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UNIVERSITY OF CALIFORNIA  
RIVERSIDE

Three Etudes for Symmetrical Cuboctahedral Speaker Array: An Exploration of Aural  
Space-Time Symmetry

A Dissertation submitted in partial satisfaction  
of the requirements for the degree of

Doctor of Philosophy

in

Music

by

Christopher Diaz

December 2019

Dissertation Committee:

Dr. Tim Labor, Co-Chairperson

Dr. Ian Dicke, Co-Chairperson

Dr. Rene Lysloff

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Christopher Diaz  
2019

The Dissertation of Christopher Diaz is approved:

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University of California, Riverside

## ABSTRACT OF THE DISSERTATION

Three Etudes for Symmetrical Cuboctahedral Speaker Array: An Exploration of Aural  
Space-Time Symmetry

by

Christopher Diaz

Doctor of Philosophy, Graduate Program in Music  
University of California, Riverside, December 2019

Dr. Tim Labor, Co-Chairperson

Dr. Ian Dicke, Co-Chairperson

Strategies of musical symmetry can partition an aural space in a manner similar to the way that scales partition an octave. In an exploration of a 12-channel diffusion system, I will compose three musical pieces expressing various methods of designing symmetrical based aural spatial premises with the goal of creating an experienceable spatial harmony. To achieve this, I will first design a cuboctahedral speaker array capable of representing multiple spatial harmonies. Second, I will develop a notation system that symmetrically visually represents the spatial harmonies for compositional intentionality. Finally, I will program a Max MSP software patch capable of integrating the array and notation for playback experiences and compositional interpretations. After these processes are outlined and described, I will then compose three etudes to demonstrate diverse methods of compositional strategies; they are dedicated to, and inspired by the works of Terrence McKenna, Nicola Tesla, and Carl Sagan.

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## **Chapter 1**

### **Introduction**

This project is inspired by a personal fascination with symmetry. The purpose of this dissertation is to explore any possible aesthetics associated with equal-tempered space, and music that is representative of aural space-time symmetry. The goal is to create a new serial compositional form that can effectively communicate intentionality between composer and listener. There are three steps that will need to take place before any new composition can occur to express the possibilities of this new system. First, a symmetrical speaker array that will allow for the playback of symmetrical compositions will be designed and constructed. Second, a form of notation that visually relates compositional input to the speaker array will be developed with a focus on usability and accessibility. Finally, a Max MSP patch that integrates the speaker array with the notation system will be coded and modified for portability (and again, accessibility). Once these steps have been outlined, described and completed, I will then compose three etudes to demonstrate three different approaches towards composing with the concept of aural space-time symmetry with the intention of exploring equal-tempered space for any possible artistic aesthetics.

Symmetry is no stranger to the history of Western music. From the technically proficient music of Bach from the Baroque period (Lederman 2008), to the atonal serialist music of Schoenberg from the 20<sup>th</sup> century (Boss 2016), symmetry has played an important role in the development of the musical canon I was presented as a music student in academia. Where pieces like *The Musical Offering (Crab Sonata)* by Bach

(Phillips 2017) or *Music for String Instruments, Percussion and Celeste* by Bartok may be more obvious examples of symmetrical inspiration (Solomon 2002), the concept can be likened to fundamental tonal musical concepts such as form, harmonic progressions (Kempf 1996), and the octave. As the only interval that by definition cannot waiver, a series of octaves represents a type of fractal-symmetry, where each successive iteration is an equal 2:1 ratio to the previous (Duffin 2008). I wanted to build on this historic tradition, but I was attracted to symmetry for another reason.

My use of a speaker array may be reminiscent of composers interested in spatialization, but this is not intended to be a spatialization project. The work of composers like Stockhausen, who experimented with speaker arrays, were influential to my work, but my ultimate ambition was to create a universal form capable of maintaining an artistic aesthetic if played or performed anywhere in the universe. The concept behind *Telemusik* was more inspirational to my idea than that of a piece like *Kontakte*, as I wanted something that could not only transcend our human cultural and language differences, but any difference between ourselves and other intelligent life beyond our planet (Wörner 1973). Where attempts at interstellar sonic communication have been made before, I have always personally struggled with some of the projects I have seen regarding this idea (Gambino 2012). For example, when I first learned about the Voyager Golden Record sent to outer space, the first questions I had were, “Why would they have record players? What if they don’t have two ears? What if they don’t listen to sound? What if it actually got played on another world, would it sound the same under a different atmosphere?” These kinds of thoughts are what led me to symmetry.

Symmetry can be thought of as a universal language. Much like the octave, by definition it cannot waiver or exist on a spectrum (Lederman 2008). It either exists or does not. Because of this, I believe it can be used to circumvent almost any communication barrier. For instance, an English speaker and a Spanish speaker could both be presented with a Chinese symbol that neither of them could read. However, they could both agree on which symbols were symmetrical and which ones were not, regardless of their abilities to understand what they meant. I hoped that this idea could be extracted and transferred to a musical form that could present symmetrically to any potential listener, anywhere in our universe.

No matter how and where the following project is constructed, the musical symmetry should remain constant. There are no measurements given in this paper, as the size of the array will have no impact on the ability to present spatial harmonies. Even if the array were built on another heavenly body, under a different atmosphere, the alteration of the playback would apply equally to all channels and still present as spatially aurally symmetrical. The sin waves used in this project, which are governed and defined by the second determined from the rotation of this planet, would still maintain their symmetrical relationships with each other if defined by a different cosmic rotation. And since the array was meant to treat the body as the listening apparatus, as opposed to the ears, any type of “listener” with alternative ear numbers and placements, who spoke and understood any kind of aural language, would still be able to perceive the symmetrical intentions of a composer when placed in the middle of the speaker array. In fact, the only

requirements are that the listener being placed within the array exists in three dimensions and can perceive sound, even if not through listening.

### The Playback Speaker Array

The desire to design a speaker array that would inherently be compatible with the concept of aural symmetry led the beginning of this project towards the Platonic solids. This is because these five shapes (tetrahedron, cube, octahedron, dodecahedron, icosahedron) can all be interpreted as three-dimensional symmetrical shapes because they are defined by having faces and angles that are all congruent/equal. A shape that could outline a symmetrical space would be ideal to use as the principal structure for a symmetrical speaker array meant to surround the body of the listener, not necessarily the ears, in a sonic bath of aural space symmetry. There are only five in existence; they are all pictured in figure 1-1.

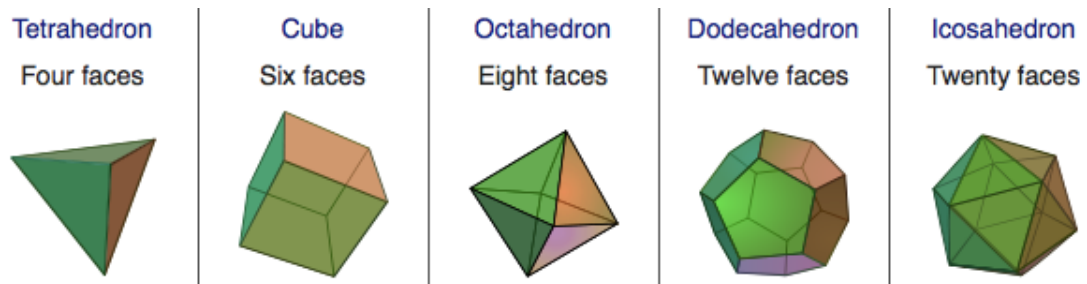


Figure 1-1 – The Platonic Solids

Having limited access to tools and woodcutting expertise, the original plan was to build a large cube with either six speakers located at the center of each face, or eight speakers located in each corner. However, this idea did not last long when the realization was made that another shape (related to the cube) would be better suited for a project based on symmetry. While not a Platonic solid, the cuboctahedron (created by bisecting

each side of a cube) does provide its own unique form of symmetry that is not found in any other three-dimensional shape. This solid shape and this unique property can be seen in figure 1-2.

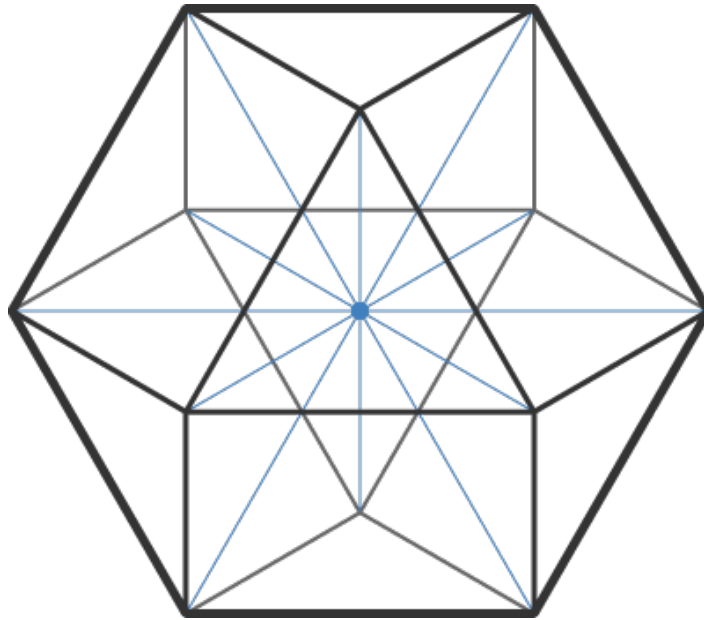


Figure 1-2 – The Cuboctahedron

The cuboctahedron is also called the vector equilibrium due to the fact that it has a unique symmetry based on the relationships of the vertices and the exact center of the shape. Just like a Platonic solid, all of its sides are equal (unlike a Platonic solid, its faces are not), but in addition to this element of symmetry, the distance between each vertex, as well as from each vertex to the exact center, are equal to each other. Top put it simply, in figure 1-2, every line shown is equal to every other line shown. If this shape was used as the outline for the array, and a speaker was placed at each vertex angled towards the direct center, the symmetrical arrangement would go beyond that of the relationship of the speakers to each other and would also apply to a listener placed in the exact center. The

decision was then made to construct a simple cube out of PVC pipes and to then place a small speaker at the center of each edge of this cube, making sure each speaker was angled equally. This would not only allow for easy entrance into and out of the array by potential listeners and a much easier physical construction process, but it would also allow a composer to perfectly adjust the angles of each speaker so that the symmetries would be as close to exact as possible. The placement of the speakers at perfect angles would have been more difficult to achieve had they been placed in the corners or on the inner faces of a cube.

While the unique symmetrical qualities of this shape initially led to its use as the shape of the speaker array, the fact that it had 12 vertices that would align with 12 speakers was also functional for a project based on symmetry. Foremost, the number 12 is a superior highly composite number. That is, it can be perfectly divided by 12, 6, 4, 3, 2, and 1. With symmetry as the primary compositional determinant, the idea of composing perfectly symmetrical duodecets, sextets, quartets, trios, duos and solos was interesting for the sake of compositional diversity. A desire to develop a compositional form that would allow for as much potential creation as possible could be supported by having more options with regard to equal speaker groupings, ultimately allowing multiple levels of spatial symmetry composition. The number 12 was also attractive because of its place in esoteric and mystical belief systems. Concepts like the 12 signs of the zodiac, or the 12 tribes of Israel, or the 12 disciples of Christ were inspirational when looking for thematically inspired material to associate with this new concept. Scientific concepts like time division, base 12 mathematics and the standard model could also provide plenty of inspiration outside of the

mystical and superstitious. After considering all of this, it was decided that the cuboctahedron was ideal due to the multi-functional nature of its properties with regard to the goals of this project.

Designing this project as a graduate student, many of the decisions made in the physical construction of the first prototype were based on budget constraints, ergo the PVC pipe. Purchasing 12 identical amplified speakers to be assigned to 12 independent outputs from a computer audio interface could get extremely expensive. With this in mind, the decision was made to use six pair of small USB powered cube shaped speakers. Their cube shape would allow for easy symmetrical placement, their USB power needs could easily be met with a hub, and their cheaper price made them reasonably affordable. The main drawback to using these speakers turned out to actually be an asset with regards to developing the numerical speaker identification model. This turned out to be important to the development of a communicative notation system.

The fact that these speakers came in stereo L/R pairs meant that each speaker in a pair could only be a limited distance from each other. This would force extra time to be taken in order to find a way where all six pair could be properly assigned to a symmetrical pattern which would still maintain their mono independence. Marking each left speaker as an odd integer and each right speaker as even, a design emerged that allowed for both a consistent flow around the cuboctahedron, that would create a visually symmetrical numerical pattern, that could be mathematically calculated to verify accuracy, all while working under the limitations of stereo paired speakers with reasonably short cabling between them. Figure 1-3 is an outline of this design. Lines and arrows represent a bound



pair of stereo L/R speakers, and their order is based on the limitations of sequential necessity and the desire to design a verifiably calculable spatially symmetrical numbering system. In this case, this is achieved by analyzing two polar opposing speakers around the array. No matter which speaker is chosen, the number of the speaker and the number of its polar opposite on the other side of the array will always equal 13 when added together. This summation concept later proved to be useful for finding and verifying all of the different types of symmetric harmonies.

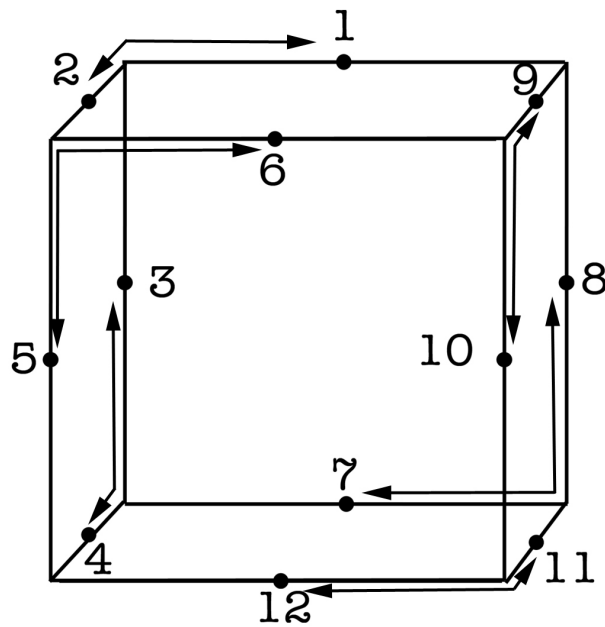


Figure 1-3 – The six stereo pairs of speakers laid out symmetrically around the array

Using a piece of standard graph paper, a pattern emerged when the speakers were listed in their sequential order, where the divisions of the speakers into equal groupings of two would consistently create a symmetric image/form across the grid. This is illustrated in figure 1-4. Examples 1, 2 and 4 can be seen as variations created from the stereo pair pattern outlined in figure 1-3. Each row represents the four ways that 12 can

be divided evenly into equal halves by sequential grouping. Groupings of 1, 2, 3, and 6 are possible (1-4, respectively), but groupings of 4 will result in 3 equal parts, which are by definition unable to represent any type of duo-based symmetry. In this text the word “harmony” will be used to represent these physical forms of spatial symmetry, as their use and completion will be at the center of composing within this form. Having two unique sounds, duos could be thought of as perfect fifths, with three unique sounds, trios could be thought of as triads, with four unique sounds, quartets could be thought of as 7<sup>th</sup> chords, with six unique sounds, sextets could be thought of as 11<sup>th</sup> chords, and with twelve unique sounds coming out of each speaker, duodecets could be thought of as the chromatic scale, and/or a complete circle of fifths. Many of the concepts developed for this project can be related to traditional musical concepts associated with tonal harmony.

	Sp.1	Sp.2	Sp.3	Sp.4	Sp.5	Sp.6	Sp.7	Sp.8	Sp.9	Sp.10	Sp.11	Sp.12
<u>1.</u>	<b>A</b>	<b>B</b>	<b>A</b>	<b>B</b>	<b>A</b>	<b>B</b>	<b>A</b>	<b>B</b>	<b>A</b>	<b>B</b>	<b>A</b>	<b>B</b>
<u>2.</u>	<b>A</b>	<b>A</b>	<b>B</b>	<b>B</b>	<b>A</b>	<b>A</b>	<b>B</b>	<b>B</b>	<b>A</b>	<b>A</b>	<b>B</b>	<b>B</b>
<u>3.</u>	<b>A</b>	<b>A</b>	<b>A</b>	<b>B</b>	<b>B</b>	<b>B</b>	<b>A</b>	<b>A</b>	<b>A</b>	<b>B</b>	<b>B</b>	<b>B</b>
<u>4.</u>	<b>A</b>	<b>A</b>	<b>A</b>	<b>A</b>	<b>A</b>	<b>A</b>	<b>B</b>	<b>B</b>	<b>B</b>	<b>B</b>	<b>B</b>	<b>B</b>

Figure 1-4 – Preliminary Duo Chart

All of the examples in the graph above create a system where each polar pair of speakers are equally represented by one “A” sound and one “B” sound. This represents on the graph down the middle of the sheet, in between the columns for speakers 6 & 7. Throughout this text this line will be referred to as the “meridian.” If you look closely you can see a pattern where each half created by the meridian represents a perfect inverse

of the other half; where every “A” on one half is paired equidistantly from the meridian with every “B” on the other half, and vice versa. Another way to say this is that in every graph above, where  $S_x + S_y = 13$ , either  $S_x = A$  and  $S_y = B$ , or  $S_y = A$  and  $S_x = B$  has to be true. This concept was inspirational to begin looking for more concrete ways to extract and define symmetries by means of the emerging graphical notation as opposed to visually inspecting the array. With the speaker array designed, the next step would be to create a notation system that was visually representative of the array.

## Chapter 2

### **The Notation System**

While trying to conceptualize a notation system for this project, consideration was given to user-friendliness, visual representation of the symmetries mapping to the physical array, and compatibility between the notation and Max MSP, which would ultimately be used to connect the array and the notation. However, before any decisions could be made regarding the development of a notation system, decisions needed to be made about what kinds of sound would actually be playing through the speakers. In order to make sure that the two platforms could communicate, the sounds that would be used as the principal compositional materials needed to be determined. Progress could not commence without knowing what kind of information would actually be going into the notation system to be interpreted by the array.

Initially, there were thoughts of presenting famous speeches in 12 different languages or presenting pieces of tonal music in all 12 possible keys of the chromatic scale simultaneously. However, the goal was to develop an aural form completely inspired by symmetry. With this in mind, the decision was made to use the most symmetrical sound of all: the sine wave. Without any overtones or partials to consider, nothing made more sense to use for the development of a perfectly symmetrical compositional form than using perfectly symmetrical sound waves. This would also be compatible with the graph inspired notation system, because instead of the place holding letters used in figure 4, integers could be used to represent the frequencies of sine waves chosen by a composer. Knowing that Max MSP could easily be programmed to create an

original patch that could convert a list of integers into a series of independent sine waves made the decision final.

After analyzing all of the goals of the project with the emerging details of the construction process, the decision was made to use Microsoft Excel as the program where the notation system would be developed. It was an ideal choice for a communicative form that needed to be based on a grid and that would allow for a composer to easily input sin wave frequency data as integers. Not only did this program meet all of the project needs, the universal spreadsheet format inspired confidence that work prepared in this format would be translatable into data that Max MSP could interpret for playback. It is also compatible with various programs, across multiple operating systems, contributing to the project goals of accessibility and usability, with regards to future potential composers.

In addition to the 12 speaker columns identified in figure 4 of the previous chapter, the decision was made to add a few more to make the notation more comprehensive. A “Master” column was added as a means of identifying the location point inside the piece. The number in this column could be seen as a sort of “measure” number from the perspective of a traditional score. It was soon after this that the need for a “Tempo” column was realized, so that composers could input information to make adjustments to the speed of the piece, from the beginning and dynamically as it progresses. Finally, the “End” column was added to provide a data input point that would allow composers to formally bring the song, and the Max MSP patch, to an ending. Below is an outline of the template, with some of the harmonies shown for

demonstration purposes. Note how the speakers are controlled with integers representing frequencies, how the tempo is controlled in microseconds, and how the end is triggered with the number 0. All of this can be seen in figure 2-1.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End	
1	111	0	0	0	0	0	0	0	0	0	0	0	1000		12
2	0	222	0	0	0	0	0	0	0	0	0	0			12
3	0	0	333	0	0	0	0	0	0	0	0	0			12
4	0	0	0	444	0	0	0	0	0	0	0	0			12
5	0	0	0	0	555	0	0	0	0	0	0	0			12
6	0	0	0	0	0	666	0	0	0	0	0	0			12
7	0	0	0	0	0	0	777	0	0	0	0	0			12
8	0	0	0	0	0	0	0	888	0	0	0	0			12
9	0	0	0	0	0	0	0	0	999	0	0	0			12
10	0	0	0	0	0	0	0	0	0	1010	0	0			12
11	0	0	0	0	0	0	0	0	0	0	1111	0			12
12	0	0	0	0	0	0	0	0	0	0	0	1212			12
13	111	111	0	0	0	0	0	0	0	0	0	0			6
14	0	0	222	222	0	0	0	0	0	0	0	0			6
15	0	0	0	0	333	333	0	0	0	0	0	0			6
16	0	0	0	0	0	0	444	444	0	0	0	0			6
17	0	0	0	0	0	0	0	0	555	555	0	0			6
18	0	0	0	0	0	0	0	0	0	0	666	666			6
19	111	111	111	0	0	0	0	0	0	0	0	0			4
20	0	0	0	222	222	222	0	0	0	0	0	0			4
21	0	0	0	0	0	0	333	333	333	0	0	0			4
22	0	0	0	0	0	0	0	0	0	444	444	444			4
23	111	111	111	111	0	0	0	0	0	0	0	0			3
24	0	0	0	0	222	222	222	222	0	0	0	0			3
25	0	0	0	0	0	0	0	0	333	333	333	333			3
26	111	111	111	111	111	111	0	0	0	0	0	0			2
27	0	0	0	0	0	0	222	222	222	222	222	222			2
28	111	111	111	111	111	111	111	111	111	111	111	111			1
29															0

12 Speakers behave as (12) individuals

6 Speakers behave as (6) equal pairings of two

4 Speakers behave as (4) equal groups of three

3 Speakers behave as (3) equal groups of four

2 Speakers behave as (2) equal groups of six

1 Speakers behave as (1) unison voice

Figure 2-1 – Open Texture Examples of Each Harmonic Form

In figure 2-1 we see one single example of a duodectet, sextet, quartet, trio, duo and solo where only one speaker or speaker grouping sounds at one time. I refer to this style of the use of the symmetric harmonies as an “open texture.” The color spectrum on the right explains the groupings as how the array is divided. All of the symmetries shown in figure 2-1 could easily be compressed down into something that fires off sound in each speaker or speaker grouping simultaneously. In this case, every speaker/grouping has

something to play for every “Master” position point. This is shown in figure 2-2. Figure 2-2 is the same version of events from figure 2-1 happening simultaneously and over less time. I refer to this style of use of the symmetries as “closed texture.” There is no data associated with the color spectrum markings on the far right or across the top; they are only there to help the composer organize their symmetrical plans, as I quickly learned how easy it is to get lost inside of a longer piece.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End	
1	111	222	333	444	555	666	777	888	999	1010	1111	1212	1000		12
2	111	111	222	222	333	333	444	444	555	555	666	666			6
3	111	111	111	222	222	222	333	333	333	444	444	444			4
4	111	111	111	111	222	222	222	222	333	333	333	333			3
5	111	111	111	111	111	111	222	222	222	222	222	222			2
6	111	111	111	111	111	111	111	111	111	111	111	111			1
7															0

Figure 2-2 – Closed Texture Examples of Each Harmonic Form

The different types of textures can be used by a composer for aural diversity within a piece. Compositional diversity and possibilities were important to the development of this project, as the goal was to create a musical system capable of as much experimentation as the diatonic models. In addition to these different types of textures, there are a large number of symmetric harmonies possible within each different grouping system (duo, trio, etc.). As we move into the comprehensive list of all possibilities, you will see that some of the harmonies do not seem to represent as visually symmetrical as others do within the notation. In order to “see” this type of symmetry, we will have to imagine a three-dimensional version of our two-dimensional grid that allows for the additional visualization of radial symmetries. It made sound odd, but with just a few simple manipulations of the two-dimensional grid system, we can create a three-

dimensional toroid that is more reflective of those symmetries better observed beyond the limitations of a two-dimensional plane. Figure 2-3 outlines how to imagine this process over a few simple steps.

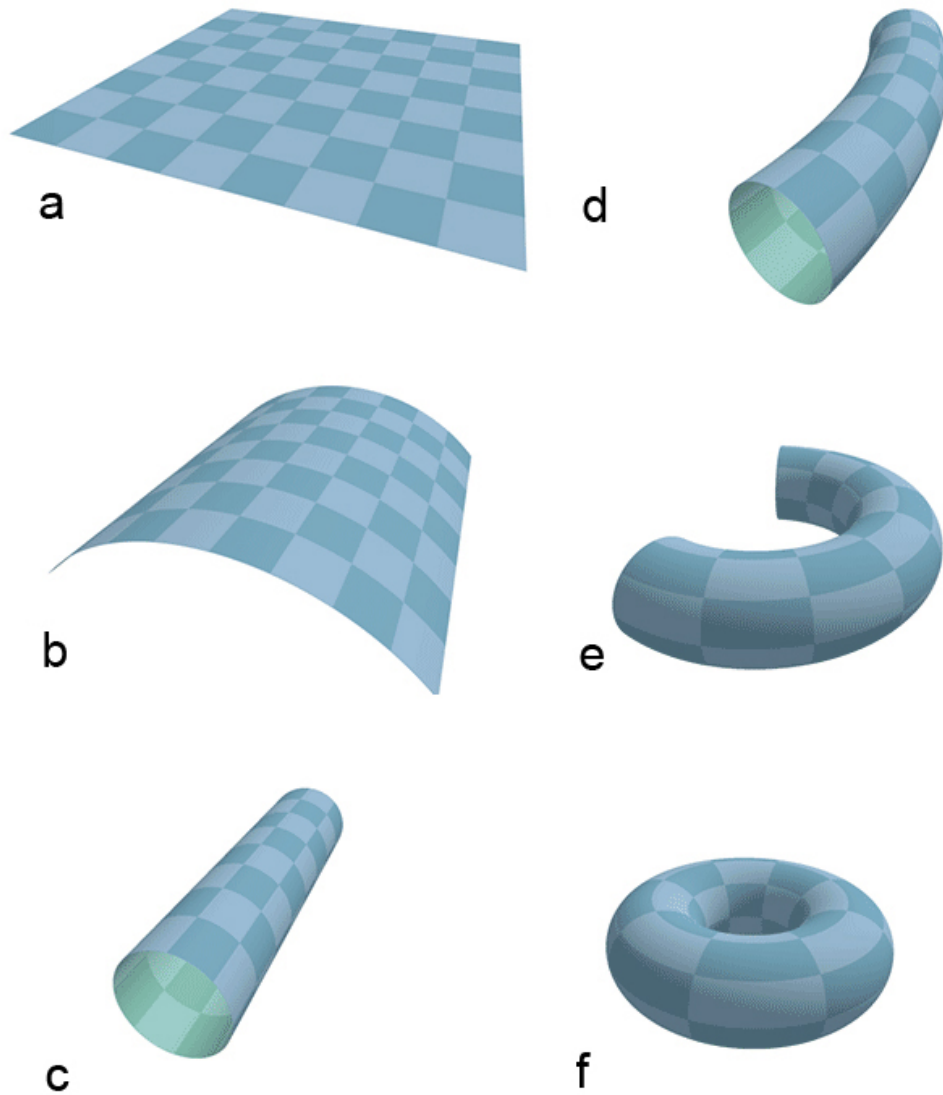


Figure 2-3 – Turning a grid into a toroid



Here we see how the two-dimensional grid can be thought of as a three-dimensional toroid. This makes the sequence-based harmonies (defined in figure 4 of the previous chapter) easier to visualize symmetrically as infinitely repeating patterns, radially symmetrical around the center. Quartet symmetries are also easier to perceive within the notation with this form in mind, as they tend to be sequential or radial. The toroidal perspective also helps display how “rotations” are truly just as equal as root position harmonies (Chapter 3), especially when considering the numbering system truly, while relatively objectively symmetrical, begins and ends at subjectively chosen points. At this point the decision was made to actively eliminate any kind of dynamic controls for each speaker. Dynamic and/or volume indicators, and their associated data columns, could always be added later, but due to the early experimental nature of this project they were intentionally omitted so that they would not impede the ability of a listener to perceive aural symmetry. While dynamic controls could provide for more compositional diversity, the first few songs are meant to explore alternative aesthetics that would be better represented with complete/full volume.

### **Chapter 3**

#### **The Harmonies – Duos, Trios, Quartets and Sextets**

Since the solo and duodectet were comprehensively covered with each of their single possibilities being outlined with the notation definition, they are not addressed in this chapter. However, the remaining duos, trios, quartets and sextets can be quite complex, so their possibilities are extensively covered. Both notation and array maps will be provided for EVERY symmetrical possibility (or “harmony”). A series of these harmonies can be thought of in the same manner as a chord progression; those that create new harmonies by holding sounds over into new relationships can be related to the traditional concept of suspensions.

#### **The Duos**

The first examples of the duos are listed in figure 1. For better visibility, each one is shown in an open texture; they could just as easily be configured to happen in half as much time using a closed texture. Figures 3-1(1-2), 3-1(3-4), 3-1(5-6), and 3-1(7-8) show how the notation relates to the array. Note, there are plenty of other duos that can be generated from these root position by rotating them (The rotation and other alterations are fully defined at the end of this chapter and visualized in figure 3-13). This is due to the toroidal perspective explained in the previous chapter, as any of these forms can slide forward or backward through the subjective number set (left or right, by one or more speaker positions) and still maintain their objective symmetrical property. Creating a long list of all of the different possible rotations would be exhaustive and overwhelming.

Instead, only all of the “root position” harmonies will be listed. This will be the case for each successive form (trios, quartets, and sextets) described in this chapter. In the case of the duos, each example demonstrates a fundamental way to divide the array into perfectly equal halves that can then be rotated and presented in open or closed textures.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
1	111		111		111		111		111		111		1000	
2		222		222		222		222		222		222		
3	111	111			111	111			111	111				
4			222	222			222	222			222	222		
5	111	111	111				111	111	111					
6				222	222	222				222	222	222		
7	111	111	111	111	111	111								
8							222	222	222	222	222	222		

Figure 3-1 – Root Position Sequence Based Duos (open texture)

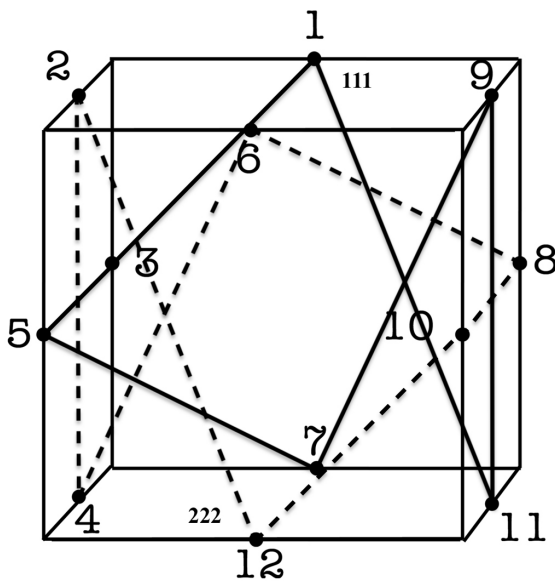


Figure 3-1(1-2)

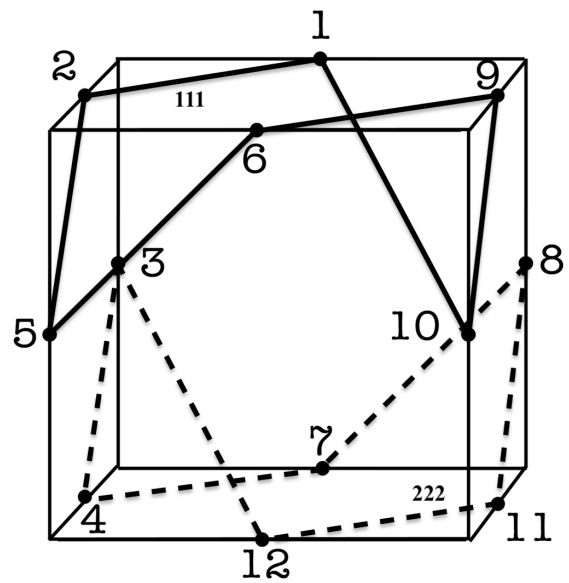


Figure 3-1(3-4)

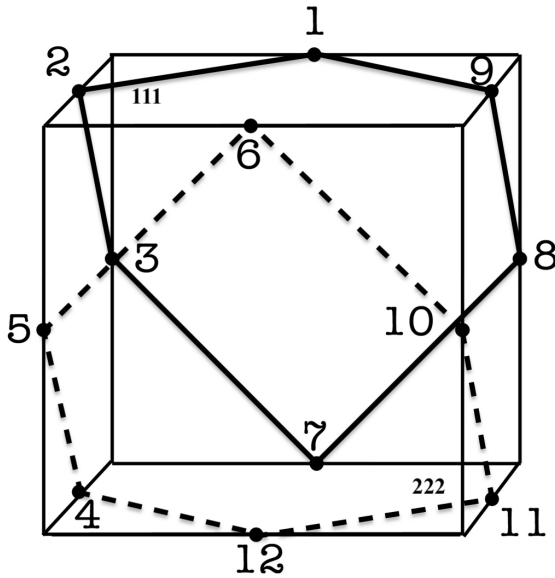


Figure 3-1(5-6)

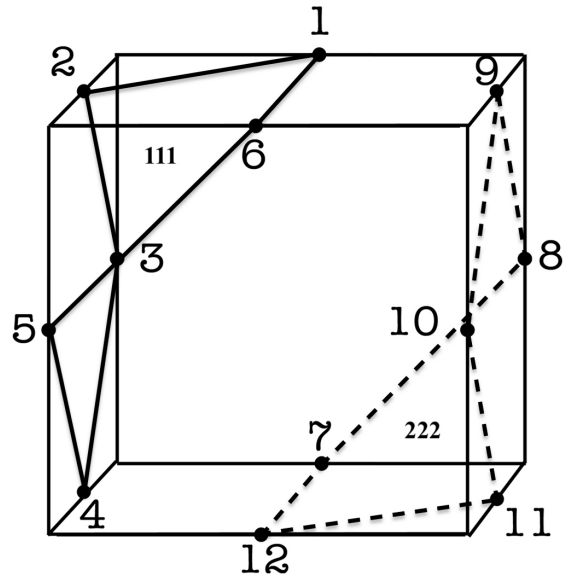


Figure 3-1(7-8)

These first few duos will be familiar, as they were briefly described at the end of chapter 1. These harmonies were discovered by using a sequence-based grouping system with each perfect divisor of 12. All of the mappings associated with these notation examples show matching frequency integers to help make them easy to match (111 and 222). Measures 1-2 and figure 3-1(1-2) represent the grouping of every other speaker or can be collectively seen as the grouping of the odd and even numbered speakers. Measures 3-4 and figure 3-1(3-4) represent the grouping of two sequenced speakers or may be better understood as the collection of the original stereo pairs of speakers. Measures 5-6 and figure 3-1(5-6) can be seen as the grouping of sequential ordered speakers by groups of three. The final example notated in measures 7-8 and figure 3-1(7-8) are based on grouped sequences of six or can be seen as an equal split of two sounds on opposing sides of the meridian.

When we create harmonies using these sequence-based groupings, their rotations are limited by their sequential grouping structure, as they eventually come back around to their root positions with just a few rotations in one direction depending on their grouping size. The harmony in measures 1-2 can be rotated one time before returning to its root position. Measures 3-4 can rotate three times before returning to its root position. Measures 5-6 can be rotated five times, and measures 7-8 can be rotated 11 times before returning back to their root positions. They kind of behave like intervals within the 12-tone system that can only be used so many times before infinitely looping (i.e. tritones, major thirds, minor thirds).

The first attempts at developing a formula to define these harmonies were based on an idea akin to, if  $S_x + S_y = 13$ , either  $S_x = A$  and  $S_y = B$ , or  $S_y = A$  and  $S_x = B$  must be true ( $A = 111$  and  $B = 222$  in these cases). However, in order for a proper duo to be identified, there are other rules which must be added to this formula to make it functional. The sequence-based duos are determined by the following rules:

$$A:B (S_1-S_{12}) = 1:1$$

$$A:B (S_1-S_6) = B:A (S_7-S_{12})$$

$$\text{If } S_X + S_Y = 13, \text{ then } X = A \text{ and } Y = B \text{ or } X = B \text{ and } Y = A$$

While these groupings are somewhat easier to conceptualize by applying sequential patterns to the numbering system, this first formula was inspirational for finding alternative ways to equally divide the notation and array into additional equal halves by way of calculation. Like those harmonies listed in figure 3-1, they also

represent in an inversely symmetrical fashion across the meridian. These are listed in figure 3-2.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End	
1	111	111	111	111	111		111						1000		2
2						222		222	222	222	222	222			2
3	111	111	111	111			111	111							2
4					222	222			222	222	222	222			2
5	111	111	111	111		111		111							2
6					222		222		222	222	222	222			2
7	111	111	111			111		111	111						2
8				222	222		222			222	222	222			2
9	111	111	111		111	111			111						2
10				222			222	222		222	222	222			2
11	111	111	111		111		111		111						2
12				222		222		222		222	222	222			2
13	111	111				111		111	111	111					2
14			222	222	222		222				222	222			2
15	111	111			111		111		111	111					2
16			222	222		222		222			222	222			2
17	111	111		111	111		111			111					2
18			222			222		222	222		222	222			2
19	111	111		111		111		111		111					2
20			222		222		222		222		222	222			2
21	111					111		111	111	111	111				2
22		222	222	222	222		222					222			2
23	111				111		111		111	111	111				2
24		222	222	222		222		222				222			2
25	111			111	111		111			111	111				2
26		222	222			222		222	222			222			2
27	111			111		111		111		111	111				2
28		222	222		222		222		222			222			2
29	111		111	111		111		111			111				2
30		222			222		222		222	222		222			2

Figure 3-2 – Additional Sequence Based Duos (open texture)

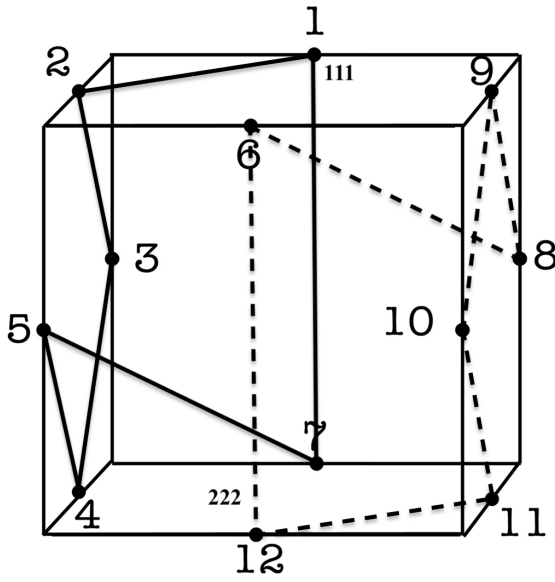


Figure 3-2(1-2)

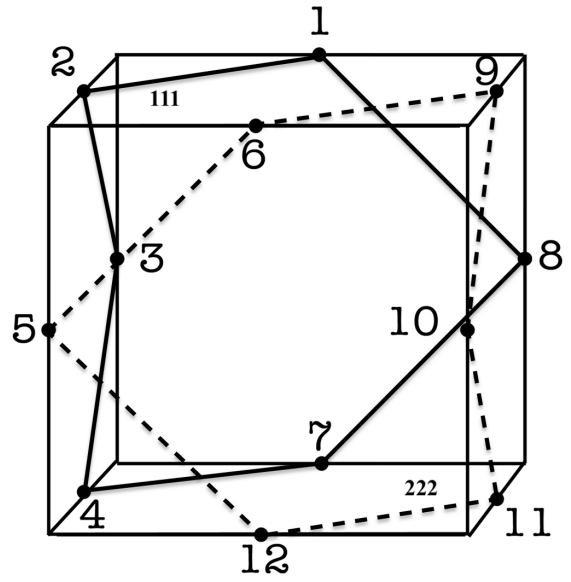


Figure 3-2(3-4)

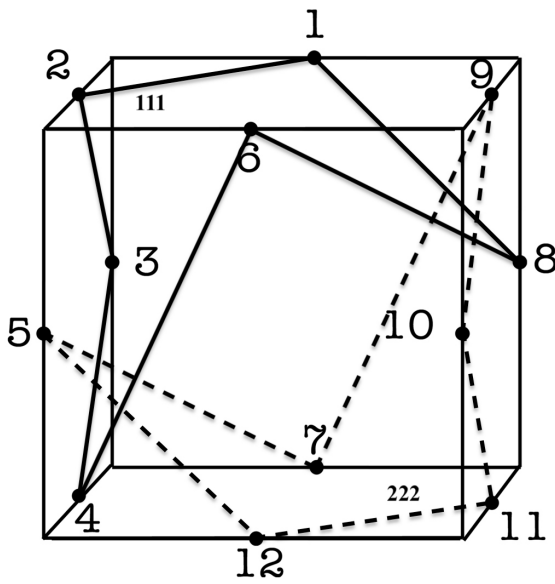


Figure 3-2(5-6)

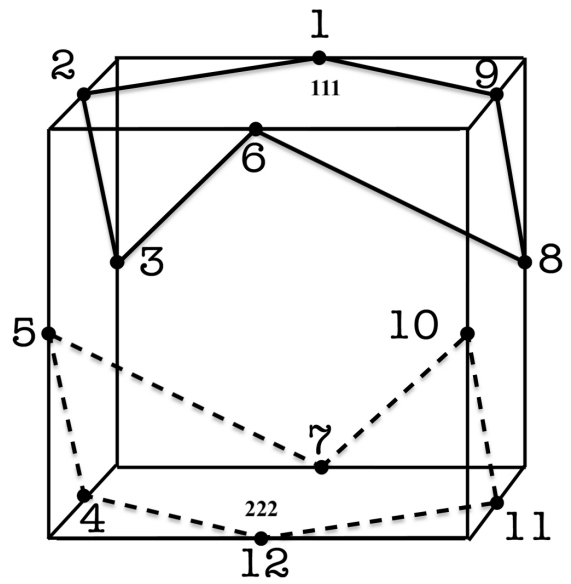


Figure 3-2(7-8)

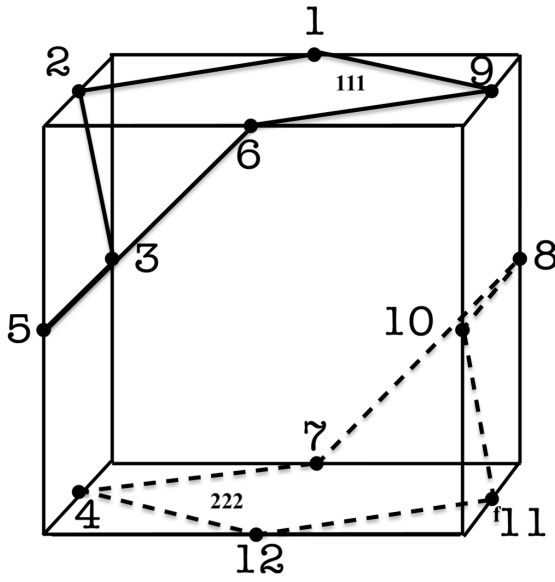


Figure 3-2(9-10)

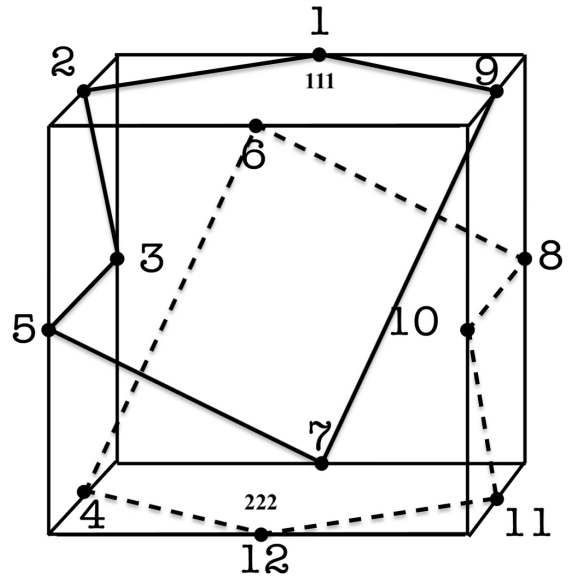


Figure 3-2(11-12)

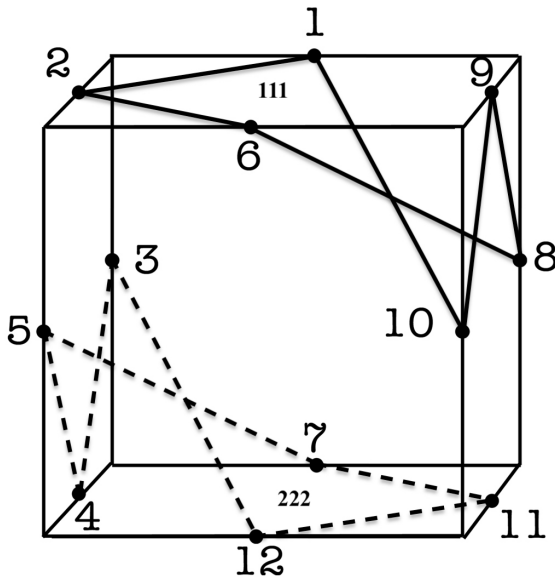


Figure 3-2(13-14)

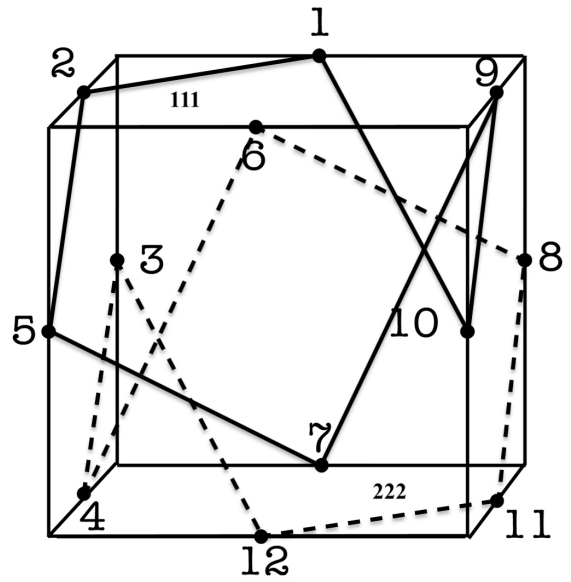


Figure 3-2(15-16)



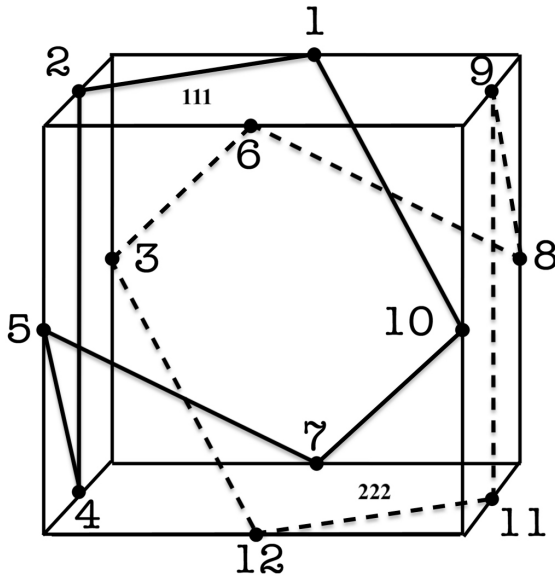


Figure 3-2(17-18)

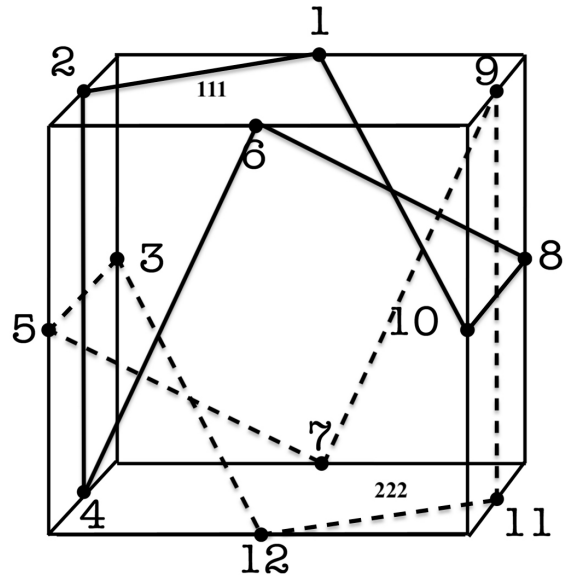


Figure 3-2(19-20)

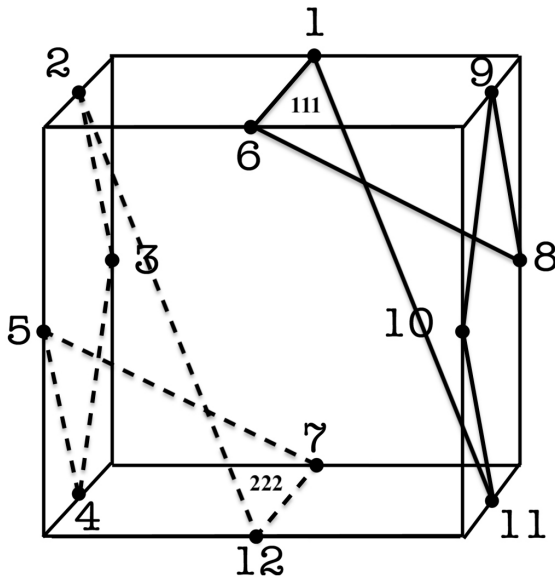


Figure 3-2(21-22)

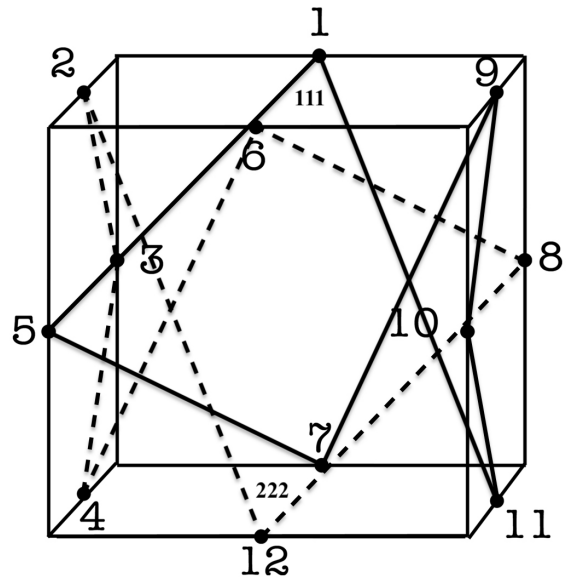


Figure 3-2(23-24)

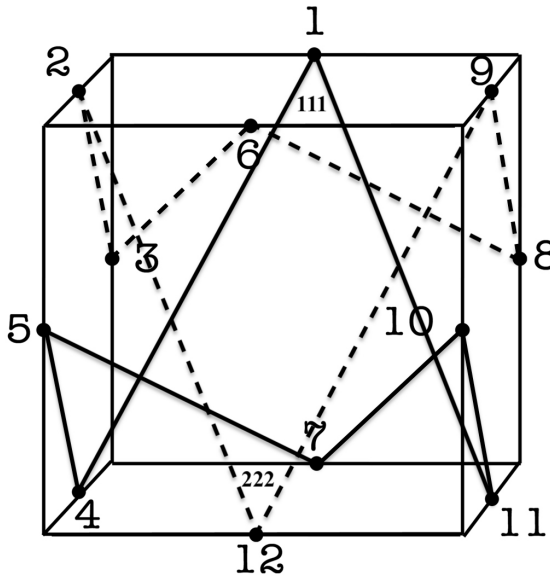


Figure 3-2(25-26)

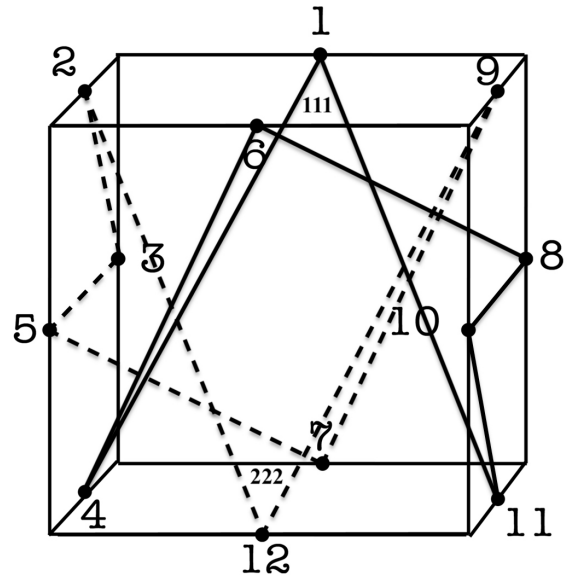


Figure 3-2(27-28)

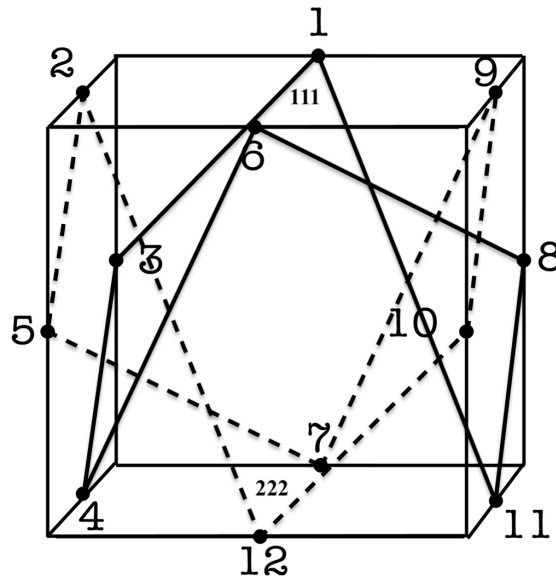


Figure 3-2(29-30)

While some of these configurations are complex, note how they consistently produce exact duplications of each other, just physically rotated around to the opposite side of the array. It is important to understand that the lines don't represent anything

more than those speakers that are grouped together. They are only used to illustrate whether a proposed harmony actually produces spatially symmetrical results when applied to the array.

Like their counterparts in figure 3-1, the duos listed in figure 3-2 can be rotated for additional compositional diversity, but these additional sequence-based duos created a new relationship to the meridian that allow for another way to alter their form while maintaining their symmetrical properties. If we take any of these types of duos in figure 3-2, we can flip them horizontally around the meridian while maintaining their order, just represented in the opposite direction. I refer to these alterations as “inversions.” Due to the sequential groupings used to define the harmonies in figure 3-1, any inversion of these duos will only create an already defined rotation. However, due to the irregular, but still symmetrical, groupings created by extending the formula into the possibilities listed in figure 3-2, inverting them around the meridian will create an alteration not achievable by simple rotation. These will be much more visible when we visit the quartets.

So far, we can alter harmonies by rotating and inverting them, but there are still other ways to diversify compositional intentions. For example, open textured harmonies can be played in any order. In my example figures they are playing in order of their values, but this is simply for comprehension. When we move into trios, quartets and sextets, this will also be true. As long as all parts of an open textured harmony are played an equal amount of times, their order of play does not affect their symmetrical value. I refer to this process as “re-sequencing.”

Since time was taken to create an array with perfect 360° symmetry regarding the speaker numbering system, and since the sum of the speaker identification numbers could be used to divide the array with perfect divisors of 78 ( $\Sigma \text{Sp.1-Sp.12} = 78$ ), this created a situation where one could theoretically create additional duo harmonies simply by dividing the number 78 in half, and then defining speaker groups where the sums of the speaker identification numbers are equal to each other. In other words, if we group speakers 1,12,6,7,3,10 and speakers 2,11,9,4,8,5, and since  $1+12+6+7+3+10 = 39$  and since  $2+11+9+4+8+5 = 39$ , these groupings will create another form of duo harmony. While this may not be as easily seen upon visual inspection of the speaker array or numbering system, this creates an interesting visual representation within the notation system, by showing as a perfect line of symmetry across the meridian. Whereas the examples listed above represented a type of inverse symmetry across the graphical notation, this approach creates a type of mirror image symmetry instead. The rules were similar to those used to define sequence-based duos, with one alteration:

$$A:B (S_1-S_{12}) = 1:1$$

$$A:B (S_1-S_6) = B:A (S_7-S_{12})$$

$$\text{If } S_X + S_Y = 13 \text{ then either } X = Y = A \text{ or } X = Y = B$$

By applying these simple rules across all twelve speakers we can calculate all of the additional possibilities notated in figure 3-3. This represents all of the possible ways to create mirror image symmetry along the meridian with two unique sounds that are not rotations or inversions of previously defined duos. Several of the possibilities discovered with this set of rules were instances of such overlap. For the remainder of this chapter,

care will be taken to avoid double-listing any rotations/inversions of previously defined harmonies. This should inspire composers to make use of all of the alterations while trying to exploit the most symmetrical possibilities, as rotations/inversions not listed do not have any less value with regards to symmetry-based harmony and their root position counterparts. All of the associated array mappings are listed as variants of figure 3-3. As I refer to those duos in figures 3-1 and 3-2 as sequence-based, I refer to those in figure 3-3 as summation-based. This label is affected by the order by which I chose to define these harmonies, as in some cases, a sequence-based harmony can become a summation-based harmony via rotation, and vice-versa. It is up to the composer to decide which of the forms are being used and for what purpose. Harmonies that can be seen as both sequence-based and summation-based depending on their rotation or inversion can be compared to the traditional concept of subjective enharmonic interpretation.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End	
<b>1</b>	111	111			111			111			111	111	1000		<b>2</b>
<b>2</b>			222	222		222	222		222	222					<b>2</b>
<b>3</b>	111	111				111	111				111	111			<b>2</b>
<b>4</b>			222	222	222			222	222	222					<b>2</b>
<b>5</b>	111		111	111					111	111		111			<b>2</b>
<b>6</b>		222			222	222	222	222			222				<b>2</b>
<b>7</b>	111		111			111	111			111		111			<b>2</b>
<b>8</b>		222		222	222			222	222		222				<b>2</b>
<b>9</b>	111			111		111	111		111			111			<b>2</b>
<b>10</b>		222	222		222			222		222	222				<b>2</b>
<b>11</b>	111				111	111	111	111				111			<b>2</b>
<b>12</b>		222	222	222					222	222	222				<b>2</b>

Figure 3-3 – Root Position Summation Based Duos (open texture)

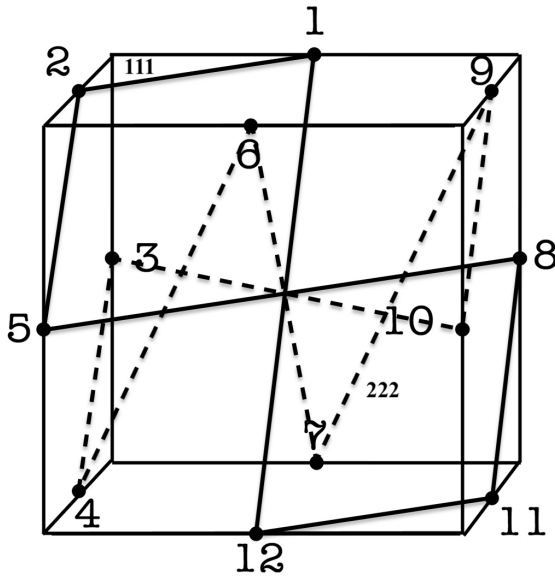


Figure 3-3(1-2)

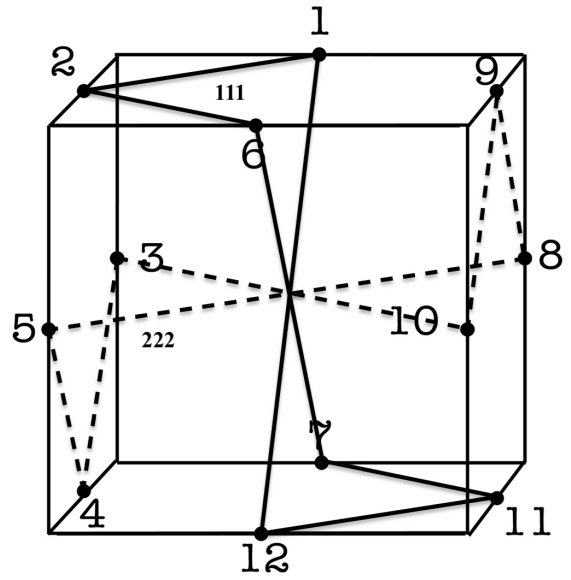


Figure 3-3(3-4)

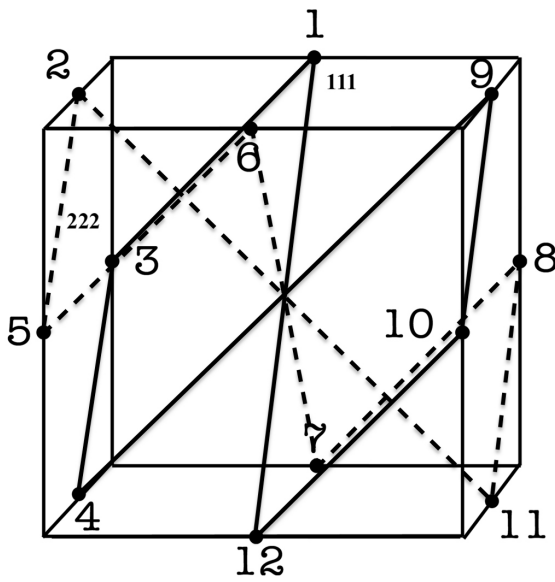


Figure 3-3(5-6)

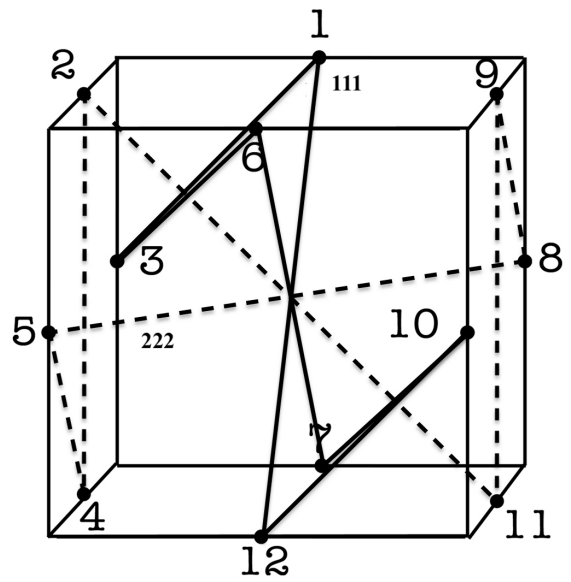


Figure 3-3(7-8)

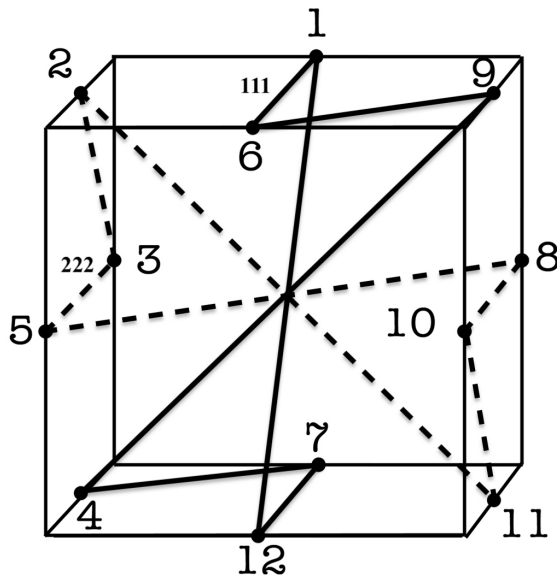


Figure 3-3(9-10)

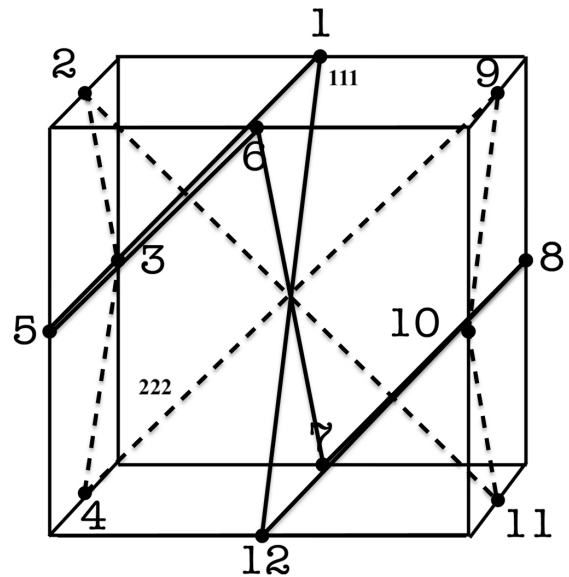


Figure 3-3(11-12)

If you look ahead in this chapter, you will find the rest of the symmetrical harmonies outlined in the same manner as we are covering the duos now. The quartets and sextets were functional in using their forms to verify if my methods were comprehensively covering all of the duo possibilities, as they could be subdivided to find further duos not discovered by the formulas used so far. I refer to harmonies discovered via this method as “compactions.” Compactions are a method of seeing a harmony by means of a different harmony with more voices, by combining its equal subdivisions to create new equal divisions. A simple formula to demonstrate this commutative property would be if  $a = b = c = d$ , then  $a + b = c + d$ ,  $a + c = b + d$ , and  $a + d = b + c$ . Almost all of these possibilities were met by figures 3-1, 3-2, and 3-3, but this compaction method demonstrated two more ways to create additional duos that did not follow the sequence-based or summation-based rules.

In figure 3-9(1-4) If we pair groups 1,2,3 and 5,4,12 you will subsequently have to create the other pair from 6,9,10 and 7,8,11. If you line these up as duo pairings on the notation, you then got a pair out of 12,1,2,3,4,5 and 6,7,8,9,10,11, respectively. This is a perfect example of a repeat by rotation. However, you can still obtain a different duo from Figure 3-7(9-12) by pairing 1,2,3 with either 6,9,10 or 7,8,11. In this case 1,2,3,6,9,10 and 4,5,12,7,8,11 as well as 1,2,3,7,8,11 and 4,5,12,6,9,10 are both additional equal duo-based harmonies. Notice how neither of these duos appear to be rotations or inversions of any previously defined root position duo. The same thing can be done for figure 3-9(5-8), however both figures only create two new duo options between them, as two of these duos are inversions of each other and one is a duplicate by rotation.

These duos do follow the ratio rules used to define the other duos, but there does not appear to be any kind of way to define the relationships of the speakers to each other across the meridian. Since they are only visible by means of compaction, I refer to these last options as “compaction-based.” There are many different ways to create something that matches this rule, but as these options were explored, they did not represent with any kind of symmetry around the array. For this reason, they have not been included in this functional list.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
1	111	111	111			111			111	111			1000	
2				222	222		222	222			222	222		
3	111	111	111				111	111			111			
4				222	222	222			222	222		222		

Figure 3-4 – Root Position Compaction Based Duos (open texture)



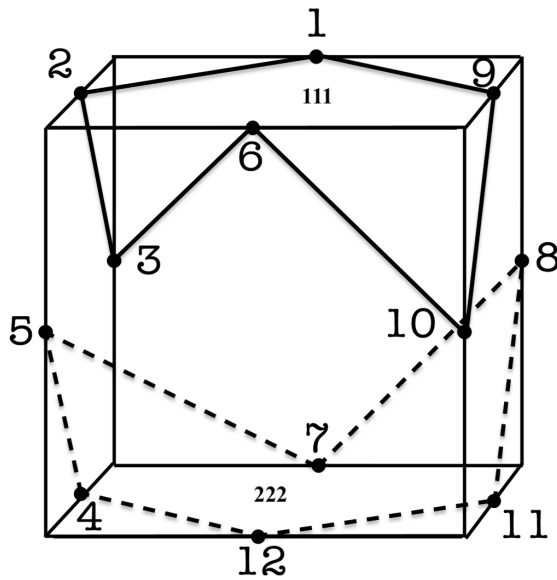


Figure 3-4(1-2)

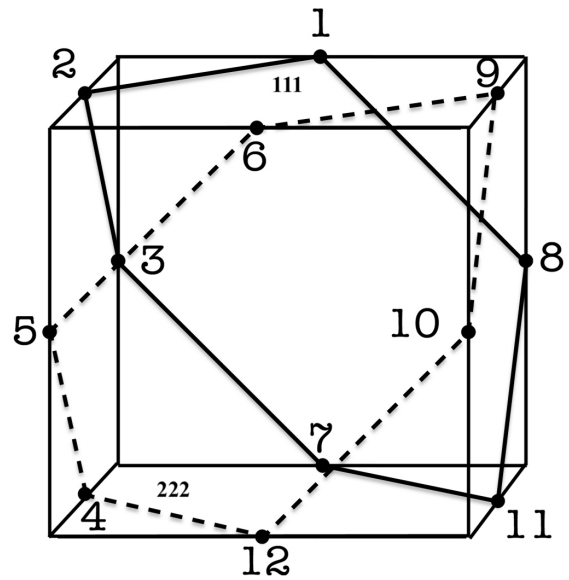


Figure 3-4(3-4)

### The Trios

After outlining the duo possibilities with sequence-based, summation-based and compaction-based systems, the trios were initially approached with the same methods that had proven successful with the previous harmony. The sequence-based methods of equally grouping and counting off were tried first; both of these options are notated in figure 3-5. While these represent nicely as notation, they don't create any kind of physical symmetry around the array. Since the goal of this project was to search for possibilities that symmetrically divide space, it was decided that these would be less useful for testing if people could perceive spatial aural symmetry. They have been listed to demonstrate how the notation and the array must be verified to present actual spatial

symmetry before proceeding with symmetrical composition intentions. These are not used in any of the etudes featured at the end of this project.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
1	111	111	111	111									1000	3
2					222	222	222	222						
3									333	333	333	333		
4	111			111			111			111				
5		222			222			222				222		
6			333			333			333			333		

Figure 3-5 – Root Position Sequence Based Trios (open texture)

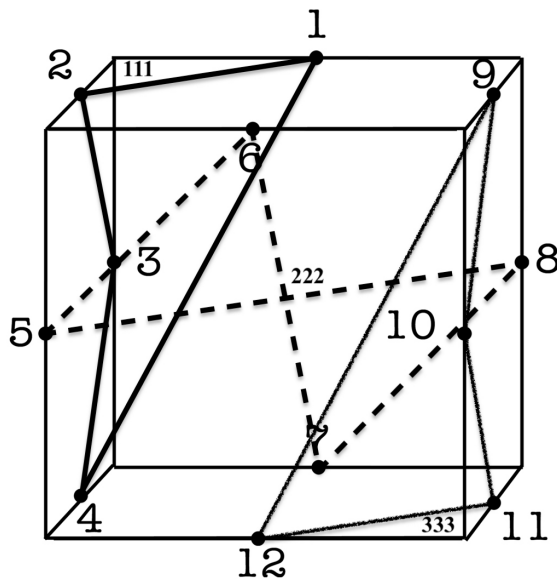


Figure 3-5(1-3)

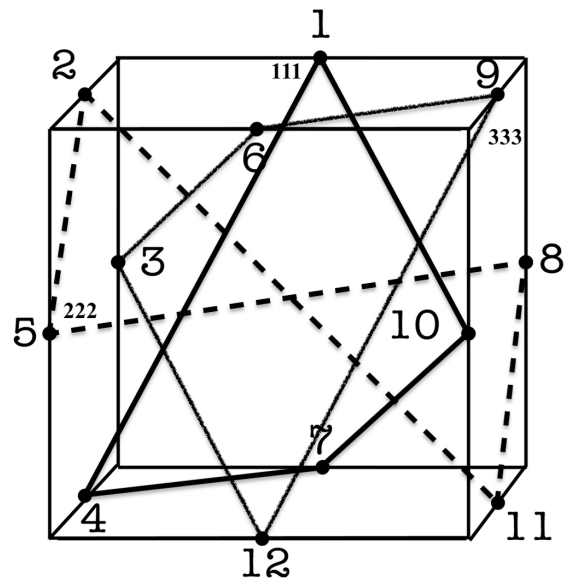


Figure 3-5(4-6)

In figure 3-5(1-3), we see how the sequential division of the notation creates “equal” groupings that don’t exactly behave the same way around the array. Groupings 1,2,3,4 and 9,10,11,12 are similar, but they don’t appear too much like 5,6,7,8. This also goes for figure 3-5(4-6) as well, as 1,4,7,10 appears equal to 3,6,9,12, but neither are like 2,5,8,11. After realizing that the sequence-based method would not yield positive results

with the trio, attention was then given to the summation based method with the hope that it would be more functional beyond the duo form.

Concerning the organizing of speakers into three equal groups of four, there was an immediate solution that became apparent when first building the array. In figure 3-6 we see the outline of this symmetrical form, where four speaker points equally represent the three planes of dimensional space. This configuration also meets the summation rule explained above. That is,  $1+6+7+12$  and  $2+9+11+4$  and  $3+5+8+10$  all add up to 26.

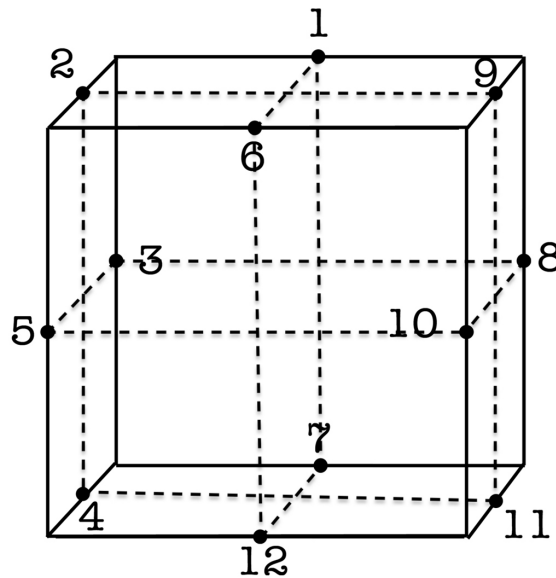


Figure 3-6 – One trio mapping (the summation rule applies)

If we take this formula,  $S_a + S_b + S_c + S_d = 26$ , we can then extract other trios that meet this variation on the summation rule. Although not as visibly symmetrical as the trio shown in figure 3-6, we can also conclude that something like 3,7,6,10 and 1,5,8,12 and 2,4,9,11 are another example of a summation-based trio. Like their duo counterparts, these forms will also represent as mirror image across the meridian. These

are listed in figure 3-7 representing the various ways that the notation can be symmetrically divided down the meridian with three unique sounds (111/222/333). Measures 40-42 represent figure 3-6 above.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End	
1	111	111									111	111	1000		3
2			222	222					222	222					
3					333	333	333	333							
4	111	111									111	111			3
5			222		222			222		222					
6				333		333	333		333						
7	111	111									111	111			3
8			222			222	222			222					
9				333	333			333	333						
10	111		111							111		111			3
11		222		222					222		222				
12					333	333	333	333							
13	111		111							111		111			3
14		222			222			222			222				
15				333		333	333		333						
16	111		111							111		111			3
17		222				222	222				222				
18				333	333			333	333						
19	111			111					111			111			3
20		222	222							222	222				
21					333	333	333	333							
22	111			111					111			111			3
23		222			222			222			222				
24			333			333	333			333					
25	111			111					111			111			3
26		222				222	222				222				
27			333		333			333		333					
28	111				111			111				111			3
29		222	222							222	222				
30				333		333	333		333						
31	111				111			111				111			3
32		222		222					222		222				
33			333			333	333			333					
34	111				111			111				111			3
35		222				222	222				222				
36			333	333					333	333					
37	111					111	111					111			3
38		222	222							222	222				
39				333	333			333	333						
40	111					111	111					111			3
41		222		222					222		222				
42			333		333			333		333					
43	111					111	111					111			3
44		222			222			222			222				
45			333	333					333	333					

Figure 3-7 – Root Position Summation Based Trios (open texture)

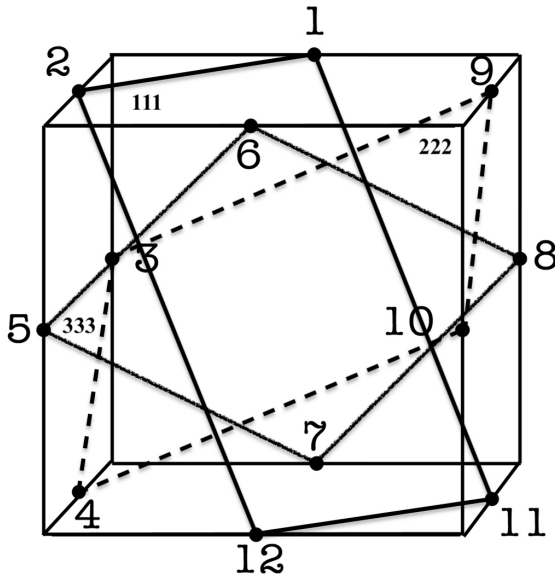


Figure 3-7(1-3)

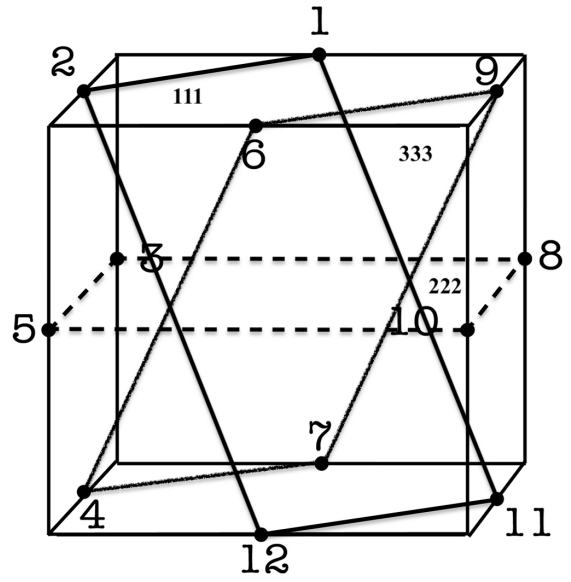


Figure 3-7(4-6)\*

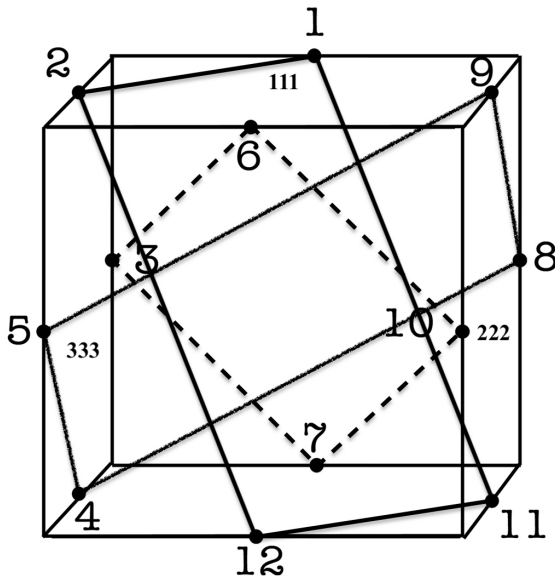


Figure 3-7(7-9)

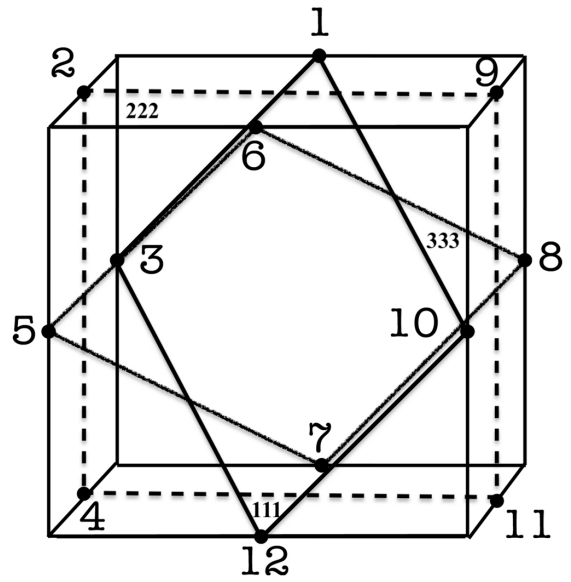


Figure 3-7(10-12)\*

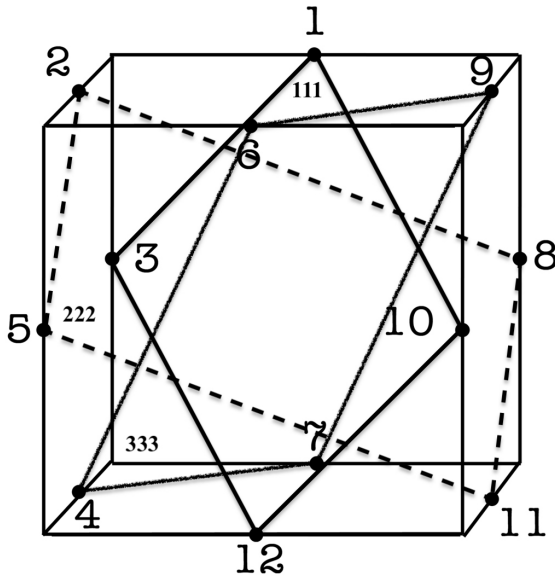


Figure 3-7(13-15)

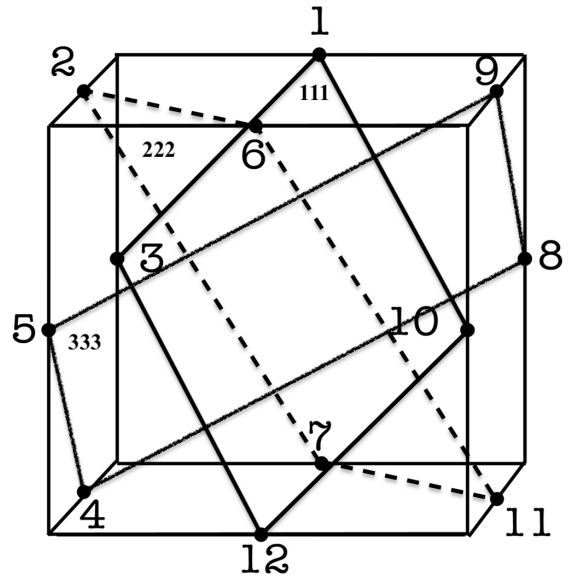


Figure 3-7(16-18)

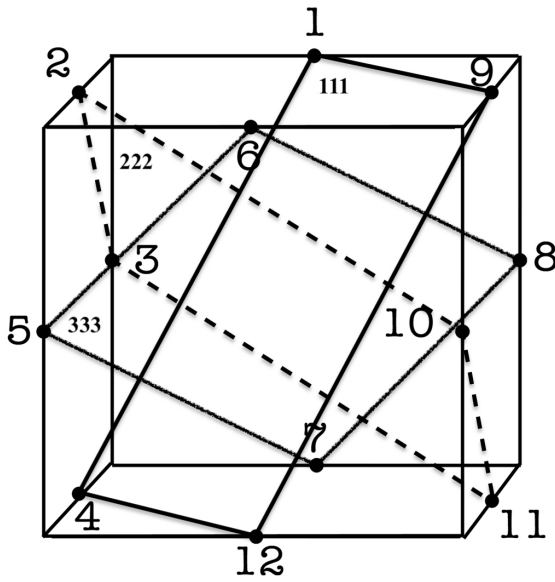


Figure 3-7(19-21)

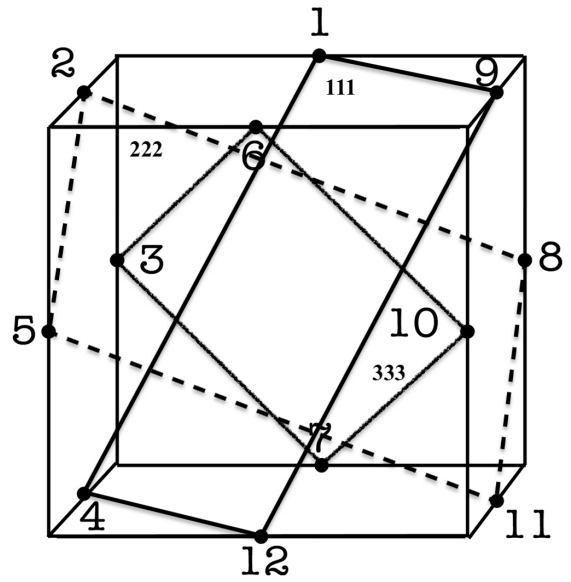


Figure 3-7(22-24)

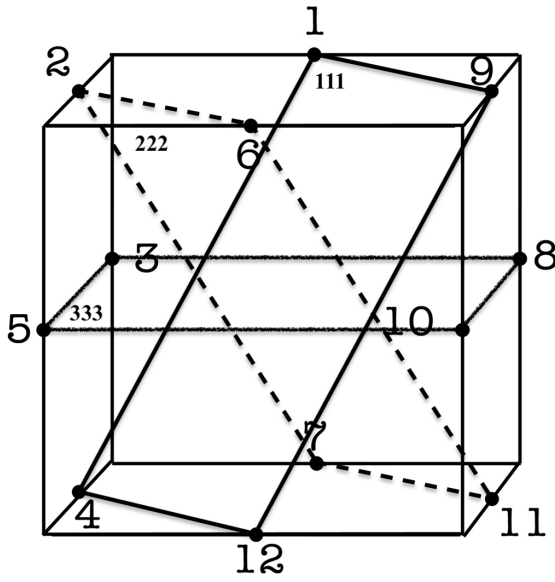


Figure 3-7(25-27)\*

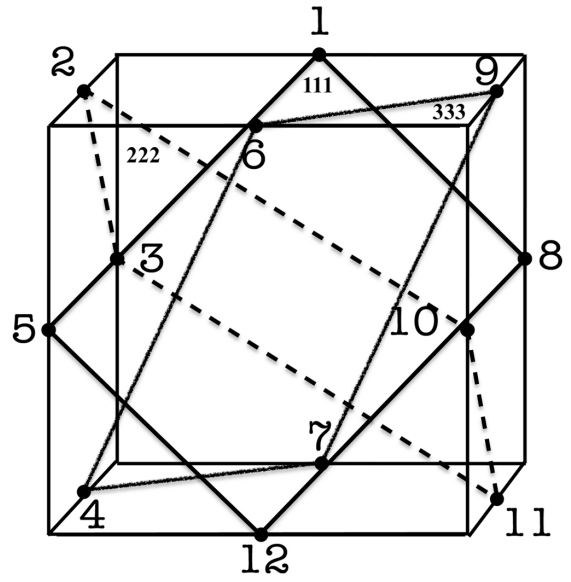


Figure 3-7(28-30)

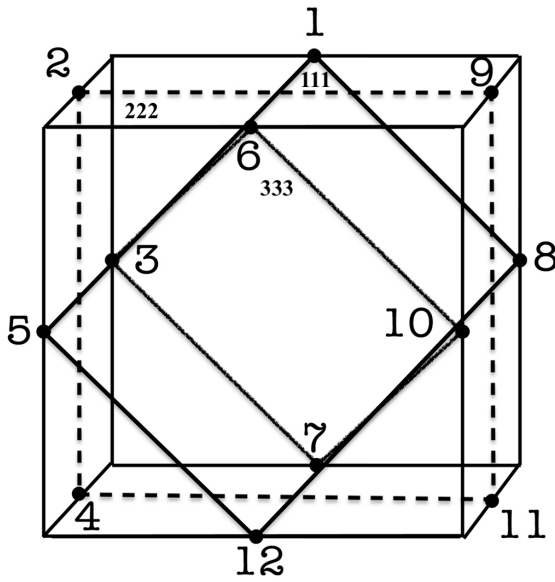


Figure 3-7(31-33)\*

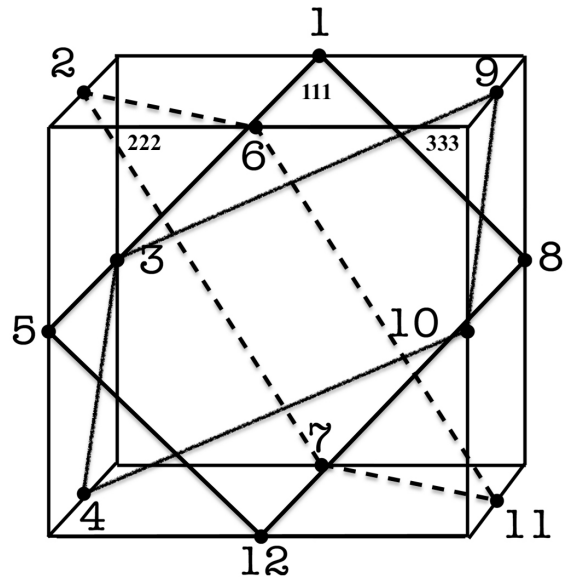


Figure 3-7(34-36)



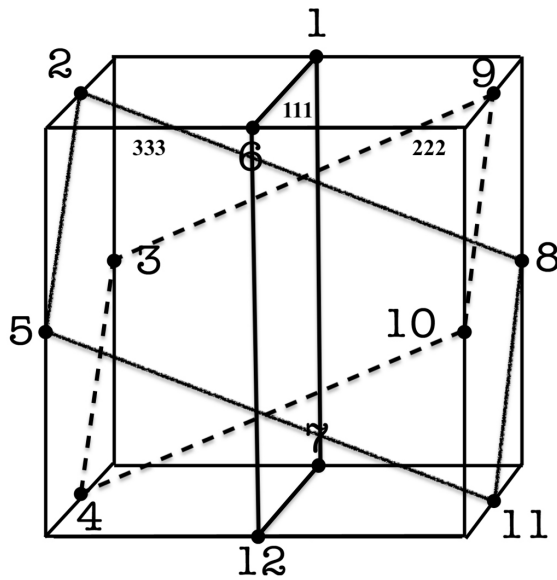


Figure 3-7(37-39)\*

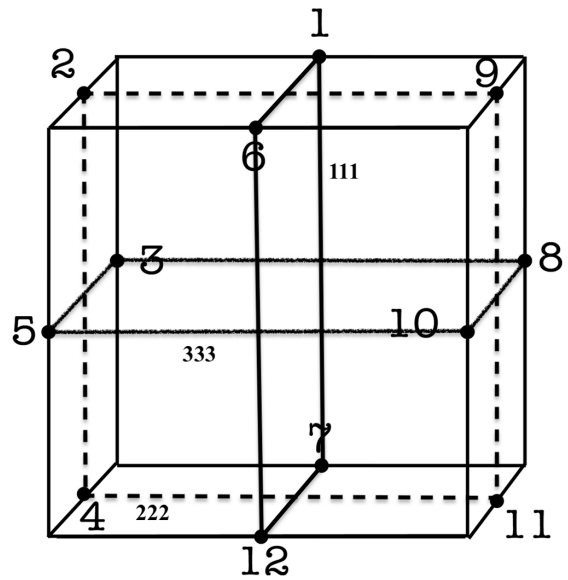


Figure 3-7(40-42)

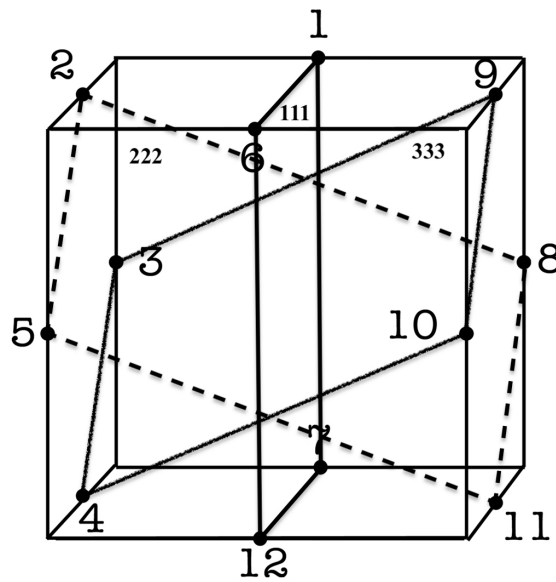


Figure 3-7(43-45)\*

The summation rule appears to be the ONLY way to find true trio-based harmonies, as these examples and their rotations comprehensively cover any ratio-based possibilities adapted to accommodate trios, however, there is something that must be

addressed in order to make sure that a true spatially symmetrical harmony is created.

Figure 3-7(40-42) represents the division of the array into the three dimensions of space.

It is the only true trio that can make use of ANY of these groupings, as using these groupings with other groupings will not result in equal groupings of speakers. Those that were calculated but did not result in a proper harmony have been marked with an asterisk. Since they do not represent symmetrically around the array, they will not be used in any of the compositions looking to find a perceivable aesthetic.

Any attempt to create a compaction from sextets must follow the summation rule to achieve spatial symmetry. This is due to a change in the relationship that the trios have to the meridian. Whereas two equal groupings of six-speakers had multiple ways to be evenly spread across the two halves of the notation (5:1, 4:2 and 3:3), three equal groupings of four speakers cannot be further equally divided into two equal parts (2:2:2 is the only available ratio). However, the addition of another voice does allow for an extra depth of composing with open textured harmonies and implementing the re-sequencing method.

The added voice of the trio helped make it more apparent that a perfectly backwards re-sequenced summation-based harmony can play simultaneously with its root position counterpart without any kind of “one speaker/two frequency interference.” This is also true of a duo that follows the summation rules. When a re-sequencing is in the exact opposite order of its original self, I refer to this specific alteration as a “horizontal inversion.” Horizontal inversions are much more limited than the traditional inversions discussed thus far, as they can only be used in open textured harmonies and they do not

contribute to the creation of true alterations. However, the ability to overlap them with themselves and run them simultaneously for a listener were inspirational to the compositional process when trying to create effects that made use of open-textured forms in a manner that caused them to be played back in a closed-texture style.

### The Quartets

After realizing the limitations of using the duo methods for identifying the trios, care was taken to make sure that the quartets were given their own unique perspective. The standard summation rule would not apply due to the fact that the number four would not evenly divide into the sum of the speaker values (78). It was also apparent that four equal groups of three speakers would create limitations with the ratio rule, since four unique sounds cannot be equally represented on each side of the meridian. With this in mind, care was taken by starting with the sequence-based approach. These are notated and mapped in figure 3-8.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
<b>1</b>	111	111	111										1000	
<b>2</b>				222	222	222								
<b>3</b>							333	333	333					
<b>4</b>										444	444	444		
<b>5</b>	111				111				111					
<b>6</b>		222				222				222				
<b>7</b>			333				333				333			
<b>8</b>				444				444				444		

Figure 3-8 – Root Position Sequence Based Quartets (open texture)

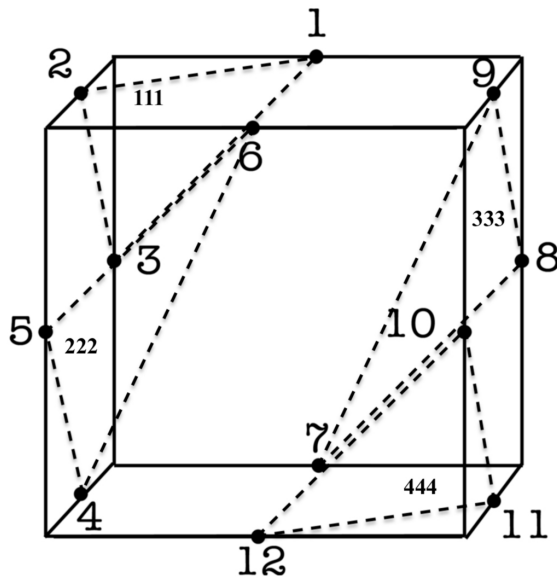


Figure 3-8(1-4)\*

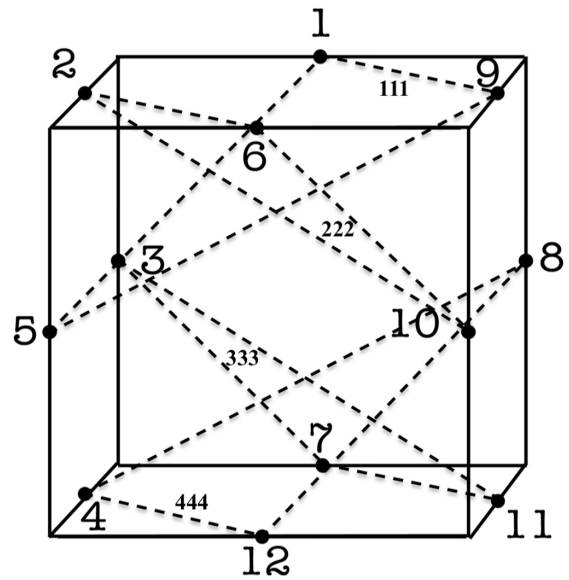


Figure 3-8(5-8)

The ability to identify the different line textures was getting difficult beyond 2-3, so from this point forward, the various harmonies within the mapping arrays for the quartets and sextets will be represented with the same dotted line texture. Hopefully, the matching integer frequencies are noted in a manner so that you can see their relationship to the associated notation example. The harmony outlined in figure 3-8(1-4) does not represent symmetrically around the array so it was relegated to a non-functional status, but the mapping in figure 3-8(5-8) did create four congruent triangles that would overlap symmetrically around potential listeners. This turned out to be unusual for this specific harmony, so this form was used extensively in the etude examples.

With the summation rule out of mind, attention was then focused to the physical array, trying to visually find various ways to create equal quartets. The first two options discovered could be created by essentially sectioning off the four corners of the cube

support structure. These are notated in figure 3-9. After identifying the first option in measures 1-4, this figure was then rotated around the center of the array to find the quartet notated in measures 5-8. Any more rotations in this manner would only lead back to the original groupings, but the emerging triangles in both figures 3-8 and 3-9 were useful guides when trying to find more possibilities that could only be found by visual inspection of the array, without mathematical calculation.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
1	111	111	111										1000	
2				222	222							222		
3						333			333	333				
4							444	444			444			
5	111							111	111					
6		222			222	222								
7			333	333			333							
8										444	444	444		

Figure 3-9 – Root Position Corner Based Quartets (open texture)

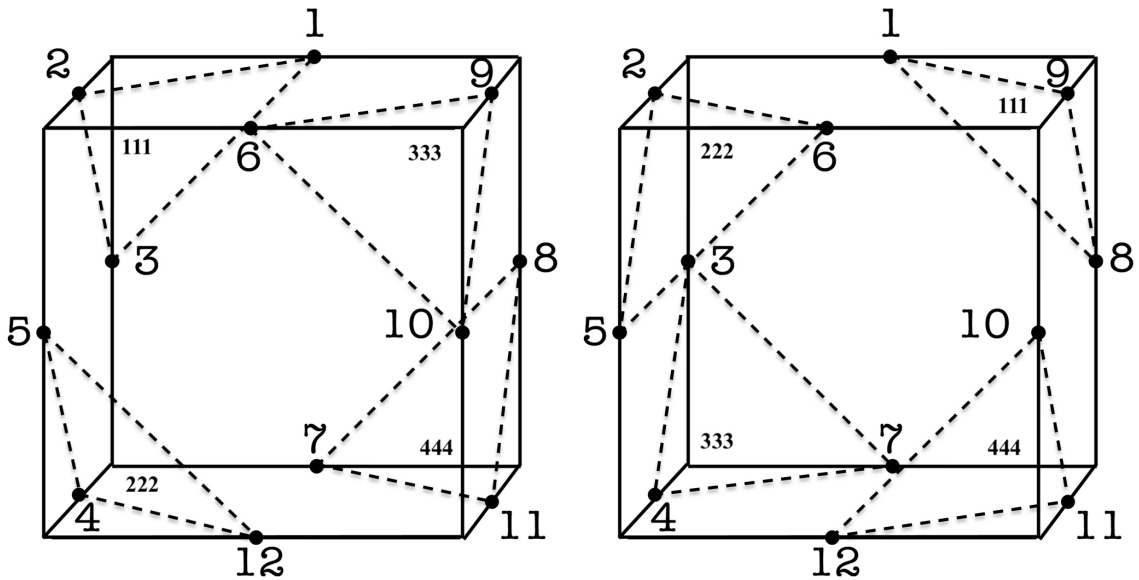


Figure 3-9(1-4)

Figure 3-9(5-8)

Since all examples of quartets would represent as triangles on the array by definition, diagrams were used to begin tracking the various ways triangles could be created out of 3 speakers, that would allow the other 9 speakers to be grouped into three more congruent triangles. By using each of the three-dimensional edges as the vertex point for the 90° angle in a series of isosceles right triangles, six more quartets emerged, with each physical dimension providing two different directions of said triangles. This may be better seen than said, so these quartets are outlined below in notation and array mapping form in the figure 3-10 images. Notice how these don't represent within the notation with much of a symmetrical display, despite their perfect radial symmetry around the array. However, after notating these quartets, it was visually verifiable that all versions of open texture quartets can take advantage of the horizontal inversions described earlier.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
<b>1</b>	111		111					111					1000	
<b>2</b>		222				222			222					
<b>3</b>				333			333				333			
<b>4</b>					444					444		444		
<b>5</b>	111	111							111					
<b>6</b>			222				222	222						
<b>7</b>				333							333	333		
<b>8</b>					444	444				444				
<b>9</b>	111		111				111							
<b>10</b>		222		222	222									
<b>11</b>						333				333		333		
<b>12</b>								444	444		444			
<b>13</b>	111						111	111						
<b>14</b>		222	222	222										
<b>15</b>					333	333						333		
<b>16</b>									444	444	444			
<b>17</b>	111	111				111								
<b>18</b>			222	222	222									
<b>19</b>							333				333	333		
<b>20</b>								444	444	444				
<b>21</b>	111					111			111					
<b>22</b>		222	222		222									
<b>23</b>				333			333					333		
<b>24</b>								444		444	444			

Figure 3-10 – Root Position Triangle Based Quartets

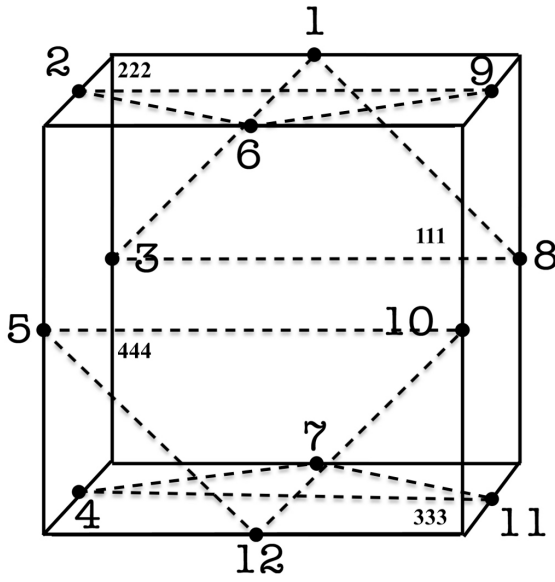


Figure 3-10(1-4)

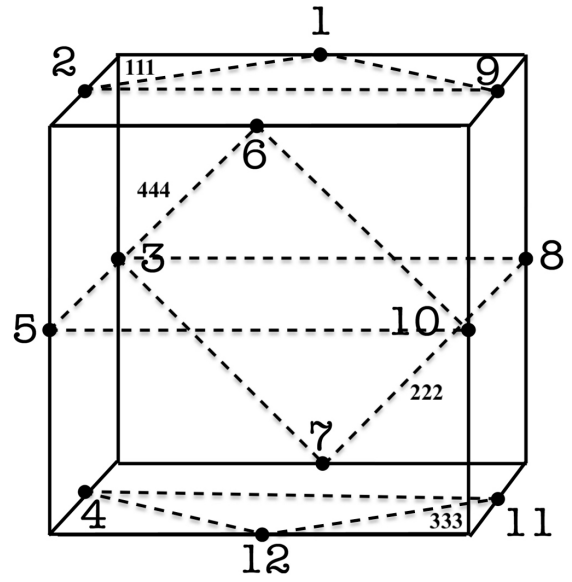


Figure 3-10(5-8)

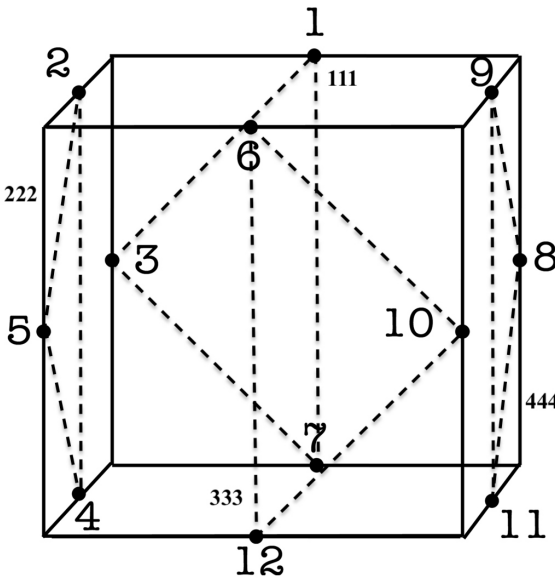


Figure 3-10(9-12)

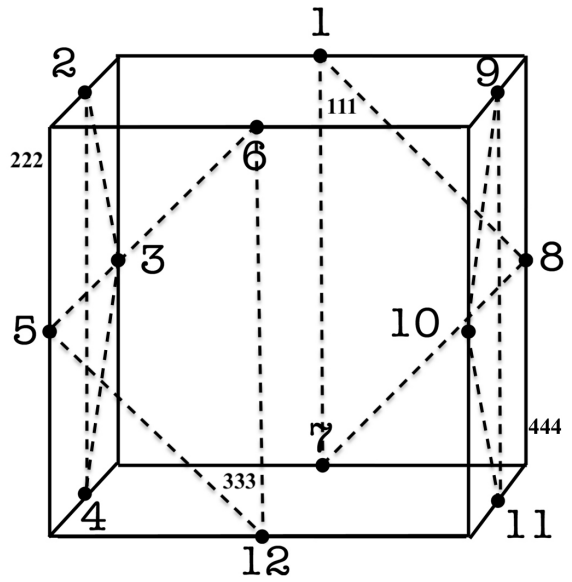


Figure 3-10(13-16)



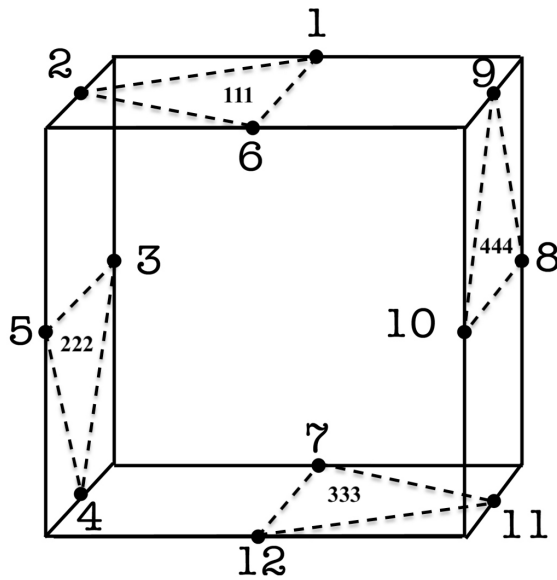


Figure 3-10(17-20)

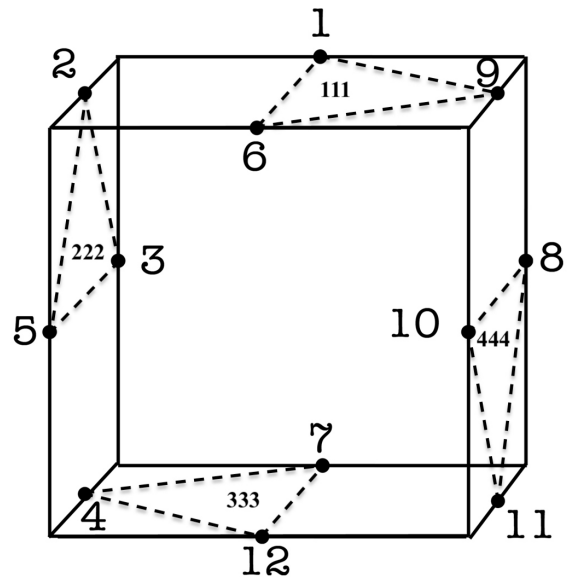


Figure 3-10(21-24)

When initially looking for quartet options, it was deduced that both the summation and ratio-based methods would not work for mathematical reasons. However, during the search for quartet triangles, an inadvertent discovery was made regarding four equal hexagon shapes that represented in four equal angles around the center axis of the array. This could be made by allowing a speaker to play more than one unique sound during a complete iteration. Until this point, all of the speaker groupings created by different harmonies have been completely independent of each other with only one speaker belonging to only one grouping. By changing this tendency and allowing for more than one sound per speaker, per harmony, a new form emerged that allowed for an additional type of quartet. By definition, this specific harmony can only be used in an open texture, but it was a unique non-triangular form that created a different type of quartet with a strong relationship to the meridian. Despite having a relationship to the

summation rule, this quartet can be both inverted and rotated for diverse compositional purposes.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
<b>1</b>	111					333	444					222	1000	
<b>2</b>			111		222			444		333				
<b>3</b>		222		111					444		333			
<b>4</b>	222					444	333					111		
<b>5</b>			333		444			222		111				
<b>6</b>		333		444					111		222			

Figure 3-11 – Root Position Hexagonal Based Quartet (open texture)

This is the only harmony notated among these comprehensive notation examples capable of using two speakers per grouping in such a perfectly symmetrical manner. In some respects, it can be seen as summation-based, sequence-based, and visually recognizable, with a strong visual relationship to the meridian, the notation and the array, respectively. Since there were so many overlapping points in this particular array mapping, different line textures and arrowheads were used to outline how it would behave around the array. This is why it appears so different from any of the previously defined harmonies. The expanded picture and the key in the middle are meant to help make sense of the associated notational content.

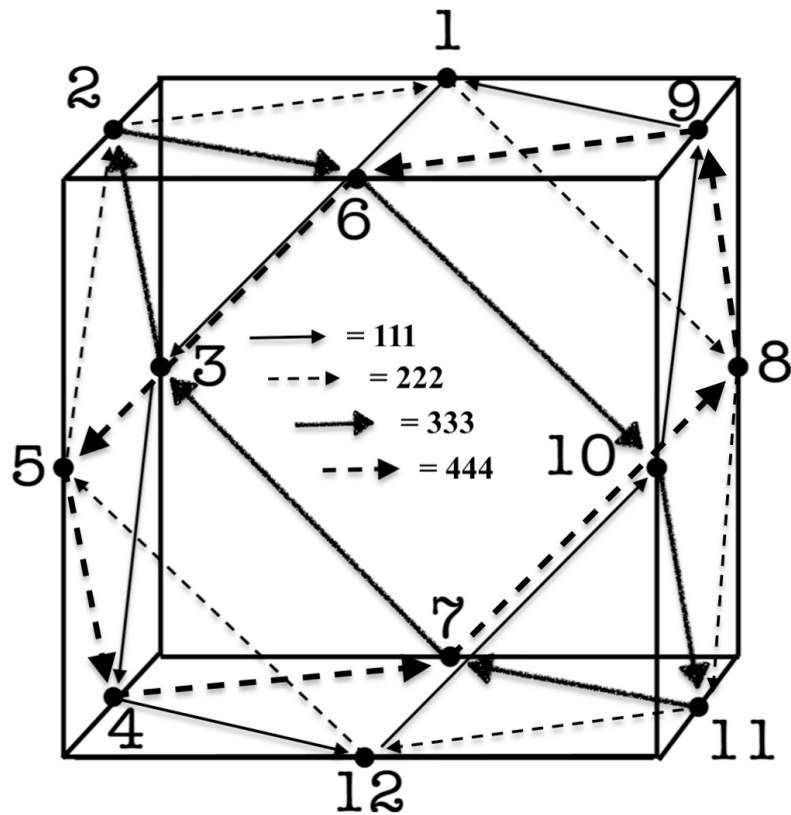


Figure 3-11(1-6)

This quartet may seem strange in comparison to the other previously defined harmonies, but the different texture of lines and arrowheads should help make sense of this unique arrangement of speaker groupings. The goal is to illustrate that 1,3,4,12,10,9 (111) and 12,5,2,1,8,11 (222) and 6,10,11,7,3,2 (333) and 7,8,9,6,5,4 (444), are all symmetrically equal, but only when played in that order. This means that this form can be re-sequenced like all polyphonic harmonies, but the order must be strictly maintained. That is, if measure 6 moved up to measure 1, measures 1-5 would all need to equally move down by one measure each, essentially keeping the order the same, just starting from a different point within the quartet harmony presentation. In addition to this visual

symmetrical representation, you can see that this arrangement completes the summation rule, as the sum of all four of these speaker arrangements all equal to 39, and the example integers used all have a 1:1 ratio across the meridian upon one complete iteration. This would complete the list of possible quartet harmonies.

### **The Sextets**

The last form that needed to be notated and mapped is the sextet. Requiring six equal groupings of two speakers, the use of the sequence and summation-based options was limited to only two symmetrical harmonies. These are notated in figure 3-12. The only way to use a sequence which results in physically symmetrical results is by pairing off the original stereo pairs as shown in measures 1-6. Any other sequence-based sextet layout will result in unequal groupings. Measures 7-12 represent the summation rule, with mirror-image symmetry across the meridian. The need for six equal pair makes this the only summation possible. Both of these options can be rotated, re-sequenced, and/or horizontally inverted. Measures 1-6 can be inverted along the meridian but measures 7-12 cannot be inverted to create any alteration.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
1	111	111											1000	
2			222	222										
3					333	333								
4							444	444						
5									555	555				
6											666	666		
7	111											111		
8		222									222			
9			333							333				
10				444					444					
11					555			555						
12						666	666							

Figure 3-12 – Root Position Sequence and Summation Based Sextets (open texture)

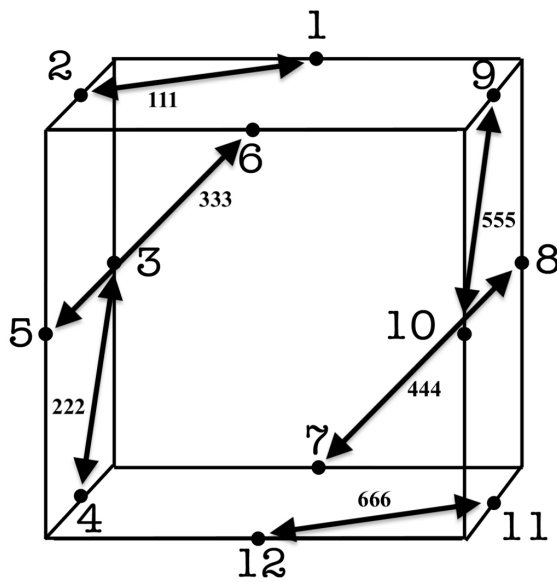


Figure 3-12(1-6)

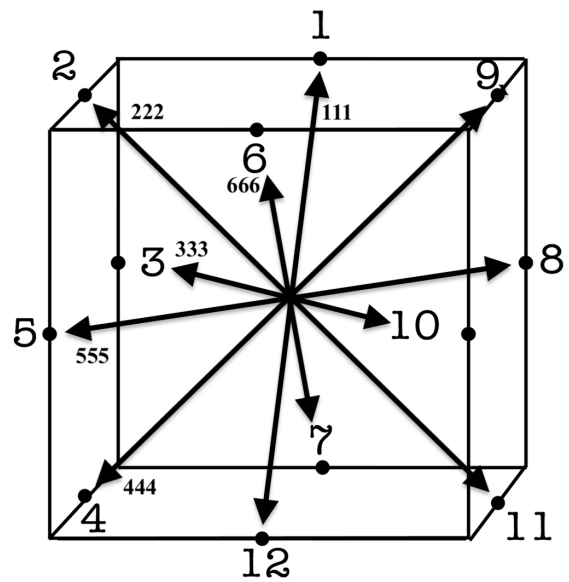


Figure 3-12(7-12)

The associated array mappings demonstrate how well the sextet notation in figure 3-12 represents as a physical symmetry around the speaker array. Figure 3-12(1-6) is the division of the array based on simple L/R pairs. Where we see the summation rule in figure 3-12(7-12) we also see interesting ways to compact things down into less obviously equal duos and trios. Remember, these have already been outlined and

described as possible alterations of already defined root position harmonies, they just may be easier to visualize in this manner. The associated frequencies from the notation example are listed with their groupings to help see which parts of the notation equate to which parts of the array.

After compiling a comprehensive list of root position harmony notation and the mappings of the harmonies related to their notation, time will now be taken to better explain all of the different types of alterations described throughout this chapter. For this demonstration a quartet will be used, since it will be so illustrative of every possible alteration. In figure 3-13 there is an example of a root position quartet that has been rotated, inverted, re-sequenced and horizontally inverted. The arrows on the side are meant to help illustrate the how the alteration was made.

Master	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Tempo	End
1	111	111				111							1000	
2			222	222	222									
3							333				333	333		
4								444	444	444				
5		111	111				111							
6				222	222	222								
7	333							333				333		
8									444	444	444			
9							111				111	111		
10								222	222	222				
11	333	333				333								
12			444	444	444									
13								444	444	444				
14	111	111				111								
15			222	222	222									
16							333				333	333		
17								444	444	444				
18							333				333	333		
19			222	222	222									
20	111	111				111								

Figure 3-13 – Alteration Examples (rotation, inversion, re-sequence, horizontal inversion)

Measures 1-4 represent the root position quartet that will be used to generate and demonstrate all of the alteration possibilities. Measures 5-8 represent the same quartet, only now it has been rotated by one position to the right. The arrows on the side are meant to illustrate how the motion appears on the notation. Measures 9-12 are an example of an inversion. Notice how the original root position form has been vertically flipped along the meridian. Measures 13-16 show a re-sequenced harmony, where the last measures now plays first, and the following measures all play one measure later. Measures 17-20 represent a horizontal inversion. It is important to remember that only open texture harmonies can be re-sequenced and/or horizontally inverted. Sequence-based harmonies tend to have limited rotation options, and summation-based harmonies cannot make use of the inversion (vertical). Alterations allow for compositional diversity while not breaking any of the rules regarding spatial symmetry. There is no difference in the symmetrical value of a harmony if it is in root position or if it has been altered. They allow harmonies to move and reposition themselves around the array.

## Chapter 4

### **The Max MSP Software Patch**

With the speaker array built, the potential symmetries mapped and a completely developed notation system ready to go, the final step in bringing all of these parts together would be the construction of a Max MSP software patch capable of properly integrating these two inventions. Before any coding could begin, the audio interface would need to be selected. A device with at least 12 independent outputs that could connect to a laptop by way of USB or firewire connection were the only essential requirements that needed to be met.

At the beginning of this project, this type of product did not exist. Most USB/firewire capable interfaces maxed out at 8 inputs, and the interfaces that had 12 and over required the installation of a PCI card on the motherboard of a desktop computer. Fortunately, the Cymatic Audio® LP-16 came to the professional audio market just a few months into the development of the speaker array and notation. With 16 independent audio outputs, zero inputs, and nothing more than a USB port for compatibility, an order was placed for this interface as soon it was made available to the public. Since the speakers used in the array were stereo pairs made for the intention of laptop monitoring, adapters would that could convert the (6) stereo 1/8” connectors into (12) independent 1/4” connections would be necessary. A USB power hub was also required so that each pair could be powered by USB connection. Other than this, the physical connecting of the array to the audio interface was a simple matter of matching numbers. The number of the



audio outputs were matched with the assigned numbers of the speakers so that control of the notation and the array mappings would maintain consistency.

Before programming the main patch that would be used for the interpretation of songs composed within the excel notation, a simple test patch was created for the purposes of verifying that everything had been labeled and connected properly. This was done to make sure that a small oversight was not made in the beginning that could lead to some difficult troubleshooting problems down the line. In figure 1 you can see an image of the patch that was programmed for this purpose. It is understood that some readers of this work may or may not be well-versed in the Max MSP programming language. Even those that are familiar with the platform, could be using a different version on a different operating system. For this reason, time will be taken to explain the underlying functions of each of the objects included in all of the patch images provided throughout this chapter.

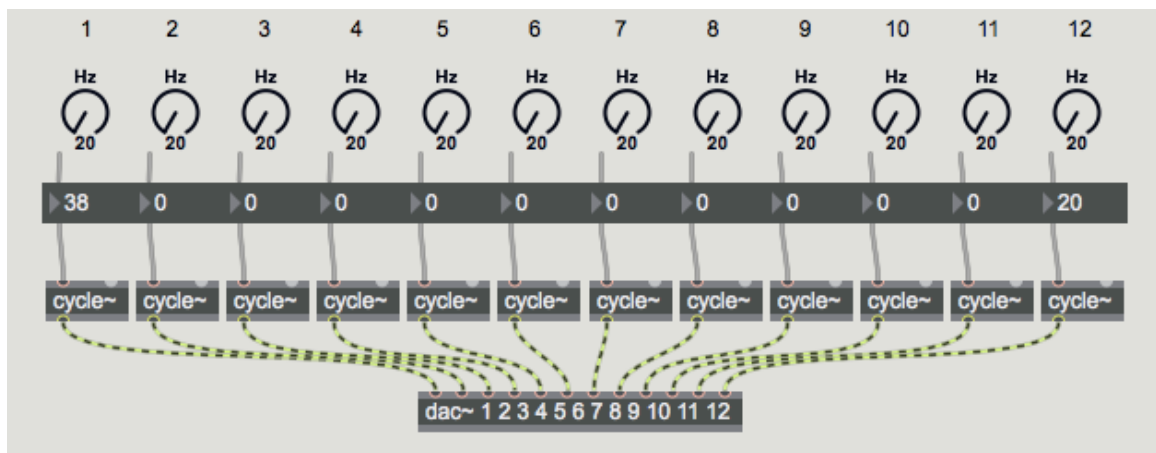


Figure 4-1 – The calibration/test patch

Figure 4-1 is a direct screenshot from the programming construct of Max MSP. The numbers across the top represent speaker number indicators. The dials beneath those

numbers represent all of the frequencies within the human aural spectrum (20Hz-20Khz). The integer boxes beneath the dials are not necessary, but they allow for better visualization of the frequencies being used. The “cycle~” object is a sine wave generator; these 12 are being controlled by their corresponding dial and/or integer box. The sine wave generators are then sent to a “dac~” object that converts digital audio data into analog audio signals. The numbers in this object relate directly to the speakers to which they are connected. This patch is useful for testing if the array is properly labeled and connected, but it is not functional for compositional purposes. However, the fundamental elements of this patch were relevant to the master composition patch that will now be described. It was discovered early on that a master volume and master kill switch would be helpful during the development process. Figure 4-2 illustrates the first objects used in the development of the master patch. Having taken several objects to complete, it has been broken up as it is described.

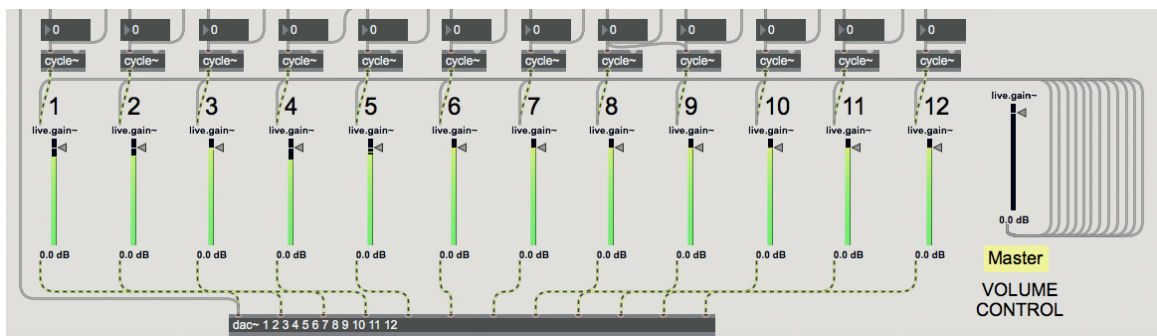


Figure 4-2 – First section of the master patch

In this screenshot we see a lot of similar objects that were used in the calibration patch shown in figure 4-1. In fact, other than the order in which these objects have been placed, the only major change from the calibration patch is the addition of “live.gain”

objects used to control the volume of each individual speaker. The “live.gain” object located to the far-right acts as a master volume as it controls all twelve speakers equally. With this first stage of coding completed, a method to integrate sequential based data extracted from an excel/pages spreadsheet needed to be conceptualized.

There are several objects in the next portion of the patch shown in figure 4-3 that are responsible for making this happen. It is not imperative to understand exactly how every object works; it is just important to understand how the design will impact compositional decisions made within the notation. The yellow and red boxes are “comment” objects that only allow for labeling purposes. The yellow box is meant to remind the composer what the current tempo of the piece is, while the red box provides instructions on how to convert the excel notation data into the “keyed json” format required to translate the data into something readable by Max MSP; this application has no problem at all interpreting data when it is formatted using this method, and the included steps make use of free online tools to maintain accessibility.

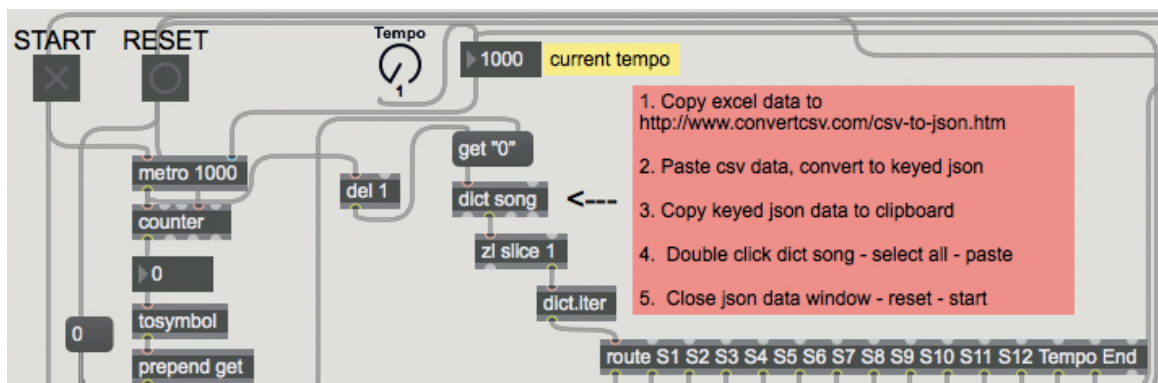


Figure 4-3 – Second section of the master patch

At the top right corner of this view we see the functional “start” and “reset” buttons that are not only important to controlling when the array turns off and on, but are

also used when there is a need to clear out all of the old data from a previously played composition, so that a new piece may be properly loaded. The “metro” object sets the default metronome speed to 1000 microseconds, or 1 second per iteration/measure. This can be dynamically updated throughout the playback of any piece with the data provided by the composer in the tempo column of the excel notation. The “counter” object provides the measure number where the piece is currently playing, and this data is used to obtain the proper sine wave frequencies for each speaker at each specific measure. The integer box after the counter is just a visual display to help keep track of where exactly the piece is currently playing. The “tosymbol” object converts the numerical related data associated with the integer object into symbol-based data, which can then be used to retrieve the data from the table originally created within the spreadsheet notation. Finally, in this chain of objects we see the “prepend get” object which simply places the “get” function in front of the symbol we just created from integer data.

As we move from left to right within this part of the patch, we see an object indicated by “del 1.” This is a delay, that is giving the patch one microsecond to perform all of the functions within the last chain of objects before grabbing the correct sine wave frequency data. The dial and integer box at the top are indicative of the current tempo being played. They also can be used to change the tempo of a piece while it is being played, however, these decisions will be overridden by any new tempo data that comes in from the spreadsheet notation. The next object is a message box that allows for dynamic updates to the corresponding measure. This is a command function that interacts directly with the next object in line. The “dict song” object is what is referred to as a

“dictionary,” and it is the data format within Max MSP that can correspond to a spreadsheet database. Since they are not perfectly compatible, there is a conversion process that is necessary. This conversion process is outlined as a five-step process within the red comment box.

While there are plenty of commercial products available to convert these different types of data in between formats, the goal of attracting new composers to use this form was always an important consideration. This is why the website that is listed in the conversion process was chosen, as it would allow anybody located anywhere in the world to continue working past this translation impediment to completing a new composition. After a piece has been completed within any spreadsheet application functioning as notation, the composer only needs to select and copy the part of the spreadsheet which contains the measure, speaker, tempo and end data, and then use a browser to visit the included website. Next, the copied excel data needs to be pasted into the window/cell where it is required on the website. The click of a button then allows one to convert the excel/csv data into Max MSP/keyed json data. This new keyed json data can now be copied to the clipboard and returned back to the master patch. At this point, one would need to double click the “dict song” dictionary object, open up the data table, and replace all of the data contained within the object with all of the newly converted json data. After closing this data table, a new song is loaded and ready for playback. A click of “reset” will dump any old data and to take the playback to the beginning; a click of “start” will begin the playback.

Since the dictionary object functions much like a data table, it still needs to be unpacked in a manner which allows for the routing of data to different objects. These are what the “zl slice” and “dict.iter” objects are doing in between the dictionary and the routing object. They are organizing a two-dimensional table of data into a single-file line of information that can be given an identifier which allows for direct control of where multiple columns of information are sent. This information is then sent to the “route” object, which routes the properly identified piece of information to the place where it needs to be used for interpretation. In the case of the (speaker) numbers 1-12, that data is the integer representing the frequency to be played as a sine wave through the proper corresponding speaker. The “tempo” channel routes the tempo data to the dynamic tempo indicator described above. This is where any tempo changes made in real time will be updated by the compositional input composed into the original notation. The end channel triggers the end of the piece when the final “0” comes through from that portion of the notation. These two portions (figure 2 and 3) of the patch make up the majority of the functionality. The last portion of the patch is outlined in figure 4-4.

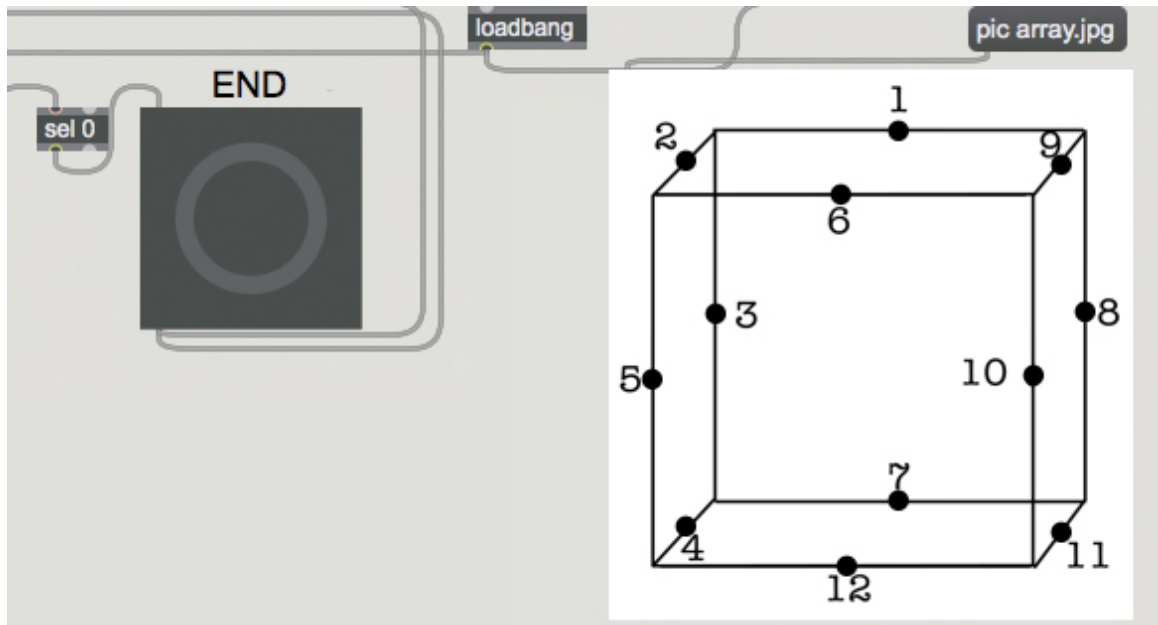


Figure 4-4 – Third section of the master patch

The final portion of the master patch contains the portions that control when the piece of music ends, as well as a visual display of the array with points that light up to indicate when a speaker is being used. On the far left, the “sel 0” is waiting to receive that “0” that needs to be placed at the end of a piece of notation in the “end” column to stop a piece. When that number is detected, the “end” bang is triggered, and the piece is told to stop when it should. On the right side of this image, we see an image of the array that has been programmed to help a composer see which speakers are being used. The “loadbang” and “pic array.jpg” objects are what cause this specific image to load when every time this patch is launched. If a speaker is not playing anything, the dot remains black, but when any frequency is detected for any speaker, the black dot turns yellow. This is more of a guide and doesn’t have much of an impact on the array. It can be

helpful to watch the playback to verify the proper use of open textured symmetries that should present themselves visually around the array.

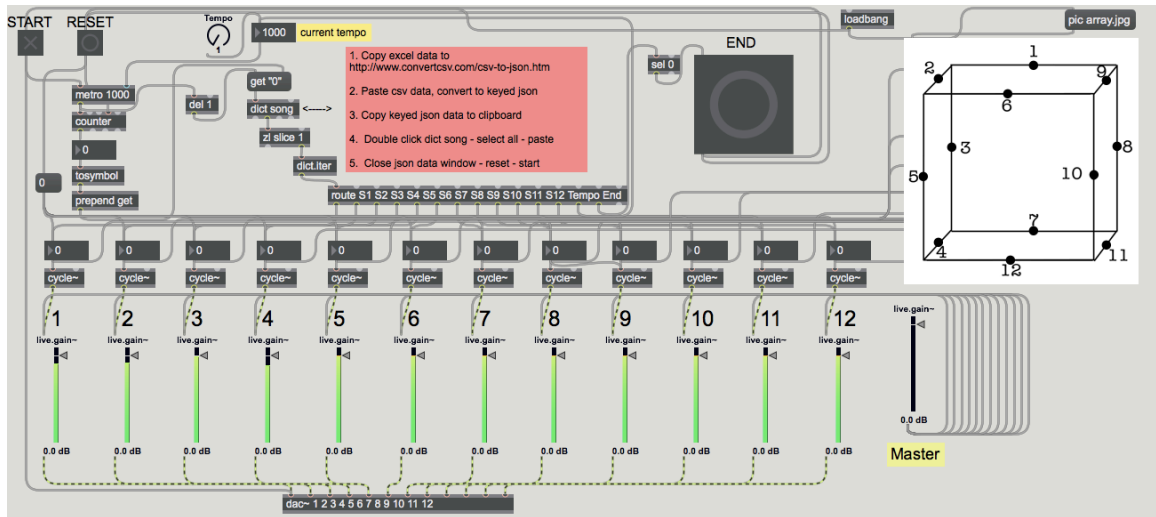


Figure 4-5 – Complete image of master patch

Since most of this information is not meant to be altered by a composer, a streamlined version of this patch in presentation mode could be extremely useful. That is, a lot of what can be seen here does not need to be known, and potential composers would be better served by a patch with a much easier perspective to relate the array mappings in chapter one, and the notation outlines in chapter two. Figure 4-6 shows this finalized version. By taking the time to provide the important compositional elements needed by a composer, we have a patch that is more likely to properly translate the intentions of a composer, as opposed to misinterpreting a programming error.



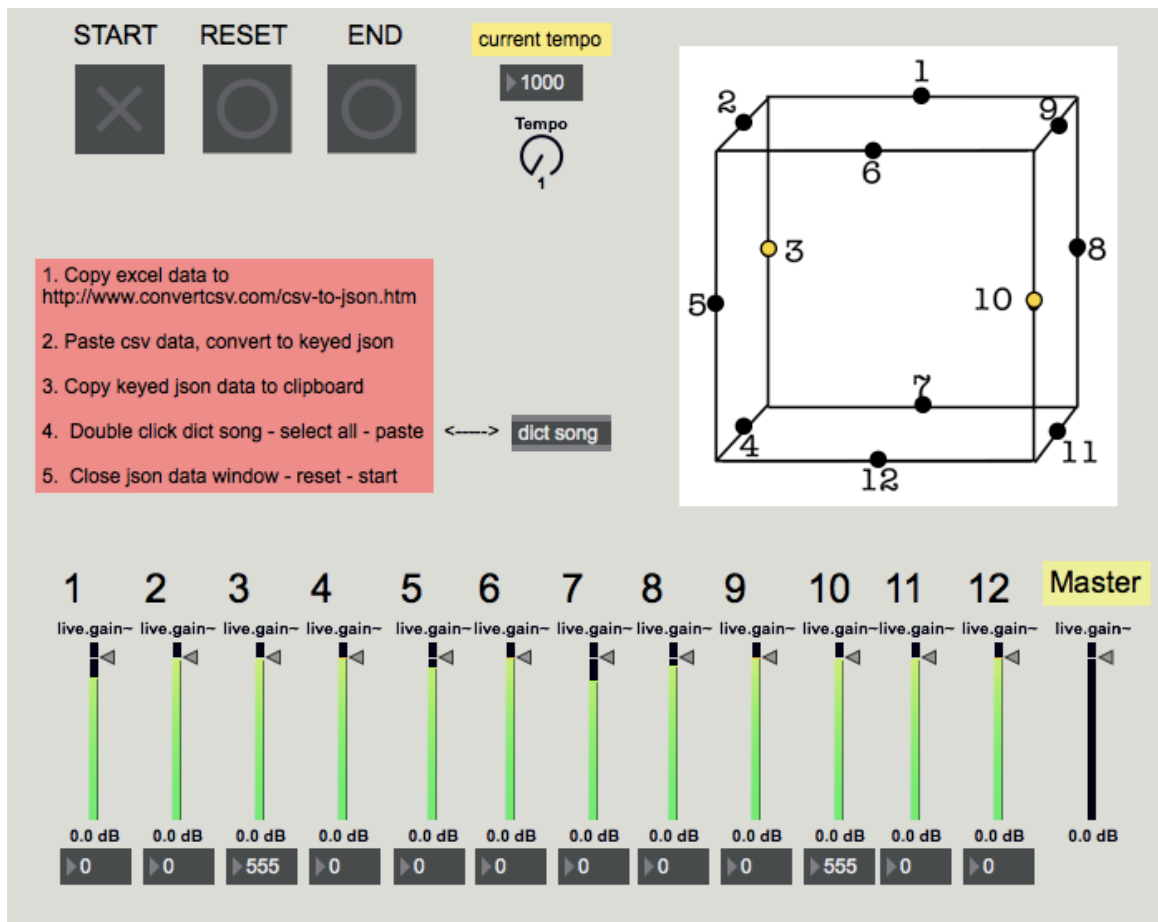


Figure 4-6 – Final master patch in presentation mode

In figure 4-6 we can see how the most important controls and visual guides are provided. By using this version of the patch, the composer has access to all of the basic control/transport functions with the “start, reset and end” objects. The tempo controller is included, despite its limited functionality in the face of programmed tempos. The directions and the “dict song” object required to import a new song have been kept in their same form. The array mapping in the upper right-hand corner shows which speakers are actively engaged (here we see speakers 3 and 10 are playing). The mixer across the bottom shows which speaker is playing which frequency (0 indicates silence),

and the individual and master sliders allow for volume control over each independent speaker, or over the entire array, respectively.

After completing the software patch, all of the technical elements of the project were finished, and the new form was ready for composition. The next three chapters outline and analyze the first three demonstration pieces I composed for this new musical form. For obvious reasons, I like to call this new form “Super-Symmetrism.” However, as a fan of science, I also gave it this name because of how similar the notation of this form resembles the graphical representations of supersymmetric algebras, known as Adinkras. There are a couple examples of these Adinkras in figure 4-7; hopefully you can see how much the notation of a completed piece of this super symmetric music resembles these mathematical images.

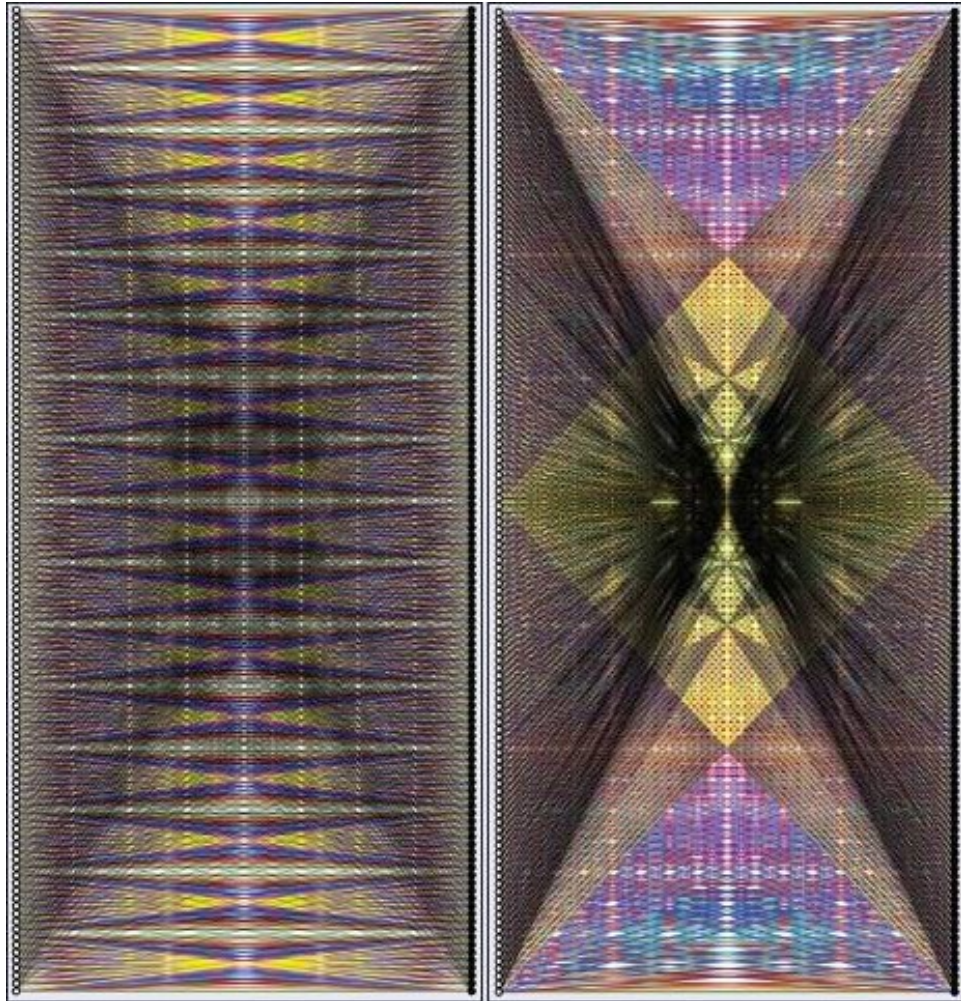


Figure 4-7 – Examples of Adinkra Images

## **Chapter 5 – Three Etudes for Cuboctahedral Array**

### **Etude I - “Synesthesia no. 9” for Terrence McKenna**

After completing all of the technical steps outlined in the first chapters of this dissertation, I knew that I wanted to compose three unique pieces to demonstrate various approaches towards composition. I wanted my first piece to use a small number set to demonstrate every possible closed texture harmony outlined in chapter two. I wanted my second piece to make use of a larger number set, but without the need to use every single possible harmony available as achieved in the first piece, and I wanted to experiment with open textures. Finally, I wanted my third piece to use an even larger number set, with just a few of the harmonic possibilities represented, in an overtly repetitive, almost algorithmic manner. The hope was that this approach would produce diverse outputs by purposely creating different compositional strategies from the beginning of their development. All three of these pieces are inspired by the work of historical researchers who were personally inspirational to the conceptualization of this project.

I decided to dedicate my first piece to the late Terrence McKenna, as a nod to the value of inner exploration by alternative means. A listening device so focused on the individual experience was analogous to the psychedelic world and the and the impact of exploring alternative spaces within the self. The use of a device meant to project a listener to an alternative space was reminiscent of the inner journey taken by way of psychedelics. As the first person to act as listener, I felt a sense of exploration, a sincere curiosity of what to expect, when I experienced the first piece. While I contend that my symmetrical definitions are universally true, I must admit that this a fine arts dissertation

meant to explore artistic dimensions that are not so easily defined. The difficulties I have in explaining and describing psychedelic experiences are not all that dissimilar from the difficulties I have when trying to explain why I prefer consonance over dissonance. The mathematics of tuning may explain how tonal harmony works, but they do not explain why I personally find tonal harmony so satisfying. This reminded of the work of McKenna, who had a tendency to allow his fascination with alternative realities to inspire research where these experiences were treated as valid (McKenna 1993).

As a fan of esoteric and ancient numerology, I decided to use some of the integers associated with concepts such as Chakra frequencies, Fibonacci sequences, Lambdomas, and sacred geometry. I looked for overlapping integers that were shared between these various esoteric concepts and mathematical anomalies, which I then selected to use as the basis of my symmetrical patterns. This ultimately resulted in the use of frequencies made up of three-digit numbers that are all divisible by 3; when each digit within the number is added together, the sum of 3, 6 or 9 always results, hence the reference in the title. The integers ultimately chosen are listed in figure 5-1.

Solos	432
Duos	144/720
Trios	144/432/720
Quartets	180/360/540/720
Sextets	396/417/528/639/741/852
Duodecets	135/270/540/144/288/576/180/360/720/216/432/864

Figure 5-1 – Integers chosen for first demonstration piece

This first piece only makes use of three-digit frequencies, so that anything below 100Hz and anything above 1000Hz will not be used. In fact, due to the small size of the speakers in the prototype array, I will not be able to compose with the lower frequencies of the aural spectrum within this project. I will not be using any 2-digit numbers as frequency indicators throughout these etude pieces because of this reason. I look forward to the lower frequency experiments when I can finance a much larger and capable speaker array.

For the solo part, I decided to use 432Hz. For the duos, I decided to use 144Hz and 720Hz; they are both equidistant from 432Hz. As I moved towards the trios, I first added the 432Hz, back to the 144Hz and 720Hz to create the first trio, treating 432 and 720 as suspended notes to create the new form. For the quartets, I maintained the suspension of the 720 and then added the 180, 360 and 540. For the sextets, I used 396, 417, 528, 639, 741, and 852Hz. Finally, for the duodeciet demonstration, I used 12 frequencies that made overlapping use of integers used throughout the earlier part of the piece. These were 135, 270, 540, 144, 288, 576, 180, 360, 720, 216, 432, and 864Hz. These values can also be grouped into four octave groups of 3 to create overlapping quadruplets.

To create this piece, I used this small set of numbers to demonstrate all of the harmonies and their alterations defined in chapter two. Each possibility and all of its associated rotations and/or inversions are represented in the same sequence in which they were presented, moving from the solo up to the duodeciet. This piece does not use any open textured options, because I wanted the first piece to be completely immersive. As a

researcher interested in the finding out if people can aesthetically perceive intentional aural spatial symmetry, I was ultimately more interested in the closed textured possibilities which constantly make simultaneous use of all 12 speakers around the listener.

The piece starts with the only possible solo running for one second. For each successive harmony, I have divided this time of one second by the number of possible rotations. Therefore, each possible harmony will run for a total of one second, with each rotational iteration being determined by how many possibilities exist. Since the first duo presented in measures 2-3 only has two possible rotations, each one only plays for 500 microseconds. The next duo has four possible rotations, so each one plays for 250 microseconds. This system is used throughout the piece to determine the tempo of various harmonies. Measures 2-25 represent the duos defined in figure 3-1 of chapter three.

Measures 26-385 represent all of the duos featured in figure 3-2 of chapter three. Unlike the duos that played before, these have rotation and inversion possibilities which need to be represented. I allowed the inversions to play as a completely new harmony so that the tempo would remain fluid throughout this section and not get too fast. Each harmony goes through its rotations, then inverts, and then goes through the rotations of the inversion before moving on to the next possibility.

I then shifted slightly from the order of the presentation and used the compaction-based duos from figure 3-4. I chose to do this because these duos also have inversion possibilities, and I wanted to exhaust those options before moving on to the summation-

based duos listed in figure 3-3. Measures 386-433 play these duos in the same fashion as the previous set, by going through their rotations first, then inverting, and then playing the rotations of the inversion. The duos from figure 3-3 are exhibited in the final duo segment stretching from measures 434-505. Since these duos cannot be inverted, they simply run through all of their rotational options before moving on the next harmony.

The trios are then introduced, and they maintain the same tempo, since they also have 12 rotations for each of their options. I chose to only use those trios that created three truly equal arrangements of speakers, so the sequence-based options (figure 3-5) and summation-based options marked with asterisks (figures 3-7) are not used. Thus, the nine possible trio harmonies that create spatial symmetry are ran between measures 506-613. Like the duos, each possibility runs its rotational options before moving on to the next harmony.

The quartets begin at measure 614. The first option used in measures 614-637 is the spatially symmetrical sequence-based quartet from figure 3-8 (measures 5-8 in figure 3-8). Again, we have the same number of rotations per harmony, so the tempo is still constant at first, but this will eventually change. The quartets also have inversions that need to be demonstrated, so like the duos that could be inverted, the quartet inversions will also be equally divided to play equal iterations over one second. Measures 638-685 represent the quartets defined in figure 3-9, following this same pattern of rotations, inversions, and then the rotations of the inversions.

Next the triangle-based quartets from figure 3-10 are then played from measures 686-829. This is the final set of quartets that will play out with this same pattern. Since



the unique quartet defined in figure 3-11 is open textured, I was not able to use it the way it is outlined in chapter two. However, I decided that I wanted to include this harmony in this composition, so I compressed it down into a two-measure format, and then rotated the two measures as if they were one around the notation. This ultimately results in the 42-microsecond tempo, which acts as an interesting segue in measures 830-853, before reaching the sextet demonstrations.

The 83-microsecond tempo returns as the typical “rotate, invert, rotate” process returns with the two sextets presented in measures 854-889. We then hit the duodectet region where the tempo formula is broken for the first time. Instead of having the 12 possible rotations play for 1/12 of a second, I then had them progressively get longer by 1/12 of a second until reaching the exact middle point of the piece where the last harmony rotation plays for an entire second. I refer to this point as the “horizon” as it is the point where the time element of symmetry is put into effect. To make this happen, the piece simply plays its inverted retrograde from the horizon point, creating a spiral-like figure that radiates outwards away from the meridian.

The notation for this piece is included. Hopefully you can trace this analysis across the multiple pages required to cover such a lengthy work. Try to see the continuation of a spreadsheet as you would see the continuation of measures and systems across pages in traditional music notation. For future composers, using “find and replace” across multiple sheets within a single workbook to copy structures and alter them is extremely helpful for streamlining the composition process.

## **Etude II - “The Magnificence of the 3, 6, and 9” for Nicola Tesla**

If you search the internet looking for stories and information about Nicola Tesla, you may soon find yourself inundated with quotes of dubious veracity. One of the more popular that I consistently have found to be attributed to the great scientist is akin to, “If you only knew the magnificence of the 3, 6 & 9, then you would have a key to the universe.” While I have struggled to prove that this is indeed a real Tesla quote, the fact that it has permeated our social consciousness to such a degree as a meme was interesting enough for me. I chose to explore this phrase to satisfy my personal fascination with an inventor who frequently discussed frequency and vibration as relevant to his groundbreaking innovations (Tesla 2005). While he used the terms to reference non-musical concepts, a composer like myself hears those words as an expression of sonic experimentation. The idea was not only fun to play with as a modern urban legend, but the numerology was a perfect segue from the last piece. To extract frequencies from this concept, I decided to use all of the three-digit integers that I could create using only the 3, 6 & 9. This list of available numbers is outlined in the grid labeled as figure 5-2.

Here we can see how all of the digits between 100 and 1000 can be created using only the numbers 3, 6 and 9. In this arrangement, they appear as 12 rows of 9, and they can even be seen as four sets of 27, when grouping the rows by 3. These 108 frequencies were a much larger number set than that which was used to create the first example etude. The list appeared daunting and unfriendly to any kind of organization with regard to the systems described in the chapters above, so I began to look for definitions of patterns that

I could use to extract groupings that would fit nicely into the array and notation. This is where the multiple factors of 12 were functional for creating diverse harmonic possibilities. I analyzed the properties of this list to try to find patterns that could be associated with these various divisors.

333	336	339	633	636	639	933	936	939
363	366	369	663	666	669	963	966	969
393	396	399	693	696	699	993	996	999
3333	3336	3339	3633	3636	3639	3933	3936	3939
3363	3366	3369	3663	3666	3669	3963	3966	3969
3393	3396	3399	3693	3696	3699	3993	3996	3999
6333	6336	6339	6633	6636	6639	6933	6936	6939
6363	6366	6369	6663	6666	6669	6963	6966	6969
6393	6396	6399	6693	6696	6699	6993	6996	6999
9333	9336	9339	9633	9636	9639	9933	9936	9939
9363	9366	9369	9663	9666	9669	9963	9966	9969
9393	9396	9399	9693	9696	9699	9993	9996	9999

Figure 5-2 – All possible integers between 100-1000 made up of 3, 6 and 9

I eventually settled on six unique identifiers: 3-based “all sames” (AAA), 4-based “all sames” (AAAA), 3-based palindromes (ABA), 4-based palindromes (ABBA), Doubles (AABB) and Repeats (ABAB). This provided me with options to meet my trio and sextet needs, and the rows would make for nice arrangements of duodectets. I still

needed to find a way to demonstrate duos and quartets, so I opted for the following options.

With regard to the duos, all of the numbers not used by the system above can be matched with their inverse pair (AAB-BAA). For example, the numbers 336 and 633 make one of these pairs. In total these account for 39 pairs, or duo iterations, in terms of the notational expression. After this, I only needed to make use of quartets, but a system based on these digits does not divide well into four. However, the 12 rows can be grouped into groups of 3, where a pattern does emerge based on the way the table was generated. Instead of opting for perfect symmetries, I thought I would use this to demonstrate an alternative to present a quartet. This would guarantee compositional diversity between these first two etudes.

In the first etude I made an effort to create numerical relationships, whereas in the second piece I was trying more to observe them. In addition to this change, I also wanted the piece to start with a denser texture that worked its way towards a less dense center horizon point. These themes run in direct contrast to the first etude, as a purposeful means of demonstrating a diverse pool of compositional tools. In other words, this piece directly experiments with the concept of creating symmetries with numbers that are not exactly the same but instead share an abstract identity within a defined set, that relate to congruent positions within each harmony. This was a different way of seeing the harmonies filled in.

I began by running each integer from the entire set as a perfect solo in their sequential order. This lasts from measure 1-108. In keeping with the theme of the

integers, all of the tempos used only use the numbers 3, 6 or 9. With so many solos to represent, I chose to run these at 33 microseconds. This piece also introduces the concept of a silent or rest measure. Each of the different sections are separated by a short rest where the new tempo for the next section is programmed. This can be observed at measure 109.

Since the last piece was so sequential in nature, I decided to jump around between the different harmonic symmetries instead of letting them develop in order of their size. This explains why the next section makes use of the duodecimals. Measures 110-127 reflect the rows created by the grid system I used to create this integer set. Each row runs as a duodecimal for 66 microseconds before reaching the next rest and tempo change at measure 128.

The next section brings in all of the duo possibilities created by pairing numbers that are reverses of each other (336 and 633, for example). Each duo harmony plays for 99 microseconds and makes use of a root position sequence-based duo that runs for six rotations, until reaching a point where the speakers ultimately switch positions from one side of the array to the other. This pattern runs from measures 129-401 running each pair of integers within the set that meet this definition, from lowest to highest. At measure 402, the duos come to an end hitting a rest measure where the tempo adjusts to 66 microseconds for the introduction of the sextets.

The sextets are an interesting point in the piece, as the 4-based palindrome groupings are introduced in an open texture. The open texture in measures 403-408 outline the closed texture that presents in measure 409. From there, the harmony rotates

two times to a new position, where the original open textured pattern then repeats itself in the opposite direction. This pattern is how the 4-based palindrome group moves from measures 403-456. At measure 457, another dramatic shift can be seen.

At this point, a 33-microsecond tempo is used to introduce the 3-based palindromes in an open texture. Measures 457-462 bring the open texture to a closed texture at measure 463, and then the open texture moves outward to the next rotation at measure 470. This pattern repeats itself until reaching its last rotational option at measure 534. When we get to measure 535, the first set of 4-based palindromes return to run through a similar open/closed texture pattern to illustrate the other sextet possibility up to measure 583.

This larger pattern (mm. 403-583) is then applied to the other sextet groupings created by identifying doubles (AABB) and repeats (ABAB), maintaining the use of 3-based palindromes as a segue between the two sextet options presented. Essentially, measures 584-764, and measures 765-945 can be seen as variations on the original from measures 403-583, where a new integer grouping has been substituted for another, and a different rotation of the same 3-based palindromes has been used as the segue between the two types of sextets. This specific section of this piece was very much inspired by a desire to create a fractal-like design.

At measure 946 we see another rest measure with a new tempo indicated of 99 microseconds, preparing us for the transition into the trio section. Measures 947-982 run a single trio harmony with all three integers shifting into all three available positions over several repeats using the grouping created from using 3-based “all sames” (AAA). This

leads to a grand pause at measure 983, where the tempo jumps to 333 microseconds, and the trio is played again one final time in a ritarding fashion. The same pattern then runs again from measures 988-1027, but this time the 4-based “all sames” (AAAA) integers are used instead.

At measure 1028 we have another rest measure and tempo change to mark the shift from trios to the final section of quartets. As the tempo shifts to 99 microseconds, we see the introduction of the multi-speaker quartet defined in figure 3-11 of chapter three. To create these forms, I used integer groupings of four that had the same last three digits. For example, 333 is grouped with 3333, 6333, and 9333. Each successive quartet then runs the next highest number created by this set. This results in 27 different quartets running from measures 1029-1190 before hitting the horizon. Again, to maintain the time symmetry element, the piece then runs its inverse retrograde until the final completion at measure 2381.

I found that this piece not only benefited from the use of find and replace, but also from the use of sum equations. Since the numbers were very easy to calculate from themselves with basic addition functions, I was able to generate much of this graph by creating equation tables based on cells with variables that could easily be changed and used to update the notation.

### Etude III - “Prime Time in 4K” for Carl Sagan

After composing and playing with subjective numerology for fun and personal expression, I wanted to move back towards the experimental nature of this project and use a number set that felt inspired by mathematically objective concepts. I wanted to use something that felt less random than the first two etudes, and even more universal than the integers extracted by using 3, 6 and 9. I had also decided that I wanted this last piece to focus on the use of a larger number set, with less harmonic options. This is what led me to prime numbers and my choice to dedicate the piece to the late Carl Sagan. The way he described the use of primes to transcend intergalactic communication limitations in the story *Contact*, was very inspirational to my desire to find a new aural aesthetic that could transcend human cultural differences, and possibly, differences between undiscovered intelligent forms of life throughout our universe. The fact that my speaker array is a geometric device in which only a single person experiences its effect was also inspired by the details of this particular sci-fi story, which uses a dodecahedron to send the protagonist on a journey only perceivable by herself (Sagan 1997).

When I began looking for different ways to group various prime numbers into meaningful systems, I stumbled upon the concept of sexy primes. Sexy primes are defined as prime numbers that differ by six, or  $(p, p + 6)$ . An example of this would be 5 and 11. When I found the sexy prime concept, I discovered that there were also sexy prime triplets  $(p, p + 6, p + 12)$  as well as sexy prime quadruplets  $(p, p + 6, p + 12, p + 18)$ . I was hoping that there would be sexy prime sextuplets, but there was no such grouping of primes that continued the trend. However, since I wasn't worried with



making sure to use all of the possible harmonies, I didn't mind moving forward with the pairs, triplets and quadruplets provided by sexy primes.

When I began to compose this piece, I realized that the number set was far too large if I used all of the prime sexy possibilities within the human hearing range (20-20,000). I wanted to make sure that my pieces were not so long that people would hesitate to let them complete, and this was reaching a size that was far larger than either of the first two pieces I had already completed. I decided to limit all of my groupings to primes that were between 100-4000. This is why the "4K" appears in the title. It is a direct reference to the range of frequencies used.

I began by running all of the primes between 100-4000 as solos, but unlike the first two pieces, this one starts at the higher end of the aural spectrum as opposed to the lower. Measures 1-525 represent all of the solos that make use of this set of prime integers. As the primes get closer to 100, they approach the onset of the duos.

The duos in this piece do not restart at their high point, but pickup from the lower end where the solos left off. I did this to systematically make sure this etude would have a different developmental flow from the first two pieces. Each duo runs as a summation-based option, where the two frequencies switch places before moving to the next sexy prime pair. This directs the composition from measures 526-897, as the duos progressively get higher as they approach the introduction of the trios.

Like the solos at the beginning, the trios start from the higher end of the set limit at 4000. Each trio makes use of the summation-based harmony that perfectly illustrates the three dimensions of space. Like the duos, each integer within the sexy prime triplet

takes one measure in each grouping position before moving on to the next lowest sexy prime triplet. This pattern continues from measures 898-1077, using all of the available options down to the lower limit of 100, before transitioning into the quartet section.

The quartets introduce the first open texture harmony, making use of what has become my favorite quartet harmony to use in my super-symmetrical compositions. The transition from trios to quartets is not only exemplified by a change in texture, but also by the fact that the lowest sexy prime quadruplet is quite distant from the lower limit of 100. Unlike the previous sections, the quartets do not rotate or playback any alterations of their root-position forms. However, the pattern of moving in sequential order from smallest to largest does give it a familiar order-based system. This is followed from measures 1078-1149 until the upper limit is hit, and the piece hits the horizon point.

It was at this point that I decided on my tempo strategy. I knew that I wanted to use prime numbers to keep the theme consistent, but I also wanted to make sure that the piece did not take too long to complete itself. After realizing that 1149 was perfectly divisible by 3, I chose to use the first 3 prime numbers after 100 to create a pattern. This was to go from low to high, and then back to low again. I copied this all the way down the tempo column until reaching the horizon point, where it is then repeated in reverse. Like the other two etudes, the piece then plays its inverse retrograde, maintaining my desire to capture the symmetry of time, in addition to those made symmetries crafted in space.

## **Chapter 6**

### **Reflection & Conclusions**

This project was not only inspired by a personal fascination with symmetry, but also by a personal struggle. This struggle was my inability to appreciate much of the experimental atonal music presented to me as the historical antecedent to the 19<sup>th</sup> century. As a young music student, I was not very inspired by the post-romantic composers my professors presented to me as the most important to the development of music beyond tonal harmony. After eventually discovering a taste for some atonal composers like Reich, Xenakis and Penderecki, I found that I personally preferred experimental music where I could perceive and understand the experiment. This is what inspired me to focus so much on symmetry; I hoped that it would succeed in communicating intentionality to any type of potential listeners in as my potential locations, as organized, composed, and intelligently designed.

Before I even heard what my first piece sounded like, I had already come to realize the value of the work I had done. During the development and completion of this project, I recognized the personal joy and satisfaction I was getting from the experimental nature of my work. I was not concerned at all with what it would sound like, or if people would like it, but rather, I found that I was only focused on discovery and problem solving. It was an epiphany that I had been looking for ever since I was exposed to the experimental music of my generation. It was the first time I felt the need to invent out of necessity. I found myself more inspired by the potential for results, than dissuaded by the possibility of negative results.

It was at this point that I understood why I had so much trouble appreciating experimental music during my studies. For so long, I was not looking at the music from the perspective of the composer, but rather, from the perspective of a listener. While I had no trouble understanding the experimental nature of tonal composers who dared to push their ideas beyond convention, I failed to realize how experimental their music was in a time period so different from my own. This was the first time that I felt in touch with the experimental nature of composition, and I understood the personal satisfaction I was getting from being creative, with no regard for the outcome or how it would be received. It was both enlightening and liberating to finally wrap my mind around the concepts of non-harmonic 20<sup>th</sup> century composers that I had previously been unable to appreciate.

From the perspective of a graduate student finishing a terminal degree, this conclusion was perhaps the most ideal way to complete my studies. After almost 20 years of MIDI sequencing and composing, this project made me feel the way I did the very first time I discovered DAW software and the interfacing of keyboards via joystick ports. Where I would almost always subconsciously ask myself, “Why am I doing this, again?” every time I would compose a traditional beat or song using standard rhythmic and harmonic concepts, I never felt that way one time throughout this project. Every step was a new problem that needed to be solved, and as I found ways to solve them, I understood the processes that guided experimental composers throughout history that brought our discipline to its current state. The joy of invention was finally a part of my compositional process.

When I listened to the pieces, I found that I preferred the outcome of my second etude. The first and third etudes were composed from a much more algorithmic perspective, and I felt as though this translated in the performance. The freedom that I used to make compositional decisions in the second piece was much more pleasing to experience when I sat in the middle of the array. In contrast to this observation, I felt that the integers used in the third etude created the most interesting effect between themselves, but that the overall pattern became dull and uninteresting. Since these were much closer together, the distance between frequencies sounded with a dissonance that was oddly interesting, but the slow rises and descents were not as interesting as the other pieces that jumped around more. It was as though the symmetrical presentation of waves with only one or a few cycles difference between them created an almost “spatial-phasing” effect, with which I would like to explore further. I think a smaller number set that makes more use of the possible harmonies creates a much more diverse experience than the larger sets that make use of less harmonies. I personally prefer to listen to everything played by the array with eyes closed in a calm trance-like state. I find that the frequencies help me meditate to a much deeper state, and without looking at any kind of graphical notation or Max MSP patch, I find it easy to get completely lost in the presentation of piece with regards to time.

Of course, these are personal preferences, and It will take time and feedback from listeners to discover any new possible aesthetics associated with equal-tempered space that are perceivable by intelligent life. I hope my etudes are useful as inspiration for other forms of compositional experimentation. This music system supports an almost

near-infinite number of compositions that can be composed within the defined parameters, and much experimentation could be done by composing pieces by algorithmic processes to streamline the research.

During the completion of my first three etudes, I found myself jotting down ideas that I wanted to explore beyond the ideas I was in the middle of completing. I did not want to get distracted, so I tried to stay focused on the example pieces that needed to be completed, but I was finding inspiration on a regular basis. As I mentioned throughout this paper, I hope that future composers are willing to join my experiment and help develop this new form. I anxiously look forward to the ideas others can inject, as I have already begun to think of ways to take this project further along than it is in this current state.

In the near future, I imagine building larger arrays with larger speakers that allow for listeners to walk in and out of the harmonic space. With only spreadsheet software needed to compose, I would like to present this to the public in a forum where anyone who is willing to take the time can create and listen to pieces meant for this process. I personally would like to take the original Hebrew Pentateuch, transform the words into three-numbered digits, and then listen to how the algorithmic composition achieved through these means would create an alternative manner of engaging with one of the most important texts to the development of human history.

I also would like to experiment with the Platonic solid shapes presented in chapter one. Different shapes would provide for different numbers of speakers, which would inherently lead to an alternative system, even if based on the system presented here. In

addition to nesting these different shapes and sizes within each other, I would like to take the original piece into higher dimensions, by using the same bisection method used here along the vertices of a tesseract. As a series of cubes, the tesseract could be treated much as the cube was treated in the outset of this project, for experimenting with alternative methods of creating spatially symmetric aural fields. I would also like to consult with mathematicians who could help me integrate the adinkras into this project far beyond the similarity in appearance they share with the notation. The idea of hearing mathematical equations and engaging with algebra as a form of real-time music seems like an interesting concept I would like to explore further.

As our multimedia progresses more and more towards the virtual reality experience, I hope that these types of arrays and their mappings will be useful in the creation of aural sound fields that are capable of producing the sonic effects of being in alternative worlds/spaces. The larger arrays will allow for individuals to walk through them and experience the compositions from different positions within the array, even allowing for people who are deaf in one ear to experience aural space. In the meantime, the smaller arrays could still be used for spectators at home to sit inside of while doing things like watching their favorite sports matches, and logging in to the 12-microphone setup mounted on his/her favorite player, and then watching from the perspective of a camera mounted on that same player. I also imagine linking an array with a MIDI controller that allows a composer to work with the speakers in real time like a musical instrument to eliminate the tedious compositional process for users who would rather learn by jumping in and playing with a hands-on device. Anything with dials and sliders

could be used to provide a tactile means of compositional input. These are the kinds of ideas that excite me with regard to the possibilities of mass popular appeal.

I would also like to merge this project with the field of cymatics. I hope to find compositions that can actively arrange and create symmetrical patterns with vapor much in the same way that sin waves can create symmetrical patterns with salt or sand on a two-dimensional membrane (Jenny 1974). The possibility of adding a third dimension to a field defined by two could provide new ways to look at physical creation by means of sonic intervention. Perhaps the array will be able to tell us how the sound of the big bang relates to the patterns and layout of the cosmos. I would also like to see the cymatics idea taken one step further, where the human body acts as the particle being acted upon by sound.

If this array could be merged with a symmetrical sensory deprivation tank, a human being could lay on the membrane of salt-water and be exposed to sound in a state of being deprived of any of their other senses. We could see how the body reacts in relation to the sand/salt particles that normally play the role of sonic absorber. We could also use this artificial state to further isolate individual subjects being used for music therapy experimentation. It would be an ideal means of eliminating variables that would otherwise impact research results. By merging these two devices, we would be able to test the most fundamental aspects of how each frequency effects a human body. I am personally interested in finding alternative means of giving human being psychedelic experiences and/or the overview effects that usually come from the profound insights reported by users who ingest psychoactive substances. Also, here in the state of



California, we have a few unique sound space installations that I would like to consult for possible collaborations. Spaces like the Integratron in Landers, CA, the Allosphere at UCSB and AUDIUM in San Francisco could provide for interesting collaborative possibilities and/or means for the development of new compositions intended for alternative specialized spaces. These locations could also be used to nest the cuboctahedral space created by the array within another space designed specifically for sonic purposes; experiments could determine if the two spaces have any effect on each other.

I have always been personally fascinated with symmetry and numerology, so it was only natural that I try to merge it with my love for music. I found it very easy to work with concepts that I am intrinsically interested in and I discovered the value of experimentation with expectation. My project was inspired by the desire to create a means for creation. I hope that others continue to merge their personal interests to this work, so that it can develop beyond the scope of these early discoveries. I would like to see the passions of others help this system evolve, much in the way that the western passion for tonal harmony impacted the second half of the second millennium in the common era.

## Appendix

Array.jpg – Image file of the array used by the included Max MSP patches

E1.maxpat – Max MSP patch that has Etude I loaded and ready for playback

E1-Audio 1.wav – Audio file for channel 1 playback from Etude I

E1-Audio 2.wav – Audio file for channel 2 playback from Etude I

E1-Audio 3.wav – Audio file for channel 3 playback from Etude I

E1-Audio 4.wav – Audio file for channel 4 playback from Etude I

E1-Audio 5.wav – Audio file for channel 5 playback from Etude I

E1-Audio 6.wav – Audio file for channel 6 playback from Etude I

E1-Audio 7.wav – Audio file for channel 7 playback from Etude I

E1-Audio 8.wav – Audio file for channel 8 playback from Etude I

E1-Audio 9.wav – Audio file for channel 9 playback from Etude I

E1-Audio 10.wav – Audio file for channel 10 playback from Etude I

E1-Audio 11.wav – Audio file for channel 11 playback from Etude I

E1-Audio 12.wav – Audio file for channel 12 playback from Etude I

E2.maxpat – Max MSP patch that has Etude II loaded and ready for playback

E2-Audio 1.wav – Audio file for channel 1 playback from Etude II

E2-Audio 2.wav – Audio file for channel 2 playback from Etude II

E2-Audio 3.wav – Audio file for channel 3 playback from Etude II

E2-Audio 4.wav – Audio file for channel 4 playback from Etude II

E2-Audio 5.wav – Audio file for channel 5 playback from Etude II

E2-Audio 6.wav – Audio file for channel 6 playback from Etude II

E2-Audio 7.wav – Audio file for channel 7 playback from Etude II

E2-Audio 8.wav – Audio file for channel 8 playback from Etude II

E2-Audio 9.wav – Audio file for channel 9 playback from Etude II

E2-Audio 10.wav – Audio file for channel 10 playback from Etude II

E2-Audio 11.wav – Audio file for channel 11 playback from Etude II

E2-Audio 12.wav – Audio file for channel 12 playback from Etude II

E3.maxpat – Max MSP patch that has Etude III loaded and ready for playback

E3-Audio 1.wav – Audio file for channel 1 playback from Etude III

E3-Audio 2.wav – Audio file for channel 2 playback from Etude III

E3-Audio 3.wav – Audio file for channel 3 playback from Etude III

E3-Audio 4.wav – Audio file for channel 4 playback from Etude III

E3-Audio 5.wav – Audio file for channel 5 playback from Etude III

E3-Audio 6.wav – Audio file for channel 6 playback from Etude III

E3-Audio 7.wav – Audio file for channel 7 playback from Etude III

E3-Audio 8.wav – Audio file for channel 8 playback from Etude III

E3-Audio 9.wav – Audio file for channel 9 playback from Etude III

E3-Audio 10.wav – Audio file for channel 10 playback from Etude III

E3-Audio 11.wav – Audio file for channel 11 playback from Etude III

E3-Audio 12.wav – Audio file for channel 12 playback from Etude III

Final Etudes.xlsx – Microsoft Excel spreadsheet with the notation for the three etudes

Master.maxpat – Max MSP patch that is empty and ready for a new piece to be loaded

Prototype 1.jpg – Image file of the first physical prototype made for this project

Prototype 2.jpg – Image file of the first physical prototype made for this project

Prototype 3.jpg – Image file of the first physical prototype made for this project

Prototype 4.jpg – Image file of the first physical prototype made for this project

Prototype 5.jpg – Image file of the first physical prototype made for this project

## References

Boss, J. (2016). *Schoenberg's twelve-tone music: symmetry and the musical idea*. Cambridge: Cambridge University Press.

The entire first chapter (p.1-24) is an explanation on how the history of symmetry in western music had evolved until the time of Schoenberg, and how he used the technique in the development of his twelve-tone system and in the form of his compositions.

Duffin, R. W. (2008). *How equal temperament ruined harmony: (and why you should care)*. New York: W.W. Norton.

The author of this text is trying to make the argument that alternative tuning systems should be used to better appreciate music from different times in history. On pages 22-23 he describes how the octave is the only interval which cannot be altered and how it remains true across different musical cultures.

Gambino, M. (2012, April 22). What Is on Voyager's Golden Record? Retrieved December 11, 2019, from <https://www.smithsonianmag.com/science-nature/what-is-on-voyagers-golden-record-73063839/?no-ist=&page=4>.

This article describes the history behind the recorded sounds and songs that were chosen to be added to discs that were sent to space along with the Voyager 1 and Voyager 2 NASA probes launched in 1977, in an effort to begin extra-terrestrial communications.

Jenny, H. (1974). *Cymatics: the structure and dynamics of waves and vibrations*.

The contents of this book describe the geometric patterns that can be created by sending sound frequencies through flat membranes containing sand or salt. The idea first began with Ernst Chladni in the late 18<sup>th</sup> century and was further developed by Jenny in the late 20<sup>th</sup> century.

Kempf, D. (1996). What Is Symmetry in Music? *International Review of the Aesthetics and Sociology of Music*, 27(2), 155-165. doi:10.2307/3108344

This article explains some of the various ways that symmetry can be perceived in traditional music examples. On page 158 the author describes how it can be applied to concepts of form. On page 159 he explains how it is related to traditional harmonic progressions.

Lederman, L. M., & Hill, C. T. (2008). *Symmetry and the beautiful universe*. Amherst, NY: Prometheus.

This text examines symmetry from various angles, including music. From pages 16-18, the author explains how symmetry impacted the output of J.S. Bach. In the appendix, on page 295, he explains the mathematical means by which symmetry can be verified.

McKenna, T. (1993). *Food of the gods: the search for the original tree of knowledge: a radical history of plants, drugs and human evolution*. New York: Bantam.

In this book, the author examines the impact of psychedelic plants, their history with human beings, and how they could have impacted the evolution of consciousness. The concepts regarding psychedelic experiences were very inspirational to my quest for alternative means of artistically impacting people.

Phillips, T. (2017, August 21). Bach and the musical Möbius strip. Retrieved December 11, 2019, from <https://plus.maths.org/content/topology-music-m-bius-strip>.

This article explains how Bach used the concept of the inverted retrograde to inspire symmetrical compositions from his *Musical Offering*, specifically, Canons 3 and 5 from the collection.

Sagan, C. (1997). *Contact*. London: Arrow.

This non-fiction work was very inspirational to this project in several ways. First, my cuboctahedral array was a direct reference to the dodecahedron used in this story. Second, the idea of projecting a person beyond that of their physical place was inspired by the “trip” taken by the protagonist of this story. Finally, the use of prime numbers to transcend intergalactic communication problems was not only used to create a piece, but to inspire the concept of symmetry as communication.

Solomon, L. J. (2002). Symmetry as a Compositional Determinant. Retrieved December 11, 2019, from <https://edisciplinas.usp.br/mod/resource/view.php?id=2321326>.

The author of this article includes diagrams and outlines that visually represent the elaborate symmetries used by Bela Bartok in his *Music for String Instruments, Percussion and Celeste*, as well as those used by Webern in his *Piano Variations, Op. 27*.

Tesla, N. (2005). *My inventions: the autobiography of Nicola Tesla*. United States: Wildside Press.

This autobiography provides insight into the ideas behind the process and inventions of Nicola Tesla. On page 44 he describes how sound waves can be used to power the globe, claiming it would only take nine months to build the “first world system” if done properly.

Wörner Karl H., & Hopkins, B. (1973). *Stockhausen: life and work*. Berkeley: University of California Press.

This text is an outline of the works done by Karlheinz Stockhausen up until the early 1970s. Kontakte is introduced and analyzed beginning on page 46. Telemusik is introduced and analyzed on page 138. Both pieces are described in construction, design and output.