponding to* $\mathrm{d}s^2_\Sigma$. Much of the work has already been done in (2.28), but now we are able to take derivatives on the left-hand side with relative ease. It also turns out quite nicely that $q^2 = (\mathcal{S} + \bar{\mathcal{S}})^2$ and $p = 6\mathcal{G}$.

2. *Match the three-form field strengths.* We can check that the equations for H_3 and F_3 in the AFPRT solution match those of the DGKU solution. As our map only involves x^2 and $\sin^2\alpha$, it does not distinguish between the signs of x or $\cos\alpha$. These signs are fixed by the sign convention,

$$s_1 = -\operatorname{sign}(x)\operatorname{sign}(\operatorname{Im}\mathcal{U}_-) \ , \qquad s_2 = -\operatorname{sign}(\cos\alpha)\operatorname{sign}(\operatorname{Re}\mathcal{U}_-) \ . \tag{2.48}$$

Then the AFPRT three-form fields strengths in Eqs. (2.5) simplify to

$$\begin{aligned} H_3 = &-\frac{2}{9(\mathcal{D}-1)}\frac{(\operatorname{Re}\mathcal{U}_-)^2+(\operatorname{Im}\mathcal{U}_-)^2}{\kappa^2\operatorname{Im}\mathcal{U}_-}\times \\ &\left[-\frac{6\,\mathrm{d}A}{\sin^2\alpha}+(1+x^2)(2\,\mathrm{d}A+\mathrm{d}(\ln\sin^2\alpha))+\mathrm{d}\phi+\mathrm{d}(x^2)\right]\wedge\mathrm{vol}_{S^2} \ , \\ F_3 = &-\frac{\mathcal{G}\sin^2\alpha}{18}\frac{\kappa^2}{\operatorname{Re}\mathcal{U}_-}\times \\ &\left[\frac{36\,\mathrm{d}A}{\sin^2\alpha}+2(x^2-7)(2\,\mathrm{d}A+\mathrm{d}(\ln\sin^2\alpha))-2(1+2x^2)\,\mathrm{d}\phi+2\,\mathrm{d}(x^2)\right]\wedge\mathrm{vol}_{S^2} \ . \end{aligned} \tag{2.49}$$

These are equivalent to those of DGKU in Eqs. (2.23). This means that in section 2.3.4, for Example 1 we take $s_2 = -1$, and for Example 2 we take $s_1 = -1$ and $s_2 = +1$.

3. *Check the two PDEs.* We can simplify the first PDE of (2.1) to

$$\mathrm{d}\left[\frac{\mathcal{G}\mathcal{D}\operatorname{Re}\mathcal{U}_-}{\operatorname{Im}\mathcal{U}_-}\,\mathrm{d}\left(\frac{\operatorname{Re}\mathcal{U}_-}{\kappa^2}\right)+\frac{1}{3(\mathcal{D}-1)}\frac{(\operatorname{Re}\mathcal{U}_-)^2\mathcal{D}+(\operatorname{Im}\mathcal{U}_-)^2}{\kappa^2\operatorname{Im}\mathcal{U}_-}\,\mathrm{d}\mathcal{G}\right]=0 \ , \tag{2.50}$$

while the second PDE has no significant simplification.

In summary we have shown that the general local DGKU solution can be mapped to the AFPRT parametrization and satisfies the PDEs (2.1) as a consequence of the holomorphy of the $\mathcal{A}_\pm(w)$.

2.4 Mapping global solutions

After constructing a map from the local DGKU to AFPRT solutions, we now look at the global solutions constructed in [80]. They constitute a class of solutions (i.e. specified $\mathcal{A}_\pm$

functions) where Σ is taken to be the upper half-plane of the w complex plane, whose boundary is the real axis. The $\partial\mathcal{A}_\pm$ have poles on the boundary. The geometry is completely regular everywhere, except for the location of the poles where the supergravity background becomes that of a (p, q) five-brane. The supergravity solution can be viewed as the conformal near-horizon limit of a (p, q) five-brane web and the poles are interpreted as the residues of the semi-infinite external five-branes framing the web. If we take (x, α) to be alternative coordinates of Σ, we can ask how these features are mapped over. We will find that while these global solutions are represented by a single coordinate patch on the w complex plane, mapping over to the (x, α)-coordinates requires multiple coordinate patches in order to have single-valued supergravity fields.

For simplicity we define

$$1/\sqrt{\mathcal{D}} = \sin\alpha\sqrt{1-x^2}\ . \tag{2.51}$$

In particular, this means $\sin\alpha \geq 0$. Then as $x^2 \leq 1$ by definition, the square,

$$-1 \leq x \leq 1 \quad \text{and} \quad 0 \leq \alpha \leq \pi\ , \tag{2.52}$$

becomes a very natural coordinate system for (x, α). We will adopt this coordinate system for this section.

2.4.1 Boundary conditions

We are mainly interested in solutions where the AdS_6 factor governs the entire non-compact part of the geometry. Therefore, we will assume that Σ is compact, with or without boundary. On a non-empty boundary $\partial\Sigma$, we enforce $f_2^2 = 0$ while keeping the other conditions the same. Physically, this corresponds to shrinking the S^2 sphere closing off the geometry and forming a regular three-cycle which carries the five brane charges. This is equivalent to the boundary conditions,

$$\kappa^2 = \mathcal{G} = 0 \text{ and } 0 < \mathcal{G}/\kappa^2 < \infty \text{ on } \partial\Sigma\ . \tag{2.53}$$

The $0 < \mathcal{G}/\kappa^2$ constraint is relaxed at isolated points on the boundary to allow for sufficiently mild singularities, such as at the poles corresponding to five-branes.

We can now make some general remarks on the boundary $\partial\Sigma$. As $|\partial\mathcal{G}|^2 \neq 0$ we have $1/\mathcal{D} = 0$. From the definition of $\mathcal{U}_-$ in (2.16), we also have $\mathrm{Re}\,\mathcal{U}_- = 0$. Therefore,

$$\begin{aligned} 1 - x^2 &= \frac{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2/\mathcal{D}}{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2} \longrightarrow 0 \ , \\ \sin^2\alpha &= \frac{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2}{(\mathrm{Re}\,\mathcal{U}_-)^2\mathcal{D} + (\mathrm{Im}\,\mathcal{U}_-)^2} \longrightarrow \frac{(\mathrm{Im}\,\mathcal{U}_-)^2}{(\mathrm{Re}\,\mathcal{U}_-)^2\mathcal{D} + (\mathrm{Im}\,\mathcal{U}_-)^2} \ . \end{aligned} \tag{2.54}$$

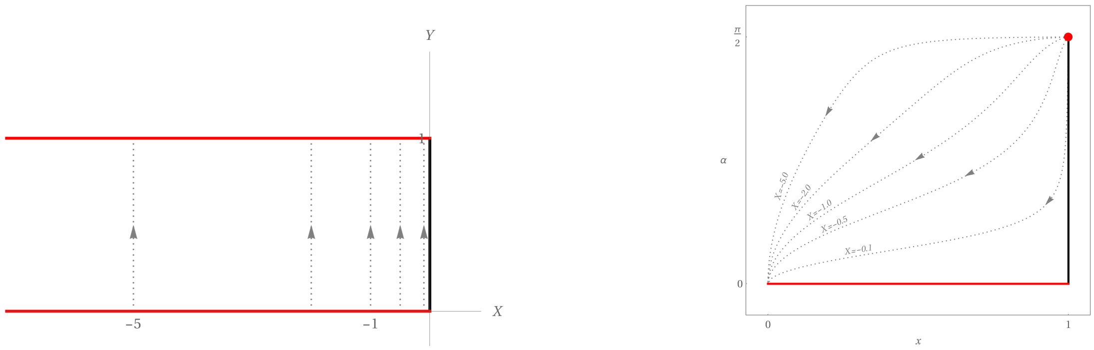
Thus $x^2 = 1$ is fixed, but because $(\mathrm{Re}\,\mathcal{U}_-)^2\mathcal{D} \sim \kappa^2/\mathcal{G}$ is non-zero and finite (away from the five-brane poles), $\sin^2\alpha$ can take generic values on the interval $[0,1]$. Therefore we can say that **the boundary $\partial\Sigma$ corresponds to (segments of) the $x = \pm 1$ edges of the (x,α) square**. Note that the boundary may not necessarily be mapped to the *entire* edge $0 \leq \alpha \leq \pi$, but can map to just a segment of the edge.

2.4.2 Example: non-Abelian T-dual

As a warm-up, let us return to the second example of section 2.3.4, with $a = 1/4$ and $b = 1/6$ for concreteness. We will first identify the Riemann surface Σ on the w complex plane, and then see how this region maps into the (x,α) square. Recall that $w = X + 3iY$.

$$\begin{aligned} \kappa^2 &= -XY \ , \\ \mathcal{G} &= -6X(1 - Y^3) \ . \end{aligned} \tag{2.55}$$

We satisfy $0 < \kappa^2, \mathcal{G}$ on the semi-infinite strip $X < 0$ and $0 < Y < 1$, which we take to be Σ. Additionally, $\kappa^2 = \mathcal{G} = 0$ on the line segment $X = 0$ and $0 \leq Y \leq 1$, which we take to be the boundary $\partial\Sigma$. The semi-infinite lines at $Y = 0$ and $Y = 1$ are then coordinate singularities,



(a) Σ in w-coordinates

(b) Σ in (x, α)-coordinates

Figure 2.1: Showing Σ in different in coordinate systems.

where various metric components blow up,

$$
\begin{aligned}
f_2^2 &= \frac{2}{3} X^2 Y^{3/4} (1-Y^3)^{5/4} (X^2 Y + (1-Y^3)^2)^{-3/4} \,, \\
f_6^2 &= 6 Y^{-1/4} (1-Y^3)^{1/4} (X^2 Y + (1-Y^3)^2)^{1/4} \,, \\
\rho^2 &= \frac{1}{6} Y^{3/4} (1-Y^3)^{-3/4} (X^2 Y + (1-Y^3)^2)^{1/4} \,.
\end{aligned}
\tag{2.56}
$$

The coordinate patch for Σ on the w complex plane is shown in Figure 2.1a. The black line represents the boundary $\partial\Sigma$, and the red lines represent the coordinate singularities.

The coordinate change, given in (2.44), maps this semi-infinite strip on the w complex plane into the quadrant $[0,1] \times [0, \pi/2]$ of the (x, α) square. Explicitly,

$$
\begin{aligned}
x &= \sqrt{\frac{(1-Y^3)^2}{X^2 Y + (1-Y^3)^2}} \,, \\
\alpha &= \sin^{-1} \sqrt{1-Y^3} \,.
\end{aligned}
\tag{2.57}
$$

Features of this map are shown in Figure 2.1. The boundary maps onto the line segment $x = +1$ as expected. We have also included contours for visual aid, represented by dotted gray lines. On the left-hand side we have drawn some contours of constant X, and on the right-hand side we show their images in the (x, α)-coordinates.

2.4.3 Example: 3-pole global solution

Let us finally turn our attention to the global solutions. We summarize the relevant details from [80] for the general case of L poles, and then specialize to a concrete example of three poles.

Take Σ to be the upper half-plane of the w complex plane, and $\partial\Sigma$ to be the real-axis. Let $p_\ell \in \mathbb{R}$ be the locations of L poles on the real-axis, and $s_n \in \mathbb{H}$ be the locations of N zeros strictly in the upper half-plane, where $L = N + 2$. Then let

$$\partial\mathcal{A}_\pm(w) = \sum_{\ell=1}^{L} \frac{Z_\pm^\ell}{w - p_\ell} \ , \tag{2.58}$$

where $Z_\pm^\ell$ for $\ell = 1, 2, \ldots, L$ are constants defined by

$$Z_+^\ell = iC_0 \prod_{n=1}^{N}(p_\ell - s_n) \prod_{\substack{\ell'=1 \\ \ell'\neq\ell}}^{L} \frac{1}{(p_\ell - p_{\ell'})} \ , \qquad Z_-^\ell = -\overline{Z_+^\ell} \ , \tag{2.59}$$

and $C_0 \in \mathbb{C}$ is a complex-valued normalization constant.[4] If we integrate the expressions for $\partial\mathcal{A}_\pm(w)$, we have

$$\mathcal{A}_+(w) = \mathcal{A}^0 + \sum_{\ell=1}^{L} Z_+^\ell \ln(w - p_\ell) \ , \tag{2.60a}$$

$$\mathcal{A}_-(w) = -\overline{\mathcal{A}^0} + \sum_{\ell=1}^{L} Z_-^\ell \ln(w - p_\ell) \ , \tag{2.60b}$$

where $\mathcal{A}^0 \in \mathbb{C}$ is a constant satisfying the equation below for $k = 1, 2, \ldots, L$,

$$\mathcal{A}^0 Z_-^k + \overline{\mathcal{A}^0} Z_+^k + \sum_{\substack{\ell=1 \\ \ell\neq k}}^{L} (Z_+^\ell Z_-^k - Z_-^\ell Z_+^k) \ln|p_\ell - p_k| = 0 \ . \tag{2.61}$$

This makes κ^2 and $\mathcal{G}$ vanish on the real-axis, according to the usual definitions in (2.7) after some consideration of branch cuts. $\mathcal{G}$ contains dilogarithms, so any quantity containing it in undifferentiated form (such as $\mathcal{D}$) will not admit a simple form. However, near a pole

[4] In terms of the original paper, $iC_0 = \omega_0\lambda_0$ where $\bar{\omega}_0 = -\omega_0$ and $|\lambda_0| = 1$.

we can look at the asymptotic behavior. Let us consider a pole p_m and take the semi-circle $w = p_m + re^{i\theta}$, where $0 \leq \theta \leq \pi$ and $0 < r \ll |p_m - p_\ell|$ for all $\ell \neq m$. Then we have the following relevant leading behaviors:

$$\mathcal{D} \approx \frac{|\ln r|}{3\sin^2\theta} \ , \tag{2.62a}$$

$$\mathrm{Re}\,\mathcal{U}_- \approx \kappa_m^2 (Z_+^m - Z_-^m) \frac{|\ln r|}{r} \sin\theta \ , \tag{2.62b}$$

$$\mathrm{Im}\,\mathcal{U}_- \approx \kappa_m^2 (Z_+^m - Z_-^m) \frac{|\ln r|}{r} \cos\theta \ , \tag{2.62c}$$

where

$$\kappa_m^2 = -2i \sum_{\substack{\ell=1 \\ \ell\neq m}}^{L} \frac{Z_+^m Z_-^\ell - Z_-^m Z_+^\ell}{p_m - p_\ell} \ , \tag{2.63}$$

so that near a pole as $r \to 0$,

$$1 - x^2 \quad = \quad \frac{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2/\mathcal{D}}{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2} \quad \longrightarrow \quad \sin^2\theta \ , \tag{2.64a}$$

$$\sin^2\alpha \quad = \quad \frac{(\mathrm{Re}\,\mathcal{U}_-)^2 + (\mathrm{Im}\,\mathcal{U}_-)^2}{(\mathrm{Re}\,\mathcal{U}_-)^2\mathcal{D} + (\mathrm{Im}\,\mathcal{U}_-)^2} \quad \longrightarrow \quad \frac{3}{|\ln r|} \quad \longrightarrow \quad 0 \ . \tag{2.64b}$$

Therefore, small semi-circles around a pole on the w complex plane map to lines of constant α on the (x, α) square, which approach either the $\alpha = 0$ or $\alpha = \pi$ edge as $r \to 0$. Because x is approximately $\pm\cos\theta$, these semi-circles necessarily map to the *entire* line segment running between $-1 \leq x \leq 1$. This can all be loosely summarized by saying **poles on the boundary $\partial\Sigma$ correspond to the $\alpha = 0, \pi$ edges of the (x, α) square**.

For concreteness let us take the 3-pole solution, which is the simplest global solution with the fewest number of poles. We pick the locations of the three poles,

$$p_1 = 1 \ , \qquad p_2 = 0 \ , \qquad p_3 = -1 \ , \tag{2.65}$$

the location of the one zero,

$$s = \frac{1}{2} + 2i \ , \tag{2.66}$$

and the normalization constant,

$$C_0 = 1 \ . \tag{2.67}$$

The relations (2.61) are solved by $\mathcal{A}^0 = iC_0 s \ln 2$.

This defines a coordinate change $(w, \bar{w}) \to (x, \alpha)$ from the upper half-plane into the square $[-1, 1] \times [0, \pi]$. This map does not admit a simple form as it contains dilogarithms, but its general features are shown in Figure 2.2. The left-hand diagrams show an unshaded region on the w complex plane, and the right-hand diagrams show the corresponding region on the (x, α) square. Solid black lines represent the boundary $\partial\Sigma$. "X" marks on the w complex plane represent locations of the five-brane poles, which map to black dashed lines on the (x, α) square. Two contours C_1, C_2 are included for visual aid.

There are three important considerations which make this map well-defined:

1. For convenience, we pick a map which obeys the sign convention (2.48) with $s_1 = s_2 = +1$. On the w complex plane, we have represented the curve where $\text{Im}\,\mathcal{U}_- = 0$ with an orange dotted line. This curve is mapped to $x = 0$. The side of the curve where $\text{Im}\,\mathcal{U}_- > 0$ gets mapped to the $x < 0$ side of the (x, α) square, and the side where $\text{Im}\,\mathcal{U}_- < 0$ gets mapped to $x > 0$. Similarly, the curve where $\text{Re}\,\mathcal{U}_- = 0$ is represented with a blue dotted line.[5]

2. In order to map the entire upper half-plane of the w complex plane into (x, α)-coordinates in a one-to-one manner, we need to introduce multiple coordinate patches. This follows from a simple counting argument: each of the three poles needs to be mapped their own $\alpha = 0$ or $\alpha = \pi$ edge, only two of which exist on the square. We can accommodate a one-to-one map at the expense of introducing multiple (x, α) squares and gluing them together.

[5] $\text{Re}\,\mathcal{U}_-$ also vanishes on the boundary $\partial\Sigma$, but we exclude this.

For instance, the $p_2 = 0$ pole maps to the $\alpha = 0$ edge, and the boundary segments $p_3 < \mathrm{Re}\, w < p_2$ and $p_2 < \mathrm{Re}\, w < p_1$ map to the $x = -1$ and $x = 1$ edges, respectively. The $p_1 = 1$ and $p_3 = -1$ poles then map to the $\alpha = \pi$ edge of two different (x, α) squares 2.2d and 2.2h, respectively. These two patches are glued together along the orange line "b". Figure 2.2b shows these two patches glued together at the expense of introducing a branch cut, represented by the red jagged line.

3. The Jacobian J of the map vanishes on the solid red line,

$$J \quad = \quad \det \begin{pmatrix} \partial x & \bar{\partial} x \\ \partial \alpha & \bar{\partial} \alpha \end{pmatrix} \quad \propto \quad \left[\partial \left(\frac{\mathrm{Re}\, \mathcal{U}_-}{\mathrm{Im}\, \mathcal{U}_-} \right) \bar{\partial} \mathcal{D} - \bar{\partial} \left(\frac{\mathrm{Re}\, \mathcal{U}_-}{\mathrm{Im}\, \mathcal{U}_-} \right) \partial \mathcal{D} \right] . \tag{2.68}$$

In the present example, if a contour on the w complex plane passes through this line, the image of the contour in (x, α)-coordinates will instead bounce off this line. This means that we need an additional coordinate patch to maintain a one-to-one map. For instance, consider the contour C_2 in (x, α)-coordinates: it starts on the coordinate patch 2.2d, hits the red line "f", and then bounces off onto another coordinate patch 2.2f.

To summarize, Σ is represented on the w complex plane by a single coordinate patch, taken to be the upper half-plane. When we map over to (x, α)-coordinates, we need at least three coordinate patches to represent the whole Σ: 2.2d, 2.2f, and 2.2h. 2.2d is glued to 2.2h along "b", 2.2d to 2.2f along "f", and 2.2f to 2.2h along "e".

2.5 Discussion

In this chapter, we have found an explicit map between the type IIB AdS_6 solutions formulated in [77] and in [78]. This mapping is given by a coordinate change $(w, \bar{w}) \to (x, \alpha)$ for the surface Σ_2 and was explicitly verified for two previously known examples. This result shows that the two solutions are indeed equivalent and that therefore the solutions of [78] are the most general type IIB solutions with an AdS_6 factor preserving sixteen supersymmetries.

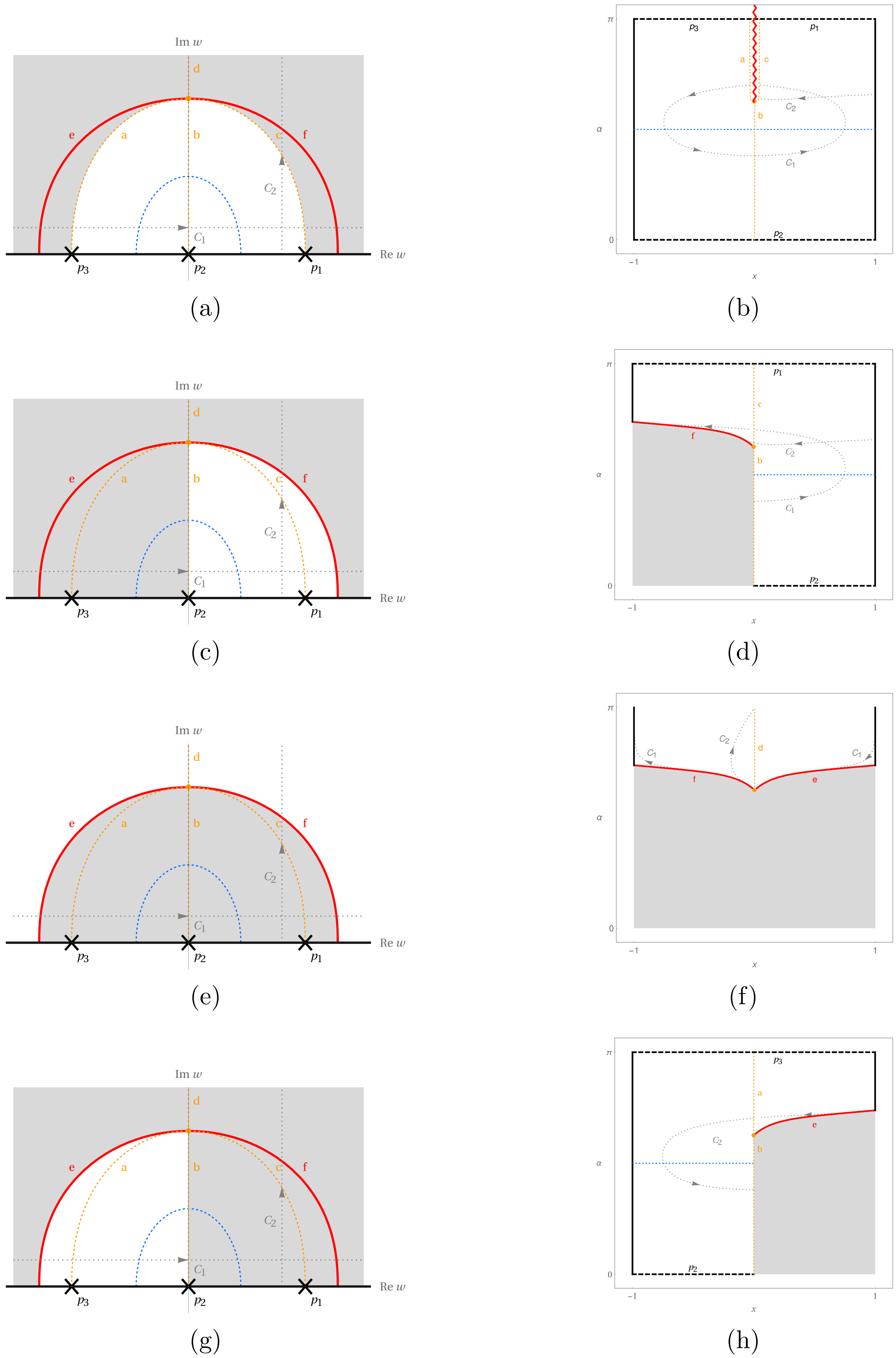


Figure 2.2: Coordinate patches needed for the 3-pole solution.

Furthermore, we mapped over the global solutions of [80] and found that multiple coordinate patches in (x, α) were necessary in order to have single-valued solutions. This arose from a simple counting argument that each five-brane pole of the global solution needs to be mapped to its own horizontal edge of the (x, α)-coordinate square, but global solutions have ≥ 3 poles whereas each (x, α) square has 2 horizontal edges. Thus, an advantage of the complex coordinate parametrization of [78] is that a global solution can be represented in a single coordinate chart. Note that in [77], the four quantities (A, ϕ, x, α) were initially treated on the same footing and subsequently (x, α) were chosen to be coordinates of the two-dimensional space Σ_2. It is an interesting open question whether the global solutions can be formulated by making different coordinate choices.

In [96,97], the AFPRT solution was reduced on AdS_6 and an effective scalar coset theory was constructed. The symmetries of the coset can be used to generate new solutions. It would be interesting to investigate how the coset transformations act on the DGKU solutions using the mapping constructed in this chapter. Another interesting direction would be to see how the alternative formulation in [98] can be related to the DKGU solutions.

CHAPTER 3

Holographic line defects in $F(4)$ gauged supergravity

In this chapter, we construct supergravity solutions that are holographically dual to line defects in five-dimensional superconformal field theories. One approach to construct such line defects is to consider probe branes. Our aim is to construct non-singular supergravity solutions that correspond to the fully back-reacted solution, which should describe the system when the number of probe branes becomes large. The fact that the ten-dimensional type IIA and IIB undeformed AdS_6 vacuum solutions [71–80, 85, 96] are already warped products makes the construction of holographic defect solutions in ten dimensions quite challenging. Here, we consider a simpler system, namely Romans' six-dimensional $F(4)$ gauged supergravity [99]. Recent results [91, 92, 100] show that any solution of this six-dimensional theory can be uplifted and embedded in the general IIB solutions of [78–80]. This implies that the solutions in this chapter lift to ten-dimensional holographic defect solutions. Recently, various supersymmetric solutions of $F(4)$ supergravity without additional matter multiplets have been constructed in [101–103]. Examples of solutions of $F(4)$ gauged supergravity with matter couplings can be found in [64, 104–106].

Five-dimensional SCFTs have a unique superconformal algebra, $F(4)$ [107, 108], and its subalgebras were classified in [109, 110]. This analysis shows that superconformal defects should exist, such as the half-BPS Janus solution found in [64]. In this chapter, we construct supergravity solutions corresponding to half-BPS superconformal line defects preserving eight of the sixteen supersymmetries of $F(4)$, falling into a $D(2,1;2) \times \mathrm{SU}(2)$ sub-superalgebra. The structure of this chapter is as follows. In section 3.1, we present the necessary background on $F(4)$ gauged supergravity. In section 3.2, we derive the non-singular line defect solution using the BPS equations first derived in [111]. In section 3.3, we perform some holographic calculations using the solution presented in section 3.2. In particular, we calculate the on-shell action and the one-point function of the stress tensor using holographic renormalization. Some implications of our solution and directions for future research are given in section 3.4. In the appendices, we present our conventions and details of the calculation of the counterterms using the method of holographic renormalization.

3.1 $F(4)$ gauged supergravity

In this section we review the features of $F(4)$ gauged supergravity [99] which will be relevant. Six-dimensional $F(4)$ gauged supergravity contains the following bosonic fields: a metric $G_{\mu\nu}$, a real scalar ϕ, a 2-form gauge potential B, a non-Abelian SU(2) vector field A^i for $i = 1, 2, 3$, and an Abelian vector field A^0. The bosonic Lagrangian of the theory takes the following form,[6]

$$\begin{aligned}\mathcal{L} = & R *_6 1 - 4X^{-2} *_6 \mathrm{d}X \wedge \mathrm{d}X - V(X) *_6 1 \\ & - \frac{1}{2}X^4 *_6 H \wedge H - \frac{1}{2}X^{-2}\big(*_6 F^i \wedge F^i + *_6 F \wedge F\big) \\ & - B \wedge \left(\frac{1}{2}\,\mathrm{d}A^0 \wedge \mathrm{d}A^0 + \frac{1}{\sqrt{2}}mB \wedge \mathrm{d}A^0 + \frac{1}{3}m^2 B \wedge B + \frac{1}{2}F^i \wedge F^i\right) ,\end{aligned} \tag{3.1}$$

where the field strengths derived from the potentials are given by

$$\begin{aligned} H &= \mathrm{d}B \ , \\ F^i &= \mathrm{d}A^i + \frac{g}{2}\varepsilon_{ijk}A^j \wedge A^k \ , \\ F &= \mathrm{d}A^0 + \sqrt{2}mB \ , \end{aligned} \tag{3.2}$$

and, for convenience, the scalar field ϕ has been redefined in terms of X by

$$X = \exp\left(-\frac{1}{2\sqrt{2}}\phi\right) . \tag{3.3}$$

[6]See appendix 3.A for our conventions regarding differential forms.

Then the potential produced by the gauging of the supergravity is given by

$$V(X) = m^2 X^{-6} - 4\sqrt{2} g m X^{-2} - 2g^2 X^2 \ , \tag{3.4}$$

which can be rewritten in terms of a superpotential $f(X)$ as

$$\begin{aligned} V(X) &= 16X^2 \big(\partial_X f(X)\big)^2 - 80 f(X)^2 \ , \\ f(X) &= \frac{1}{8}(m X^{-3} + \sqrt{2} g X) \ . \end{aligned} \tag{3.5}$$

The equations of motion following from the variation of the Lagrangian (3.1) are

$$\begin{aligned} R_{\mu\nu} =& 4X^{-2}\partial_\mu X \partial_\nu X + \frac{1}{4}V(X) G_{\mu\nu} + \frac{1}{4}X^4 \left(H_\mu{}^{\alpha\beta} H_{\nu\alpha\beta} - \frac{1}{6} H^{\alpha\beta\gamma} H_{\alpha\beta\gamma} G_{\mu\nu} \right) \\ &+ \frac{1}{2}X^{-2}\left(F_\mu{}^\alpha F_{\nu\alpha} - \frac{1}{8}F^{\alpha\beta}F_{\alpha\beta}G_{\mu\nu} + F^i_\mu{}^\alpha F^i_{\nu\alpha} - \frac{1}{8}F^{i\alpha\beta}F^i_{\alpha\beta}G_{\mu\nu} \right) \ , \\ \mathrm{d}\big(X^4 *_6 H\big) =& -\frac{1}{2}F\wedge F - \frac{1}{2}F^i \wedge F^i - \sqrt{2} m X^{-2} *_6 F \ , \\ \mathrm{d}\big(X^{-2} *_6 F\big) =& - F\wedge H \ , \\ \mathrm{D}\big(X^{-2} *_6 F^i\big) =& - F^i \wedge H \ , \\ \mathrm{d}\big(X^{-1} *_6 \mathrm{d}X\big) =& \frac{1}{8}X^{-2}\big(*_6 F\wedge F + *_6 F^i \wedge F^i \big) - \frac{1}{4}X^4 *_6 H \wedge H - \frac{1}{8} X\partial_X V(X) *_6 1 \ , \end{aligned} \tag{3.6}$$

where D is the gauge covariant derivative,

$$\mathrm{D}F^i = \mathrm{d}F^i + g\varepsilon_{ijk} A^j \wedge F^k \ . \tag{3.7}$$

The supersymmetry variations of the fermionic fields can be expressed in terms of an SU(2)-doublet of symplectic-Majorana-Weyl Killing spinors ζ^a for $a = 1, 2$ as

$$\begin{aligned} \delta\psi^a_\mu =& \nabla_\mu \zeta^a + g A^i_\mu (T^i)^a{}_b \zeta^b - i f(X)\Gamma_\mu \Gamma_* \zeta^a + \frac{X^2}{48} H_{\nu\rho\sigma}\Gamma^{\nu\rho\sigma}\Gamma_\mu\Gamma_*\zeta^a \\ &+ i\frac{X^{-1}}{16\sqrt{2}}\big(\Gamma_\mu{}^{\nu\rho} - 6 e_\mu{}^\nu \Gamma^\rho\big)\big(F_{\nu\rho}\delta^a{}_b - 2\Gamma_* F^i_{\nu\rho}(T^i)^a{}_b\big)\zeta^b \ , \end{aligned} \tag{3.8}$$

$$\begin{aligned} \delta\chi^a =& X^{-1}\Gamma^\mu \partial_\mu X \zeta^a + 2iX\partial_X f(X)\Gamma_*\zeta^a - \frac{X^2}{24} H_{\mu\nu\rho}\Gamma^{\mu\nu\rho}\Gamma_*\zeta^a \\ &- i\frac{X^{-1}}{8\sqrt{2}}\Gamma^{\mu\nu}\big(F_{\mu\nu}\delta^a{}_b - 2\Gamma_* F^i_{\mu\nu}(T^i)^a{}_b\big)\zeta^b \ , \end{aligned} \tag{3.9}$$

where Γ^m for $m = 1, 2, \ldots, 6$ generate the $(5+1)$-dimensional Clifford algebra in an orthonormal frame and $\Gamma_* = \Gamma^{123456}$. The $T^i = -i\sigma^i/2$ are the generators of SU(2) satisfying $[T^i, T^j] = \varepsilon_{ijk} T^k$.

The space of inequivalent theories are labeled by the couplings m and g, modulo the parameter rescaling $g \to a^{-1} g$, $m \to a^3 m$ accompanied by appropriate field redefinitions. The choice,

$$g = \frac{3m}{\sqrt{2}} \, , \tag{3.10}$$

is a canonical choice, so that in the supersymmetric AdS_6 vacuum the scalar takes the value $X = 1$. We will make this choice throughout this chapter, using m in lieu of g. The potential then takes the form,

$$V(X) = m^2 \big(X^{-6} - 12 X^{-2} - 9 X^2 \big) \; . \tag{3.11}$$

In [112] it was shown that six-dimensional $F(4)$ gauged supergravity is a consistent non-linear Kaluza-Klein reduction of the warped AdS_6 solutions of type IIA massive supergravity. Recently an analogous statement has been shown [91,92] for the warped $\mathrm{AdS}_6 \times S_2$ solutions of type IIB supergravity found in [78–80]. The fact that such a consistent truncation exists implies that any solution of $F(4)$ gauged supergravity can be lifted to ten-dimensional solutions, which have precise holographic duals. For example, the massive type IIA solution is dual to a $d = 5$, USp(N) gauge theory for large N [71]. Consequently, the defect solution we construct in section 3.2 also exists in the AdS_6 solutions of massive type IIA and type IIB and corresponds to a line defect in the dual CFT.

3.2 Line defect solution

In this section we find a non-singular line defect solution by solving the BPS equations. An appropriate ansatz can be obtained by considering the unbroken sub-superalgebra of the superconformal algebra $F(4)$ suitable for a conformal line defect, namely $D(2,1;2) \times \mathrm{SU}(2)$, which has a bosonic part $\mathrm{SO}(2,1)\times\mathrm{SU}(2)^3$. We can associate the $\mathrm{SO}(2,1)$ with the global isometry of an AdS_2 factor. The three $\mathrm{SU}(2)$ factors can be interpreted as the isometry $\mathrm{SO}(4)\sim\mathrm{SU}(2)\times\mathrm{SU}(2)$ of a three sphere S^3 and unbroken $\mathrm{SU}(2)$ R-symmetry. Consequently, the isometries are realized by an $\mathrm{AdS}_2\times S^3$ geometry warped over an interval I_α,

$$\mathrm{d}s^2 = e^{2U(\alpha)}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + e^{2V(\alpha)}\,\mathrm{d}\alpha^2 + e^{2W(\alpha)}\,\mathrm{d}s^2_{S^3}\ , \tag{3.12}$$

where $\mathrm{d}s^2_{\mathrm{AdS}_2}$ and $\mathrm{d}s^2_{S^3}$ are unit-radius metrics. Note that the warp factor V is non-dynamical, but it is introduced because its gauge-fixing will turn out to simplify the BPS equations drastically. The isometries and unbroken R-symmetry imply that all gauge fields have to vanish, but there can be a non-vanishing B potential along the AdS_2 factor and a non-trivial scalar profile,

$$B = b(\alpha)\,\mathrm{vol}_{\mathrm{AdS}_2}\ , \qquad X = X(\alpha)\ , \qquad A^0 = A^i = 0\ , \tag{3.13}$$

where $\mathrm{vol}_{\mathrm{AdS}_2}$ is a unit-radius volume 2-form.

3.2.1 BPS equations

The BPS equations for the ansatz (3.12) and (3.13) have been derived in [111], where it was shown that solutions which preserve eight of the sixteen supercharges satisfy the following system of first-order ordinary differential equations (ODEs),[7]

$$\begin{aligned}
\theta' &= -e^V \sin(2\theta)(f - X\partial_X f)\ ,\\
X' &= -\frac{1}{4}e^V X\cos(2\theta)^{-1}\big(e^{-U}\sin(2\theta) + 2\sin(2\theta)^2 f + (7+\cos(4\theta))X\partial_X f\big)\ ,\\
U' &= \frac{1}{4}e^V\cos(2\theta)^{-1}\big(e^{-U}\sin(2\theta) + (5+3\cos(4\theta))f + 6\sin(2\theta)^2 X\partial_X f\big)\ ,\\
W' &= -\frac{1}{4}e^V\cos(2\theta)^{-1}\big(-e^{-U}\sin(2\theta) + (-9+\cos(4\theta))f + 2\sin(2\theta)^2 X\partial_X f\big)\ ,\\
b' &= -\frac{e^{V+2U}}{X^2}\cos(2\theta)^{-1}\big(e^{-U} + 2\sin(2\theta)(f + 3X\partial_X f)\big)\ ,\\
Y' &= \frac{Y}{8}e^V\cos(2\theta)^{-1}\big(e^{-U}\sin(2\theta) + (5+3\cos(4\theta))f + 6\sin(2\theta)^2 X\partial_X f\big)\ ,
\end{aligned} \tag{3.14}$$

[7] We have set $L = 1$ in the equations of [111].

where $'$ denotes the derivative with respect to α. Y and θ are functions related to the spinor parameters ζ^a.

The first three equations for θ', X', and U' should be treated as a coupled system of ODEs. Once these are solved, the last three equations for W', b', and Y' should be treated as three independent ODEs, the right-hand sides acting as inhomogenous terms. In fact, assuming we have a solution of the first three equations, the solution for $W(\alpha)$ and $b(\alpha)$ is

$$\begin{aligned} b &= b_0 - \frac{e^{2U}}{m} X \big(e^{-U} + 2\sin(2\theta)(f - X\partial_X f)\big) \ , \\ e^{-W} &= mr\big(e^{-U}\cos(2\theta)^{-1} + 2\tan(2\theta)(3f + X\partial_X f)\big) \ , \end{aligned} \tag{3.15}$$

where b_0 and r are (real) integration constants. b_0 is set to zero in order to satisfy the equations of motion. r can be interpreted as the S^3 radius. The solution for $Y(\alpha)$ is inconsequential for our considerations in this chapter, but for completion is

$$Y = Y_0 e^{U/2} \ , \tag{3.16}$$

where Y_0 is a constant.

To simplify the first three equations in (3.14), we pick a gauge on the warp factor V [111],

$$e^{-V} = \sin(2\theta)(f - X\partial_X f) \ , \tag{3.17}$$

so that the first equation in (3.14) becomes $\theta' = -1$. The associated integration constant involves constant shifts of α, which has no physical consequence. So we can set

$$\theta(\alpha) = -\alpha \ . \tag{3.18}$$

Then the two remaining equations become

$$\begin{aligned} X' &= \frac{X}{4m\sin(2\alpha)\cos(2\alpha)}\big(m(-5-\cos(4\alpha)) - 2e^{-U}\sin(2\alpha)X^3 + 6mX^4\big) \ , \\ -U' &= \frac{1}{4m\sin(2\alpha)\cos(2\alpha)}\big(m(-1+3\cos(4\alpha)) - 2e^{-U}\sin(2\alpha)X^3 + 6mX^4\big) \ . \end{aligned} \tag{3.19}$$

We can note that

$$U' + \frac{X'}{X} + \frac{2\cos(2\alpha)}{\sin(2\alpha)} = 0 \ , \tag{3.20}$$

so if we set

$$e^{-U(\alpha)} = mpX(\alpha)\sin(2\alpha) \ , \tag{3.21}$$

for some (real) integration constant p, which can be interpreted as the curvature radius of the AdS_2 factor, then (3.19) is equivalent to solving a single ODE for $X(\alpha)$,

$$X' = \frac{X}{4\sin(2\alpha)\cos(2\alpha)}\big(-5 - \cos(4\alpha) + 2\big(3 - p\sin(2\alpha)^2\big)X^4\big) \ . \tag{3.22}$$

The solution to this equation is

$$X = \cos(2\alpha)^{1/2}\big(1 - p\sin(2\alpha)^2 + q\sin(2\alpha)^3\big)^{-1/4} \ , \tag{3.23}$$

for some (real) integration constant q. Then using (3.15), (3.17), and (3.21), we have a family of solutions to the BPS equations, labeled by real numbers p, q, and r,

$$\begin{aligned}
e^{2U} &= \frac{1}{p^2m^2}\big(1 - p\sin(2\alpha)^2 + q\sin(2\alpha)^3\big)^{1/2}\sin(2\alpha)^{-2}\cos(2\alpha)^{-1} \ , \\
e^{2W} &= \frac{1}{r^2(3-p)^2m^2}\big(1 - p\sin(2\alpha)^2 + q\sin(2\alpha)^3\big)^{1/2}\sin(2\alpha)^{-2}\cos(2\alpha) \ , \\
e^{2V} &= \frac{4}{m^2}\big(1 - p\sin(2\alpha)^2 + q\sin(2\alpha)^3\big)^{-3/2}\sin(2\alpha)^{-2}\cos(2\alpha)^3 \ , \\
b &= \frac{1 - p + q\sin(2\alpha)^3}{p^2m^2}\sin(2\alpha)^{-1}\cos(2\alpha)^{-2} \ , \\
X &= \big(1 - p\sin(2\alpha)^2 + q\sin(2\alpha)^3\big)^{-1/4}\cos(2\alpha)^{1/2} \ .
\end{aligned} \tag{3.24}$$

In the next section we analyze how the regularity of the solutions depends on the integration constants.

3.2.2 Defect solution

The positivity of the metric factors in (3.24) implies that the maximal range for the coordinate α is the interval I_α is $\alpha \in [0, \pi/4]$. Matching the metric to that of AdS_6 asymptotically at the conformal boundary $\alpha \to 0$ requires equating the e^{2U} and e^{2W} factors, which implies

$$r^2(3-p)^2 = p^2 \ . \tag{3.25}$$

This should be viewed as a condition fixing r in terms of p. Near the conformal boundary, the AdS_6 radius is $\ell = m^{-1}$. We can also observe that $X \to 1$, which is the appropriate value for the global AdS_6 vacuum.

The solutions (3.24) with the condition (3.25) give a family of half-BPS solutions with AdS_6 asymptotics, labeled by two constants p and q. The $q = 0$ solutions coincide with those given in [111]. Incidentally, the $q = 0$, $p = 1$ case describes global AdS_6,

$$\begin{aligned} e^{2U} &= \frac{1}{m^2}\sin(2\alpha)^{-2} \ , \qquad e^{2W} = \frac{1}{m^2}\sin(2\alpha)^{-2}\cos(2\alpha)^2 \ , \\ e^{2V} &= \frac{4}{m^2}\sin(2\alpha)^{-2} \ , \qquad b = 0 \ , \qquad X = 1 \ . \end{aligned} \tag{3.26}$$

Under the coordinate transformation,

$$\cosh\rho = \sin(2\alpha)^{-1} \ , \tag{3.27}$$

the metric becomes

$$\mathrm{d}s^2 = \frac{1}{m^2}\left[\cosh^2\rho \, \mathrm{d}s^2_{\mathrm{AdS}_2} + \sinh^2\rho \, \mathrm{d}s^2_{S^3} + \mathrm{d}\rho^2\right] \ . \tag{3.28}$$

Let us now describe the behavior of these solutions at $\alpha \to \pi/4$, which corresponds to the center of the space. The $\cos(2\alpha)$ in the metric factors vanishes here so we generically expect to have a singularity. However, the factor,

$$\Delta(\alpha) \equiv 1 - p\sin(2\alpha)^2 + q\sin(2\alpha)^3 \ , \tag{3.29}$$

may also vanish at some $0 < \alpha_0 < \pi/4$ by tuning the constants p and q, in which case we can expect to have a singularity located at $\alpha_0 < \pi/4$. If we call $\beta \equiv \alpha_0 - \alpha$, then we have enough freedom to arrange for Δ to vanish as either $\mathcal{O}(\beta^1)$ or $\mathcal{O}(\beta^2)$. We can also have $\alpha_0 = \pi/4$, in which case Δ vanishes as either $\mathcal{O}(\beta^2)$ or $\mathcal{O}(\beta^4)$ and we have to consider the $\cos(2\alpha)$ factors.

This gives us five distinct cases, which are characterized by the behavior of the metric and fields as $\beta \to 0$. These are summarized in Table 3.1, and the corresponding regions on the pq-plane are illustrated in Figure 3.1. A "1" denotes approaching a constant, i.e. $\mathcal{O}(\beta^0)$.

	Region on pq-plane	e^{2U}	e^{2W}	e^{2V}	B	X	R
I: Δ does not vanish	region I	β^{-1}	β	β^3	β^{-2}	$\beta^{1/2}$	β^{-5}
II: $\Delta \sim \beta$, $\alpha_0 < \pi/4$	region II	$\beta^{1/2}$	$\beta^{1/2}$	$\beta^{-3/2}$	1	$\beta^{-1/4}$	$\beta^{-1/2}$
III: $\Delta \sim \beta^2$, $\alpha_0 < \pi/4$	$q = 2(p/3)^{3/2}$ for $p > 3$	β	β	β^{-3}	1	$\beta^{-1/2}$	β^{-1}
IV: $\Delta \sim \beta^2$, $\alpha_0 = \pi/4$	$q = p - 1$ for $p < 3$	1	β^2	1	1	1	β^{-2}
IV′: $\Delta \sim \beta^2$, $\alpha_0 = \pi/4$	$(p,q) = (1,0)$ or $(-3,-4)$	1	β^2	1	1	1	1
V: $\Delta \sim \beta^4$, $\alpha_0 = \pi/4$	$(p,q) = (3,2)$	β	β^3	β^{-3}	1	$\beta^{-1/2}$	β^{-3}

Table 3.1: Leading-order behavior of metric factors, 2-form potential, scalar field, and Ricci scalar as $\beta \to 0$ for each distinct case.

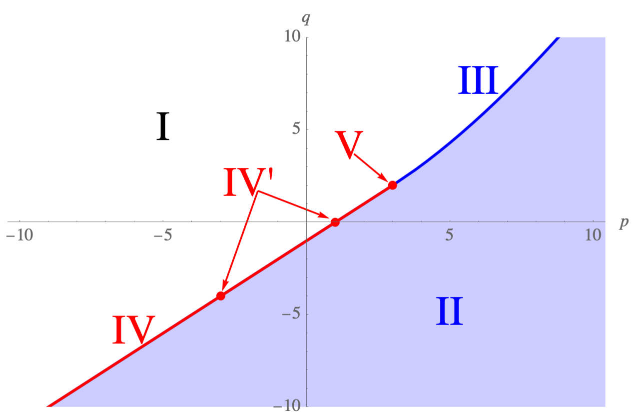


Figure 3.1: Distinct cases shown on the pq-plane.

Case IV looks the most promising, so we will start there. In the limit $\beta \equiv \pi/4 - \alpha \to 0$, the metric has the following leading behavior,

$$\mathrm{d}s^2 \approx \frac{32}{(6-2p)^{3/2} m^2} \left[\mathrm{d}\beta^2 + \frac{(6-2p)^2}{16p^2} \beta^2 \, \mathrm{d}s^2_{S^3} + \frac{(6-2p)^2}{64p^2} \, \mathrm{d}s^2_{\mathrm{AdS}_2} \right] . \tag{3.30}$$

We can avoid an angular deficit/excess at $\beta = 0$ when $(6-2p)^2/16p^2 = 1$, i.e. when $p = 1$ or $p = -3$. These two special cases are denoted IV$'$ on the table. The former is just global AdS_6, so this leaves a single non-trivial defect solution which remains finite as $\alpha \to \pi/4$, corresponding to substituting $(p, q) = (-3, -4)$ into (3.24),

$$\begin{aligned}
\mathrm{d}s^2 &= f_1^2\,\mathrm{d}\alpha^2 + f_2^2\,\mathrm{d}s^2_{\mathrm{AdS}_2} + f_3^2\,\mathrm{d}s^2_{S^3}\ , \\
f_1^2 &= \frac{4}{m^2}\big(1+3\sin(2\alpha)^2 - 4\sin(2\alpha)^3\big)^{-3/2}\sin(2\alpha)^{-2}\cos(2\alpha)^3\ , \\
f_2^2 &= \frac{1}{9m^2}\big(1+3\sin(2\alpha)^2 - 4\sin(2\alpha)^3\big)^{1/2}\sin(2\alpha)^{-2}\cos(2\alpha)^{-1}\ , \\
f_3^2 &= \frac{1}{9m^2}\big(1+3\sin(2\alpha)^2 - 4\sin(2\alpha)^3\big)^{1/2}\sin(2\alpha)^{-2}\cos(2\alpha)\ , \\
b &= \frac{4}{9m^2}\big(1-\sin(2\alpha)^3\big)\sin(2\alpha)^{-1}\cos(2\alpha)^{-2}\ , \\
X &= \big(1+3\sin(2\alpha)^2 - 4\sin(2\alpha)^3\big)^{-1/4}\cos(2\alpha)^{1/2}\ .
\end{aligned} \tag{3.31}$$

As a check, we have verified that the equations of motion (3.6) hold for this solution. In summary, we have found a new non-singular solution in case IV$'$, whereas all other cases I-V are singular. We will focus our analysis on the non-singular solution (3.31) in the rest of the chapter.

3.2.3 Asymptotics

We will now calculate the asymptotic behavior of the defect solution (3.31) near the conformal boundary $\alpha \to 0$. Recall that the AdS_6 radius is $\ell = m^{-1}$, which we will set to unity from here on. Following a prescription similar to [21, 22], we want to put the metric into the Fefferman-Graham (FG) form,

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{z^2}\big(\mathrm{d}z^2 + g_{ij}(x,z)\,\mathrm{d}x^i\,\mathrm{d}x^j\big)\ , \\
g(x,z) &= g_0(x) + z g_1(x) + z^2 g_2(x) + \cdots\ ,
\end{aligned} \tag{3.32}$$

where $i, j = 1, 2, \ldots, 5$ run over the AdS_2 and S^3 indices, and $z \to 0$ is the conformal boundary. This is done by taking $z = z(\alpha)$ so that the appropriate coordinate change is obtained by a solution to the ODE,

$$f_1(\alpha)\,\mathrm{d}\alpha = \frac{\mathrm{d}z}{z}\ . \tag{3.33}$$

Expanding in α and integrating term by term gives a perturbative expansion,

$$z(\alpha) = 3\alpha - 17\alpha^3 + 24\alpha^4 + \frac{722}{5}\alpha^5 - \frac{2504}{5}\alpha^6 - \frac{103009}{105}\alpha^7 + \cdots\ , \tag{3.34}$$

which can be inverted,

$$\alpha(z) = \frac{1}{3}z + \frac{17}{81}z^3 - \frac{8}{81}z^4 + \frac{241}{1215}z^5 - \frac{752}{3645}z^6 - \frac{12275}{45927}z^7 + \cdots\ . \tag{3.35}$$

This gives the following expansions in the z coordinate,

$$\begin{aligned} f_2^2 &= \frac{1}{z^2}\left(\frac{1}{4} - \frac{1}{18}z^2 + \frac{1}{324}z^4 + \frac{16}{1215}z^5 + \frac{56}{2187}z^6 + \cdots\right)\ , \\ f_3^2 &= \frac{1}{z^2}\left(\frac{1}{4} - \frac{1}{6}z^2 - \frac{31}{324}z^4 + \frac{32}{405}z^5 - \frac{184}{2187}z^6 + \cdots\right)\ , \\ b &= \frac{2}{3}z^{-1} - \frac{2}{27}z + \frac{16}{81}z^3 - \frac{896}{3645}z^4 + \frac{2768}{6561}z^5 + \cdots\ , \\ X &= 1 - \frac{4}{9}z^2 + \frac{8}{27}z^3 - \frac{16}{81}z^4 + \frac{56}{243}z^5 - \frac{172}{729}z^6 + \frac{1072}{3645}z^7 - \frac{34304}{98415}z^8 + \cdots\ . \end{aligned} \tag{3.36}$$

For the metric, we see that $g_1 = g_3 = 0$ as expected and g_5 will be related to the expectation value of the stress tensor. We do not have to worry about the gravitational conformal anomaly as $d = 5$ is odd, which is consistent with the fact that no terms which are logarithmic in the FG coordinate z appear in the expansion.

The conformal dimensions of the dual operators in the CFT corresponding to the scalar ϕ and tensor field B are determined by the linearized bulk equations of motion (3.6) near the AdS boundary. For instance, we can plug $\phi \sim z^{\Delta_\phi}$ into the linearized equation of motion for the scalar in AdS_6 to obtain the relation,

$$\Delta_\phi(\Delta_\phi - 5) = -6\ , \tag{3.37}$$

where the -6 is the mass-squared of the ϕ field from expanding the potential $V(X)$, with $m = 1$. The mass is within the window where both standard and alternative quantization is possible [20], which implies that the scaling dimension of dual can be either $\Delta_\phi = 2$ or $\Delta_\phi = 3$. However, we can argue that because the dual operators in the gravity multiplet fall into a superconformal multiplet with the stress tensor as the top component [113], we should have $\Delta_\phi = 3$ for the bottom scalar operator dual to the scalar ϕ. It follows from the near boundary expansion (3.36) that the defect solution has a non-trivial source as well as expectation value for the scalar operator.

Similarly, plugging $B = z^{\Delta_B - 2}\,\mathrm{d}x^1 \wedge \mathrm{d}x^2$ into the linearized equation of motion for the B-field gives

$$(\Delta_B - 2)(\Delta_B - 3) = 2 \ , \tag{3.38}$$

and so we have $\Delta_B = 4$ for the operator dual to 2-form potential B. It follows from (3.36) that the defect solution turns on a source for the operator dual to B.

3.3 Holographic calculations

In this section we use the formalism of holographic renormalization [21, 22] to calculate two quantities: (i) the on-shell action of the solution, which gives the expectation value of the dual defect operator, and (ii) the expectation value of the boundary stress tensor in the presence of the line defect.

3.3.1 Counterterms

For a well-defined variational principle of the metric, we need to add to the bulk action given by the Lagrangian (3.1) the Gibbons-Hawking boundary term,

$$\begin{aligned} I_{\text{bulk}} &= \frac{1}{16\pi G_{\text{N}}} \int_M \mathcal{L} \ , \\ I_{\text{GH}} &= \frac{1}{8\pi G_{\text{N}}} \int_{\partial M} \mathrm{d}^5 x \sqrt{-h}\ \mathrm{Tr}\big(h^{-1} K\big) \ , \end{aligned} \tag{3.39}$$

where h_{ij} is the induced metric on the boundary and K_{ij} is the extrinsic curvature. In the FG coordinates (3.32) these take the form,

$$h_{ij} = \frac{1}{z^2} g_{ij} \ , \qquad K_{ij} = -\frac{z}{2} \partial_z h_{ij} \ . \tag{3.40}$$

This action diverges due to the infinite volume of integration. To regulate the theory, we restrict the bulk integral to the region $z \geq \varepsilon$ and evaluate the boundary term at $z = \varepsilon$. Divergences in the action then appear as $1/\varepsilon^k$ poles.[8] Counterterms are added on the boundary which subtract these divergent terms, leaving a renormalized action. In all,

$$I_{\text{ren}} = I_{\text{bulk}} + I_{\text{GH}} + I_{\text{ct}} \ . \tag{3.41}$$

The counterterms can be expressed in terms of local quantities on the boundary. They have been explicitly worked out in appendix 3.B, which mirrors the derivation in [114].[9]

$$\begin{aligned} I_{\text{ct}} = \frac{1}{8\pi G_{\text{N}}} \int_{\partial M} \mathrm{d}x^5 \sqrt{-h} \bigg(& -4 - \frac{1}{6} R[h] + \frac{1}{8} B^{ij} B_{ij} - 4(1-X)^2 \\ & + \frac{5}{288} R[h]^2 - \frac{1}{18} R^{ij}[h] R_{ij}[h] - \frac{7}{192} R[h] B^{ij} B_{ij} \\ & - \frac{1}{6} R^i{}_j[h] B^j{}_k B^k{}_i + \frac{13}{512} (B^{ij} B_{ij})^2 - \frac{1}{8} B^i{}_j B^j{}_k B^k{}_\ell B^\ell{}_i \bigg) \ , \end{aligned} \tag{3.42}$$

where the inverse boundary metric h^{ij} is used to raise all indices and construct $R[h]$ and $R_{ij}[h]$, and B_{ij} is the induced 2-form on the boundary. Note that this is only a subset of the most general counterterms; we have only included the terms which are non-zero for our defect solution.

Having a renormalized action allows us to obtain a finite result when computing the on-shell action of a solution. Using the equations of motion (3.6), we can put the on-shell "bulk" action into the more convenient form,

$$I_{\text{bulk}} \bigg|_{\text{on-shell}} = -\frac{1}{8\pi G_{\text{N}}} \int_M X^{-2} (2 + 3X^4) *_6 1 + \frac{1}{8\pi G_{\text{N}}} \int_{\partial M} \left(\frac{1}{6} X^4 *_6 H \wedge B + \frac{1}{3} X^{-1} *_6 \mathrm{d}X \right) . \tag{3.43}$$

[8]In even boundary dimensions, a logarithmic term proportional to $\log \varepsilon$ also appears.

[9]This fixes a typo in Eq. (5.37), where the coefficient $+9/32\sqrt{2}$ should be $+7/32\sqrt{2}$ instead.

The second integral over the boundary can be written more explicitly using the boundary metric h_{ij} as

$$\int_{\partial M} \mathrm{d}^5x \sqrt{-h}\, z \left(\frac{1}{12} X^4 B^{ij} H_{ijz} + \frac{1}{3} X^{-1} \partial_z X \right) . \tag{3.44}$$

The bulk integral can be performed for $\alpha \in [0, \pi/4]$ and the boundary integral, including I_{GH} and I_{ct}, can be evaluated at $z = 0$. All divergences should cancel out, by construction of the counterterms. The on-shell action was calculated for both the global AdS_6 (3.26) and defect (3.31) solutions.

$$\begin{aligned} I_{\mathrm{ren}}(\mathrm{AdS}_6) &= -\frac{2}{3} \cdot \frac{1}{8\pi G_{\mathrm{N}}} \mathrm{Vol}(\mathrm{AdS}_2)\, \mathrm{Vol}(S^3) \ , \\ I_{\mathrm{ren}}(\mathrm{defect}) &= \frac{2}{81} \cdot \frac{1}{8\pi G_{\mathrm{N}}} \mathrm{Vol}(\mathrm{AdS}_2)\, \mathrm{Vol}(S^3) \ , \end{aligned} \tag{3.45}$$

where $\mathrm{Vol}(S^3) = 2\pi^2$ and $\mathrm{Vol}(\mathrm{AdS}_2) = -2\pi$ is the regularized volume of AdS_2 [115, 116].

3.3.2 Stress tensor

Given the renormalized action, we can calculate the expectation value of the boundary stress tensor. This contains two parts, one coming from the regularized action and one coming from the counterterms,

$$T_{ij}[h] = T^{\mathrm{reg}}_{ij}[h] + T^{\mathrm{ct}}_{ij}[h] \ . \tag{3.46}$$

As usual [117], the former is given by

$$T^{\mathrm{reg}}_{ij}[h] = -\frac{2}{\sqrt{-h}} \frac{\delta(I_{\mathrm{bulk}} + I_{\mathrm{GH}})}{\delta h^{ij}} = -\frac{1}{8\pi G_{\mathrm{N}}} \Big(K_{ij} - h_{ij}\, \mathrm{Tr}\big(h^{-1} K\big) \Big) \ . \tag{3.47}$$

The latter can be calculated by taking the variation of the counterterms in (3.42), which is straightforward to compute [23],

$$T^{\mathrm{ct}}_{ij}[h] = -\frac{2}{\sqrt{-h}} \frac{\delta I_{\mathrm{ct}}}{\delta h^{ij}} \ . \tag{3.48}$$

The expectation value of the boundary stress tensor is then related to $T_{ij}[h]$ by taking the leading term in z, or more concretely,

$$\langle T_{ij}\rangle \equiv -\frac{2}{\sqrt{-g_0}}\frac{\delta I_{\text{ren}}}{\delta g_0^{ij}} = \lim_{\varepsilon\to 0}\left(\varepsilon^{-3}\, T_{ij}[h]\Big|_{z=\varepsilon}\right) . \tag{3.49}$$

By construction of the counterterms, this limit exists and we are left with a finite result, which we are able to write in terms of FG expansion coefficients. Taking the following expansion of fields,

$$\begin{aligned} z^2 h = g &= g_0 + z^2 g_2 + z^4 g_4 + z^5 g_5 + \mathcal{O}\big(z^6\big) \ , \\ B &= z^{-1}B_{-1} + zB_1 + z^2 B_2 + \mathcal{O}\big(z^3\big) \ , \\ X &= 1 + z^2 X_2 + z^3 X_3 + \mathcal{O}\big(z^4\big) \ , \end{aligned} \tag{3.50}$$

where B_{-1}, B_1, and B_2 are 2-forms on the $x^1, x^2, \ldots, x^5$ coordinates excluding z, the expectation value of the boundary stress tensor is

$$\begin{aligned} \langle T_{ij}\rangle = \frac{1}{8\pi G_{\text{N}}}\bigg[&\frac{5}{2}g_{5ij} - \frac{5}{2}g_{0ij}\,\text{Tr}\big(g_0^{-1}g_5\big) - \frac{1}{4}g_{0ij}\,\text{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_2\big) \\ &+ \frac{1}{2}B_{-1ik}g_0^{k\ell}B_{2\ell j} + \frac{1}{2}B_{2ik}g_0^{k\ell}B_{-1\ell j} - 8g_{0ij}X_2X_3\bigg] . \end{aligned} \tag{3.51}$$

This quantity depends on the FG coefficients left undetermined by the equations of motion, namely g_5, B_2, and X_3, as expected. Taking the trace with the conformal boundary metric g_0 gives,

$$\langle T^i_i\rangle = \frac{1}{8\pi G_{\text{N}}}\left[-10\,\text{Tr}\big(g_0^{-1}g_5\big) - \frac{1}{4}\,\text{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_2\big) - 40X_2X_3\right] . \tag{3.52}$$

This result is accompanied by a Ward identity encoding the spontaneous breaking of scale invariance,

$$\frac{5}{2}\,\text{Tr}\big(g_0^{-1}g_5\big) + \frac{1}{4}\,\text{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_2\big) - \frac{1}{4}X_3\,\text{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_{-1}\big) + 12X_2X_3 = 0 \ , \tag{3.53}$$

which comes from the bulk Einstein equation (3.6), expanded in FG coordinates to order $\mathcal{O}(z^3)$. Explicitly evaluating these two expectation values for our defect solution, using the expansion coefficients in (3.36), yields

$$\langle T_{ij} \rangle = \begin{pmatrix} -\frac{88}{243} g_{\mathrm{AdS}_2} & 0 \\ 0 & -\frac{16}{81} g_{S^3} \end{pmatrix} , \qquad \langle T_i^i \rangle = -\frac{1280}{243} , \tag{3.54}$$

where g_{AdS_2} and g_{S^3} are unit radius.

3.4 Discussion

In this chapter, we found a non-singular solution of $F(4)$ gauged supergravity, which is of the form $\mathrm{AdS}_2 \times S^3$ warped over an interval. It preserves eight of the sixteen supersymmetries and represents a holographic dual of a half-BPS superconformal line defect. This solution is uniquely determined by the symmetries of the ansatz and the fact that it is half-BPS. Solutions of $F(4)$ gauged supergravity can be consistently lifted to AdS_6 solutions of massive type IIA [112] or type IIB solutions [91, 92]. Consequently, the solution found in this chapter lifts to a holographic line defect for the ten-dimensional theories. The ten-dimensional warped AdS_6 solutions have a holographic field theory dual, such as $\mathrm{USp}(N)$ gauge theories for massive type IIA and long quiver theories coming from (p, q) five-brane webs for type IIB.

The lifted solution should correspond to a heavy line defect in these ten-dimensional theories and is universal in the sense that it exists in all of the ten-dimensional AdS_6 solutions. However, unlike the holographic Wilson line solutions for $N = 4$ SYM found in [49], we do not know which representation the line defect corresponds to and we do not have families of solutions corresponding to different representations in a given AdS_6 vacuum. One way to obtain such solutions is to start in the ten-dimensional theory, but since even the AdS_6 vacuum has the form of a warped product this is considerably harder than in the $\mathrm{AdS}_5 \times S^5$ case. The form of the lifted solution may give hints on how a more general ansatz should look like. Furthermore, generalizing the solution found in this chapter to theories which include additional vector multiplets may be useful, since a consistent truncation in some cases was found recently [100]. We leave these interesting questions for future work.

3.A Conventions

	AdS_2		S^3			I_α
$\mu =$	1	2	3	4	5	6
$x^\mu =$	t	r	ψ	θ	ϕ	α or z

Table 3.2: Choice of coordinate ordering.

The six-dimensional Hodge dual is given by

$$*_6(\mathrm{d}x^{\mu_1} \wedge \cdots \wedge \mathrm{d}x^{\mu_r}) = \frac{\sqrt{-G}}{(6-r)!}\varepsilon_{\nu_1 \dots \nu_{D-r}}{}^{\mu_1 \dots \mu_r}\, \mathrm{d}x^{\nu_1} \wedge \cdots \mathrm{d}x^{\nu_{D-r}} \ , \tag{3.55}$$

where $\varepsilon_{123456} = 1$. More concretely, we use the coordinates give in Table 3.2,

$$\begin{aligned} \mathrm{d}s^2_{\mathrm{AdS}_2} &= -(1+r^2)\,\mathrm{d}t^2 + (1+r^2)^{-1}\,\mathrm{d}r^2 \ , \\ \mathrm{d}s^2_{S^3} &= \mathrm{d}\psi^2 + \sin^2\psi\,\mathrm{d}\theta^2 + \sin^2\psi\sin^2\theta\,\mathrm{d}\phi^2 \ . \end{aligned} \tag{3.56}$$

The norm of a p-form is defined as

$$\|F\|^2_g = \frac{1}{p!}F^{\mu_1 \dots \mu_p}F_{\mu_1 \dots \mu_p} \ , \tag{3.57}$$

where all indices are raised using the specified metric g. For the Riemann curvature tensor, we use the sign convention,

$$\begin{aligned} R^\rho{}_{\sigma\mu\nu} &= \partial_\mu \Gamma^\rho_{\nu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - (\mu \leftrightarrow \nu) \ , \\ R_{\mu\nu} &= R^\rho{}_{\mu\rho\nu} \ . \end{aligned} \tag{3.58}$$

3.B Counterterms

Here we briefly outline the calculation for obtaining the counterterms in (3.42). We will follow the steps in [114], making some simplifications suited for our purposes.

Using the Einstein equation (3.6) we can write the on-shell bulk action as

$$I_{\text{bulk}}\Big|_{\text{on-shell}} = \frac{1}{16\pi G_{\mathrm{N}}}\int_M \left[\frac{1}{2}V(X) *_6 1 - \frac{1}{2}X^4 *_6 H\wedge H - \frac{1}{2}X^{-2} *_6 B\wedge B - \frac{1}{3}B\wedge B\wedge B\right], \tag{3.59}$$

where we have set $m = 1$ and ignored terms involving A^i and A^0. The on-shell action also includes the Gibbons-Hawking term,

$$I_{\text{GH}} = \frac{1}{16\pi G_{\mathrm{N}}}\int_{\partial M} \mathrm{d}^5x\left(-2z\partial_z\sqrt{-h}\right). \tag{3.60}$$

We assume the following expansions of the fields,

$$\begin{aligned} g &= g_0 + z^2 g_2 + z^4 g_4 + \mathcal{O}\big(z^5\big)\,,\\ B &= z^{-1}B_{-1} + \mathrm{d}z\wedge A_0 + zB_1 + \mathcal{O}\big(z^2\big)\,,\\ H &= -z^{-2}\,\mathrm{d}z\wedge B_{-1} + z^{-1}\,\mathrm{d}B_{-1} - \mathrm{d}z\wedge \mathrm{d}A_0 + \mathrm{d}z\wedge B_1 + \mathcal{O}(z)\,,\\ X &= 1 + z^2X_2 + \mathcal{O}\big(z^3\big)\,, \end{aligned} \tag{3.61}$$

where B_{-1} and B_1 are 2-forms on the $x^1, x^2, \ldots, x^5$ coordinates excluding z, and A_0 is a 1-form on the same coordinates. The general strategy is to plug these expansions into the on-shell action, integrate the bulk terms over $z \geq \varepsilon$, and evaluate the boundary terms at $z = \varepsilon$. We will have order $\mathcal{O}(\varepsilon^{-5})$, $\mathcal{O}(\varepsilon^{-3})$, and $\mathcal{O}(\varepsilon^{-1})$ divergences, which are worked out order-by-order and then canceled out by appropriate counterterms. It is important to remember that the counterterm added to cancel the $\mathcal{O}(\varepsilon^{-5})$ divergence will also contribute to the $\mathcal{O}(\varepsilon^{-3})$ divergence, and so forth.

Along the way, we will need to use the equations of motion (3.6) expanded in the FG coordinates (3.32). This requires the expansion of the six-dimensional Ricci tensor in these coordinates,

$$
\begin{aligned}
R_{zz} &= \frac{1}{4}\operatorname{Tr}\big(g^{-1}g'g^{-1}g'\big) - \frac{1}{2}\operatorname{Tr}\big(g^{-1}g''\big) + z^{-1}\frac{1}{2}\operatorname{Tr}\big(g^{-1}g'\big) - 5z^{-2} \ , \\
R_{iz} &= \frac{1}{2}g^{jk}\nabla_k g'_{ij} - \frac{1}{2}g^{jk}\nabla_i g'_{jk} \ , \\
R_{ij} &= \frac{1}{2}g'_{ik}g^{k\ell}g'_{\ell j} - \frac{1}{4}g'_{ij}\operatorname{Tr}\big(g^{-1}g'\big) - \frac{1}{2}g''_{ij} + R_{ij}[g] + z^{-1}\left(2g'_{ij} + \frac{1}{2}g_{ij}\operatorname{Tr}\big(g^{-1}g'\big)\right) - 5z^{-2}g_{ij} \ ,
\end{aligned}
\tag{3.62}
$$

where $R_{ij}[g]$ and ∇_i are constructed using the five-dimensional metric,

$$
g = g_0 + g_1 z + g_2 z^2 + \cdots \ , \tag{3.63}
$$

and $'$ denotes the derivative with respect to z. For instance, the order $\mathcal{O}(z^0)$ Einstein equation implies that

$$
\begin{aligned}
g_{2ij} &= -\frac{1}{3}\left(R_{ij}[g_0] - \frac{1}{8}g_{0ij}R[g_0]\right) - \frac{3}{16}g_{0ij}\|B_{-1}\|^2_{g_0} - \frac{1}{2}B_{-1ik}{g_0}^{k\ell}B_{-1\ell j} \ , \\
\operatorname{Tr}\big(g_0^{-1}g_2\big) &= -\frac{1}{8}R[g_0] + \frac{1}{16}\|B_{-1}\|^2_{g_0} \ .
\end{aligned}
\tag{3.64}
$$

Another useful expansion is the determinant,

$$
\begin{aligned}
\sqrt{-g} = \sqrt{-g_0}\ \Bigg[&1 + \frac{1}{2}z^2\operatorname{Tr}\big(g_0^{-1}g_2\big) \\
&+ \frac{1}{2}z^4\left(\operatorname{Tr}\big(g_0^{-1}g_4\big) - \frac{1}{2}\operatorname{Tr}\big(g_0^{-1}g_2g_0^{-1}g_2\big) + \frac{1}{4}\operatorname{Tr}^2\big(g_0^{-1}g_2\big)\right) + \cdots\Bigg] \ .
\end{aligned}
\tag{3.65}
$$

For each order in ε, we will give the contributing divergence from each term in the action (3.59, 3.60), omitting an implicit $\sqrt{-g_0}/16\pi G_{\mathrm{N}}$ factor.

Order $\mathcal{O}(\varepsilon^{-5})$:

$$
\begin{aligned}
&\frac{1}{2}V(X) *_6 1 : && -2 \\
&-2z\partial_z\sqrt{-h} : && 10
\end{aligned}
$$

Adding these two contributions and restoring the $\sqrt{-g_0}/16\pi G_{\mathrm{N}}$ factor, the $\mathcal{O}(\varepsilon^{-5})$ divergence of the on-shell action is

$$
I_5 = \frac{\varepsilon^{-5}}{16\pi G_{\mathrm{N}}}\int_{z=\varepsilon} \mathrm{d}^5x\,\sqrt{-g_0}\,8 \ . \tag{3.66}
$$

A suitable counterterm which cancels this at leading order is

$$I_{\mathrm{ct},5} = \frac{1}{16\pi G_{\mathrm{N}}} \int_{\partial M} \mathrm{d}^5 x \sqrt{-h}\,(-8) \ . \tag{3.67}$$

Order $\mathcal{O}(\varepsilon^{-3})$:

$$\begin{aligned}
\frac{1}{2} V(X) *_6 1 : &\qquad -\frac{5}{3}\operatorname{Tr}\big(g_0^{-1} g_2\big) \\
-\frac{1}{2} X^4 *_6 H \wedge H : &\qquad -\frac{1}{6}\|B_{-1}\|^2_{g_0} \\
-\frac{1}{2} X^{-2} *_6 B \wedge B : &\qquad -\frac{1}{6}\|B_{-1}\|^2_{g_0} \\
-2z\partial_z \sqrt{-h} : &\qquad 3\operatorname{Tr}\big(g_0^{-1} g_2\big) \\
-8\sqrt{-h} : &\qquad -4\operatorname{Tr}\big(g_0^{-1} g_2\big)
\end{aligned}$$

$$I_3 = \frac{\varepsilon^{-3}}{16\pi G_{\mathrm{N}}} \int_{z=\varepsilon} \mathrm{d}^5 x \sqrt{-g_0} \left(\frac{1}{3} R[g_0] - \frac{1}{2}\|B_{-1}\|^2_{g_0} \right) , \tag{3.68}$$

where we used (3.64). Thus,

$$I_{\mathrm{ct},3} = \frac{1}{16\pi G_{\mathrm{N}}} \int_{\partial M} \mathrm{d}^5 x \sqrt{-h} \left(-\frac{1}{3} R[h] + \frac{1}{2}\|B\|^2_h \right) . \tag{3.69}$$

In order to write down the $\mathcal{O}(\varepsilon^{-1})$ divergences, we need the FG expansion of $R[h] = z^2 R[g]$. A particularly convenient expansion is obtained from the order $\mathcal{O}(z^2)$ Einstein equation, which implies that

$$\begin{aligned}
R[g] = R[g_0] + z^2 \bigg(& -8\operatorname{Tr}\big(g_0^{-1} g_4\big) + 5\operatorname{Tr}\big(g_0^{-1} g_2 g_0^{-1} g_2\big) + \operatorname{Tr}^2\!\big(g_0^{-1} g_2\big) \\
& - 20 {X_2}^2 - X_2 \|B_{-1}\|^2_{g_0} - \frac{1}{2}\operatorname{Tr}\big(g_0^{-1} B_{-1} g_0^{-1} B_1\big) \\
& + \frac{1}{2}\operatorname{Tr}\big(g_0^{-1} B_{-1} g_0^{-1} B_{-1} g_0^{-1} g_2\big) + \frac{1}{2}\|A_0\|^2_{g_0} \bigg) + \mathcal{O}\big(z^4\big) \ , \\
\operatorname{Tr}\big(g_0^{-1} g_4\big) = \frac{1}{4} & \operatorname{Tr}\big(g_0^{-1} g_2 g_0^{-1} g_2\big) - \frac{5}{2} {X_2}^2 - \frac{3}{8} X_2 \|B_{-1}\|^2_{g_0} - \frac{1}{8}\operatorname{Tr}\big(g_0^{-1} B_{-1} g_0^{-1} B_1\big) \\
& + \frac{1}{16}\operatorname{Tr}\big(g_0^{-1} B_{-1} g_0^{-1}\,\mathrm{d}A_0\big) - \frac{3}{16}\|A_0\|^2_{g_0} + \frac{1}{16}\|\mathrm{d}B_{-1}\|^2_{g_0} \ .
\end{aligned} \tag{3.70}$$

We will also further assume $A_0 = 0$ and $\mathrm{d}B_{-1} = 0$, which is not true in general but is true for our solution and vastly simplifies calculations.

Order $\mathcal{O}(\varepsilon^{-1})$:

$$
\begin{aligned}
\frac{1}{2}V(X) *_6 1 : &\quad -5\,\mathrm{Tr}\big(g_0^{-1}g_4\big) + \frac{5}{2}\,\mathrm{Tr}\big(g_0^{-1}g_2g_0^{-1}g_2\big) - \frac{5}{4}\,\mathrm{Tr}^2(g_0^{-1}g_2) - 12{X_2}^2 \\
-\frac{1}{2}X^4 *_6 H\wedge H : &\quad -\frac{1}{4}\big(\mathrm{Tr}\big(g_0^{-1}g_2\big) + 8X_2\big)\|B_{-1}\|^2_{g_0} - \frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_1\big) \\
&\quad -\frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_{-1}g_0^{-1}g_2\big) \\
-\frac{1}{2}X^{-2} *_6 B\wedge B : &\quad -\frac{1}{4}\big(\mathrm{Tr}\big(g_0^{-1}g_2\big) - 4X_2\big)\|B_{-1}\|^2_{g_0} + \frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_1\big) \\
&\quad -\frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_{-1}g_0^{-1}g_2\big) \\
-2z\partial_z\sqrt{-h} : &\quad \mathrm{Tr}\big(g_0^{-1}g_4\big) - \frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}g_2g_0^{-1}g_2\big) + \frac{1}{4}\,\mathrm{Tr}^2(g_0^{-1}g_2) \\
-8\sqrt{-h} : &\quad -4\,\mathrm{Tr}\big(g_0^{-1}g_4\big) + 2\,\mathrm{Tr}\big(g_0^{-1}g_2g_0^{-1}g_2\big) - \mathrm{Tr}^2(g_0^{-1}g_2) \\
-\frac{1}{3}R[h]\sqrt{-h} : &\quad \frac{8}{3}\,\mathrm{Tr}\big(g_0^{-1}g_4\big) - \frac{5}{3}\,\mathrm{Tr}\big(g_0^{-1}g_2g_0^{-1}g_2\big) + \mathrm{Tr}^2(g_0^{-1}g_2) \\
&\quad + \frac{20}{3}{X_2}^2 + \frac{1}{3}X_2\|B_{-1}\|^2_{g_0} - \frac{1}{12}\|B_{-1}\|^2_{g_0}\,\mathrm{Tr}\big(g_0^{-1}g_2\big) \\
&\quad -\frac{1}{6}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_{-1}g_0^{-1}g_2\big) + \frac{1}{6}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_1\big) \\
\frac{1}{2}\|B\|^2_h\sqrt{-h} : &\quad \frac{1}{4}\|B_{-1}\|^2_{g_0}\,\mathrm{Tr}\big(g_0^{-1}g_2\big) - \frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_1\big) \\
&\quad +\frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_{-1}g_0^{-1}g_2\big)
\end{aligned}
$$

$$
\begin{aligned}
I_1 = \frac{\varepsilon^{-1}}{16\pi G_\mathrm{N}} \int_{z=\varepsilon} \mathrm{d}^5x\,\sqrt{-g_0}\,\Bigg(& 8{X_2}^2 - \frac{5}{144}R[g_0]^2 + \frac{1}{9}\,\mathrm{Tr}\big(g_0^{-1}\mathrm{Ric}[g_0]g_0^{-1}\mathrm{Ric}[g_0]\big) \\
& + \frac{7}{48}\|B_{-1}\|^2_{g_0}R[g_0] + \frac{1}{3}\,\mathrm{Tr}\big(g_0^{-1}\mathrm{Ric}[g_0]g_0^{-1}B_{-1}g_0^{-1}B_{-1}\big) - \frac{13}{64}\|B_{-1}\|^4_{g_0} + \frac{1}{4}\,\mathrm{Tr}\big[(g_0^{-1}B_{-1})^4\big] \\
& - \frac{2}{3}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_{-1}g_0^{-1}g_2\big) + \frac{1}{3}\,\mathrm{Tr}\big(g_0^{-1}B_{-1}g_0^{-1}B_1\big) - \frac{1}{3}\|B_{-1}\|^2_{g_0}\big(\mathrm{Tr}\big(g_0^{-1}g_2\big) - 4X_2\big)\Bigg)\,,
\end{aligned}
\tag{3.71}
$$

where we used (3.64) and (3.70). The terms on the last line cancel out using the order $\mathcal{O}(z^{-1})$ B-field equation of motion,

$$
B_{1ij} = 2X_2B_{-1ij} - \frac{1}{2}\,\mathrm{Tr}\big(g_0^{-1}g_2\big)B_{-1ij} + g_{2ik}g_0^{k\ell}B_{-1\ell j} + B_{-1ik}g_0^{k\ell}g_{2\ell j}\,. \tag{3.72}
$$

Thus, a suitable choice of counterterms is

$$I_{\mathrm{ct},1} = \frac{1}{16\pi G_{\mathrm{N}}} \int_{\partial M} \mathrm{d}^5x \, \sqrt{-h} \, \bigg(- 8(1-X)^2 + \frac{5}{144} R[h]^2 - \frac{1}{9} \,\mathrm{Tr}\big(h^{-1}\mathrm{Ric}[h] h^{-1}\mathrm{Ric}[h]\big) \\ - \frac{7}{48} \|B\|_h^2 R[h] - \frac{1}{3} \,\mathrm{Tr}\big(h^{-1}\mathrm{Ric}[h] h^{-1} B h^{-1} B\big) + \frac{13}{64} \|B\|_h^4 - \frac{1}{4} \,\mathrm{Tr}\big[(h^{-1}B)^4\big]\bigg) \, . \quad (3.73)$$

This fixes a typo in Eq. (5.37) of [114], where the coefficient $+9/32\sqrt{2}$ should be $+7/32\sqrt{2}$ instead.

CHAPTER 4

Holographic line defects in 4d $N = 2$ gauged supergravity

In this chapter, we construct supergravity solutions that are holographically dual to half-BPS line defects in three-dimensional $\mathcal{N} = 2$ superconformal field theories. We consider four-dimensional $N = 2$ gauged supergravity, which has been used in the past to describe condensed matter systems in three dimensions in order to find holographic models for superfluids and superconductors, see e.g. [118–120]. We generalize the analysis of [121], which considered pure gauged supergravity, to the case of matter couplings. The structure of this chapter is as follows. In section 4.1, we review our conventions for four-dimensional $N = 2$ gauged supergravity coupled to vector multiplets. In section 4.2, we give a general solution describing a half-BPS line defect, obtained by a double analytic continuation of the black hole solutions first found by Sabra [122]. Since the behavior of the vector multiplet scalars can only be determined implicitly, we consider three examples, namely a single scalar model, the gauged STU model, and the $\mathrm{SU}(1, n)$ coset model to obtain explicit solutions. In section 4.3, we use the machinery of holographic renormalization to calculate holographic observables for the solutions, namely the on-shell action and the expectation values of operators dual to the supergravity fields. In section 4.4, we explore the conditions for a regular geometry and calculate their consequences. In section 4.5, we discuss our results and possible directions for future research. Our conventions and some details of the calculations presented in the main body of the chapter are relegated to several appendices.

4.1 Four-dimensional $N = 2$ gauged supergravity

In this section, we review four-dimensional $N = 2$ gauged supergravity coupled to n vector multiplets. We use the conventions and notations of [123–125].

The field content of the gauged supergravity theory is as follows. The supergravity contains one graviton e_μ^a, two gravitinos ψ_μ^i, and one graviphoton A_μ^0. The gravity multiplet can be coupled to $N = 2$ matter, and in particular we consider n vector multiplets, which are labeled by an index $\alpha = 1, 2, \ldots, n$. Each vector multiplet contains one vector field A_μ^α, two gauginos λ_i^α, and one complex scalar τ^α. In this chapter we do not consider adding $N = 2$ hypermultiplets.

It is convenient to introduce a new index $I = 0, 1, \ldots, n$ and include the graviphoton with the other vector fields as A_μ^I. The complex scalars τ^α parametrize a special Kähler manifold equipped with a holomorphic symplectic vector,

$$v(\tau) = \begin{pmatrix} Z^I(\tau) \\ \mathcal{F}_I(\tau) \end{pmatrix} , \tag{4.1}$$

where the Kähler potential $\mathcal{K}(\tau, \bar{\tau})$ is determined by

$$e^{-\mathcal{K}(\tau,\bar{\tau})} = -i \langle v, \bar{v} \rangle \equiv -i(Z^I \bar{\mathcal{F}}_I - \mathcal{F}_I \bar{Z}^I) \ . \tag{4.2}$$

In the models we will consider, there exists a holomorphic function $\mathcal{F}(Z)$, called the prepotential, that is homogeneous of second order in Z such that

$$\mathcal{F}_I(\tau) = \frac{\partial}{\partial Z^I} \mathcal{F}\big(Z(\tau)\big) \ . \tag{4.3}$$

The supergravity theory is fully specified by the prepotential $\mathcal{F}(Z)$ and the choice of gauging of the SU(2) R-symmetry. We will choose the U(1) Fayet-Iliopoulos (FI) gauging. The only charged fields of the theory are the gravitinos, which couple to the gauge fields through the linear combination $\xi_I A^I$, for some real constants ξ_I. The two gravitinos have opposite charges $\pm g \xi_I$ for each U(1) gauge factor, where g is the gauge coupling.

The bosonic action is[10]

$$e^{-1}\mathcal{L}_{\rm bos} = \frac{1}{2}R - g_{\alpha\bar\beta}\partial^\mu\tau^\alpha\partial_\mu\bar\tau^{\bar\beta} - V(\tau,\bar\tau) + \frac{1}{4}({\rm Im}\,\mathcal{N})_{IJ}F^{I\mu\nu}F^J_{\mu\nu} - \frac{1}{8}({\rm Re}\,\mathcal{N})_{IJ}e^{-1}\varepsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma}\ ,$$

where $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$ are the field strengths and $g_{\alpha\bar\beta} = \partial_\alpha\partial_{\bar\beta}\mathcal{K}$ is the Kähler metric of the scalar manifold. We use $G_{\mu\nu}$ to denote the four-dimensional metric, so $e = \sqrt{-\det G}$. The scalar potential is

$$V(\tau,\bar\tau) = -2g^2\xi_I\xi_J\Big(({\rm Im}\,\mathcal{N})^{-1|IJ} + 8e^{\mathcal{K}}\bar Z^I Z^J\Big)\ , \tag{4.4}$$

where the kinetic matrix $\mathcal{N}_{IJ}$ is given by

$$\mathcal{N}_{IJ}(\tau,\bar\tau) = \bar{\mathcal{F}}_{IJ} + 2i\frac{({\rm Im}\,\mathcal{F}_{IL})({\rm Im}\,\mathcal{F}_{JK})Z^L Z^K}{({\rm Im}\,\mathcal{F}_{MN})Z^M Z^N}\ , \qquad \mathcal{F}_{IJ} \equiv \frac{\partial}{\partial Z^I}\frac{\partial}{\partial Z^J}\mathcal{F}(Z)\ . \tag{4.5}$$

This is equivalently defined as the matrix which solves the equations,

$$\mathcal{F}_I = \mathcal{N}_{IJ}Z^J\ , \qquad \mathcal{D}_{\bar\alpha}\bar{\mathcal{F}}_I = \mathcal{N}_{IJ}\mathcal{D}_{\bar\alpha}\bar Z^J\ , \tag{4.6}$$

where $\mathcal{D}$ is the Kähler covariant derivative,

$$\begin{aligned}
\mathcal{D}_\alpha v &= (\partial_\alpha + \partial_\alpha\mathcal{K})v\ ,\\
\mathcal{D}_{\bar\alpha}\bar v &= (\partial_{\bar\alpha} + \partial_{\bar\alpha}\mathcal{K})\bar v\ ,\\
\mathcal{D}_\alpha\bar v &= \partial_\alpha\bar v = 0\ ,\\
\mathcal{D}_{\bar\alpha}v &= \partial_{\bar\alpha}v = 0\ .
\end{aligned} \tag{4.7}$$

The equations of motion are obtained by varying the Lagrangian (4.4),

$$\begin{aligned}
R_{\mu\nu} &= 2g_{\alpha\bar\beta}\partial_\mu\tau^\alpha\partial_\nu\bar\tau^{\bar\beta} + VG_{\mu\nu} + ({\rm Im}\,\mathcal{N})_{IJ}\left(-F^I_{\ \mu}{}^{\rho}F^J_{\ \nu\rho} + \frac{1}{4}F^{I\rho\sigma}F^J_{\ \rho\sigma}G_{\mu\nu}\right)\ ,\\
\partial_\mu\Big(eg_{\alpha\bar\beta}\partial^\mu\bar\tau^{\bar\beta}\Big) &= e\bigg((\partial_\alpha g_{\beta\bar\gamma})\partial^\mu\tau^\beta\partial_\mu\bar\tau^{\bar\gamma} - \frac{1}{4}\partial_\alpha({\rm Im}\,\mathcal{N})_{IJ}F^{I\mu\nu}F^J_{\mu\nu} + \partial_\alpha V\bigg)\\
&\quad + \frac{1}{8}\partial_\alpha({\rm Re}\,\mathcal{N})_{IJ}\varepsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma}\ ,\\
0 &= \partial_\mu\left(e({\rm Im}\,\mathcal{N})_{IJ}F^{J\mu\nu} - \frac{1}{2}({\rm Re}\,\mathcal{N})_{IJ}\varepsilon^{\mu\nu\rho\sigma}F^J_{\rho\sigma}\right)\ .
\end{aligned} \tag{4.8}$$

[10] We set $8\pi G_N = 1$.

The supersymmetry transformations are given in appendix 4.A.

4.2 Line defect solutions

In this section, we give a general solution describing a half-BPS line defect in four-dimensional $N = 2$ gauged supergravity, and then construct the solution for three specific choices of the prepotential.

A conformal line defect in three dimensions is a codimension-two defect which breaks the three-dimensional conformal group $\mathrm{SO}(3,2)$ down to an $\mathrm{SO}(2,1) \times \mathrm{SO}(2)$ subgroup. The subgroup factors represent the unbroken conformal symmetry along the defect and transverse rotations about the defect, respectively. Minkoswski space $\mathbb{R}^{1,2}$ is related by a Weyl transformation to $\mathrm{AdS}_2 \times S^1$, namely

$$- \mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \, \mathrm{d}\phi^2 = \Omega(r) \left(\frac{- \mathrm{d}t^2 + \mathrm{d}r^2}{r^2} + \mathrm{d}\phi^2 \right) \, . \tag{4.9}$$

Hence in the holographic dual, the $\mathrm{SO}(2,1) \times \mathrm{SO}(2)$ symmetry can be realized as the isometries of $\mathrm{AdS}_2 \times S^1$, which we choose as the boundary of the four-dimensional asymptotically anti-de Sitter space. Therefore we consider a metric ansatz with $\mathrm{AdS}_2 \times S^1$ warped over a radial coordinate. We note that the location of the defect at $r = 0$ in Minkowski space gets mapped to the boundary of AdS_2 in the $\mathrm{AdS}_2 \times S^1$ geometry. Secondly, the absence of a conical singularity on the boundary fixes the periodicity of the angle ϕ to be 2π.

The superconformal algebras in three dimensions are $\mathrm{OSp}(\mathcal{N}|4)$, where $\mathcal{N} = 1, 2, \ldots, 6, 8$. For the CFT dual of four-dimensional $N = 2$ gauged supergravity, the relevant superalgebra is $\mathrm{OSp}(2|4)$ which has four Poincaré and four conformal supercharges. A conformal line defect is called superconformal if it preserves some supersymmetry. In the present chapter, we will consider half-BPS defects which preserve an $\mathrm{OSp}(2|2)$ superalgebra and hence four of the eight supersymmetries.

4.2.1 General solution

Four-dimensional $N = 2$, U(1) FI gauged supergravity admits half-BPS black hole solutions first found in [122]. The line defect solutions with $\mathrm{AdS}_2 \times S^1$ geometry are constructed by a double analytic continuation of the black hole solution. The metric and gauge fields are given by

$$\begin{aligned}
\mathrm{d}s^2 &= r^2\sqrt{H(r)}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f(r)}{\sqrt{H(r)}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H(r)}}{f(r)}\,\mathrm{d}r^2 \ , \\
f(r) &= -1 + 8g^2 r^2 H(r) \ , \\
H(r)^{1/4} &= \frac{1}{\sqrt{2}} e^{\mathcal{K}/2} Z^I H_I(r) \ , \\
H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0, 1, \dots, n \ , \\
A^I &= \left(-2H(r)^{-1/4} e^{\mathcal{K}/2} Z^I + \mu^I\right) \mathrm{d}\theta \ , \qquad I = 0, 1, \dots, n \ ,
\end{aligned} \tag{4.10}$$

for some real constants q_I and μ_I, where $Z^I = \bar{Z}^I$. Given a prepotential $\mathcal{F}(Z)$ and choice of parametrization of the symplectic sections $Z^I(\tau)$, the scalars τ^α are given implicitly by the equation,

$$iH^{1/4} e^{\mathcal{K}/2}(\mathcal{F}_I - \bar{\mathcal{F}}_I) = \frac{1}{\sqrt{2}} H_I \ . \tag{4.11}$$

At the conformal boundary where $r \to \infty$, in order to have asymptotic AdS_4 we need $2\sqrt{2}g\theta$ to be 2π-periodic, i.e. $\theta \sim \theta + \pi/\sqrt{2}g$. The AdS_4 length scale is then given by

$$L^{-2} = 8g^2 H(r = \infty)^{1/2} \ . \tag{4.12}$$

We will set $8g^2 = 1$ to obtain the usual S^1 periodicity $\theta \sim \theta + 2\pi$.

The center of the space[11] $r = r_+$ corresponds to the largest value of r where $f(r) = 0$. We consider radii taking values in the range $r \in [r_+, \infty)$. Demanding a regular geometry also requires $r_+ > 0$ and the absence of a conical singularity at the center of the space, both

[11] For the black hole geometry this is the location of the horizon.

of which can be done by tuning the q_I and ξ_I parameters. This is explored in further detail in section 4.4.

For a general prepotential, the equation (4.11) is very complicated and can only be solved numerically. Consequently, we will explicitly work out the line defect solution for three specific prepotentials, for which we can find explicit expressions for the scalars. An important requirement is the existence of an AdS_4 vacuum, which not all prepotentials admit, see e.g. [125, 126].

4.2.2 Single scalar model

Consider a single ($n = 1$) vector multiplet with the prepotential $\mathcal{F}(Z) = -iZ^0Z^1$. This theory has a single complex scalar τ and the scalar manifold is $\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$. Using the parametrization $(Z^0, Z^1) = (1, \tau)$, we can calculate the Kähler potential, kinetic matrix, and scalar potential,

$$\begin{aligned} e^{\mathcal{K}(\tau,\bar\tau)} &= \frac{1}{2(\tau+\bar\tau)} \ , \\ \mathcal{N}(\tau,\bar\tau) &= -i\begin{pmatrix} \tau & 0 \\ 0 & 1/\tau \end{pmatrix} \ , \\ V(\tau,\bar\tau) &= -\frac{1}{2(\tau+\bar\tau)}\left(\xi_0^2 + 2\xi_0\xi_1(\tau+\bar\tau) + \xi_1^2\tau\bar\tau\right) \ . \end{aligned} \tag{4.13}$$

The potential has extrema at $\tau = \pm\xi_0/\xi_1$, but only $\tau = \xi_0/\xi_1$ maintains $e^{\mathcal{K}} > 0$ for $\xi_I > 0$. The cosmological constant at this extremum gives the AdS_4 length scale,

$$L^{-2} = \frac{1}{2}\xi_0\xi_1. \tag{4.14}$$

We choose $\xi_1 = 2/\xi_0$ to set the AdS$_4$ length scale to unity. The line defect solution (4.10) has the explicit form,

$$\begin{aligned}
\mathrm{d}s^2 &= r^2\sqrt{H}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f}{\sqrt{H}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H}}{f}\,\mathrm{d}r^2 \ , \\
f(r) &= -1 + r^2 H(r) \ , \\
\sqrt{H(r)} &= \frac{1}{2}H_0 H_1 \ , \\
H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0,1 \ , \\
A^I &= \left(-\frac{\sqrt{2}}{H_I} + \mu^I\right)\mathrm{d}\theta \ , \qquad I = 0,1 \ .
\end{aligned} \tag{4.15}$$

The scalar is given by

$$\tau = \frac{H_0}{H_1} \ . \tag{4.16}$$

We have verified that the above fields obey the equations of motion (4.8).

4.2.3 Gauged STU model

The STU model is given by considering $n = 3$ vector multiplets with the prepotential,

$$\mathcal{F}(Z) = -2i\sqrt{Z^0 Z^1 Z^2 Z^3} \ . \tag{4.17}$$

This theory has three complex scalars τ^1, τ^2, τ^3 and the scalar manifold is three copies of $\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$. When all $\xi_I = \xi > 0$ are equal, this theory is a consistent truncation of $N = 8$ gauged supergravity [127, 128]. For reference on this model, see [129]. Using the parametrization $(Z^0, Z^1, Z^2, Z^3) = (1, \tau^2\tau^3, \tau^1\tau^3, \tau^1\tau^2)$, the Kähler potential is

$$e^{\mathcal{K}(\tau,\bar{\tau})} = \frac{1}{(\tau^1 + \bar{\tau}^1)(\tau^2 + \bar{\tau}^2)(\tau^3 + \bar{\tau}^3)} \ . \tag{4.18}$$

The expressions for the kinetic matrix and scalar potential are complicated, but simplify for real scalars $\tau^\alpha = \bar{\tau}^{\bar{\alpha}}$, which will be the case for the line defect solution.

$$\begin{aligned}
\mathcal{N}(\tau, \bar{\tau} = \tau) &= -i\,\mathrm{diag}\left(\tau^1\tau^2\tau^3, \frac{\tau^1}{\tau^2\tau^3}, \frac{\tau^2}{\tau^1\tau^3}, \frac{\tau^3}{\tau^1\tau^2}\right) \ , \\
V(\tau, \bar{\tau} = \tau) &= -\frac{1}{2}\left(\xi_0\left(\frac{\xi_1}{\tau^1} + \frac{\xi_2}{\tau^2} + \frac{\xi_3}{\tau^3}\right) + \left(\tau^1\xi_2\xi_3 + \xi_1\tau^2\xi_3 + \xi_1\xi_2\tau^3\right)\right) \ .
\end{aligned} \tag{4.19}$$

The potential has extrema at

$$\tau^1 = \pm\sqrt{\frac{\xi_0\xi_1}{\xi_2\xi_3}} \ , \qquad \tau^2 = \pm\sqrt{\frac{\xi_0\xi_2}{\xi_1\xi_3}} \ , \qquad \tau^3 = \pm\sqrt{\frac{\xi_0\xi_3}{\xi_1\xi_2}} \ . \tag{4.20}$$

Positivity of $e^{\mathcal{K}}$ requires us to choose the positive root. The cosmological constant at this extremum gives the AdS_4 length scale,

$$L^{-2} = \sqrt{\xi_0\xi_1\xi_2\xi_3} \ . \tag{4.21}$$

We pick the non-zero constants ξ_I in a way that sets the AdS_4 length scale to unity. The line defect solution (4.10) has the explicit form,

$$\begin{aligned}
\mathrm{d}s^2 &= r^2\sqrt{H}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f}{\sqrt{H}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H}}{f}\,\mathrm{d}r^2 \ , \\
f(r) &= -1 + r^2 H(r) \ , \\
H(r) &= H_0H_1H_2H_3 \ , \\
H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0,1,2,3 \ , \\
A^I &= \left(-\frac{1}{\sqrt{2}H_I} + \mu^I\right)\mathrm{d}\theta \ , \qquad I = 0,1,2,3 \ .
\end{aligned} \tag{4.22}$$

The scalars are

$$\tau^1 = \sqrt{\frac{H_0H_1}{H_2H_3}} \ , \qquad \tau^2 = \sqrt{\frac{H_0H_2}{H_1H_3}} \ , \qquad \tau^3 = \sqrt{\frac{H_0H_3}{H_1H_2}} \ . \tag{4.23}$$

This solution is also the double analytic continuation of the hyperbolic black hole solution in [130]. As consistency checks, we have verified that the above solution obeys the equations of motion (4.8) and is half-BPS. The latter was done by a direct calculation, independent of [122], which can be found in appendix 4.A.

4.2.4 $\mathrm{SU}(1,n)$ coset model

Another model which admits an AdS_4 vacuum has the prepotential $\mathcal{F}(Z) = \frac{i}{4}Z^I\eta_{IJ}Z^J$, and can be formulated with any number of vector multiplets. η_{IJ} is a Minkowski metric, which we will take to be $\eta = \mathrm{diag}(-1, +1, \ldots, +1)$. The scalar manifold of this theory is $\frac{\mathrm{SU}(1,n)}{\mathrm{U}(1)\times\mathrm{SU}(n)}$. Using the parametrization $(Z^0, Z^\alpha) = (1, \tau^\alpha)$, the Kähler potential is

$$e^{\mathcal{K}(\tau,\bar{\tau})} = \frac{1}{1 - \sum_\alpha \tau^\alpha \bar{\tau}^\alpha} \ . \tag{4.24}$$

Once again, the kinetic matrix and scalar potential have simpler forms for real scalars $\tau^\alpha = \bar{\tau}^{\bar{\alpha}}$. The matrix η_{IJ} is used to lower indices, e.g. $Z_I = \eta_{IJ} Z^J$.

$$\begin{aligned} \mathcal{N}_{IJ}(\tau, \bar{\tau} = \tau) &= -\frac{i}{2}\eta_{IJ} - i e^{\mathcal{K}(\tau,\tau)} Z_I Z_J \ , \\ V(\tau, \bar{\tau} = \tau) &= \frac{1}{2}\xi_I \eta^{IJ} \xi_J - \frac{(\xi_0 + \sum_\alpha \xi_\alpha \tau^\alpha)^2}{1 - \sum_\alpha (\tau^\alpha)^2} \ . \end{aligned} \tag{4.25}$$

This potential has an extremum at $\tau^\alpha = -\xi_\alpha/\xi_0$.[12] The cosmological constant at this extremum gives us the AdS_4 length scale,

$$L^{-2} = -\xi^2/2 \ , \tag{4.26}$$

where $\xi^2 = \xi_I \eta^{IJ} \xi_J$. We pick a time-like ξ_I with $\xi^2 = -2$ that will set the AdS_4 length scale to unity. The line defect solution (4.10) has the explicit form,

$$\begin{aligned} \mathrm{d}s^2 &= r^2 \sqrt{H}\, \mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f}{\sqrt{H}}\, \mathrm{d}s^2_{S^1} + \frac{\sqrt{H}}{f}\, \mathrm{d}r^2 \ , \\ f(r) &= -1 + r^2 H(r) \ , \\ \sqrt{H(r)} &= -\frac{1}{2} H_I \eta^{IJ} H_J \ , \\ H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0, 1, \ldots, n \ , \\ A^I &= \left(\frac{\sqrt{2}\eta^{IJ} H_J}{\sqrt{H}} + \mu^I \right) \mathrm{d}\theta \ , \qquad I = 0, 1, \ldots, n \ . \end{aligned} \tag{4.27}$$

The scalars are

$$\tau^\alpha = -\frac{H_\alpha}{H_0} \ . \tag{4.28}$$

We have verified that the above fields obey the equations of motion (4.8).

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[12] The other extrema at $\xi_0 + \sum_\alpha \xi_\alpha \tau^\alpha = 0$ do not admit AdS_4 vacua while maintaining $e^{\mathcal{K}}$ positive.

4.3 Holographic calculations

In this section, we use the machinery of holographic renormalization [21, 22] to calculate the on-shell action and the one-point functions of dual operators of the boundary CFT in the presence of the defect, namely the stress tensor, scalar, and currents. This is done explicitly for the three examples in sections 4.2.2–4.2.4.

4.3.1 General procedure

First, we put the metric into the Fefferman-Graham (FG) form,

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(\mathrm{d}z^2 + g_{ij}(x,z)\,\mathrm{d}x^i\,\mathrm{d}x^j\right) , \tag{4.29}$$

where $i, j = 1, 2, 3$ run over the AdS_2 and S^1 indices and $z \to 0$ is the conformal boundary. This is done by taking $z = z(r)$ so that the appropriate coordinate change is obtained by the solution to the ordinary differential equation,

$$-\frac{H(r)^{1/4}}{f(r)^{1/2}}\,\mathrm{d}r = \frac{\mathrm{d}z}{z} , \tag{4.30}$$

which can be integrated perturbatively in $1/r$. This coordinate change gives the FG expansions of the fields, which we assume will take the form,

$$\begin{aligned}
g_{ij} &= g_{0ij} + z^2 g_{2ij} + z^3 g_{3ij} + \mathcal{O}\left(z^4\right) , \\
A^I &= A_0^I + z A_1^I + \mathcal{O}\left(z^2\right) , \\
\tau^\alpha &= \tau_0^\alpha + z\tau_1^\alpha + z^2\tau_2^\alpha + \mathcal{O}\left(z^3\right) , \\
\bar{\tau}^{\bar{\alpha}} &= \tau_0^\alpha + z\tau_1^\alpha + z^2\tau_2^\alpha + \mathcal{O}\left(z^3\right) ,
\end{aligned} \tag{4.31}$$

where A_0^I and A_1^I are 1-forms on the x^1, x^2, x^3 coordinates. The constants τ_0^α are the AdS_4 vacuum values of the scalars, which depend on the model. There is no gravitational conformal anomaly (i.e. a term proportional to $z^3 \log z$ in the expansion of g_{ij}) since $d = 3$ is odd.

In the three-dimensional boundary CFT, the conformal dimensions of the dual operators corresponding to the scalars τ^α and vector fields A^I are determined by the linearized bulk equations of motion near the AdS boundary. For instance, using the expansion $\tau^\alpha \sim \tau_0^\alpha + z^{\Delta_\tau}$ in the linearized equation of motion for the scalar, we find that the scaling dimension of the dual operator is related to the mass-squared of the field by the equation,

$$\Delta_\tau(\Delta_\tau - 3) = -2 \ . \tag{4.32}$$

The mass-squared is -2 for all scalars of the three examples considered in this chapter. This mass-squared is within the window where both standard and alternative quantization are possible [20], which implies that the scaling dimension of the dual operator can be either $\Delta_\tau = 1$ or $\Delta_\tau = 2$. Similarly, using the expansion $A^I \sim z^{\Delta_A - 1}\, \mathrm{d}\theta$ in the linearized equation of motion for the vector field gives us

$$(\Delta_A - 1)(\Delta_A - 2) = 0 \ . \tag{4.33}$$

We must have $\Delta_A = 2$ as the vector field sources a conserved current of the boundary CFT.

These scaling dimensions naturally fit into the flavor current multiplet $A_2\overline{A}_2[0]_1^{(0)}$ of the $d = 3$, $\mathcal{N} = 2$ boundary CFT, using the notation of [131]. This short multiplet contains, in addition to the spin-1 operator $[2]_2^{(0)}$ with scaling dimension $\Delta = 2$, two scalar operators $[0]_1^{(0)}$ and $[0]_2^{(0)}$ as bottom and top components with scaling dimensions $\Delta = 1$ and 2 respectively. The stress tensor multiplet $A_1\overline{A}_1[2]_2^{(0)}$ is also present, as usual.

In the four-dimensional gauged supergravity, for a well-defined variational principle of the metric we need to add to the bulk action given by the Lagrangian (4.4) the Gibbons-Hawking boundary term,

$$\begin{aligned} I_{\text{bulk}} &= \int_M \mathrm{d}^4x\, \mathcal{L}_{\text{bos}} \ , \\ I_{\text{GH}} &= \int_{\partial M} \mathrm{d}^3x\, \sqrt{-h}\ \mathrm{Tr}\big(h^{-1}K\big) \ , \end{aligned} \tag{4.34}$$

where h_{ij} is the induced metric on the boundary and K_{ij} is the extrinsic curvature. In FG coordinates, these take the form,

$$h_{ij} = \frac{1}{z^2} g_{ij} \ , \qquad K_{ij} = -\frac{z}{2} \partial_z h_{ij} \ . \tag{4.35}$$

The action $I_{\text{bulk}} + I_{\text{GH}}$ diverges due to the infinite volume of integration. To regulate the theory, we restrict the bulk integral to the region $z \geq \varepsilon$ and evaluate the boundary term at $z = \varepsilon$. Divergences in the action then appear as $1/\varepsilon^k$ poles.[13] Counterterms I_{ct} are added on the boundary which subtract these divergent terms. The counterterms have been constructed in [129] and are compatible with supersymmetry. They are

$$I_{\text{ct}} = \int_{\partial M} \mathrm{d}^3 x \sqrt{-h} \left(\mathcal{W} - \frac{1}{2} R[h] \right) \ , \qquad \mathcal{W} \equiv -\sqrt{2} e^{\mathcal{K}/2} \left| \xi_I Z^I \right| \ , \tag{4.36}$$

where $R[h]$ is the Ricci scalar of the boundary metric and $\mathcal{W}$ is the superpotential. In all, the renormalized action,

$$I_{\text{ren}} = I_{\text{bulk}} + I_{\text{GH}} + I_{\text{ct}} \ , \tag{4.37}$$

is finite. We can then take functional derivatives to obtain finite expectation values of the dual CFT operators. Let T_{ij} be the boundary stress tensor, $\mathcal{O}_\alpha$ be the operators dual to τ^α, and J_{Ii} be the current operators dual to A^I_μ.

4.3.1.1 Stress tensor expectation value

The expectation value of the boundary stress tensor is defined to be [23]

$$\langle T_{ij} \rangle \equiv \frac{-2}{\sqrt{-g_0}} \frac{\delta I_{\text{ren}}}{\delta g_0^{ij}} \ . \tag{4.38}$$

The variation decomposes into two contributions: one coming from the regularized action and one coming from the counterterms. As usual [117], the former is given by

$$T^{\text{reg}}_{ij}[h] \equiv \frac{-2}{\sqrt{-h}} \frac{\delta(I_{\text{bulk}} + I_{\text{GH}})}{\delta h^{ij}} = -K_{ij} + h_{ij} \operatorname{Tr}\big(h^{-1} K\big) \ . \tag{4.39}$$

[13]In even boundary dimensions, a term proportional to $\log \varepsilon$ may also appear.

The latter is straightforward to compute, and is given by

$$T_{ij}^{\rm ct}[h] \equiv \frac{-2}{\sqrt{-h}}\frac{\delta I_{\rm ct}}{\delta h^{ij}} = h_{ij}\left(\mathcal{W} - \frac{1}{2}R[h]\right) + R_{ij}[h] \ . \tag{4.40}$$

Therefore,

$$\langle T_{ij}\rangle = \lim_{\varepsilon\to 0}\left[\varepsilon^{-1}\left(T_{ij}^{\rm reg}[h] + T_{ij}^{\rm ct}[h]\right)\Big|_{z=\varepsilon}\right] \ . \tag{4.41}$$

By construction of the counterterms, this limit exists.

4.3.1.2 Scalar expectation values

The expectation value of the operator $\mathcal{O}_\alpha$ is similarly defined by

$$\langle\mathcal{O}_\alpha\rangle \equiv \frac{1}{\sqrt{-g_0}}\frac{\delta I_{\rm ren}}{\delta \tau_1^\alpha} = \lim_{\varepsilon\to 0}\left[\varepsilon^{-2}\ \frac{1}{\sqrt{-h}}\frac{\delta I_{\rm ren}}{\delta\tau^\alpha}\Big|_{z=\varepsilon}\right] \ . \tag{4.42}$$

The variation has contributions from the bulk action and the counterterms, and is

$$\frac{1}{\sqrt{-h}}\frac{\delta I_{\rm ren}}{\delta\tau^\alpha} = g_{\alpha\bar\beta}z\partial_z\bar\tau^{\bar\beta} + \partial_\alpha\mathcal{W} \ . \tag{4.43}$$

For real scalars, supersymmetry implies $\langle\mathcal{O}_\alpha\rangle = 0$. A proof of this statement can be found in appendix 4.B.

4.3.1.3 Current expectation values

The expectation value of the current operator J_I is defined by

$$\left\langle J_I^i\right\rangle \equiv \frac{1}{\sqrt{-g_0}}\frac{\delta I_{\rm ren}}{\delta A_{0i}^I} = \lim_{\varepsilon\to 0}\left[\varepsilon^{-3}\ \frac{1}{\sqrt{-h}}\frac{\delta I_{\rm ren}}{\delta A_i^I}\Big|_{z=\varepsilon}\right] \ . \tag{4.44}$$

The only contribution to the variation comes from the bulk action, and is

$$\frac{1}{\sqrt{-h}}\frac{\delta I_{\rm ren}}{\delta A_i^I} = -({\rm Im}\,\mathcal{N})_{IJ}h^{ij}z\partial_z A_j^J \ . \tag{4.45}$$

4.3.1.4 On-shell action

We can evaluate the on-shell action for the line defect solution by further simplifying the bulk action to a total derivative [132],

$$I_{\text{bulk}}\bigg|_{\text{on-shell}} = \text{Vol}(\text{AdS}_2)\,\text{Vol}(S^1)\,\left[-\frac{H'(r)}{4H(r)}r^2 f(r) - r\big(f(r)+1\big)\right]\Bigg|_{r_+}^{\infty} \ , \tag{4.46}$$

where $\text{Vol}(S^1) = 2\pi$ and $\text{Vol}(\text{AdS}_2) = -2\pi$ is the regularized volume of AdS_2 [115, 116].

We will now use the general expressions derived in this section to compute observables for the three examples considered in this chapter.

4.3.2 Single scalar model

Let us consider the defect solution (4.15, 4.16) for the single scalar model. The FG expansion of the radial coordinate r from solving the ordinary differential equation (4.30) is

$$\frac{1}{r} = z + \frac{1}{2}\left(\sum_{I=0}^{1}\frac{q_I}{\xi_I}\right)z^2 + \frac{-16 + (3q_1\xi_0 + q_0\xi_1)(3q_0\xi_1 + q_1\xi_0)}{64}z^3 \tag{4.47}$$

$$+\frac{(q_1\xi_0 + q_0\xi_1)(-16 + 12q_0q_1\xi_0\xi_1 + 3(q_0\xi_1 + q_1\xi_0)^2)}{384}z^4 + \mathcal{O}\big(z^5\big) \ . \tag{4.48}$$

Using this coordinate change, the metric, gauge fields, and scalar can be expanded in FG coordinates. The one-point functions in the presence of the line defect can then be evaluated by computing the limits (4.41, 4.42, 4.44) directly. For the renormalized on-shell action (4.37), the finite terms at the conformal boundary cancel, leaving just the term obtained by evaluating (4.46) at $r = r_+$. In the end, we obtain the following expectation values:

$$I_{\text{ren}} = \text{Vol}(\text{AdS}_2)\,\text{Vol}(S^1) r_+ \ ,$$

$$\langle T_{ij}\rangle = \frac{1}{2}\left(\sum_{I=0}^{1}\frac{q_I}{\xi_I}\right)\begin{pmatrix}-g_{\text{AdS}_2} & 0\\ 0 & 2g_{S^1}\end{pmatrix}_{ij} \ ,$$

$$\langle T_i^i\rangle = 0 \ ,$$

$$\langle \mathcal{O}\rangle = 0 \ ,$$

$$\langle J_{Ii} \rangle = \frac{q_I}{\sqrt{2}} \delta_{i\theta} \ . \tag{4.49}$$

4.3.3 Gauged STU model

Let us consider the defect solution (4.22, 4.23) for the gauged STU model. Some of the calculations for this model are identical to those found in [130]. The FG expansion of the radial coordinate r from solving the ODE (4.30) is

$$\frac{1}{r} = z + \frac{A}{4} z^2 + \frac{-16 + B_1 + 10B_2}{64} z^3 + \frac{-16A + C_1 + 11C_2 + 62C_3}{384} z^4 + \mathcal{O}\big(z^5\big) \ , \tag{4.50}$$

where we have defined the constants,

$$\begin{aligned} A &= \sum_{I=0}^{3} \frac{q_I}{\xi_I} \ , & B_1 &= \sum_{I=0}^{3} \left(\frac{q_I}{\xi_I}\right)^2 \ , & B_2 &= \sum_{I<J} \frac{q_I q_J}{\xi_I \xi_J} \ , \\ C_1 &= \sum_{I=0}^{3} \left(\frac{q_I}{\xi_I}\right)^3 \ , & C_2 &= \sum_{I \neq J} \left(\frac{q_I}{\xi_I}\right)^2 \frac{q_J}{\xi_J} \ , & C_3 &= \sum_{I<J<K} \frac{q_I q_J q_K}{\xi_I \xi_J \xi_K} \ . \end{aligned} \tag{4.51}$$

Using this coordinate change, the fields of the defect solution can be expanded in FG coordinates. We obtain the following on-shell action and one-point functions,

$$\begin{aligned} I_{\text{ren}} &= \text{Vol}(\text{AdS}_2)\, \text{Vol}(S^1) r_+ \ , \\ \langle T_{ij} \rangle &= \frac{1}{4} \left(\sum_{I=0}^{3} \frac{q_I}{\xi_I} \right) \begin{pmatrix} -g_{\text{AdS}_2} & 0 \\ 0 & 2g_{S^1} \end{pmatrix}_{ij} \ , \\ \left\langle T^i_i \right\rangle &= 0 \ , \\ \langle \mathcal{O}_1 \rangle = \langle \mathcal{O}_2 \rangle = \langle \mathcal{O}_3 \rangle &= 0 \ , \\ \langle J_{Ii} \rangle &= \frac{q_I}{\sqrt{2}} \delta_{i\theta} \ . \end{aligned} \tag{4.52}$$

Note that the expression for I_{ren} is identical to that of the single scalar model, but the radius $r_+ = r_+ \left(\xi_I, q_I\right)$ will be different.

4.3.4 $\mathrm{SU}(1,n)$ coset model

For the defect solution (4.27, 4.28) of the $\mathrm{SU}(1,n)$ coset model, the FG expansion of the radial coordinate r is

$$\begin{aligned}\frac{1}{r} &= z - \frac{1}{2}q_I\xi^I z^2 - \frac{1}{4}\left[1 + \frac{1}{2}q_I q^I - \frac{3}{4}(q_I\xi^I)^2\right] z^3 \\ &\quad + \frac{1}{12}q_I\xi^I\left[1 + \frac{3}{2}q_I q^I - \frac{3}{4}(q_I\xi^I)^2\right] z^4 + \mathcal{O}\big(z^5\big)\ , \end{aligned} \tag{4.53}$$

where η^{IJ} is used to raise the indices of ξ_I and q_I. Using this coordinate change and expanding the fields in FG coordinates, the on-shell action and one-point functions are

$$\begin{aligned} I_{\text{ren}} &= \mathrm{Vol}(\mathrm{AdS}_2)\,\mathrm{Vol}(S^1) r_+\ , \\ \langle T_{ij}\rangle &= -\frac{q_I\xi^I}{2}\begin{pmatrix} -g_{\mathrm{AdS}_2} & 0 \\ 0 & 2g_{S^1}\end{pmatrix}_{ij}\ , \\ \big\langle T^i_i\big\rangle &= 0\ , \\ \langle \mathcal{O}_\alpha\rangle &= 0\ , \\ \langle J_{Ii}\rangle &= \frac{q_I}{\sqrt{2}}\delta_{i\theta}\ . \end{aligned} \tag{4.54}$$

4.4 Regularity

In this section, we impose two regularity conditions on the solutions. First, we demand that the geometry smoothly closes off at the largest positive zero of $f(r)$ without a conical singularity in the bulk spacetime. This condition is analogous to the regularity condition imposed on Euclidean black hole solutions. Second, we fix the periodicity of the S^1 at the conformal boundary such that when the $\mathrm{AdS}_2 \times S^1$ boundary is conformally mapped to $\mathbb{R}^{1,2}$ there is no conical deficit on the boundary. This condition is different from the one imposed in the holographic calculation of supersymmetric Rényi entropies [133–136], which use solutions that are related by double analytic continuation. For these solutions, the periodicity is related to the Rényi index n.

The regularity conditions will impose constraints on the parameters of the solutions. Since the general solution is only implicit, a detailed analysis is performed for the examples presented in this chapter. We will show that for the single scalar and coset models, these conditions imply a bound on the expectation value of the boundary stress tensor.

4.4.1 General statements

Given the metric,

$$\mathrm{d}s^2 = r^2\sqrt{H(r)}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f(r)}{\sqrt{H(r)}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H(r)}}{f(r)}\,\mathrm{d}r^2 \ , \tag{4.55}$$

the center of the space $r = r_+$ is defined to be the largest zero of $f(r) = -1 + r^2 H(r)$. We can identify four criteria a regular geometry should satisfy:

(a) positivity of the zero, $r_+ > 0$,

(b) $0 < H(r) < \infty$ on $r \in [r_+, \infty)$,

(c) $0 < f(r) < \infty$ on $r \in (r_+, \infty)$, and

(d) no conical singularity at $r = r_+$.

Criteria (b) and (c) are satisfied if $H(r)$ is continuous: the AdS length scale (4.12) is well-defined if and only if the limit $H(r = \infty)$ is positive and finite. Since a zero of $H(r)$ occurs at $f(r) < 0$, positivity of $H(r)$ at large r and continuity imply that the spacetime closes off before a zero of $H(r)$ is ever encountered.

By expanding the metric around the center of the space, criterion (d) is satisfied when

$$f'(r_+)^2 = 4H(r_+) \ . \tag{4.56}$$

This can be simplified to

$$H'(r_+)(r_+^2 f'(r_+) + 2r_+) = 0 \ . \tag{4.57}$$

As the second factor is the sum of two positive quantities, a conical singularity can be avoided if we satisfy the condition $H'(r_+) = 0$. As r_+ is determined implicitly in terms of the q_I, ξ_I constants through the equation $f(r_+) = 0$, this condition can be viewed as a constraint on the possible values q_I, ξ_I can take. Additionally, we will see that criterion (a) manifests as an inequality on q_I, ξ_I that we must satisfy.

4.4.2 Single scalar model

The single scalar model is simple enough that the conditions for a regular geometry can be solved exactly. Let us define $x_I \equiv q_I/\xi_I$, but still pick the AdS length scale to be unity, i.e. keep $\xi_0\xi_1 = 2$. The metric functions become

$$\begin{aligned} H(r) &= \left(1 + \frac{x_0}{r}\right)^2 \left(1 + \frac{x_1}{r}\right)^2 , \\ f(r) &= -1 + \frac{1}{r^2}(r + x_0)^2 (r + x_1)^2 . \end{aligned} \tag{4.58}$$

Let us first satisfy the criterion $r_+ > 0$. Solving $f(r) = 0$,

$$0 = \left(r^2 + r(x_0 + x_1 - 1) + x_0 x_1\right)\left(r^2 + r(x_0 + x_1 + 1) + x_0 x_1\right) . \tag{4.59}$$

When the first factor is zero, we have a solution,

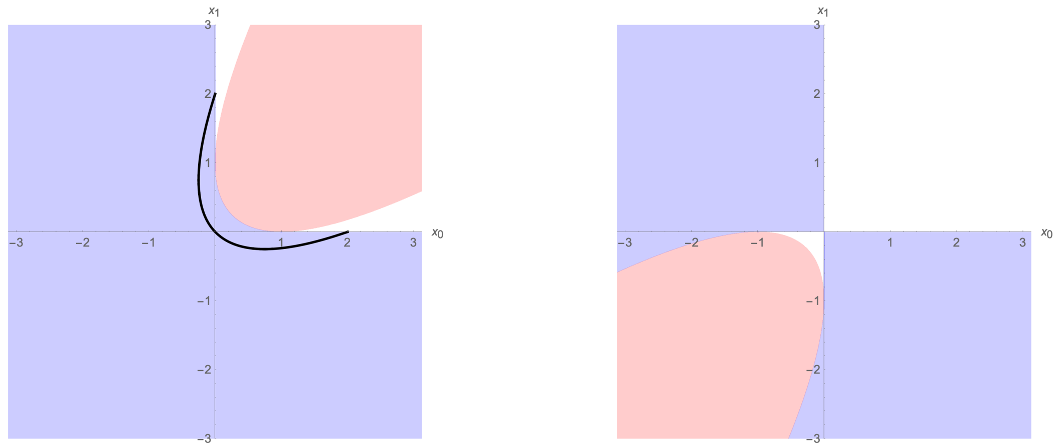
$$r_1 = \frac{1}{2}\left(-(x_0 + x_1 - 1) + \sqrt{(x_0 + x_1 - 1)^2 - 4x_0x_1}\right) , \tag{4.60}$$

where we took the + sign to get the largest root. This solution exists when $(x_0 + x_1 - 1)^2 - 4x_0x_1 \geq 0$, which is a region on the x_0x_1-plane bounded by a parabola, shown in Figure 4.1a. The red shaded region indicates where r_1 does not exist and the blue shaded region indicates where $r_1 > 0$. When the second factor of (4.59) is zero, we have another solution,

$$r_2 = \frac{1}{2}\left(-(x_0 + x_1 + 1) + \sqrt{(x_0 + x_1 + 1)^2 - 4x_0x_1}\right) , \tag{4.61}$$

where we also took the + sign. We have also marked regions where this solution exists and is positive in Figure 4.1b. In regions where r_1 and r_2 both exist and $r_1 > 0$, we have $r_1 > r_2$.

Therefore, we can take $r_+ = r_1$ and restrict the (x_0, x_1) parameter space to the blue shaded region of Figure 4.1a.



(a) r_1 (b) r_2

Figure 4.1: Candidate r_+ for the single scalar model.

Let us now avoid the conical singularity by satisfying $H'(r_+) = 0$. Calculating the derivative of $H(r)$ in (4.58) and plugging in $r_+ = r_1$ from (4.60), we get the condition,

$$0 = (x_0 - x_1)^2 - 2(x_0 + x_1) \ . \tag{4.62}$$

This is a parabola, marked by the black curve in Figure 4.1a in the region where $r_+ > 0$. For the single scalar model to admit a regular geometry, the parameters $x_I = q_I/\xi_I$ must satisfy this condition. As a corollary, we can note that

$$0 \leq x_0 + x_1 < 2 \ . \tag{4.63}$$

This implies that the components of the boundary stress tensor (4.49) have bounded expectation value. Additionally, the pure AdS$_4$ vacuum ($x_0 = x_1 = 0$) is the only solution with regular geometry and $\langle T_{ij} \rangle = 0$.

4.4.3 SU$(1, n)$ coset model

The coset model is also simple enough that the conditions for a regular geometry can be solved exactly. We can note that

$$H(r) = \left(1 - \frac{q_I \xi^I}{r} - \frac{q_I q^I}{2r^2}\right)^2 , \tag{4.64}$$

actually has the same form as (4.58), where

$$x_0 = \frac{-q_I\xi^I - \sqrt{(q_I\xi^I)^2 + 2q_I q^I}}{2} , \qquad x_1 = \frac{-q_I\xi^I + \sqrt{(q_I\xi^I)^2 + 2q_I q^I}}{2} . \tag{4.65}$$

This map is always well-defined as $(q_I\xi^I)^2 + 2q_I q^I \geq 0$, which can be checked by rotating to the frame where $\xi_I = (\sqrt{2}, 0, 0, \dots)$. Thus all our results for the single scalar model can be carried over. The bound (4.63) for the single scalar model translates to the same bound on $\langle T_{ij} \rangle$ for the coset model,

$$0 \leq -q_I \xi^I < 2 . \tag{4.66}$$

The condition (4.62) for a regular geometry translates to

$$0 = (q_I\xi^I)^2 + 2q_I q^I + 2q_I\xi^I . \tag{4.67}$$

We can show that the only regular geometry with vanishing $\langle T_{ij} \rangle$ is the AdS$_4$ vacuum. If we rotate to the frame where $\xi_I = (\sqrt{2}, 0, 0, \dots)$, the only q which satisfies $q_I\xi^I = 0$ and $q_I q^I = 0$ is $q_I = 0$. A general ξ then has a q in the orbit of $q_I = 0$, which is still the zero vector.

4.4.4 Gauged STU model

For the gauged STU model, it is not practical to solve $f(r) = 0$ to find r_+ as f is a quartic polynomial. However, we still expect the criterion $r_+ > 0$ to impose an inequality on the four-dimensional parameter space (x_0, x_1, x_2, x_3) and the condition of avoiding a conical singularity to reduce this to a three-dimensional hypersurface. However, note that unlike the single scalar and coset models, the expectation value $\langle T_{ij}\rangle$ is not bounded. In appendix 4.C we give special cases of the STU model with regular geometry which can have arbitrarily large $x_0 + x_1 + x_2 + x_3$.

4.5 Discussion

In this chapter, we constructed solutions of four-dimensional $N = 2$ gauged supergravity by a double analytic continuation of the half-BPS black hole solutions first found by Sabra [122]. While the black hole solutions exist for arbitrary prepotentials, explicit expressions for the scalars fields involve algebraic equations which in general can only be solved numerically. We considered three explicit examples of matter-coupled gauged supergravities, namely the single scalar model, the gauged STU model, and the $\mathrm{SU}(1,n)/\mathrm{U}(1)\times\mathrm{SU}(n)$ coset model to find solutions and calculate holographic observables.

The solutions we find are holographic duals to line defects in three-dimensional SCFTs. The defect is characterized by a non-trivial expectation value of the R-symmetry and flavor currents along the S^1 factor in the $\mathrm{AdS}_2 \times S^1$ description of the defect. After conformally mapping to Minkowski space, this corresponds to a holonomy when encircling the line defect. The expectation values of the real scalar operators vanish for general models as a consequence of supersymmetry.

For a conformal defect on $\mathrm{AdS}_2 \times S^1$, the expectation value of the stress tensor can be parameterized by a single coefficient h,

$$\langle T_{ab}\rangle = h\, g_{ab}^{\mathrm{AdS}_2}\ , \qquad \langle T_{\theta\theta}\rangle = -2h\, g_{\theta\theta}\ , \tag{4.68}$$

in analogy to the scaling dimension of local operators [137,138]. However, there are in general no unitarity bounds on h which follow from the superconformal algebra. For line operators in $N = 4$ SYM and ABJM theories, h can be related to the so-called Bremsstrahlung function B [139–143] which has been used in the application of conformal booostrap techniques to the study of defects [41,144–146]. For the single scalar and coset models studied in this chapter, we find that $-2 < h \leq 0$, where the upper bound is saturated only by the AdS_4 vacuum. However, such a bound does not seem to generally hold, since for the gauged STU model, h can become arbitrarily negative. Based on numerical searches, we conjecture that only the AdS_4 vacuum has vanishing h. Note that recently, the relation of h and B, as well as the negativity of h has been established on the SCFT side for various defect theories [147–150] and the arguments should carry over to the defects dual to the solutions studied in this chapter.[14]

The solutions we find are related to supergravity solutions [130, 134–136] which are holographic duals for a supersymmetric version of Rényi entropy first formulated in [133]. We note two differences. First, the solutions we find in Minkowski time signature have real gauge fields, unlike the duals cited above.[15] Second, we impose the condition that the periodicity of the circle in $\mathrm{AdS}_2 \times S^1$ boundary is such that after a conformal map, we obtain flat space without a conical singularity. On the other hand, in the holographic duals to the Super-Rényi entropy, the conical singularity is related to the Rényi index n. We note that in [130, 134–136], the holographic calculation of the Rényi entropy was compared to a localization calculation and agreement was found, and it would be interesting to see whether such a calculation can be performed for the holonomy defects described in this chapter.

Another interesting question is whether more general solutions going beyond the examples discussed in this chapter can be found. First, it would be interesting to study (numerical) solutions for more complicated superpotentials. Second, it would be interesting to see whether one can go beyond the gauged supergravity approximation and find solutions dual to holonomy defects in ten- or eleven-dimensional duals of three-dimensional $\mathcal{N} = 2$ SCFTs. Uplifting the solutions found in this chapter might prove to be a useful guide in this direction [128].

---

[14] We thank Marco Meineri and Lorenzo Bianchi for a useful correspondence regarding these matters.

[15] After analytic continuation to Euclidean signature, the gauge fields in both cases are real.

4.A Supersymmetry

We use the metric conventions $\eta = (-+++)$ and $\varepsilon_{0123} = -\varepsilon^{0123} = 1$. The gamma matrices are defined as usual, e.g.

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab} \ , \qquad \gamma_{ab} = \frac{1}{2}[\gamma_a, \gamma_b] \ , \qquad \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \ . \tag{4.69}$$

The two chiral gravitinos can be written in terms of a single complex (Dirac) spinor ψ_μ, and likewise for the gauginos λ^α. The supersymmetry transformations of the four-dimensional gauged supergravity are [124]

$$\begin{aligned}\delta\psi_\mu = \ & \bigg(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + \frac{i}{2}Q_\mu\gamma_5 + ig\xi_I A_\mu^I + ge^{\mathcal{K}/2}\gamma_\mu\xi_I\big(\operatorname{Im} Z^I + i\gamma_5 \operatorname{Re} Z^I\big) \\ & + \frac{i}{4}e^{\mathcal{K}/2}\gamma^{ab}(\operatorname{Im}\mathcal{N})_{IJ}\big(\operatorname{Im}(F_{ab}^{-I}Z^J) - i\gamma_5\operatorname{Re}(F_{ab}^{-I}Z^J)\big)\gamma_\mu\bigg)\varepsilon \ , \\ \delta\lambda^\alpha = \ & \bigg(\gamma^\mu\partial_\mu(\operatorname{Re} z^\alpha - i\gamma_5 \operatorname{Im} z^\alpha) + 2ge^{\mathcal{K}/2}\xi_I\Big(\operatorname{Im}(\mathcal{D}_{\bar\beta}\bar Z^I g^{\alpha\bar\beta}) - i\gamma_5\operatorname{Re}(\mathcal{D}_{\bar\beta}\bar Z^I g^{\alpha\bar\beta})\Big) \\ & + \frac{i}{2}e^{\mathcal{K}/2}\gamma^{ab}(\operatorname{Im}\mathcal{N})_{IJ}\Big(\operatorname{Im}(F_{ab}^{-I}\mathcal{D}_{\bar\beta}\bar Z^J g^{\alpha\bar\beta}) - i\gamma_5\operatorname{Re}(F_{ab}^{-I}\mathcal{D}_{\bar\beta}\bar Z^J g^{\alpha\bar\beta})\Big)\bigg)\varepsilon \ ,\end{aligned} \tag{4.70}$$

where ε is a complex spinor, and we have defined

$$F_{ab}^{\pm I} \equiv \frac{1}{2}(F_{ab}^I \pm \tilde F_{ab}^I) \ , \qquad \tilde F_{ab}^I \equiv -\frac{i}{2}\varepsilon_{abcd}F^{cd} \ . \tag{4.71}$$

The Kähler connection Q_μ is

$$Q_\mu = -\frac{i}{2}(\partial_\mu\tau^\alpha\partial_\alpha\mathcal{K} - \partial_\mu\bar\tau^{\bar\alpha}\partial_{\bar\alpha}\mathcal{K}) \ . \tag{4.72}$$

For the gauged STU model defect solution (4.22), we can work with the explicit coordinates $(x^0, x^1, x^2, x^3) = (t, \eta, \theta, r)$ and the metric,

$$\mathrm{d}s^2 = r^2\sqrt{H}\bigg(\frac{-\mathrm{d}t^2 + \mathrm{d}\eta^2}{\eta^2}\bigg) + \frac{f}{\sqrt{H}}\,\mathrm{d}\theta^2 + \frac{\sqrt{H}}{f}\,\mathrm{d}r^2 \ . \tag{4.73}$$

The non-vanishing spin connection 1-forms of the metric are

$$\begin{aligned}\omega^{01} &= -\frac{\mathrm{d}t}{\eta} \ , & \omega^{03} &= \frac{f^{1/2}}{H^{1/4}}\frac{\mathrm{d}}{\mathrm{d}r}(rH^{1/4})\frac{\mathrm{d}t}{\eta} \ , \\ \omega^{13} &= \frac{f^{1/2}}{H^{1/4}}\frac{\mathrm{d}}{\mathrm{d}r}(rH^{1/4})\frac{\mathrm{d}\eta}{\eta} \ , & \omega^{23} &= \frac{f^{1/2}}{H^{1/4}}\frac{\mathrm{d}}{\mathrm{d}r}\bigg(\frac{f^{1/2}}{H^{1/4}}\bigg)\mathrm{d}\theta \ .\end{aligned} \tag{4.74}$$

For the following calculations, we use the parametrization $(Z^0, Z^1, Z^2, Z^3) = (i, iz^2z^3, iz^1z^3, iz^1z^2)$. The BPS equations (4.70) simplify to

$$\begin{aligned} 0 = \delta\psi_\mu &= \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + ig\xi_I A_\mu^I + \sqrt{2}g\gamma_\mu \frac{\mathrm{d}}{\mathrm{d}r}(rH^{1/4}) - \frac{i}{2}\gamma_{23}\gamma_\mu \frac{\mathrm{d}}{\mathrm{d}r}(H^{-1/4})\right) , \\ 0 = \delta\lambda^\alpha &= \frac{\mathrm{d}z^\alpha}{\mathrm{d}r}\left(\frac{f^{1/2}}{H^{1/4}}\gamma_3 + 2\sqrt{2}grH^{1/4} + \frac{i}{H^{1/4}}\gamma_{23}\right)\varepsilon \ . \end{aligned} \tag{4.75}$$

The gaugino equation implies the projector,

$$0 = \left(1 + \frac{2\sqrt{2}gr\sqrt{H}}{\sqrt{f}}\gamma_3 - \frac{i}{\sqrt{f}}\gamma_2\right)\varepsilon \ . \tag{4.76}$$

The $\mu = t, \eta, \theta$ components of the gravitino equation then simplify to

$$\begin{aligned} 0 &= \left(\partial_t - \frac{1}{2\eta}\gamma_{01} - \frac{i}{2\eta}\gamma_{023}\right)\varepsilon \ , \\ 0 &= \left(\partial_\eta - \frac{i}{2\eta}\gamma_{123}\right)\varepsilon \ , \\ 0 &= \left(\partial_\theta + i\sqrt{2}g\left(-1 + \frac{1}{\sqrt{2}}\xi_I\mu^I\right)\right)\varepsilon \ . \end{aligned} \tag{4.77}$$

These can be integrated to

$$\varepsilon = \exp\left(-i\sqrt{2}g\theta\left(-1 + \frac{1}{\sqrt{2}}\xi_I\mu^I\right)\right)\exp\left(\frac{i}{2}\gamma_{123}\ln\eta\right)\exp\left(\frac{t}{2}(\gamma_{01} + i\gamma_{023})\right)\tilde{\varepsilon}(r) \ . \tag{4.78}$$

We can see that we need $\xi_I\mu^I \in 2\sqrt{2}\mathbb{Z}$ in order for ε to be anti-periodic under the identification $\theta \sim \theta + \pi/\sqrt{2}g$. The $\mu = r$ component of the gravitino equation simplifies to

$$\left(\partial_r + \frac{1}{8}\frac{H'}{H} + \frac{f'}{8\sqrt{2}gr\sqrt{H}\sqrt{f}}\gamma_3\right)\varepsilon \ . \tag{4.79}$$

The gaugino projector (4.76) and the radial equation (4.79) take the form of the equation solved in the appendix of [151], by identifying

$$\begin{aligned} x &\equiv \frac{2\sqrt{2}gr\sqrt{H}}{\sqrt{f}} \ , & y &\equiv \frac{-i}{\sqrt{f}} \ , \\ \Gamma_1 &\equiv \gamma_3 \ , & \Gamma_2 &\equiv \gamma_2 \ . \end{aligned} \tag{4.80}$$

The solution is

$$\tilde{\varepsilon}(r) = \frac{1}{H^{1/8}}\left(\sqrt{\sqrt{f} + 2\sqrt{2}gr\sqrt{H}} - \gamma_2\sqrt{\sqrt{f} - 2\sqrt{2}gr\sqrt{H}}\right)(1 - \gamma_3)\varepsilon_0 \ , \tag{4.81}$$

where ε_0 is a constant spinor.

4.B Vanishing of scalar one-point functions from supersymmetry

The scalar one-point function is given by

$$\langle \bar{\mathcal{O}}_{\bar{\alpha}} \rangle = \lim_{\varepsilon\to 0}\left[\frac{1}{\varepsilon^2}\left(z g_{\beta\bar{\alpha}}\partial_z \tau^\beta + \partial_{\bar{\alpha}}\mathcal{W}\right)\right] \ . \tag{4.82}$$

The derivative of the superpotential $\mathcal{W}$ simplifies to

$$\partial_{\bar{\alpha}}\mathcal{W} = \partial_{\bar{\alpha}}\left(-\sqrt{2}e^{\mathcal{K}/2}\xi_I|Z^I|\right) \tag{4.83}$$

$$= -\frac{1}{\sqrt{2}}e^{\mathcal{K}/2}\xi_I\left(\sqrt{\frac{Z^I}{\bar{Z}^I}}\partial_{\bar{\alpha}}\bar{Z}^I + (\partial_{\bar{\alpha}}\mathcal{K})|Z^I|\right) \ , \tag{4.84}$$

where $|Z^I|^2 = Z^I(\tau)\bar{Z}^I(\bar{\tau})$. For real scalars, we can choose a parameterization such that $\bar{Z}^I = Z^I$. This implies

$$\partial_{\bar{\alpha}}\mathcal{W} = -\frac{1}{\sqrt{2}}e^{\mathcal{K}/2}\xi_I\left(\partial_{\bar{\alpha}}\bar{Z}^I + (\partial_{\bar{\alpha}}\mathcal{K})\bar{Z}^I\right) = -\frac{1}{\sqrt{2}}e^{\mathcal{K}/2}\xi_I\mathcal{D}_{\bar{\alpha}}\bar{Z}^I \ , \tag{4.85}$$

so that

$$\langle \bar{\mathcal{O}}_{\bar{\alpha}} \rangle = \lim_{\varepsilon\to 0}\left[\frac{1}{\varepsilon^2}\left(z g_{\beta\bar{\alpha}}\partial_z\tau^\beta - \frac{1}{\sqrt{2}}e^{\mathcal{K}/2}\xi_I\mathcal{D}_{\bar{\alpha}}\bar{Z}^I\right)\Bigg|_{z=\varepsilon}\right] \ . \tag{4.86}$$

The gaugino BPS variation in FG coordinates is

$$\left(z\gamma_3\partial_z\tau^\beta - 2ige^{\mathcal{K}/2}\xi_I g^{\beta\bar{\alpha}}\mathcal{D}_{\bar{\alpha}}\bar{Z}^I\gamma_5\right)\varepsilon + \mathcal{O}(z^3)\varepsilon = 0 \ , \tag{4.87}$$

since $F_{ab} \sim 1/r^2 \sim \mathcal{O}(z^2)$. At $\mathcal{O}(z^2)$, the BPS equations imply

$$z\partial_z\tau^\beta = \pm 2ige^{\mathcal{K}/2}\xi_I g^{\beta\bar{\alpha}}\mathcal{D}_{\bar{\alpha}}\bar{Z}^I \ . \tag{4.88}$$

Without loss of generality, we can choose the upper sign by sending $g \to -g$ if necessary. After setting $g^2 = 1/8$ we have

$$\langle \bar{\mathcal{O}}_{\bar{\alpha}} \rangle = \langle \mathcal{O}_\alpha \rangle = 0 \ . \tag{4.89}$$

4.C STU model special cases

Here we give a construction for STU models with regular geometry and arbitrarily large $x_0 + x_1 + x_2 + x_3$. The approach we took to find these models was different than that of section 4.4.2. Instead of solving the condition $f = 0$ and then $H' = 0$, we first solved $H' = 0$ and then $f = 0$. The benefit is that H' is a lower-degree polynomial and is technically simpler to solve. The downside is that this generates spurious solutions: it is possible that r_+ does not satisfy the equation $H' = 0$, so the r we obtain from this analysis do not contain the largest root r_+. These spurious solutions then need to be removed by hand.

To summarize our findings, consider the following construction:

1. Let x_0 be any positive number.

2. Numerically solve the equation,

$$27x_1(x_0 - x_1)^4 = -16x_0(x_0 + 3x_1)^2 \ . \tag{4.90}$$

   Let x_1 be the unique solution satisfying $-x_0/3 < x_1 < 0$.

3. Consider an STU model with unit AdS_4 length scale where

$$x_0 = \frac{q_0}{\xi_0} \ , \qquad x_1 = \frac{q_1}{\xi_1} = \frac{q_2}{\xi_2} = \frac{q_3}{\xi_3} \ . \tag{4.91}$$

   Numerically solve the equation $f(r) = 0$ for r,

$$(r + x_0)(r + x_1)^3 = r^2 \ . \tag{4.92}$$

   There exist exactly two solutions: a positive solution greater than $-x_1$, and a negative solution less than $-x_0$. Let r_+ be the positive solution.

4. Check that $H'(r_+) = 0$. This is guaranteed by the following argument. Consider $r^* = -4x_0x_1/(x_0 + 3x_1) > 0$ which satisfies $H'(r^*) = 0$. This also satisfies $f(r^*) = 0$, as plugging $r = r^*$ into (4.92) simplifies to (4.90), which is satisfied by construction of x_1. But as the positive solution to $f = 0$ is unique, we must have $r_+ = r^*$.

The steps above give a STU model with regular geometry. To prove that x_0+3x_1 is arbitrarily large, we need a better bound than $-x_0/3 < x_1 < 0$. To satisfy (4.90) for large x_0, we have

$$x_1 \sim -\frac{16}{27x_0}. \tag{4.93}$$

Therefore $x_0 + x_1 + x_2 + x_3 \approx x_0$ for large x_0, and can be arbitrarily large.

CHAPTER 5

Janus solutions in 3d $\mathcal{N} = 8$ gauged supergravity

In this chapter, we construct Janus solutions in three-dimensional $\mathcal{N} = 8$ gauged supergravity. Such supergravity theories are naturally related to $\text{AdS}_3 \times S^3 \times M_4$ compactifications of type IIB, where M_4 is either T_4 or $K3$. We consider one of the simplest non-trivial settings where we find half-BPS solutions that preserve eight of the sixteen supersymmetries of the AdS_3 vacuum and only two scalars in the coset have a non-trivial profile. One interesting feature of these solutions is that one scalar is dual to a $\Delta = 2$ marginal operator with a source term that has a different value on the two sides of the interface. This behavior is the main feature of the original Janus solution [50]. On the other hand, the second scalar is dual to a $\Delta = 1$ relevant operator with a vanishing source term and a position-dependent expectation value. This behavior is a feature of the Janus solution in M-theory [59]. The structure of this chapter is as follows. In section 5.1, we review $\mathcal{N} = 8$ gauged supergravity in three dimensions. In section 5.2, we construct the half-BPS Janus solutions and investigate some of their properties using the AdS/CFT dictionary, including the calculation of the holographic entanglement entropy. We discuss some generalizations and directions for future research in section 5.3. Some technical details are relegated to appendix 5.A.

5.1 Three-dimensional $\mathcal{N} = 8$ gauged supergravity

In this section, we review the $\mathcal{N} = 8$ gauged supergravity first constructed in [152]. The theory is characterized by the number n of vector multiplets. The bosonic field content consists of a graviton $g_{\mu\nu}$, Chern-Simons gauge fields $B_\mu^{\mathcal{M}}$, and scalars fields living in a

$G/H = \mathrm{SO}(8,n)/\,\mathrm{SO}(8) \times \mathrm{SO}(n)$ coset, which has $8n$ degrees of freedom before gauging. This theory can be obtained by a truncation of six-dimensional $\mathcal{N} = (2,0)$ supergravity on $\mathrm{AdS}_3 \times S^3$ coupled to $n_T \geq 1$ tensor multiplets, where $n_T = n - 3$. The cases $n_T = 5$ and 21 correspond to compactifications of ten-dimensional type IIB on T^3 and $K3$, respectively. See [153] for a discussion of consistent truncations of six-dimensional $\mathcal{N} = (1,1)$ and $\mathcal{N} = (2,0)$ using exceptional field theory.

For future reference, we use the following index conventions:

- $I, J, \ldots = 1, 2, \ldots, 8$ for SO(8),
- $r, s, \ldots = 9, 10, \ldots, n+8$ for SO(n),
- $\bar{I}, \bar{J}, \ldots = 1, 2, \ldots, n+8$ for SO($8, n$), and
- $\mathcal{M}, \mathcal{N}, \ldots$ for generators of SO($8, n$).

Let the generators of G be $\{t^{\mathcal{M}}\} = \{t^{\bar{I}\bar{J}}\} = \{X^{IJ}, X^{rs}, Y^{Ir}\}$, where Y^{Ir} are the non-compact generators. Explicitly, the generators of the vector representation are given by

$$(t^{\bar{I}\bar{J}})^{\bar{K}}{}_{\bar{L}} = \eta^{\bar{I}\bar{K}}\delta^{\bar{J}}_{\bar{L}} - \eta^{\bar{J}\bar{K}}\delta^{\bar{I}}_{\bar{L}} \;, \tag{5.1}$$

where $\eta^{\bar{I}\bar{J}} = \mathrm{diag}(++++++++-\cdots)$ is the SO($8, n$)-invariant tensor. These generators satisfy the usual commutation relations,

$$[t^{\bar{I}\bar{J}}, t^{\bar{K}\bar{L}}] = 2\Big(\eta^{\bar{I}[\bar{K}}t^{\bar{L}]\bar{J}} - \eta^{\bar{J}[\bar{K}}t^{\bar{L}]\bar{I}}\Big) \;, \tag{5.2}$$

The scalars fields can be parametrized by a G-valued matrix $L(x)$ in the vector representation, which transforms under H and the gauge group $G_0 \subseteq G$ by

$$L(x) \to g_0(x)L(x)h^{-1}(x) \;, \tag{5.3}$$

for $g_0 \in G_0$ and $h \in H$. The Lagrangian is invariant under such transformations. We can pick a SO(8) $\times$ SO(n) gauge to put the coset representative into symmetric gauge,

$$L = \exp\big(\phi_{Ir}Y^{Ir}\big) \;, \tag{5.4}$$

for scalar fields ϕ_{Ir}.

The gauging of the supergravity is accomplished by an embedding tensor $\Theta_{\mathcal{MN}}$ (which has to satisfy various identities [154]) that determines which isometries are gauged, the coupling to the Chern-Simons fields, and additional terms in the supersymmetry variations and action depending on the gauge coupling. In the following, we will make one of the simplest choices and gauge a $G_0 = \mathrm{SO}(4)$ subset of SO(8). Explicitly, we further divide the I, J indices into

- $i, j, \ldots = 1, 2, 3, 4$ for $G_0 = \mathrm{SO}(4)$, and
- $\bar{\imath}, \bar{\jmath}, \ldots = 5, 6, 7, 8$ for the remaining ungauged $\mathrm{SO}(4) \subset \mathrm{SO}(8)$.

The embedding tensor we will employ in the following has the non-zero entries,

$$\Theta_{ij,k\ell} = \varepsilon_{ijk\ell} \ . \tag{5.5}$$

As discussed in [152], this choice of embedding tensor produces a supersymmetric AdS_3 ground state with a $\mathrm{SU}(2|1,1)_L \times \mathrm{SU}(2|1,1)_R$ superalgebra of isometries.

From the embedding tensor, the G_0-covariant currents can be obtained,

$$L^{-1}(\partial_\mu + g\Theta_{\mathcal{MN}} B_\mu^{\mathcal{M}} t^{\mathcal{N}})L = \frac{1}{2}\mathcal{Q}_\mu^{IJ} X^{IJ} + \frac{1}{2}\mathcal{Q}_\mu^{rs} X^{rs} + \mathcal{P}_\mu^{Ir} Y^{Ir} \ . \tag{5.6}$$

It is convenient to define the ${\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}}$ tensors,

$$L^{-1} t^{\mathcal{M}} L = {\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}} t^{\mathcal{A}} = \frac{1}{2}{\mathcal{V}^{\mathcal{M}}}_{IJ} X^{IJ} + \frac{1}{2}{\mathcal{V}^{\mathcal{M}}}_{rs} X^{rs} + {\mathcal{V}^{\mathcal{M}}}_{Ir} Y^{Ir} \ , \tag{5.7}$$

and the T-tensor,

$$T_{\mathcal{A}|\mathcal{B}} = \Theta_{\mathcal{MN}} {\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}} {\mathcal{V}^{\mathcal{N}}}_{\mathcal{B}} \ . \tag{5.8}$$

The T-tensor is used to construct the tensors $A_{1,2,3}$ which will appear in the scalar potential and the supersymmetry variations,

$$\begin{aligned}
A_1^{AB} &= -\frac{1}{48}\Gamma^{IJKL}_{AB} T_{IJ|KL} \ , \\
A_2^{A\dot{A}r} &= -\frac{1}{12}\Gamma^{IJK}_{A\dot{A}} T_{IJ|Kr} \ , \\
A_3^{\dot{A}r\dot{B}s} &= \frac{1}{48}\delta^{rs}\Gamma^{IJKL}_{\dot{A}\dot{B}} T_{IJ|KL} + \frac{1}{2}\Gamma^{IJ}_{\dot{A}\dot{B}} T_{IJ|rs} \ ,
\end{aligned} \tag{5.9}$$

where A, B and $\dot{A}, \dot{B}$ are SO(8)-spinor indices. Our conventions for the SO(8) Gamma matrices are presented in appendix 5.A.1.

We take the spacetime signature $\eta^{ab} = \mathrm{diag}(+--)$ to be mostly negative. The bosonic Lagrangian is

$$
\begin{aligned}
e^{-1}\mathcal{L} &= -\frac{1}{4}R + \frac{1}{4}\mathcal{P}_\mu^{Ir}\mathcal{P}^{\mu\, Ir} + W - \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho}g\Theta_{\mathcal{M}\mathcal{N}}B_\mu^{\mathcal{M}}\left(\partial_\nu B_\rho^{\mathcal{N}} + \frac{1}{3}g\Theta_{\mathcal{K}\mathcal{L}}f^{\mathcal{N}\mathcal{K}}{}_{\mathcal{P}}B_\nu^{\mathcal{L}}B_\rho^{\mathcal{P}}\right) , \\
W &= \frac{1}{4}g^2\left(A_1^{AB}A_1^{AB} - \frac{1}{2}A_2^{A\dot{A}r}A_2^{A\dot{A}r}\right) . \qquad (5.10)
\end{aligned}
$$

The supersymmetry variations are

$$
\begin{aligned}
\delta\chi^{\dot{A}r} &= \frac{1}{2}i\Gamma^I_{A\dot{A}}\gamma^\mu\varepsilon^A\mathcal{P}_\mu^{Ir} + gA_2^{A\dot{A}r}\varepsilon^A , \\
\delta\psi_\mu^A &= \left(\partial_\mu\varepsilon^A + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\varepsilon^A + \frac{1}{4}\mathcal{Q}_\mu^{IJ}\Gamma^{IJ}_{AB}\varepsilon^B\right) + igA_1^{AB}\gamma_\mu\varepsilon^B . \qquad (5.11)
\end{aligned}
$$

5.1.1 The $n = 1$ case

In this section we will consider the $n = 1$ theory, i.e. the scalar fields lie in a SO(8, 1)/ SO(8) coset. The reason for this is that the resulting expressions for the supersymmetry variations and BPS conditions are compact and everything can be worked out in detail. Furthermore, we believe that this case illustrates the important features of more general solutions.

As the index $r = 9$ takes only one value in this case, the scalar fields in the coset representative (5.4) are denoted by $\phi_I \equiv \phi_{I9}$ for $I = 1, 2, \ldots, 8$. We define the following quantities for notational convenience,

$$
\begin{aligned}
\Phi^2 &\equiv \phi_I\phi_I = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 + \phi_5^2 + \phi_6^2 + \phi_7^2 + \phi_8^2 , \\
\phi^2 &\equiv \phi_i\phi_i = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 , \\
\bar{\phi}^2 &\equiv \phi_{\bar{\imath}}\phi_{\bar{\imath}} = \phi_5^2 + \phi_6^2 + \phi_7^2 + \phi_8^2 . \qquad (5.12)
\end{aligned}
$$

The components of the $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ tensor are, with no summation over repeated indices and

I, J, K, L being unique indices,

$$
\begin{aligned}
&\mathcal{V}^{IJ}{}_{IJ} = 1 + (\phi_I^2 + \phi_J^2)\frac{\cosh\Phi - 1}{\Phi^2} \ , &&\mathcal{V}^{IJ}{}_{IK} = \phi_J\phi_K\frac{\cosh\Phi - 1}{\Phi^2} \ , \\
&\mathcal{V}^{IJ}{}_{KL} = 0 \ , &&\mathcal{V}^{I9}{}_{I9} = \cosh\Phi - \phi_I^2\frac{\cosh\Phi - 1}{\Phi^2} \ , \\
&\mathcal{V}^{I9}{}_{J9} = -\phi_I\phi_J\frac{\cosh\Phi - 1}{\Phi^2} \ , &&\mathcal{V}^{IJ}{}_{I9} = \mathcal{V}^{I9}{}_{IJ} = \phi_J\frac{\sinh\Phi}{\Phi} \ , \\
&\mathcal{V}^{IJ}{}_{K9} = \mathcal{V}^{K9}{}_{IJ} = 0 \ .
\end{aligned}
\tag{5.13}
$$

The u-components of the $\mathcal{Q}_\mu^{IJ}$ and $\mathcal{P}_\mu^I$ currents are

$$
\begin{aligned}
\mathcal{Q}_u^{IJ} &= (\phi_I'\phi_J - \phi_I\phi_J')\frac{\cosh\Phi - 1}{\Phi^2} + g\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}_{IJ}^{\mathcal{N}} \ , \\
\mathcal{P}_u^I &= \phi_I'\frac{\sinh\Phi}{\Phi} - \phi_I\Phi'\frac{\sinh\Phi - \Phi}{\Phi^2} + g\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}_{I9}^{\mathcal{N}} \ ,
\end{aligned}
\tag{5.14}
$$

where the prime $' \equiv \partial/\partial u$ denotes the derivative with respect to u. The terms involving the gauge field have different forms depending on whether I, J are in i or $\bar{\imath}$,

$$
\begin{aligned}
\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}_{ij}^{\mathcal{N}} &= \varepsilon_{ijk\ell}\left[\frac{1}{2}B_u^{k\ell}\left(1 + (\phi_i^2 + \phi_j^2)\frac{\cosh\Phi - 1}{\Phi^2}\right) + \left(\phi_i B_u^{ik}\phi_\ell + \phi_j B_u^{jk}\phi_\ell\right)\frac{\cosh\Phi - 1}{\Phi^2}\right] \ , \\
\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}_{i\bar{\imath}}^{\mathcal{N}} &= \frac{1}{2}\varepsilon_{ijk\ell}\phi_{\bar{\imath}}\phi_j B_u^{k\ell}\frac{\cosh\Phi - 1}{\Phi^2} \ , \\
\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}_{\bar{\imath}\bar{\jmath}}^{\mathcal{N}} &= 0 \ , \\
\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}_{i9}^{\mathcal{N}} &= \frac{1}{2}\varepsilon_{ijk\ell}\phi_j B_u^{k\ell}\frac{\sinh\Phi}{\Phi} \ , \\
\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}_{\bar{\imath}9}^{\mathcal{N}} &= 0 \ .
\end{aligned}
\tag{5.15}
$$

The T-tensor has non-zero components,

$$
\begin{aligned}
T_{ij|k\ell} &= \varepsilon_{ijk\ell}\left(\phi^2\frac{\cosh\Phi - 1}{\Phi^2} + 1\right) \ , \\
T_{ij|k\bar{\imath}} &= \varepsilon_{ijk\ell}\phi_\ell\phi_{\bar{\imath}}\frac{\cosh\Phi - 1}{\Phi^2} \ , \\
T_{ij|k9} &= \varepsilon_{ijk\ell}\phi_\ell\frac{\sinh\Phi}{\Phi} \ .
\end{aligned}
\tag{5.16}
$$

Taking $\varepsilon_{1234} = 1$, we can use the T-tensor to compute

$$
\begin{aligned}
A_1^{AB} &= -\frac{1}{2}\Gamma^{1234}_{AC}\left[\left(\phi^2\frac{\cosh\Phi - 1}{\Phi^2} + 1\right)\delta_{CB} + (\Gamma^i_{C\dot{A}}\phi_i)(\Gamma^{\bar{\imath}}_{\dot{A}B}\phi_{\bar{\imath}})\frac{\cosh\Phi - 1}{\Phi^2}\right] , \\
A_2^{A\dot{A}} &= -\frac{1}{2}\Gamma^{1234}_{AB}(\Gamma^i_{B\dot{A}}\phi_i)\frac{\sinh\Phi}{\Phi} , \\
A_3^{\dot{A}\dot{B}} &= -A_1^{AB}\delta_{A\dot{A}}\delta_{B\dot{B}} .
\end{aligned}
\tag{5.17}
$$

Note that $A_1^{AB} = A_1^{BA}$ and

$$
\begin{aligned}
A_1^{AC}A_1^{BC} &= \frac{1}{4}\delta_{AB}\left(\frac{\phi^2\sinh^2\Phi}{\Phi^2} + 1\right) , \\
A_2^{A\dot{A}}A_2^{B\dot{A}} &= \frac{1}{4}\delta_{AB}\frac{\phi^2\sinh^2\Phi}{\Phi^2} ,
\end{aligned}
\tag{5.18}
$$

so the scalar potential (5.10) becomes

$$
W = \frac{g^2}{4}\left(\frac{\phi^2\sinh^2\Phi}{\Phi^2} + 2\right) . \tag{5.19}
$$

5.2 Janus solutions

In this section, we construct Janus solutions which preserve eight of the sixteen supersymmetries of the AdS_3 vacuum. Our strategy is to use an AdS_2 slicing of AdS_3 and make the scalar fields as well as the metric functions only dependent on the slicing coordinate. One complication is given by the presence of the gauge fields; due to the Chern-Simons action, the only consistent Janus solution will have vanishing field strength. We show that the gauge fields can be consistently set to zero for our solutions.

5.2.1 Janus ansatz

We take the Janus ansatz for the metric, scalar fields and Chern-Simons gauge fields,

$$
\begin{aligned}
\mathrm{d}s^2 &= e^{2B(u)}\left(\frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2}\right) - e^{2D(u)}\,\mathrm{d}u^2 \ , \\
\phi_I &= \phi_I(u) \ , \\
B^{\mathcal{M}} &= B^{\mathcal{M}}(u)\,\mathrm{d}u \ .
\end{aligned} \tag{5.20}
$$

The AdS$_3$ vacuum solution given by $\phi_I \equiv 0$ and $e^B = e^D = L\sec u$ has a curvature radius related to the coupling constant by $L^{-1} = g$. The spin connection 1-forms are

$$
\omega^{01} = \frac{\mathrm{d}t}{z} \ , \qquad \omega^{02} = -\frac{B' e^{B-D}}{z}\,\mathrm{d}t \ , \qquad \omega^{12} = -\frac{B' e^{B-D}}{z}\,\mathrm{d}z \ , \tag{5.21}
$$

so the gravitino supersymmetry variation $\delta\psi^A_\mu = 0$ is

$$
\begin{aligned}
0 &= \partial_t\varepsilon + \frac{1}{2z}\gamma_0\big(\gamma_1 - B' e^{B-D}\gamma_2 + 2ige^B A_1\big)\varepsilon \ , \\
0 &= \partial_z\varepsilon + \frac{1}{2z}\gamma_1\big(-B' e^{B-D}\gamma_2 + 2ige^B A_1\big)\varepsilon \ , \\
0 &= \partial_u\varepsilon + \frac{1}{4}\mathcal{Q}^{IJ}_u\Gamma^{IJ}\varepsilon + ige^D\gamma_2 A_1\varepsilon \ ,
\end{aligned} \tag{5.22}
$$

where we have suppressed the SO(8)-spinor indices. As shown in appendix 5.A.2, the integrability conditions are

$$
\begin{aligned}
0 &= \big(1 - (2ge^B A_1)^2 + (B' e^{B-D})^2\big)\varepsilon \ , \\
0 &= 2ige^B\left(A_1' - \frac{1}{4}[A_1, \mathcal{Q}^{IJ}_u\Gamma^{IJ}]\right)\varepsilon + \left(-\frac{\mathrm{d}}{\mathrm{d}u}(B' e^{B-D}) + (2ge^B A_1)^2 e^{D-B}\right)\gamma_2\varepsilon \ .
\end{aligned} \tag{5.23}
$$

The first integrability condition gives a first-order equation which must be true for all ε, using the replacement for A_1^2 in (5.18),

$$
0 = 1 - g^2 e^{2B}\left(\frac{\phi^2\sinh^2\Phi}{\Phi^2} + 1\right) + (B' e^{B-D})^2 \ . \tag{5.24}
$$

The derivative of this simplifies the second integrability condition to

$$
0 = \left(A_1' - \frac{1}{4}[A_1, \mathcal{Q}^{IJ}_u\Gamma^{IJ}]\right)\varepsilon + \frac{ige^D}{4B'}\frac{\mathrm{d}}{\mathrm{d}u}\left(\frac{\phi^2\sinh^2\Phi}{\Phi^2}\right)\gamma_2\varepsilon \ . \tag{5.25}
$$

The BPS equation $\delta\chi^{\dot{A}} = 0$ is

$$\left(-\frac{i}{2}e^{-D}\Gamma^I\mathcal{P}_u^I\gamma_2 + gA_2\right)_{A\dot{A}}\varepsilon^A = 0 \ . \tag{5.26}$$

When $A_2 \neq 0$, this equation can be rearranged into the form of a projector,

$$0 = (iM_{AB}\gamma_2 + \delta_{AB})\varepsilon^A \ , \tag{5.27}$$

where M_{AB} is given by

$$M_{AB} = \frac{e^{-D}}{g}\frac{\Phi}{\phi^2\sinh\Phi}(\Gamma^I_{A\dot{A}}\mathcal{P}_u^I)(\Gamma^i_{\dot{A}C}\phi_i)\Gamma^{1234}_{CB} \ . \tag{5.28}$$

For consistency of the projector, we must have

$$M_{AB}M_{BC} = \delta_{AC} \ . \tag{5.29}$$

As $M^2 = 1$, every generalized eigenvector of rank ≥ 2 is automatically an eigenvector, so M is diagonalizable and has eight eigenvectors with eigenvalues ± 1. M is traceless as it is a sum of products of 2 or 4 Gamma matrices, so it has an equal number of $+1$ and -1 eigenvectors. The operator $iM_{AB}\gamma_2$ in the projector (5.27) squares to one and is traceless, and projects onto an eight-dimensional space of unbroken supersymmetry generators. If this is the only projection imposed on the solution, it will be half-BPS and hence preserve eight of the sixteen supersymmetries of the vacuum.

The condition $M^2 = 1$ gives an equation first-order in derivatives of scalars.

$$M^2 = \left(\frac{e^{-D}\Phi}{g\phi^2\sinh\Phi}\right)^2\Big(\phi^2(-\mathcal{P}_u^i\mathcal{P}_u^i + \mathcal{P}_u^{\bar{\imath}}\mathcal{P}_u^{\bar{\imath}}) - 2\phi^2(\Gamma^{\bar{\imath}}\mathcal{P}_u^{\bar{\imath}})(\Gamma^i\mathcal{P}_u^i) + 2(\mathcal{P}_u^j\phi_j)(\Gamma^{\bar{\imath}}\mathcal{P}_u^{\bar{\imath}} + \Gamma^i\mathcal{P}_u^i)(\Gamma^k\phi_k)\Big) \ . \tag{5.30}$$

For this to be proportional to the identity, we need all $\Gamma^{\bar{\imath}}\Gamma^i$ and $\Gamma^i\Gamma^j$ terms to vanish. Vanishing of the latter requires us to impose the condition,

$$\mathcal{P}_u^i\phi_j = \mathcal{P}_u^j\phi_i \ . \tag{5.31}$$

As the ratio $\mathcal{P}^i_u/\phi_i$ is the same for all i, this implies

$$\sum_i \mathcal{P}^i_u\phi_i = \sum_i \frac{\mathcal{P}^i_u}{\phi_i}\phi_i^2 = \frac{\mathcal{P}^1_u}{\phi_1}\phi^2 \qquad \Longrightarrow \qquad -\phi^2\mathcal{P}^i_u + \phi_i\sum_j \mathcal{P}^j_u\phi_j = 0 \ . \tag{5.32}$$

This means that imposing (5.31) also ensures that the $\Gamma^{\bar{\imath}}\Gamma^i$ terms vanish. Note that

$$\sum_i \mathcal{P}^i_u\mathcal{P}^i_u = \sum_i \frac{\mathcal{P}^i_u}{\phi_i}\frac{\mathcal{P}^i_u}{\phi_i}\phi_i^2 = \left(\frac{\mathcal{P}^1_u}{\phi_1}\right)^2\phi^2 \ , \tag{5.33}$$

so the $M^2 = 1$ condition becomes

$$M^2 = \left(\frac{e^{-D}\Phi}{g\phi^2\sinh\Phi}\right)^2\phi^2(\mathcal{P}^i_u\mathcal{P}^i_u + \mathcal{P}^{\bar{\imath}}_u\mathcal{P}^{\bar{\imath}}_u) = 1 \ . \tag{5.34}$$

We now give the argument why the Chern-Simons gauge fields can be set to zero. Since we demand that the $B^{\mathcal{M}}_\mu$ only has a component along the u direction and only depends on u, the field strength vanishes, consistent with the equation of motion coming from the variation of the Chern-Simons term in the action (5.10) with respect to the gauge field. However, there is another term which contains the gauge field, namely the kinetic term of the scalars via (5.14). For the gauge field to be consistently set to zero, we have to impose

$$\left.\frac{\delta\mathcal{L}}{\delta B^{k\ell}_u}\right|_{B^{\mathcal{M}}_u=0} = 0 \ . \tag{5.35}$$

For the Janus ansatz, we find

$$\left.\frac{\delta\mathcal{L}}{\delta B^{k\ell}_u}\right|_{B^{\mathcal{M}}_u=0} = eg\varepsilon_{ijk\ell}\mathcal{P}^{i\,u}\phi_j\frac{\sinh\Phi}{\Phi} \ , \tag{5.36}$$

which indeed vanishes due to (5.31) imposed by the half-BPS condition.

For a half-BPS solution, the second integrability condition (5.25) should be identical to the projector (5.27). Indeed, we have the simplification,

$$A_1' - \frac{1}{4}[A_1, \mathcal{Q}^{IJ}_u\Gamma^{IJ}] = -\frac{1}{2}\frac{\phi^2\sinh^2\Phi}{\Phi^2}M^\top \ , \tag{5.37}$$

so the Gamma matrix structures of the two equations match. Equating the remaining scalar magnitude gives us an equation for the metric factor e^B,

$$-B' = \frac{\mathrm{d}}{\mathrm{d}u}\ln\frac{\phi\sinh\Phi}{\Phi} \ . \tag{5.38}$$

We can now solve for the metric. Let us define

$$\alpha(u) \equiv \frac{\phi \sinh \Phi}{\Phi} \ , \tag{5.39}$$

and set the integration constant for B to be

$$e^B = \frac{|C|}{g\alpha} \ . \tag{5.40}$$

Plugging this into the first integrability condition (5.24) and picking the gauge $e^{-D} \equiv g$, we have a first-order equation for α,

$$0 = \alpha^2 - C^2(\alpha^2 + 1 - \alpha'^2/\alpha^2) \ . \tag{5.41}$$

The solution depends on the value of $C \in [0,1]$ and up to translations in u is

$$\begin{aligned} \alpha &= e^{\pm u} \ , && \text{if } C = 1 \ , \\ \alpha &= \frac{|C|}{\sqrt{1-C^2}} \operatorname{sech} u \ , && \text{if } 0 \leq C < 1 \ . \end{aligned} \tag{5.42}$$

We will take the case $0 \leq C < 1$. This implies that the metric is

$$\mathrm{d}s^2 = g^{-2}\left[(1-C^2)\cosh^2 u \left(\frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2}\right) - \mathrm{d}u^2\right] \ . \tag{5.43}$$

The choice $C = 0$ corresponds to the AdS_3 vacuum.

5.2.2 ϕ_4, ϕ_5 truncation

We have yet to fully solve the half-BPS conditions (5.31) and (5.34). For simplicity, let us consider the case where only ϕ_4, ϕ_5 are non-zero and the other scalars are identically zero, which trivially satisfies (5.31). It turns out that the important features of the Janus solution are captured by this truncation.

We introduce the following abbreviations,

$$\Phi^2 = \phi_4^2 + \phi_5^2 \ , \qquad \phi = |\phi_4| \ , \qquad \bar{\phi} = |\phi_5| \ . \tag{5.44}$$

Let us define

$$\beta(u) \equiv \frac{\phi_5 \sinh \Phi}{\Phi} \ , \tag{5.45}$$

so that

$$\begin{aligned} \alpha^2 + \beta^2 &= \sinh^2 \Phi \ , \\ \mathcal{P}_u^4 &= \alpha' + \alpha \Phi' \frac{1 - \cosh \Phi}{\sinh \Phi} \ , \\ \mathcal{P}_u^5 &= \beta' + \beta \Phi' \frac{1 - \cosh \Phi}{\sinh \Phi} \ . \end{aligned} \tag{5.46}$$

Plugging these into (5.34) simplifies to

$$\alpha'^2 + \beta'^2 - \frac{(\alpha'\alpha + \beta'\beta)^2}{1 + \alpha^2 + \beta^2} = \alpha^2 \ . \tag{5.47}$$

This can be rearranged into a first-order equation in $f \equiv \beta/\sqrt{1+\alpha^2}$,

$$f' = \frac{\alpha^2/C}{1+\alpha^2}\sqrt{1+f^2} \ , \tag{5.48}$$

where a sign ambiguity from taking a square-root has been absorbed into C, which is now extended to $C \in (-1, 1)$. Using the explicit solution (5.42) for α, by noting that

$$\frac{\mathrm{d}}{\mathrm{d}u} \tanh^{-1}(C \tanh u) = \frac{C \operatorname{sech}^2 u}{1 - C^2 \tanh^2 u} = \frac{\alpha^2/C}{1+\alpha^2} \ , \tag{5.49}$$

the general solution is

$$\begin{aligned} f(u) &= \frac{\sinh p + C \cosh p \tanh u}{\sqrt{1 - C^2 \tanh^2 u}} \ , \\ \beta(u) &= \frac{1}{\sqrt{1-C^2}}(\sinh p + C \cosh p \tanh u) \ , \end{aligned} \tag{5.50}$$

for some constant $p \in \mathbb{R}$. For later convenience, we also redefine $C = \tanh q$ for $q \in \mathbb{R}$.

In summary, we have solved for the scalars ϕ_4, ϕ_5 implicitly through the functions α, β,

$$\begin{aligned} \frac{|\phi_4| \sinh \Phi}{\Phi} &= |\sinh q| \operatorname{sech} u \ , \\ \frac{\phi_5 \sinh \Phi}{\Phi} &= \sinh p \cosh q + \cosh p \sinh q \tanh u \ , \end{aligned} \tag{5.51}$$

for real constants p, q. Note that the reflection $\phi_4 \to -\phi_4$ also gives a valid solution. We have explicitly checked that the Einstein equation and scalar equations of motion are satisfied.

The ϕ_4 scalar goes to zero at $u = \pm\infty$ as it is a massive scalar degree of freedom, and has a sech-like profile near the defect. The ϕ_5 scalar interpolates between two boundary values at $u = \pm\infty$, and has a tanh-like profile. The constant p is related to the boundary values of the ϕ_5 scalar, as we can note that

$$\phi_5(\pm\infty) = p \pm q \ . \tag{5.52}$$

The constant q is then related to the jump value of the ϕ_5 scalar. The defect location $u = 0$ can also be freely translated to any point along the axis. Figure 5.1 below gives a plot of the solution for the choice $(p, q) = (0, 1)$.

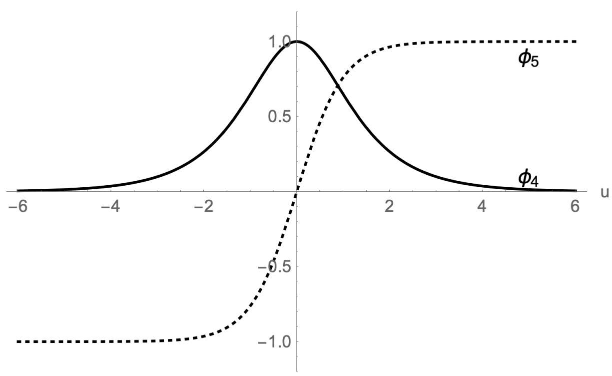


Figure 5.1: Plot of ϕ_4 and ϕ_5 for $(p, q) = (0, 1)$.

5.2.3 Holography

In our AdS-sliced coordinates, the boundary is given by the two AdS_2 components at $u = \pm\infty$, which are joined together at the $z = 0$ interface. Using $C = \tanh q$, the metric (5.43) becomes

$$\mathrm{d}s^2 = g^{-2}\left[\operatorname{sech}^2 q \cosh^2 u \left(\frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2}\right) - \mathrm{d}u^2\right] . \tag{5.53}$$

Note that this is not AdS_3 unless $q = 0$, which corresponds to the vacuum solution with all scalars vanishing. The spacetime is, however, asymptotically AdS_3. In the limit of $u \to \pm\infty$, the $\operatorname{sech}^2 q$ can be eliminated from the leading $e^{\pm 2u}$ term in the metric (5.53) by a coordinate shift. In the following, we will set the AdS length scale to unity for notational simplicity, i.e. $g \equiv 1$.

According to the AdS/CFT correspondence, the mass m^2 of a supergravity scalar field is related to the scaling dimension Δ of the dual $d = 2$ CFT operator by

$$m^2 = \Delta(\Delta - 2) \ . \tag{5.54}$$

This relation comes from the linearized equations of motion for the scalar field near the asymptotic AdS_3 boundary. Expanding the supergravity action (5.10) to quadratic order around the AdS_3 vacuum shows that the ϕ_4 field has mass $m^2 = -1$, so the dual operator is relevant with $\Delta = 1$ and saturates the Breitenlohner-Freedman (BF) bound [18,19]. Note that we choose the standard quantization [20], which is the correct one for a supersymmetric solution. The ϕ_5 field is massless, so the dual CFT operator is marginal with scaling dimension $\Delta = 2$.

The coordinates (z, u) can be mapped to Fefferman-Graham (FG) coordinates (ρ, x) where the asymptotic AdS_3 boundary is located at $\rho = 0$.[16] In FG coordinates, the general expansion for a scalar field near the boundary is

$$\begin{aligned} \phi_{\Delta=1} &\sim \psi_0\, \rho \ln \rho + \phi_0\, \rho + \cdots \ , \\ \phi_{\Delta\neq 1} &\sim \tilde{\phi}_0\, \rho^{2-\Delta} + \tilde{\phi}_2\, \rho^{\Delta} + \cdots \ . \end{aligned} \tag{5.55}$$

[16]Recall that the AdS_3 metric in Poincaré coordinates,

$$\mathrm{d}s^2 = \frac{-\,\mathrm{d}\rho^2 + \mathrm{d}t^2 - \mathrm{d}x^2}{\rho^2} \ ,$$

is related to an AdS_2-sliced metric by the coordinate change,

$$z = \sqrt{x^2 + \rho^2} \ , \qquad \sinh u = x/\rho \ .$$

Since the $\Delta = 1$ scalar saturates the BF bound, holographic renormalization and the holographic dictionary are subtle due to the presence of the logarithm [155]. As we show below for the solution (5.51), there is no logarithmic term present and ϕ_0 can be identified with the expectation value of the dual operator [155, 156]. For the $\Delta = 2$ scalar, we can identify $\tilde{\phi}_0$ with the source and $\tilde{\phi}_2$ with the expectation value of the dual operator.

It is difficult to find a global map which puts the metric (5.53) in FG form. Here, we limit our discussion to the coordinate region away from the defect, where we take $u \to \pm\infty$ and keep z finite [157, 158]. This limit probes the region away from the interface on the boundary. The coordinate change suitable for the $u \to \infty$ limit can be expressed as a power series,

$$\begin{aligned} z &= x + \frac{\rho^2}{2x} + \mathcal{O}(\rho^4) \ , \\ e^u &= \cosh q \left(\frac{2x}{\rho} + \frac{\rho}{2x} + \mathcal{O}(\rho^3) \right) \ . \end{aligned} \tag{5.56}$$

The metric becomes

$$\mathrm{d}s^2 = \frac{1}{\rho^2} \left[-\,\mathrm{d}\rho^2 + \left(1 - \frac{\rho^2 \tanh^2 q}{2x^2} \right) (\mathrm{d}t^2 - \mathrm{d}x^2) + \mathcal{O}(\rho^3) \right] \ . \tag{5.57}$$

In the $u \to -\infty$ limit, the asymptotic form of the metric is the same and the coordinate change is (5.56) with the replacements $e^u \to e^{-u}$ and $x \to -x$.

Using this coordinate change, the expansions of the scalar fields near the boundary are

$$\begin{aligned} |\phi_4| &= |\tanh q| \frac{p + \tilde{q}}{\sinh(p + \tilde{q})} \cdot \frac{\rho}{|x|} + \mathcal{O}(\rho^3) \ , \\ \phi_5 &= (p + \tilde{q}) - \frac{1}{2\sinh(p + \tilde{q})} \left(\frac{p + \tilde{q}}{\sinh(p + \tilde{q})} \tanh^2 q + \frac{\sinh p \tanh \tilde{q}}{\cosh q} \right) \cdot \frac{\rho^2}{x^2} + \mathcal{O}(\rho^4) \ , \end{aligned} \tag{5.58}$$

where $\tilde{q} \equiv qx/|x|$ (see appendix 5.A.3 for details). The defect is located on the boundary at $x = 0$. We can see that the relevant operator corresponding to ϕ_4 has no term proportional to $\rho \ln \rho$ in the expansion. This implies that the source is zero and the dual operator has a position-dependent expectation value. The marginal operator corresponding to ϕ_5 has a source term which takes different values on the two sides of the defect, corresponding to a

Janus interface where the modulus associated with the marginal operator jumps across the interface.

Another quantity which can be calculated holographically is the entanglement entropy for an interval A using the Ryu-Takanayagi prescription [25],

$$S_{\text{EE}} = \frac{\text{Length}(\Gamma_A)}{4G_N^{(3)}} \ , \tag{5.59}$$

where Γ_A is the minimal curve in the bulk which ends on ∂A.

There are two qualitatively different choices for location of the interval in an interface CFT, as shown in Figure 5.2. First, the interval can be chosen symmetrically around the defect [159, 160]. The minimal surface for such a symmetric interval is particularly simple in the AdS-sliced coordinates (5.53), and is given by $z = z_0$ and $u \in (-\infty, \infty)$. The regularized length is given by

$$\text{Length}(\Gamma_A) = \int \mathrm{d}u = u_\infty - u_{-\infty} \ . \tag{5.60}$$

We can use (5.56) to relate the FG cutoff $\rho = \varepsilon$, which furnishes the UV cutoff on the CFT side, to the cutoff $u_{\pm\infty}$ in the AdS-sliced metric,

$$u_{\pm\infty} = \pm\big(-\log\varepsilon + \log(2z_0) + \log(\cosh q)\big) \ . \tag{5.61}$$

Putting this together and using the expression for the central charge in terms of $G_N^{(3)}$ gives

$$S_{\text{EE}} = \frac{c}{3}\log\frac{2z_0}{\varepsilon} + \frac{c}{3}\log(\cosh q) \ . \tag{5.62}$$

Note that the first logarithmically divergent term is the standard expression for the entanglement entropy for a CFT without an interface present [161], since $2z_0$ is the length of the interval. The constant term is universal in the presence of an interface and can be interpreted as the defect entropy (sometimes called g-factor [162]) associated with the interface.

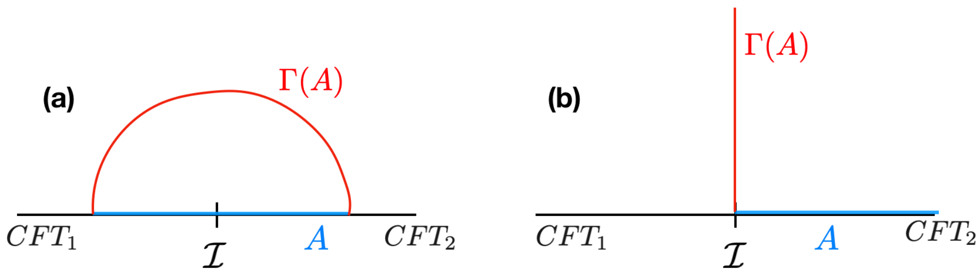


Figure 5.2: (a) The entangling surface A is symmetric around the interface $\mathcal{I}$, (b) The entangling surface A is ends at the interface $\mathcal{I}$.

Second, we can consider an interval which lies on one side of the interface and borders the interface [163,164]. As shown in [165], the entangling surface is located at $u = 0$ and the entanglement entropy for an interval of length l bordering the interface is given by

$$S'_{\rm EE} = \frac{c}{6}\,\mathrm{sech}\,q \log\frac{l}{\varepsilon}\ . \tag{5.63}$$

5.2.4 All scalars

For completeness, we also present the general solution with all ϕ_I scalars turned on. Let us define

$$\begin{aligned}\alpha_i(u) &\equiv \frac{\phi_i \sinh\Phi}{\Phi}\ , \qquad\qquad i = 1,2,3,4\ ,\\ \beta_{\bar{\imath}}(u) &\equiv \frac{\phi_{\bar{\imath}} \sinh\Phi}{\Phi}\ , \qquad\qquad \bar{\imath} = 5,6,7,8\ .\end{aligned} \tag{5.64}$$

As a consequence of (5.31), the ratio ϕ_i'/ϕ_i is the same for all i so all the ϕ_i scalars are proportional to each other. In other words, we have $\alpha_i = n_i\alpha$ for constants n_i satisfying $n_i n_i = 1$, where α is given in (5.42). Then (5.34) becomes

$$\alpha'^2 + \beta'_{\bar{\imath}}\beta'_{\bar{\imath}} - \frac{(\alpha'\alpha + \beta'_{\bar{\imath}}\beta_{\bar{\imath}})^2}{1+\alpha^2+\beta_{\bar{\imath}}\beta_{\bar{\imath}}} = \alpha^2\ . \tag{5.65}$$

We can note that there exists a family of solutions where all $\beta_{\bar{\imath}}$ functions satisfy

$$\beta_{\bar{\imath}} = n_{\bar{\imath}}\beta\ , \tag{5.66}$$

for some function β and constants $n_{\bar{\imath}}$ satisfying $n_{\bar{\imath}}n_{\bar{\imath}} = 1$. When this is the case, (5.65) then further simplifies to

$$\alpha'^2 + \beta'^2 - \frac{(\alpha'\alpha + \beta'\beta)^2}{1 + \alpha^2 + \beta^2} = \alpha^2 \ , \tag{5.67}$$

which has already been solved in the previous section. We can prove that these are the only solutions to (5.65) which satisfy the equations of motion. The scalar dependence of the Lagrangian is

$$\begin{aligned} e^{-1}\mathcal{L} &\supset -\frac{g^2}{4}\mathcal{P}_u^I\mathcal{P}_u^I + W \\ &= -\frac{g^2}{4}\left(\alpha'^2 + \beta'_{\bar{\imath}}\beta'_{\bar{\imath}} - \frac{(\alpha'\alpha + \beta'_{\bar{\imath}}\beta_{\bar{\imath}})^2}{1 + \alpha^2 + \beta_{\bar{\imath}}\beta_{\bar{\imath}}} - (\alpha^2 + 2)\right) \ . \end{aligned} \tag{5.68}$$

If we write the $\beta_{\bar{\imath}}$ in spherical coordinates, where we call the radius β, this becomes

$$= -\frac{g^2}{4}\left(\alpha'^2 + \beta'^2 + \beta^2 K^2 - \frac{(\alpha'\alpha + \beta'\beta)^2}{1 + \alpha^2 + \beta^2} - (\alpha^2 + 2)\right) \ , \tag{5.69}$$

where K^2 is the kinetic energy of the angular coordinates.[17] We can treat α, β, and the three angles as the coordinates of this Lagrangian. The equation of motion from varying the Lagrangian with respect to α will only involve α and β and their derivatives. Plugging-in (5.42) for α, satisfying this equation of motion fixes the form of β to be what was found previously in (5.50). This means that (5.65) simplifies to $\beta^2 K^2 = 0$ and the three angles must be constant.

Therefore, the general solution is

$$\begin{aligned} \frac{\phi \sinh \Phi}{\Phi} &= |\sinh q| \operatorname{sech} u \ , \\ \beta &= \sinh p \cosh q + \cosh p \sinh q \tanh u \ , \\ \phi_i &= n_i \phi \ , \qquad n_i n_i = 1 \ , \\ \frac{\phi_{\bar{\imath}} \sinh \Phi}{\Phi} &= n_{\bar{\imath}} \beta \ , \qquad n_{\bar{\imath}} n_{\bar{\imath}} = 1 \ . \end{aligned} \tag{5.70}$$

[17] Explicitly, let $K^2 = \theta'^2 + \sin^2\theta\,\phi'^2 + \sin^2\theta\sin^2\phi\,\psi'^2$.

5.3 Discussion

In this chapter, we have presented Janus solutions for three-dimensional $\mathcal{N} = 8$ gauged supergravity. We constructed the simplest solutions with the smallest number of scalars, namely the $\mathrm{SO}(8,1)/\,\mathrm{SO}(8)$ coset. The solutions we found have only two scalars displaying a non-trivial profile. One scalar is dual to a marginal operator $\mathcal{O}_2$ with scaling dimension $\Delta = 2$ and the other scalar is dual to a relevant operator $\mathcal{O}_1$ with scaling dimension $\Delta = 1$. We used the holographic correspondence to find the dual CFT interpretation of these solutions. It is given by a superconformal interface, with a constant source of the operator $\mathcal{O}_2$ that jumps across the interface. For the operator $\mathcal{O}_1$, the source vanishes, but there is an expectation value that depends on the distance from the interface. It would be interesting to study whether half-BPS Janus interfaces that display these characteristics can be constructed in the two-dimensional $\mathcal{N} = (4,4)$ SCFTs.

We considered solutions for the $\mathrm{SO}(8,1)/\,\mathrm{SO}(8)$ coset, but these solutions can be trivially embedded into the $\mathrm{SO}(8,n)/\big(\mathrm{SO}(8)\times\mathrm{SO}(n)\big)$ cosets with $n > 1$. Constructing solutions with more scalars with non-trivial profiles is in principle possible, but the explicit expressions for the quantities involved in the BPS equations are becoming very complicated. We also believe that the $n = 1$ case already illustrates the important features of the more general $n > 1$ cosets. Another possible generalization is given by considering more general gaugings. One important example is given by replacing the embedding tensor (5.5) with

$$\Theta_{IJ,KL} = \alpha\varepsilon_{ijk\ell} + \varepsilon_{\bar{\imath}\bar{\jmath}\bar{k}\bar{\ell}} \ . \tag{5.71}$$

This is a deformation that produces an AdS_3 vacuum which is dual to a SCFT with a large $D(2,1;\alpha)\times D(2,1;\alpha)$ superconformal algebra. As discussed in [152], this gauging is believed to be a truncation type II supergravity compactified on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ [166, 167]. It should be straightforward to adapt the methods for finding solutions developed in this chapter to this case.

We calculated the holographic defect entropy for our solution. It would be interesting to investigate whether this quantity can be related to the Calabi diastasis function following [168,169]. For this identification to work, we would have to consider the case $n = 2$ for which the scalar coset is a Kähler manifold.

We leave these interesting questions for future work.

5.A Technical details

In this appendix, we present various technical details which are used in the main part of the chapter.

5.A.1 SO(8) Gamma matrices

We are working with 8×8 Gamma matrices $\Gamma^I_{A\dot{A}}$ and their transposes $\Gamma^I_{\dot{A}A}$, which satisfy the Clifford algebra,

$$\Gamma^I_{A\dot{A}}\Gamma^J_{\dot{A}B} + \Gamma^J_{A\dot{A}}\Gamma^I_{\dot{A}B} = 2\delta^{IJ}\delta_{AB} \ . \tag{5.72}$$

Explicitly, we use the basis in [170],

$$\begin{aligned}
\Gamma^8_{A\dot{A}} &= 1 \otimes 1 \otimes 1 \ , & \Gamma^1_{A\dot{A}} &= i\sigma_2 \otimes i\sigma_2 \otimes i\sigma_2 \ . \\
\Gamma^2_{A\dot{A}} &= 1 \otimes \sigma_1 \otimes i\sigma_2 \ , & \Gamma^3_{A\dot{A}} &= 1 \otimes \sigma_3 \otimes i\sigma_2 \ . \\
\Gamma^4_{A\dot{A}} &= \sigma_1 \otimes i\sigma_2 \otimes 1 \ , & \Gamma^5_{A\dot{A}} &= \sigma_3 \otimes i\sigma_2 \otimes 1 \ . \\
\Gamma^6_{A\dot{A}} &= i\sigma_2 \otimes 1 \otimes \sigma_1 \ , & \Gamma^7_{A\dot{A}} &= i\sigma_2 \otimes 1 \otimes \sigma_3 \ .
\end{aligned} \tag{5.73}$$

The matrices Γ^{IJ}_{AB}, $\Gamma^{IJ}_{\dot{A}\dot{B}}$ and similar are defined as unit-weight antisymmetrized products of Gamma matrices with the appropriate indices contracted. For instance,

$$\Gamma^{IJ}_{AB} \equiv \frac{1}{2}(\Gamma^I_{A\dot{A}}\Gamma^J_{\dot{A}B} - \Gamma^J_{A\dot{A}}\Gamma^I_{\dot{A}B}) \ . \tag{5.74}$$

5.A.2 Integrability conditions

For BPS equations of the form,

$$
\begin{aligned}
\partial_t \varepsilon &= -\frac{1}{2z}\gamma_0\big(\gamma_1 + f(u) + g(u)\gamma_2\big)\varepsilon \ , \\
\partial_z \varepsilon &= -\frac{1}{2z}\gamma_1\big(f(u) + g(u)\gamma_2\big)\varepsilon \ , \\
\partial_u \varepsilon &= \big(F(u) + G(u)\gamma_2\big)\varepsilon \ ,
\end{aligned}
\tag{5.75}
$$

where f, g, F, G are matrices acting on ε that commute with γ_a, the integrability conditions are

$$
t, z : \qquad 0 = (1 + f^2 + g^2)\varepsilon + [f, g]\gamma_2\varepsilon \ , \tag{5.76}
$$

$$
t, u : \qquad 0 = (f' + [f, F] - \{g, G\})\varepsilon + (g' + [g, F] + \{f, G\})\gamma_2\varepsilon \ , \tag{5.77}
$$

$$
z, u : \qquad \text{same as for } t, u \ .
$$

5.A.3 Scalar asymptotics

The asymptotic expansions of the ϕ_4 and ϕ_5 scalar fields, as given in (5.51), in the limits $u \to \pm\infty$ are

$$
\begin{aligned}
|\phi_4| &= 2|\sinh q| \frac{p \pm q}{\sinh(p \pm q)} e^{\mp u} \\
&\quad - \frac{2|\sinh q|}{\sinh^2(p \pm q)} \left(\frac{p \pm q}{\sinh(p \pm q)} (\sinh^2 p + \sinh^2 q) \pm 2 \sinh p \sinh q \right) e^{\mp 3u} + \mathcal{O}(e^{\mp 5u}) \ , \\
\phi_5 &= (p \pm q) - \frac{2}{\sinh(p \pm q)} \left(\frac{p \pm q}{\sinh(p \pm q)} \sinh^2 q \pm \sinh p \sinh q \right) e^{\mp 2u} + \mathcal{O}(e^{\mp 4u}) \ .
\end{aligned}
\tag{5.78}
$$

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